Efficiency in DMP Model

704a Macroeconomic Theory II
Topic 5

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Is Unemployment Efficient?

- We built a model of unemployment and studied the positive implications
- Today, we focus on the normative implications
- Is the equilibrium efficient? Is unemployment too high or too low?

Constrained Efficient Allocation

Constrained Efficiency

- Consider the canonical DMP model as in lecture 2
- We want to study how a benevolent planner would allocate resources
- If the planner could get rid of search frictions, would do so
  - neither interesting nor realistic
- Instead, we treat search friction as part of technology
- Use the concept of **constrained efficiency**:
  Planner's problem taking search friction as given

Planning Problem

$$\max_{\{C_t, v_t, u_{t+1}\}} \sum_{t=0}^{\infty} \beta^t C_t$$

$$\text{s.t.} \qquad C_t = z_t(1 - u_t) + bu_t - cv_t$$

$$u_{t+1} - u_t = s(1 - u_t) - f(v_t/u_t)u_t, \quad u_0 \text{ given}$$

- Here, b is treated as home production
- With linear preferences, maximizing consumption = maximizing output
  - Transfers immaterial: everyone has the same marginal utility of consumption
- The last constraint captures "constrained" efficiency
  - Without it, $u_t = v_t = 0$ iff $z_t > b$ (again, neither interesting nor realistic)

Reducing the Constraints

- Planner's problem simplifies to a standard dynamic optimization:

$$\max_{\{v_t, u_{t+1}\}} \sum_{t=0}^{\infty} \beta^t [z_t(1-u_t) + bu_t - cv_t]$$

s.t. $\quad u_{t+1} - u_t = s(1-u_t) - M(v_t, u_t), \quad u_0$ given

- Can solve using
  - Lagrangian method
  - Dynamic programming

Recursive Formulation

- The Bellman equation is

$$\Omega(u,z) = \max_{u',v} z(1-u) + bu - cv + \beta\mathbb{E}\Omega(u',z')$$

$$\text{s.t.} \quad u' - u = s(1-u) - M(v,u)$$

- FOC:

$$c = -\beta\frac{\partial M(u,v)}{\partial v}\mathbb{E}\left[\frac{\partial\Omega(u',z')}{\partial u'}\right]$$

- Envelope:

$$\frac{\partial\Omega(u,z)}{\partial u} = -z + b + \beta\left(1 - s - \frac{\partial M(u,v)}{\partial u}\right)\mathbb{E}\left[\frac{\partial\Omega(u',z')}{\partial u'}\right]$$

Algebra

- We rewrite

$$\frac{\partial M(u,v)}{\partial u} = \underbrace{\frac{\partial \ln M(u,v)}{\partial \ln u}}_{\equiv \alpha} \underbrace{\frac{M(u,v)}{u}}_{\equiv f(\theta)}$$

$$= \alpha f(\theta)$$

- Under CRS matching function, $M = (\partial_u M)u + (\partial_v M)v$, so

$$\frac{\partial M(u,v)}{\partial v} = \frac{1}{v} M(u,v) - \frac{\partial M(u,v)}{\partial u} u \frac{1}{v}$$

$$= \frac{1}{v} M(u,v) - \underbrace{\frac{\partial \ln M(u,v)}{\partial \ln u}}_{\equiv \alpha} \frac{M(u,v)}{u} u \frac{1}{v}$$

$$= q(\theta)(1-\alpha)$$

Planner's Solution vs. Equilibrium

- Defining the planner's surplus from a job as $S_t^{SP} \equiv -\partial_u \Omega(u_t, z_t)$

- The planner's solution $\{S_t^{SP}, \theta_t^{SP}\}$ solves

$$S_t^{SP} = z_t - b + \beta(1 - s - \alpha_t f(\theta_t^{SP}))\mathbb{E}S_{t+1}^{SP}$$

$$c = (1 - \alpha_t)\beta q(\theta_t^{SP})\mathbb{E}_t S_{t+1}^{SP}$$

- Recall in the decentralized equilibrium, $\{S_t^{DE}, \theta_t^{DE}\}$ solves

$$S_t^{DE} = z_t - b + \beta(1 - s - \gamma f(\theta_t^{DE}))\mathbb{E}S_{t+1}^{DE}$$

$$c = (1 - \gamma)\beta q(\theta_t^{DE})\mathbb{E}_t S_{t+1}^{DE}$$

- Planner and eqm share the same stock-flow equation.

- Find the difference?

Hosios Condition

Hosios (1990) Condition

Decentralized equilibrium is constrained efficient if and only if $\alpha_t = \gamma$

- Under Cobb-Douglas, $M(u, v) = \bar{m}u^{\alpha}v^{1-\alpha}$, efficiency is achieved when $\alpha = \gamma$
- Holds only in a knife-edge case
- To understand, it is useful to break down into two margins
    1. Investment margin:
       Is vacancy creation incentive efficient given the value of matches?
    2. Valuation margin:
       Given market tightness, are the matches valued correctly?

Investment Margin

$$c = (1-\alpha_t)\beta q(\theta_t^{SP})\mathbb{E}_t S_{t+1}^{SP} \qquad\qquad c = (1-\gamma)\beta q(\theta_t^{DE})\mathbb{E}_t S_{t+1}^{DE}$$

- If matches are valued correctly ($S_{t+1}^{SP} = S_{t+1}^{DE}$), is market tightness θ_t efficient?
- When a firm creates a vacancy, it creates a social surplus of $\frac{\partial M(u,v)}{\partial v} S = (1-\alpha)q(\theta)S$
  - Less than $q(\theta)S$ because it lowers the meeting prob. of other firms
  - Impose negative externality $\Rightarrow$ force toward too many vacancy creations
- Firm's private incentive to create a job is $(1-\gamma)q(\theta)S$
  - Less than $q(\theta)S$ because workers capture part of rents (hold-up problem)
  - Firms cannot capture full surplus $\Rightarrow$ force toward too little vacancy creations
- When $1-\gamma = 1-\alpha$, these two forces exactly cancel

Valuation Margin

$$S_t^{SP} = z_t - b + \beta(1 - s - \alpha_t f(\theta_t^{SP}))\mathbb{E}S_{t+1}^{SP}$$

$$S_t^{DE} = z_t - b + \beta(1 - s - \gamma f(\theta_t^{DE}))\mathbb{E}S_{t+1}^{DE}$$

- If market tightness is the same ($\theta_t^{SP} = \theta_t^{DE}$), is the valuation of the job S_t efficient?
- When the match separates, it creates a social surplus of $\frac{\partial M(u,v)}{\partial u}S = \alpha f(\theta)S$
  - Lower than $f(\theta)S$ because it congests the market
- When the match separates, it creates a private surplus of $\gamma f(\theta)S$
  - Lower than $f(\theta)S$ because it workers can only get a fraction of surplus
- When $\alpha = \gamma$, private and social valuation are aligned

Magic of Hosios Condition

- Despite there being **two** sources of inefficiency, **one** condition ensures efficiency
- This is magical to me

Unemployment Too High or Too Low?

- Focus on the steady state.
- Then

$$c = \beta q(\theta^{DE})\frac{z - b}{1 - \beta(1 - s - \gamma f(\theta^{DE}))} \qquad \text{vs.} \qquad c = \beta q(\theta^{SP})\frac{z - b}{1 - \beta(1 - s - \alpha f(\theta^{SP}))}$$

- One can show

$$\gamma < \alpha \Leftrightarrow \theta^{DE} > \theta^{SP} \Leftrightarrow u^{DE} < u^{SP}$$

- No clear empirical guidance on the choice of γ and α
- Often suggested: $\gamma \ll \alpha$, which means unemployment is too low!

Optimal Policy

Implementation

- Now return to the decentralized equilibrium.
- Ask: what type of policies should the government implement?
- Introduce a labor tax τ_t per worker paid by the firm (or the worker)
- The decentralized equilibrium $\{S_t^{DE}, \theta_t^{DE}\}$ now solves

$$S_t^{DE} = z_t - \tau_t - b + \beta(1 - s - \alpha_t f(\theta_t^{DE}))\mathbb{E}S_{t+1}^{DE}$$

$$c = (1 - \gamma)\beta q(\theta_t^{DE})\mathbb{E}_t S_{t+1}^{DE}$$

- Ask: what τ_t would achieve $\theta_t^{DE} = \theta_t^{SP}$?

Optimal Income Tax

- Solving for τ_t yields

$$\tau_t = \frac{\alpha_{t-1} - \gamma}{1 - \gamma} \left(z_t - b + \theta_t^{SP} c \right)$$

  - When $\gamma > \alpha_t$ and u too high, subsidize labor ($\tau_t < 0$)
  - When $\gamma < \alpha_t$ and u too low, tax labor ($\tau_t > 0$)

- Implementing such a tax requires the knowledge of $(\alpha_t, \gamma, z_t, \theta_t^{SP}, b, c)$

- High informational requirements

Composition Externalities: “Good Jobs versus Bad Jobs”

Acemoglu (2001)

Heterogenous Firms

- No uncertainty
- Suppose there are many types of jobs/firms, $i \in \{1,2,\ldots,J\}$
  - Jobs with higher i are good jobs in the sense $z_{i+1} > z_i$ for all i, t
- Workers are homogenous
- Suppose that all firm types face random matching in a single pooled market
  - If firm i posts v_i vacancies, it meets $q_t v_i$ of workers
  - $q_t = \frac{M(u_t, v_t)}{v_t}$ and $v_t = \sum_i v_{i,t}$
- Important here is that all firm types are bundled in the same matching function

Convex Vacancy Cost

- Assume each firm type i incur $c(v_{it})$ of costs in creating v_{it} vacancies
- Assume the cost function is convex: $c' > 0$ and $c'' > 0$
- Convexity is important to ensure both types are active in equilibrium
  - If $c(v_{it}) = cv_{it}$, only the most productive creates jobs
- Assume vacancy only lasts for one period after created
- Note the difference from MacCall + DMP model
  - There, all firms are homogenous when created
  - Heterogeneity comes in after meeting (idiosyncratic match quality)
- Here, jobs are ex-ante heterogenous

Values

- Value of unemployment is now

$$U_t = b + \beta \left[f_t \sum_j \omega_j(\mathbf{v}_t) E_{j,t+1} + (1 - f_t) U_{t+1} \right]$$

where $\omega_j(\mathbf{v}_t) \equiv v_{j,t} / \sum_k v_{k,t}$.

- Note we ignored the possibility of workers declining the offer.
  - This is without loss of generality since such jobs are never created: $v_{j,t} = 0$
- Other Bellman equations remain unchanged (except adding i subscript)

$$E_{i,t} = w_{i,t} + \beta[(1-s)E_{i,t+1} + sU_{t+1}]$$
$$J_{i,t} = z_i - w_{i,t} + \beta(1-s)J_{i,t+1}$$

- Nash bargaining: $E_{i,t} - U_t = \gamma S_{i,t}, \quad J_{i,t} = (1-\gamma)S_{i,t}$

Equilibrium Conditions

- Surplus from match i

$$S_{i,t} = z_i - b + \beta(1-s)S_{i,t+1} - \beta\gamma f_t \sum_j \omega_j(\mathbf{v}_t)S_{j,t+1}$$

- Optimal vacancy solves:

$$\max_{v_{i,t}} \beta q_{i,t} J_{i,t+1} v_{i,t} - c(v_{i,t})$$

which results in the following optimality condition:

$$c'(v_{i,t}) = (1-\gamma)\beta q_t S_{i,t+1}$$

Planner's Problem

- Planner's state variable is now the stock of workers employed at each type
  - Denote vectors with bold font: $(n_1, \ldots, n_J) = \mathbf{n}$ and $(v_1, \ldots, v_J) = \mathbf{v}$
- Value function of the planner

$$\Omega(\mathbf{n}) = \max_{\mathbf{v},\mathbf{n}'} \sum_i z_i n_i + b(1 - \sum_i n_i) - \sum_i c(v_i) + \beta\Omega(\mathbf{n}')$$

$$\text{s.t.} \quad n_i' = (1-s)n_i + v_i q\left(\frac{\sum_j v_j}{1 - \sum_j n_j}\right)$$

- Optimality:

$$c'(v_i) = \beta q(\theta)\partial_{n_i}\Omega(\mathbf{n}) + \beta \sum_j (v_j/u) q'(\theta)\partial_{n_j}\Omega(\mathbf{n})$$

$$\partial_{n_i}\Omega(\mathbf{n}) = z_i - b + \beta(1-s)\partial_{n_i}\Omega(\mathbf{n}') + \beta\sum_j (v_j/u)q'(\theta)\theta\partial_{n_j}\Omega(\mathbf{n}')$$

Planner vs. Equilibrium: Valuation Margin

- Define the planner's marginal value of a job at firm i as

$$S_{it}^{sp} = \partial_{n_i}\Omega(\mathbf{n}_t, \mathbf{z}_t)$$

- Compare match surplus in the decentralized eqm and in the planner's solution:

$$S_{i,t}^{DE} = z_{i,t} - b + \beta(1-s)S_{i,t+1}^{DE} - \beta\gamma f_t^{DE}\left[\sum_j \omega_j(\mathbf{v}_t^{DE})S_{j,t+1}^{DE}\right]$$

$$S_{i,t}^{SP} = z_{i,t} - b + \beta(1-s)S_{i,t+1}^{SP} - \beta\alpha_t f_t^{SP}\left[\sum_j \omega_j(\mathbf{v}_t^{SP})S_{j,t+1}^{SP}\right]$$

- Under Hosios condition, $\alpha_t = \gamma$, both coincide if $(f_t, \mathbf{v}_t)$ are the same
- Are $(f_t, \mathbf{v}_t)$ the same?

Planner vs. Equilibrium: Investment Margin

- Now compare the vacancy creation conditions:

$$c'(v_{i,t}^{DE}) = \beta q_t^{DE}(1-\gamma)S_{i,t+1}^{DE}$$

$$c'(v_{i,t}^{SP}) = \beta q_t^{SP}\left((1-\alpha_t)S_{i,t+1}^{SP}+\alpha_t\left[S_{i,t+1}^{SP}-\sum_j \omega_j S_{j,t+1}^{SP}\right]\right)$$

- Even with Hosios condition, equilibrium is inefficient
- Heterogeneity $\Rightarrow$ **composition externality (Acemoglu, 2001)**
  - Planner internalizes that creation of job i congests matching market for job j
  - Private agents do not
  - In the decentralized eqm…,
    too many "bad jobs" (low $S_{i,t}$) and too few "good jobs" (high $S_{i,t}$)

Policy Implications

- Workers randomly meet firms $\Rightarrow$ low-prod. firms "free-ride" labor market
- Planner would like to divert job creation away from low-prod. firms
- How? Ideally, tax different jobs at a different rates.
- Minimum wages and UI benefits can be crude policy tools to address the inefficiency