Efficiency in DMP Model

704a Macroeconomic Theory II Topic 5

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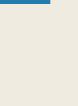






- We built a model of unemployment and studied the positive implications
- Today, we focus on the normative implications
- Is the equilibrium efficient? Is unemployment too high or too low?

Is Unemployment Efficient?





Constrained Efficient Allocation



Constrained Efficiency

- Consider the canonical DMP model as in lecture 2
- We want to study how a benevolent planner would allocate resources
- If the planner could get rid of search frictions, would do so
 - neither interesting nor realistic
- Instead, we treat search friction as part of technology
- Use the concept of constrained efficiency: Planner's problem taking search friction as given



Planning Problem

- $\max_{\substack{\{C_t, v_t, u_{t+1}\}}} \sum_{t=1}^{t}$
- s.t. $C_t = z_t(1 u_t)$ $u_{t+1} u_t = s(1 u_t)$
- Here, b is treated as home production
- With linear preferences, maximizing consumption = maximizing output
 - Transfers immaterial: everyone has the same marginal utility of consumption
- The last constraint captures "constrained" efficiency
 - Without it, $u_t = v_t = 0$ iff $z_t > b$ (again, neither interesting nor realistic)

$$\sum_{t=0}^{\infty} \beta^{t} C_{t}$$

$$-u_t + bu_t - cv_t$$

$$-u_t - f(v_t/u_t)u_t, \quad u_0 \text{ given}$$



Reducing the Constraints

Planner's problem simplifies to a standard dynamic optimization:

$$\max_{\{v_t, u_{t+1}\}} \sum_{t=0}^{\infty} \beta^t [z_t(1 - u_t) + bu_t - cv_t]$$

s.t.
$$u_{t+1} - u_t = s(1)$$

Can solve using

- Lagrangian method
- Dynamic programming

 $-u_t) - M(v_t, u_t), \quad u_0 \text{ given}$





The Bellman equation is

$$\Omega(u, z) = \max_{u', v} z(1 - u)$$



 $c = -\beta \frac{\partial M(\iota)}{\partial t}$



$$\frac{\partial \Omega(u,z)}{\partial u} = -z + b + \beta \left(1 - s - \frac{\partial M(u,v)}{\partial u}\right) \mathbb{E}\left[\frac{\partial \Omega(u',z')}{\partial u'}\right]$$

Recursive Formulation

u) + $bu - cv + \beta \mathbb{E} \Omega(u', z')$

s.t. u' - u = s(1 - u) - M(v, u)

$$\frac{u,v)}{v} \mathbb{E} \left[\frac{\partial \Omega(u',z')}{\partial u'} \right]$$





■ Under CRS matching function, $M = (\partial_{A})$

$$\frac{\partial M(u,v)}{\partial v} = \frac{1}{v} M(u,v) - \frac{\partial M(u,v)}{\partial u} u \frac{1}{v}$$
$$= \frac{1}{v} M(u,v) - \frac{\partial \ln M(u,v)}{\partial \ln u} \frac{M(u,v)}{u} u \frac{1}{v}$$
$$\underbrace{\frac{\partial \ln u}{\exists \alpha}}_{\equiv \alpha}$$

 $= q(\theta)(1 - \alpha)$

Algebra

$\partial M(u,v)$ $\partial \ln M(u,v) M(u,v)$ ר 1

<i>d</i> ln <i>u</i>	U
$\equiv \alpha$	$\equiv f(\theta)$
$f(\theta)$	

$$\partial_u M$$
) $u + (\partial_v M)v$, so



Planner's Solution vs. Equilibrium • Defining the planner's surplus from a job as $S_t^{SP} \equiv -\partial_u \Omega(u_t, z_t)$

- The planner's solution $\{S_t^{SP}, \theta_t^{SP}\}$ solves
- Recall in the decentralized equilibrium, $\{S_t^{DE}, \theta_t^{DE}\}$ solves
- Planner and eqm share the same stock-flow equation.
- Find the difference?

 $S_t^{SP} = z_t - b + \beta(1 - s - \alpha_t f(\theta_t^{SP})) \mathbb{E}S_{t+1}^{SP}$ $c = (1 - \alpha_t)\beta q(\theta_t^{SP})\mathbb{E}_t S_{t+1}^{SP}$ $S_t^{DE} = z_t - b + \beta(1 - s - \gamma f(\theta_t^{DE})) \mathbb{E}S_{t+1}^{DE}$ $c = (1 - \gamma)\beta q(\theta_t^{DE}) \mathbb{E}_t S_{t+1}^{DE}$



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Hosios (1990) Condition Decentralized equilibrium is constrained efficient if and only if $\alpha_t = \gamma$

- Under Cobb-Douglas, $M(u, v) = \bar{m}u^{\alpha}v^{1-\alpha}$, efficiency is achieved when $\alpha = \gamma$
- Holds only in a knife-edge case
- To understand, it is useful to break down into two margins
 - 1. Investment margin: Is vacancy creation incentive efficient given the value of matches?
 - 2. Valuation margin: Given market tightness, are the matches valued correctly?







Investment Margin

$c = (1 - \alpha_t)\beta q(\theta_t^{SP})\mathbb{E}_t S_{t+1}^{SP}$

- If matches are valued correctly ($S_{t+1}^{SP} = S_{t+1}^{DE}$), is market tightness θ_t efficient?
- When a firm creates a vacancy, it creates a social surplus of $\frac{\partial M(u,v)}{\partial v}S = (1 \alpha)q(\theta)S$ • Less than $q(\theta)S$ because it lowers the meeting prob. of other firms

- Firm's private incentive to create a job is $(1 \gamma)q(\theta)S$
 - Less than $q(\theta)S$ because workers capture part of rents (hold-up problem)
 - Firms cannot capture full surplus ⇒ force toward too little vacancy creations
- When $1 \gamma = 1 \alpha$, these two forces exactly cancel

$$c = (1 - \gamma)\beta q(\theta_t^{DE}) \mathbb{E}_t S_{t+1}^{DE}$$





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Valuation Margin $S_t^{SP} = z_t - b + \beta(1 - s - \alpha_t f(\theta_t^{SP})) \mathbb{E}S_{t+1}^{SP}$ $S_t^{DE} = z_t - b + \beta(1 - s - \gamma f(\theta_t^{DE})) \mathbb{E}S_{t+1}^{DE}$

- When the match separates, it creates a social surplus of $\frac{\partial M(u,v)}{\partial u}S = \alpha f(\theta)S$
 - Lower than $f(\theta)S$ because it congests the market
- When the match separates, it creates a private surplus of $\gamma f(\theta) S$
 - Lower than $f(\theta)S$ because it workers can only get a fraction of surplus
- When $\alpha = \gamma$, private and social valuation are aligned

If market tightness is the same ($\theta_t^{SP} = \theta_t^{DE}$), is the valuation of the job S_t efficient?





Despite there being two sources of inefficiency, one condition ensures efficiency

This is magical to me

Magic of Hosios Condition



Unemployment Too High or Too Low?

Focus on the steady state.

Then

$$c = \beta q(\theta^{DE}) \frac{z - b}{1 - \beta(1 - s - \gamma f(\theta^{DE}))}$$

 $\gamma < \alpha \Leftrightarrow \theta^{DE}$

- **No clear empirical guidance on the choice of** γ and α
- Often suggested: $\gamma \ll \alpha$, which means unemployment is too low!

vs.
$$c = \beta q(\theta^{SP}) \frac{z - b}{1 - \beta(1 - s - \alpha f(\theta^{SP}))}$$

$$> \theta^{SP} \Leftrightarrow u^{DE} < u^{SP}$$



Optimal Policy







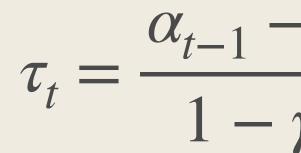


Implementation

- Now return to the decentralized equilibrium.
- Ask: what type of policies should the government implement?
- Introduce a labor tax τ_t per worker paid by the firm (or the worker)
- The decentralized equilibrium $\{S_t^{DE}, \theta_t^{DE}\}$ now solves $S_t^{DE} = z_t - \tau_t - b + \beta(1 - s - \alpha_t f(\theta_t^{DE})) \mathbb{E}S_{t+1}^{DE}$ $c = (1 - \gamma)\beta q(\theta_t^{DE}) \mathbb{E}_t S_{t+1}^{DE}$
- Ask: what τ_t would achieve $\theta_t^{DE} = \theta_t^{SP}$?



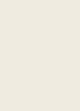
Solving for τ_t yields



- When $\gamma > \alpha_t$ and u too high, subsidize labor ($\tau_t < 0$)
- When $\gamma < \alpha_t$ and u too low, tax labor ($\tau_t > 0$)
- Implementing such a tax requires the knowledge of $(\alpha_t, \gamma, z_t, \theta_t^{SP}, b, c)$
- High informational requirements

Optimal Income Tax

$$\frac{-\gamma}{\gamma}\left(z_t - b + \theta_t^{SP}c\right)$$





Composition Externalities: "Good Jobs versus Bad Jobs" Acemoglu (2001)







Heterogenous Firms

No uncertainty

- Suppose there are many types of jobs/firms, $i \in \{1, 2, ..., J\}$
 - Jobs with higher *i* are good jobs in the sense $z_{i+1} > z_i$ for all *i*, *t*
- Workers are homogenous
- Suppose that all firm types face random matching in a single pooled market
 If firm *i* posts v_i vacancies, it meets q_tv_i of workers
 - If firm *i* posts v_i vacancies, it meets • $q_t = \frac{M(u_t, v_t)}{v_t}$ and $v_t = \sum_i v_{i,t}$
- Important here is that all firm types are bundled in the same matching function





- Assume each firm type *i* incur $c(v_{it})$ of costs in creating v_{it} vacancies
- Assume the cost function is convex: c' > 0 and c'' > 0
- Convexity is important to ensure both types are active in equilibrium
 - If $c(v_{it}) = cv_{it}$, only the most productive creates jobs
- Assume vacancy only lasts for one period after created
- Note the difference from MacCall + DMP model
 - There, all firms are homogenous when created
 - Heterogeneity comes in after meeting (idiosyncratic match quality)
- Here, jobs are ex-ante heterogenous

Convex Vacancy Cost





Nash bargaining: $E_{i,t} - U_t = \gamma S_{i,t}$, $J_{i,t} = (1 - \gamma)S_{i,t}$

Values

- r.
- created: $v_{j,t} = 0$
- *i* subscript)



Equilibrium Conditions

Surplus from match i

$$S_{i,t} = z_i - b + \beta(1-s)S_{i,t+1} - \beta\gamma f_t \sum_j \omega_j(\mathbf{v}_t)S_{j,t+1}$$

Optimal vacancy solves:

 $\max_{v_{i,t}} \beta q_{i,t}$

which results in the following optimality condition:

 $c'(v_{i,t}) =$

$$J_{i,t+1}v_{i,t} - c(v_{i,t})$$

$$(1 - \gamma)\beta q_t S_{i,t+1}$$



Planner's Problem

- Planner's state variable is now the stock of workers employed at each type
 - Denote vectors with bold font: (n_1, \dots, n_n)
- Value function of the planner

$$\Omega(\mathbf{n}) = \max_{\mathbf{v},\mathbf{n}'} \sum_{i} z_{i} n_{i} + b(1 - \sum_{i} n_{i}) - \sum_{i} c(v_{i}) + \beta \Omega(\mathbf{n}')$$

s.t. $n_{i}' = (1 - s)n_{i} + v_{i}q \left(\frac{\sum_{j} v_{j}}{1 - \sum_{j} n_{j}}\right)$

$$\max_{\mathbf{v},\mathbf{n}'} \sum_{i} z_{i} n_{i} + b(1 - \sum_{i} n_{i}) - \sum_{i} c(v_{i}) + \beta \Omega(\mathbf{n})$$

s.t. $n_{i}' = (1 - s)n_{i} + v_{i}q \left(\frac{\sum_{j} v_{j}}{1 - \sum_{j} n_{j}}\right)$

Optimality:

$$..., n_J) = \mathbf{n} \text{ and } (v_1, ..., v_J) = \mathbf{v}$$

 $c'(v_i) = \beta q(\theta) \partial_{n_i} \Omega(\mathbf{n}) + \beta \sum_j (v_j/u) q'(\theta) \partial_{n_j} \Omega(\mathbf{n})$

 $\partial_{n_i} \Omega(\mathbf{n}) = z_i - b + \beta(1 - s) \partial_{n_i} \Omega(\mathbf{n}') + \beta \sum_i (v_j / u) q'(\theta) \theta \partial_{n_i} \Omega(\mathbf{n}')$



Planner vs. Equilibrium: Valuation Margin

Define the planner's marginal value of a job at firm i as

$$S_{it}^{sp} = \partial_{n_i}$$

Compare match surplus in the decentralized eqm and in the planner's solution:

$$S_{i,t}^{DE} = z_{i,t} - b + \beta(1-s)S_{i,t+1}^{DE} - \beta\gamma f_t^{DE} \left[\sum_j \omega_j(\mathbf{v}_t^{DE})S_{j,t+1}^{DE}\right]$$
$$S_{i,t}^{SP} = z_{i,t} - b + \beta(1-s)S_{i,t+1}^{SP} - \beta\alpha_t f_t^{SP} \left[\sum_j \omega_j(\mathbf{v}_t^{SP})S_{j,t+1}^{SP}\right]$$

$$S_{i,t}^{DE} = z_{i,t} - b + \beta(1-s)S_{i,t+1}^{DE} - \beta\gamma f_t^{DE} \left[\sum_j \omega_j(\mathbf{v}_t^{DE})S_{j,t+1}^{DE}\right]$$
$$S_{i,t}^{SP} = z_{i,t} - b + \beta(1-s)S_{i,t+1}^{SP} - \beta\alpha_t f_t^{SP} \left[\sum_j \omega_j(\mathbf{v}_t^{SP})S_{j,t+1}^{SP}\right]$$

• Under Hosios condition, $\alpha_t = \gamma$, both coincide if (f_t, \mathbf{v}_t) are the same • Are (f_t, \mathbf{v}_t) the same?

- $\Omega(\mathbf{n}_t, \mathbf{z}_t)$



Planner vs. Equilibrium: Investment Margin

Now compare the vacancy creation conditions:

$$\begin{aligned} c'(v_{i,t}^{DE}) &= \beta q_t^{DE} (1-\gamma) S_{i,t+1}^{DE} \\ c'(v_{i,t}^{SP}) &= \beta q_t^{SP} \left((1-\alpha_t) S_{i,t+1}^{SP} + \alpha_t \left[S_{i,t+1}^{SP} - \sum_j \omega_j S_{j,t+1}^{SP} \right] \right) \end{aligned}$$

- Even with Hosios condition, equilibrium is inefficient
- Heterogeneity ⇒ composition externality (Acemoglu, 2001)
 - Planner internalizes that creation of job *i* congests matching market for job *j*
 - Private agents do not
 - In the decentralized eqm..., too many "bad jobs" (low $S_{i,t}$) and too few "good jobs" (high $S_{i,t}$)



Policy Implications

- Workers randomly meet firms ⇒ low-prod. firms "free-ride" labor market
- Planner would like to divert job creation away from low-prod. firms
- How? Ideally, tax different jobs at a different rates.
- Minimum wages and UI benefits can be crude policy tools to address the inefficiency



