Directed Seach Models
(a.k.a. Competitive Search Models)

704a Macroeconomics
Lecture 6

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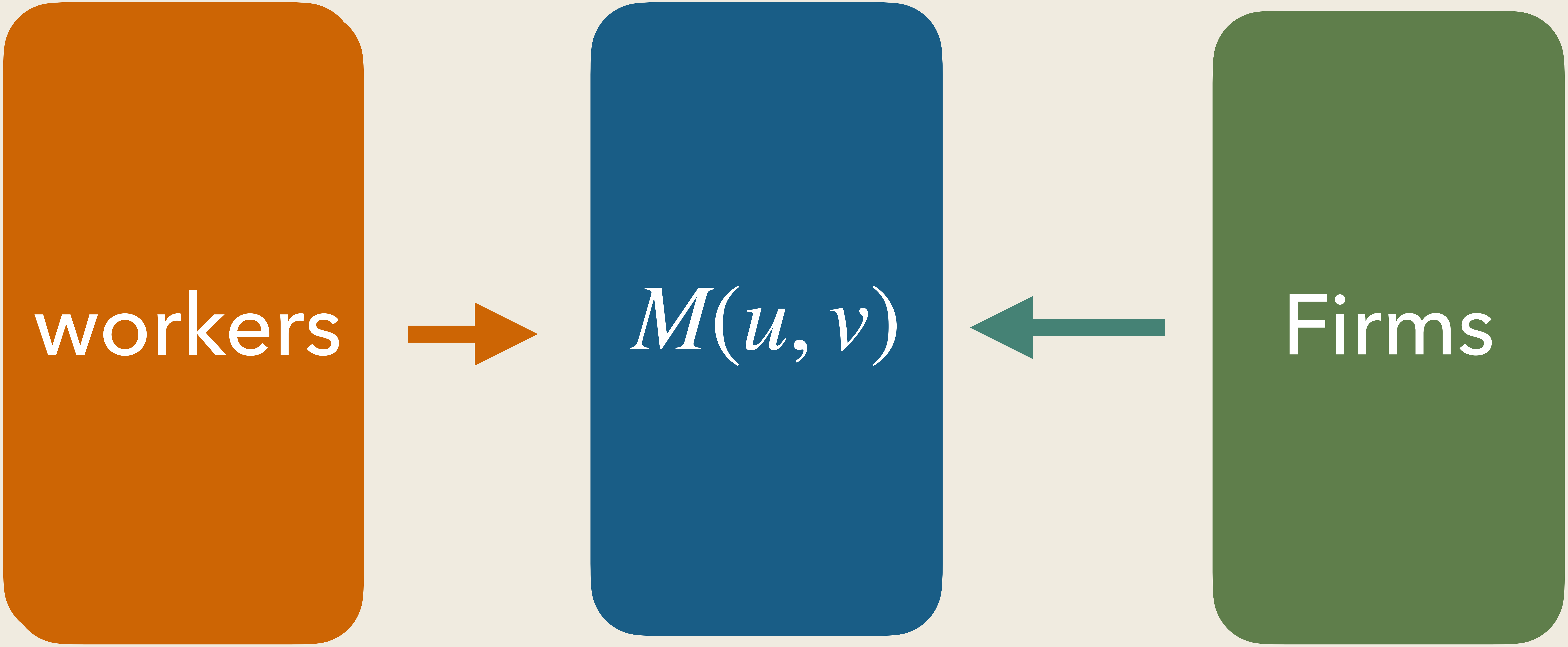
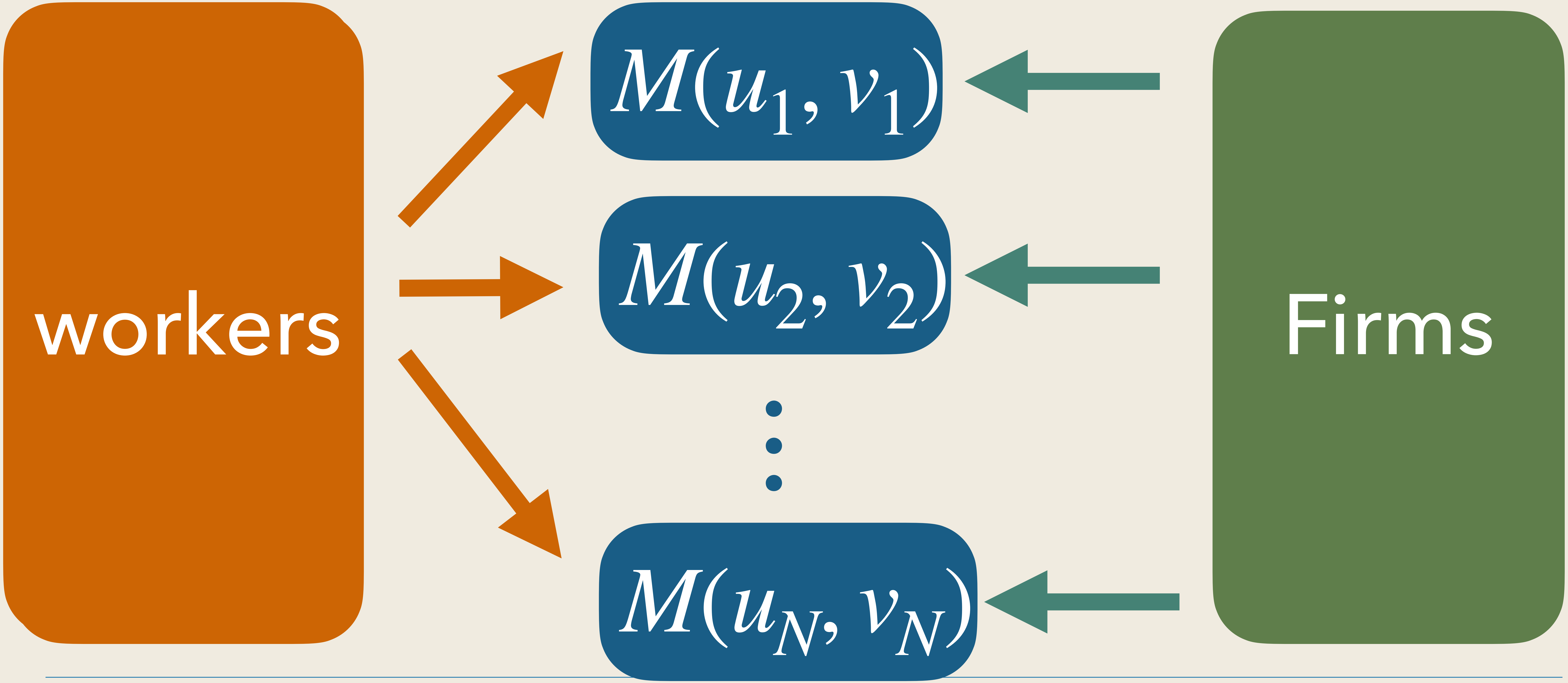
Directed Search Model

- So far, we considered the environment with random search
  - Workers randomly bump into job postings
- In reality, workers can (at least partially) direct their search
  - Workers decide which jobs to apply for
- We will consider an environment where workers can perfectly direct the search
- Called **directed search** (**competitive search**) model
  - Pioneered by Moen (1997), and popularized by Menzio and Shi (2011)

Random Search



Directed Search



Environment

Environment

- Start from no heterogeneity
- Populated by a unit measure of workers and firms
  - Both have discount factor β and linear preferences: $\sum_{t=0}^{\infty} \beta^t c_t$
- Worker is endowed with a unit of labor
  - earn b when unemployed
- Firm operates linear technology in labor with productivity z
  - job exogenously separates with prob s

Submarkets

- There is a continuum of **submarkets** indexed by w
  - Matching function within each submarket, $M(u, v)$
  - Let $\theta(w) \equiv v(w)/u(w)$ denote the market tightness for submarket w
  - When matched, firms offer the wage w
- Firms can post vacancy in each submarket at cost c
  - Find a worker with prob. $q(\theta(w))$ where $q(\theta) \equiv M(1/\theta, 1) = M(u, v)/v$
- Workers can choose which submarket to search (can choose only one)
  - Find a job with prob. $f(\theta(w))$ where $f(\theta) \equiv M(1, \theta)$
- Timing:
  firms post vacancy → workers apply → match → produce → separate

Interpretation

- Firms post and **commit** to wage offer w
  - Post a job ad saying “we will pay $15/hr”
- Workers see all available job postings and decide which job to apply for
  - again, can apply for only one job
- Note the contrast to DMP: communication and commitment
  - In DMP, workers had no ex-ante info about wage offers
  - In DMP, firms had no commitment to future wages

Equilibrium

Worker's Problem

- Throughout, we focus on the steady state
- Value of unemployed workers searching in a submarket w:

$$U(w) = b + \beta[f(\theta(w))E(w) + (1 - f(\theta(w)))U]$$

where

$$E(w) = w + \beta[(1 - s)E(w) + sU]$$

- Workers arbitrage between markets implies:

$$U(w) = U \quad \Rightarrow \quad f(\theta(w))[E(w) - U] = [(1 - \beta)U - b]/\beta \equiv \Lambda \quad \text{for all } w \text{ s.t. } E(w) > U$$

  - $E(w)$ is increasing in $w \Rightarrow f(\theta(w))$ is decreasing in w
  - Better jobs are harder to find in equilibrium

Firm's Problem

- Firms decide which submarket to post a vacancy (what wage to post)

$$\max_{w} q(\theta(w))J(w) \tag{4}$$

$$\text{s.t.} \quad f(\theta(w))[E(w) - U] = \Lambda \tag{IC}$$

  where

$$J(w) = z - w + \beta[(1-s)J(w) + sV]$$

$$V = -c + \beta[q(w^*)J(w^*) + (1 - q(w^*))V] = 0$$

- The (IC) constraint captures subgame perfection
  - Firms rationally anticipate how many workers will apply when posting w
- Tradeoff: higher wage (i) attracts more workers (ii) but is costly.

Equilibrium Definition

A competitive search equilibrium is a tuple $\{U, E(w), V, J(w), \theta(w), w^*\}$ such that

1. U solves
$$U = b + \beta[f(\theta(w))E(w) + (1 - f(\theta(w)))U] \quad \text{for all } w$$
2. $E(w)$ solves
$$E(w) = w + \beta[(1 - s)E(w) + sU]$$
3. w^* is the solution to
$$\max_w q(\theta(w))J(w) \qquad \text{s.t.} \quad f(\theta(w))[E(w) - U] = \Lambda, \quad \text{for all } w \text{ s.t. } E(w) > U$$
4. $J(w)$ solves
$$J(w) = z - w + \beta[(1 - s)J(w)]$$
5. $\theta(w)$ satisfies
$$c = \beta q(\theta(w))J(w) \quad \text{for } w = w^*$$
$$c > \beta q(\theta(w))J(w) \quad \text{for } w \neq w^*$$

Wage Determination

- The first order condition w.r.t. w gives

$$q'(\theta(w))\theta'(w)J(w) + J'(w)q(\theta(w)) = 0$$

- Totally differentiating (IC),

$$f'(\theta(w))\theta'(w)[E(w) - U] + f(\theta(w))E'(w) = 0$$

- Combining the above two conditions and manipulating,

$$\alpha J(w^*) = (1-\alpha)[E(w^*) - U]$$

where $\alpha = -\frac{d\ln q}{d\ln\theta}|_{\theta=\theta(w^*)} = \frac{\partial \ln M}{\partial \ln u}$ and $1-\alpha = \frac{d\ln f}{d\ln\theta}|_{\theta=\theta(w^*)} = \frac{\partial \ln M}{\partial \ln v}$

- Defining $S = J(w) + E(w) - U$ (note S independent of wage) as the match surplus,

$$J(w^*) = (1-\alpha)S, \qquad E(w^*) - U = \alpha S$$

- Looks familiar?

Equilibrium is Efficient

$$J(w^*) = \alpha S, \qquad E(w^*) - U = (1-\alpha)S$$

- Hosios condition is endogenously achieved in equilibrium!
- As a result, the investment margin is efficient

$$c = \beta q(\theta(w)) \underbrace{J(w)}_{(1-\alpha)S} \quad \text{for } w = w^*$$

  and no vacancy is created for $w \neq w^*$

- The valuation margin is also efficient:

$$S = z - b + \beta[(1-s)S - \alpha f(\theta(w^*)S]$$

- Result: Competitive search equilibrium is efficient
- Collorary:
  Competitive search equilibrium results in the same allocation as DMP with Hosios

Reason for Efficiency

- In DMP, eqm was not efficient
  1. Planner cares about how much additional vacancy congests the market
  2. Firms care about how much an additional vacancy generates profit
- With ex-post bargaining, no reason 1 and 2 coincide
- Here, firms set wages facing the same trade-off just as the planner does
  1. To hire more workers, firms have to be in a less congested market
  2. But a less congested market generates lower profits

Firm Heterogeneity

Firm Heterogeneity

- Now introduce heterogeneous firm types, $\{z_1, z_2, \ldots, z_J\}$
- Each firm type decides which submarket w to post with vacancy cost $c(v_i)$
- Workers decide which submarket to search for a job (same as before)
- The firm's optimal choice of w solves

$$\max_{w_i} q(\theta(w_i))J_i(w)$$

where $\quad f(\theta(w_i))[E(w_i) - U] = \Lambda$

$$J_i(w_i) = z_i - w_i + \beta[(1-s)J_i(w_i)]$$

- Solution: $E(w_i) - U = \alpha_i S_i, \quad J(w_i) = (1-\alpha_i)S_i$, where $\alpha_i = \partial \ln M(u_i, v_i)/\partial \ln u_i$
- Optimal vacancy creation: $\quad c'(v_i) = \beta q(\theta(w_i))J(w_i)$

Planner's Problem

$$\Omega(\mathbf{n},\mathbf{z}) = \max_{\mathbf{v},\mathbf{n}',\mathbf{u}} \sum_i z_i n_i + b(1-\sum_i n_i) - \sum_i c(v_i) + \beta\Omega(\mathbf{n}',\mathbf{z}')$$

$$\text{s.t.} \quad n_i' = (1-s)n_i + v_i q\left(\frac{v_i}{u_i}\right)$$

$$\sum_i n_i = 1 - \sum_i u_i \qquad (\star)$$

- Optimality (Λ: Lagrangian multiplier on $(\star)$):

$$c'(v_i) = \beta\left(q(\theta_i) + \theta q'(\theta_i)\right)\partial_{n_i}\Omega(\mathbf{n},\mathbf{z})$$

$$\Lambda = \beta\partial_{n_i}\Omega(\mathbf{n}',\mathbf{z}')\left(-\theta_i^2 q'(\theta_i)\right)$$

$\alpha_i f(\theta_i)$

$$\partial_{n_i}\Omega(\mathbf{n},\mathbf{z}) = z_i - b + \Lambda + \beta(1-s)\partial_{n_i}\Omega(\mathbf{n}',\mathbf{z}')$$

Efficiency with Job Heterogeneity

- In equilibrium,

$$S_i^{DE} = z_i - b + \beta(1 - s - \alpha_i f_i^{DE}) S_i^{DE}$$

$$c'(v_i^{DE}) = (1 - \alpha)\beta q_i^{DE} S_i^{DE}$$

$$\Lambda^{DE} = \alpha_i f_i S_i^{DE}$$

- In the planner's solution ($S_i^{SP} \equiv \partial_{n_i}\Omega(\mathbf{n}, \mathbf{z})$)

$$S_i^{SP} = z_i - b + \beta(1 - s - \alpha_i f_i^{SP}) S_i^{SP}$$

$$c'(v_i^{SP}) = (1 - \alpha)\beta q_i^{SP} S_i^{SP}$$

$$\Lambda^{SP} = \alpha_i f_i S_i^{SP}$$

- Equilibrium is efficient even with heterogeneity (firms endogenously segment)
- Productive firms endogenously sort into less-tight but high-wage submarket

Random or Directed?

- Normative implications strikingly differ between random vs. directed search
- Reality is clearly a mix of random and directed search
  - Workers do not randomly apply to a job
  - Workers face uncertainty about what type of job they are getting
- How much are searches in the real world directed? How can we tell from the data?
- See Lentz, Maibom and Moen (2024) for a recent attempt