
Monoposony Models of Frictional Labor Market

704 Macroeconomic Theory II
Topic 7

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Firms as Wage-Setters

- Now we go back to random search models
- In DMP, workers and firms bargain over wages
- Are wages really bargained in the data?

Hall and Krueger (2012)

- Survey 1,300 workers
- **Q:** When you were offered your job, did your employer make a “take-it-or-leave-it” offer or was there some bargaining that took place over the pay?

A: 33% bargained

- 25% for women. 85% for professional degree. 6% for blue-color workers.

- **Q:** At the time that you were first interviewed for your job, did you already know exactly how much it would pay?

A: 23% yes

- 23% for women. 14% for professional degree. 57% for blue-color workers.

Wage Posting

- In the data, the majority of workers receive “take-it-or-leave-it” offers
- Now let us replace wage bargaining with wage posting in DMP

	Wage Bargaining	Wage Posting
Random Search	DMP	Today
Directed Search		Competitive Search (Moen, 1997)

Diamond (1971) Paradox

DMP with Wage Posting

- Consider the DMP model in continuous time with discount rate $r > 0$
- To focus on the wage settings, let us assume q and f are both exogenous

- The unemployed workers value function:

$$rU = b + f \max\{E(w) - U, 0\}$$

- The employed workers:

$$rE(w) = w + s(U - E(w))$$

- Workers accept the job offer if $w \geq w^R$, where $E(w^R) = U$

Extreme Monopsony

- Firms decide what wages to offer to workers:

$$rV = -c + \max_{w \geq w^R} q \mathbb{1}(w \geq w^R) J(w)$$
$$rJ(w) = z - w + s(V - J(w))$$

- What is the firm's optimal wage setting? Clearly,

$$w = w^R$$

since there is no reason to offer $w > w^R$

- Solving for w^R , the unique equilibrium features all firms offering

$$w = w^R = b$$

- Firms set wages so that workers are exactly indifferent to unemployment
– an extreme form of “monopsony”

Heterogenous Firms

- The result extends even when firms have differing productivity $z_i \in \{z_1, z_2, \dots, z_J\}$
- Workers problem unchanged
- Firms with productivity z_i solves

$$rV_i = -c + \max_{w_i} q \mathbb{1}(w_i \geq w^R) J_i(w_i)$$
$$rJ_i(w_i) = z_i - w_i + s(V_i - J_i(w_i))$$

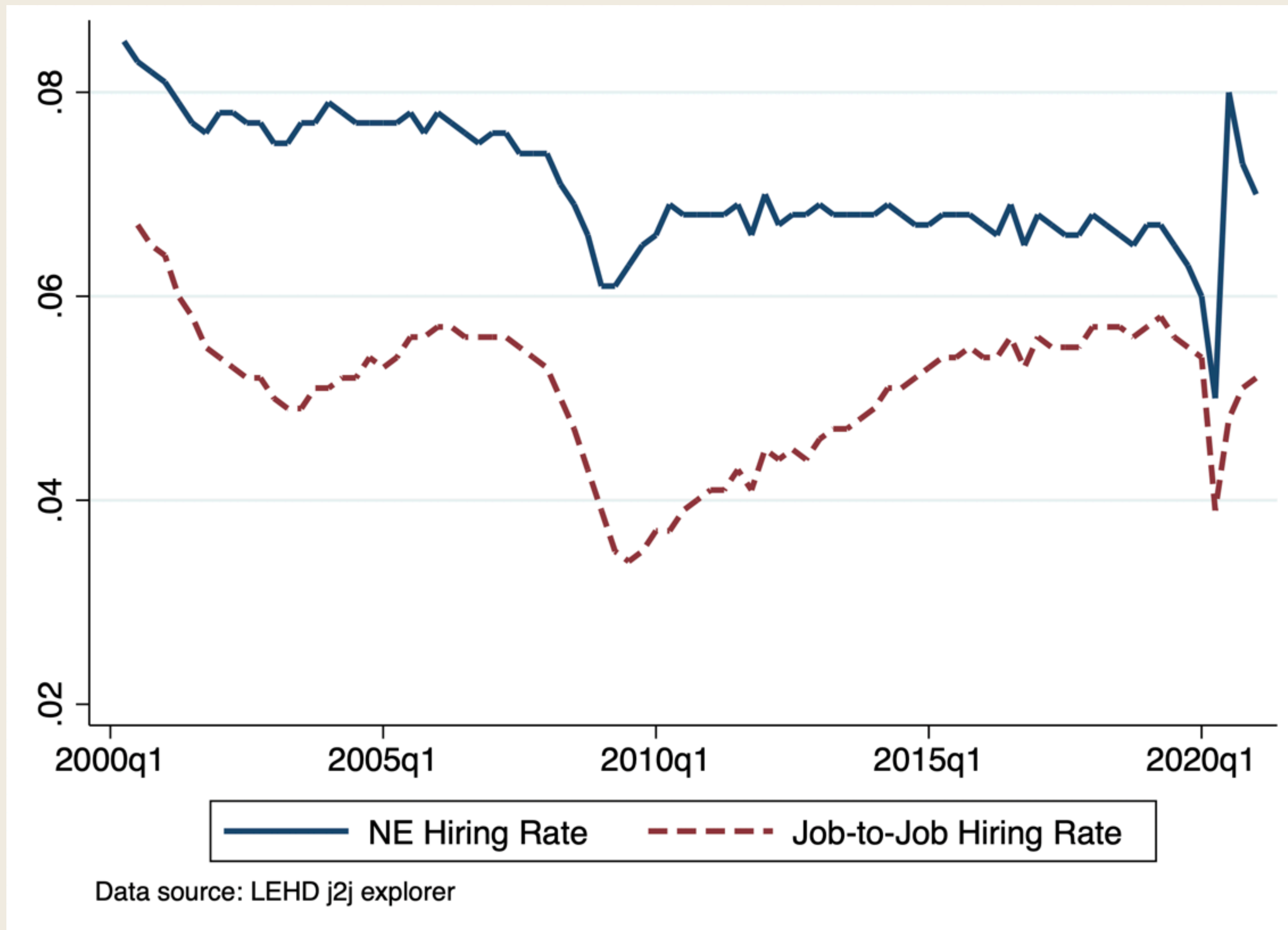
- Despite firm heterogeneity,

$$w_i = w^R = b \quad \text{for all } i$$

Diamond Paradox

- This is called Diamond (1971) Paradox
- Why is this a paradox? Why is this surprising?
- A tiny deviation from perfect competition results in an extreme form of monopsony!
- Firms capture all the rents even when $f, q \rightarrow \infty$
- This spurred subsequent research. Solutions to the paradox:
 1. Heterogenous workers (Albrecht and Axell, 1984)
 2. Multiple job applications at a time (Burdett and Judd, 1983)
 3. On-the-job search (Burdett and Mortensen, 1998)
- We focus on Burdett and Mortensen (1998)

On-the-Job Search in the Data



Burdett and Mortensen (1998) Model

Environment

- Let us introduce on-the-job to the previous model
- Unemployed workers receive job-offer at the arrival rate f^U
- Employed workers receive at a rate $f^E \equiv \zeta f^U$
- Firms with measure $m \equiv 1$ post v vacancy (exogenous) and meets worker at rate q
- Firms post wage w that applies to all employees ("firm-wage")
- Start from homogenous firm case with common productivity z
- In the background, think of a matching function that determines (f^U, f^E, q) :

$$f^U = \frac{M(u + \zeta(1 - u), v)}{u + \zeta(1 - u)}, \quad f^E = \zeta \frac{M(u + \zeta(1 - u), v)}{u + \zeta(1 - u)}, \quad q = \frac{M(u + \zeta(1 - u), v)}{v}$$

Worker's Problem

- Unemployed workers value function:

$$rU = b + f^U \int [\max\{E(w) - U, 0\}] dG(w) \quad (1)$$

- Employed workers with wage w :

$$rE(w) = w + f^E \int \max\{E(w') - E(w), 0\} dG(w') - s(E(w) - U) \quad (2)$$

- Worker's policy:

- Unemployed: accept job offer iff $w \geq w^R$ where $E(w^R) = U$
- Employed: accept job offer iff $w' \geq w$

Reservation Wage w

- Combining $E(w^R) = U$, (1), and (2),

$$\begin{aligned}w^R - b &= (f^U - f^E) \int_{w^R} (E(w) - U) dG(w) \\ &= (f^U - f^E) \int_{w^R} E'(w)(1 - G(w)) dw \\ &= (f^U - f^E) \int_{w^R} \frac{1 - G(w)}{r + s + f^E(1 - G(w))} dw\end{aligned}$$

where the second line uses integration by parts

- When $f^U = f^E$, $w^R = b$
- When $f^U > f^E$, $w^R > b$ because accepting a job offer lowers future job opportunity

Worker Flow

- The unemployment flow equation is

$$\dot{u} = -f^U u + s(1 - u)$$

- In the steady state,

$$u = \frac{s}{s + f^U}$$

- Let $\hat{H}(w)$ be the mass of employed workers with wages below w , which follows

$$\dot{\hat{H}}(w) = f^U G(w)u - [s + f^E(1 - G(w))]\hat{H}(w)$$

- In the steady state,

$$\hat{H}(w) = \frac{f^U G(w)u}{s + f^E(1 - G(w))}$$

- Let $H(w)$ be the cdf of wage distribution among employed.

$$H(w) = \frac{\hat{H}(w)}{1 - u} = \frac{sG(w)}{s + f^E(1 - G(w))}$$

Labor Supply Function

- Employment at a firm offering wage $w \geq w^R$ evolves

$$\dot{l}(w) = qv(\chi + (1 - \chi)H(w)) - sl(w) - f^E l(w)(1 - G(w))$$

where $\chi \equiv u/(u + \zeta(1 - u)) = s/(s + f^E)$ is prob. of meeting u conditional on meeting

- In the steady state

$$\begin{aligned} l(w) &= \frac{qv(\chi + (1 - \chi)H(w))}{s + f^E(1 - G(w))} \\ &= \frac{qvs}{(s + f^E(1 - G(w)))^2} \end{aligned}$$

- $l(w)$ is increasing in w : higher $w \Rightarrow$ poach more and poached less

Search Friction as a source of Monopsony Power

- To simplify our life, let $r \rightarrow 0$ so that the firms maximize the steady-state profit
- The firms solve

$$\max_w (z - w)l(w)$$

- At this point, this is a typical “monoposny” problem:
⇒ firms face an upward-sloping labor supply curve.
- Search frictions give you a full microfoundation of $l(w)$
- Other microfoundations:
 - Job differentiation
 - A finite number of firms (“Oligopsony”)

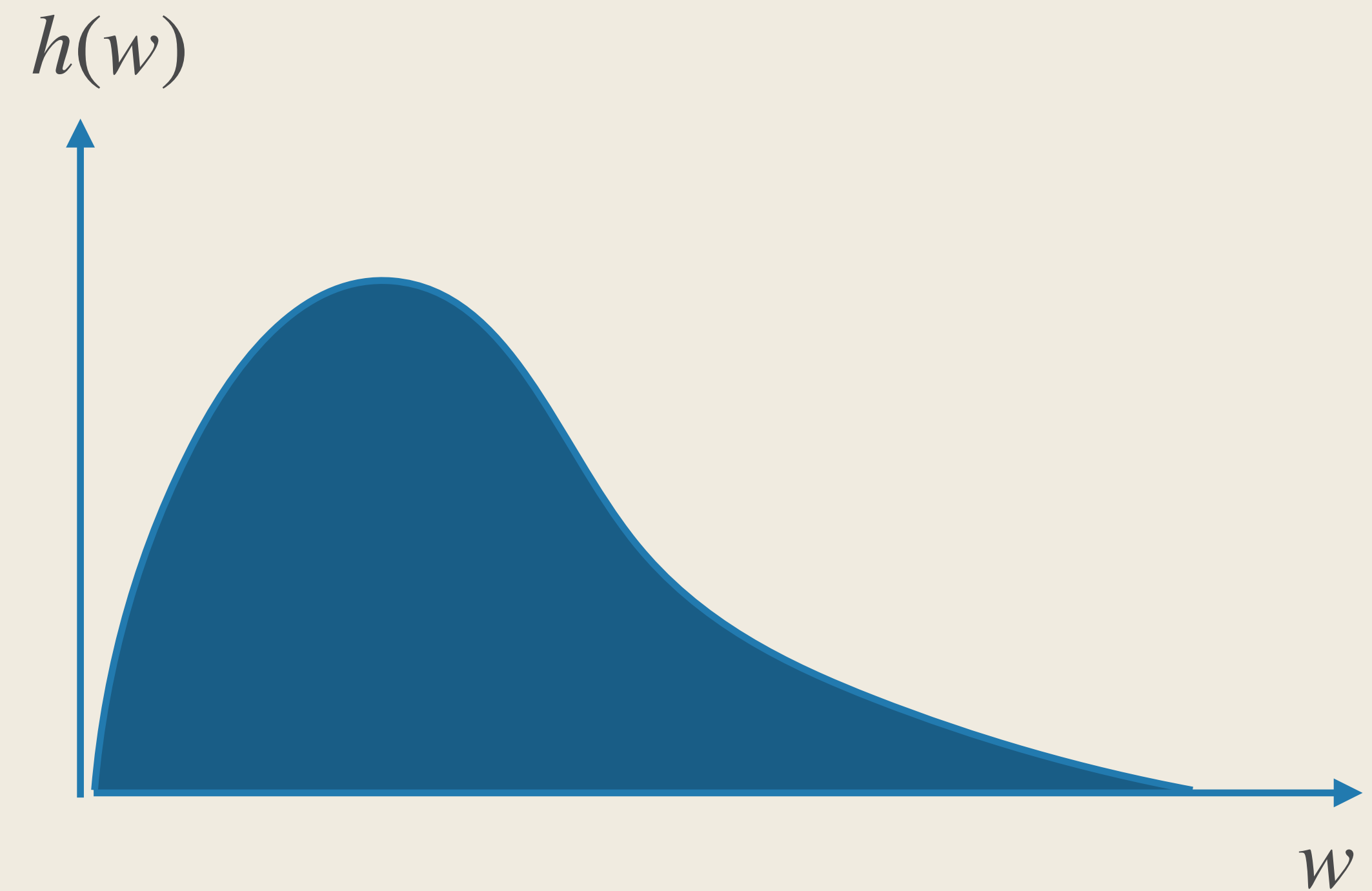
Frictional Wage Dispersion

- Since all firms are homogenous, tempted to think we have a symmetric eqm
- Suppose all firms offer $w = \hat{w} \in [w^R, z)$
- Then, a firm can profitably deviate by offering $\hat{w} + \epsilon$
 - The cost of doing so is continuous in ϵ
 - But it attracts a discontinuously larger amount of workers
 - The firm can poach all workers
 - No other firms can poach workers from the firm
- All firms offering $\hat{w} \geq z$ cannot be an eqm because $w = z - \epsilon$ gives higher profits
- Therefore, equilibrium has to be a mixed-strategy equilibrium
⇒ “Frictional wage dispersion”

Smooth Wage Distribution

More generally,

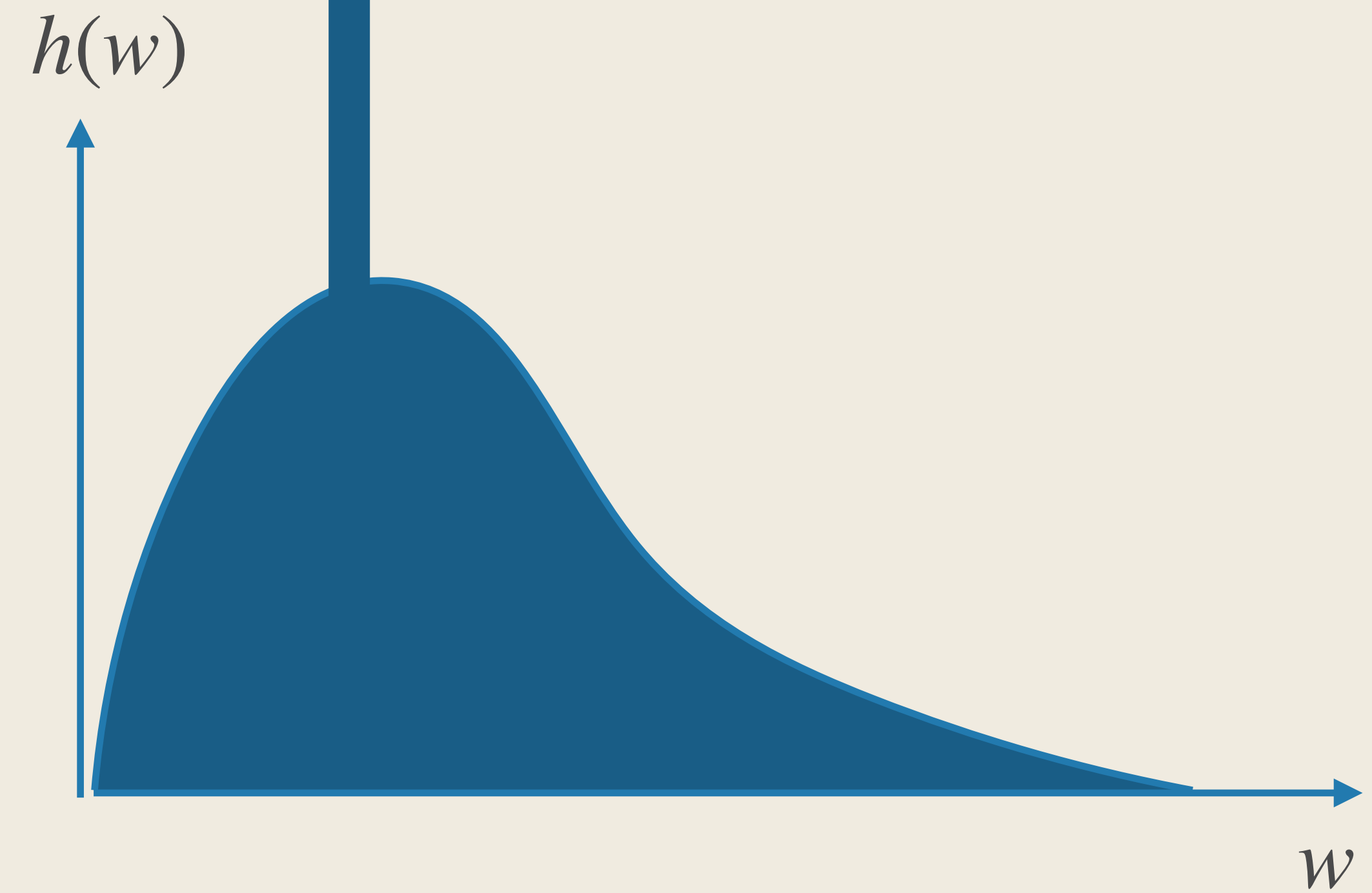
1. There cannot be a mass point
2. There cannot be a gap



Smooth Wage Distribution

More generally,

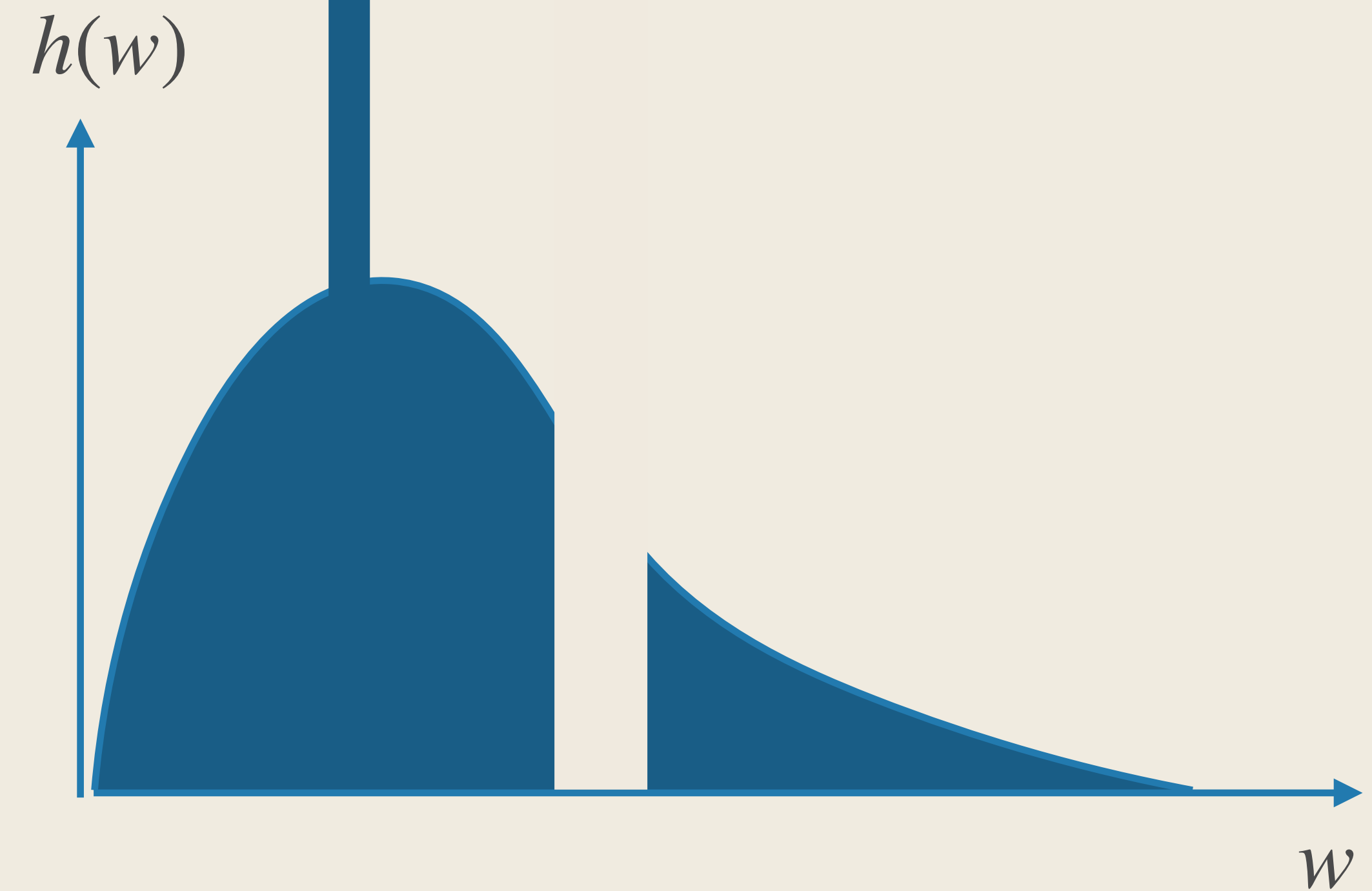
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Smooth Wage Distribution

More generally,

1. There cannot be a mass point
2. There cannot be a gap



Wage Offer Distribution

- Firms with the lowest wage offer must find it optimal to offer w^R

- All the other firms must be indifferent to offering w^R , implying

$$(z - w)l(w) = (z - w^R)l(w^R) \quad \text{for all } w \text{ in the support of } G$$

- Since $l(w^R) = qvs/(f^E + s)^2$, $G(w)$ must satisfy

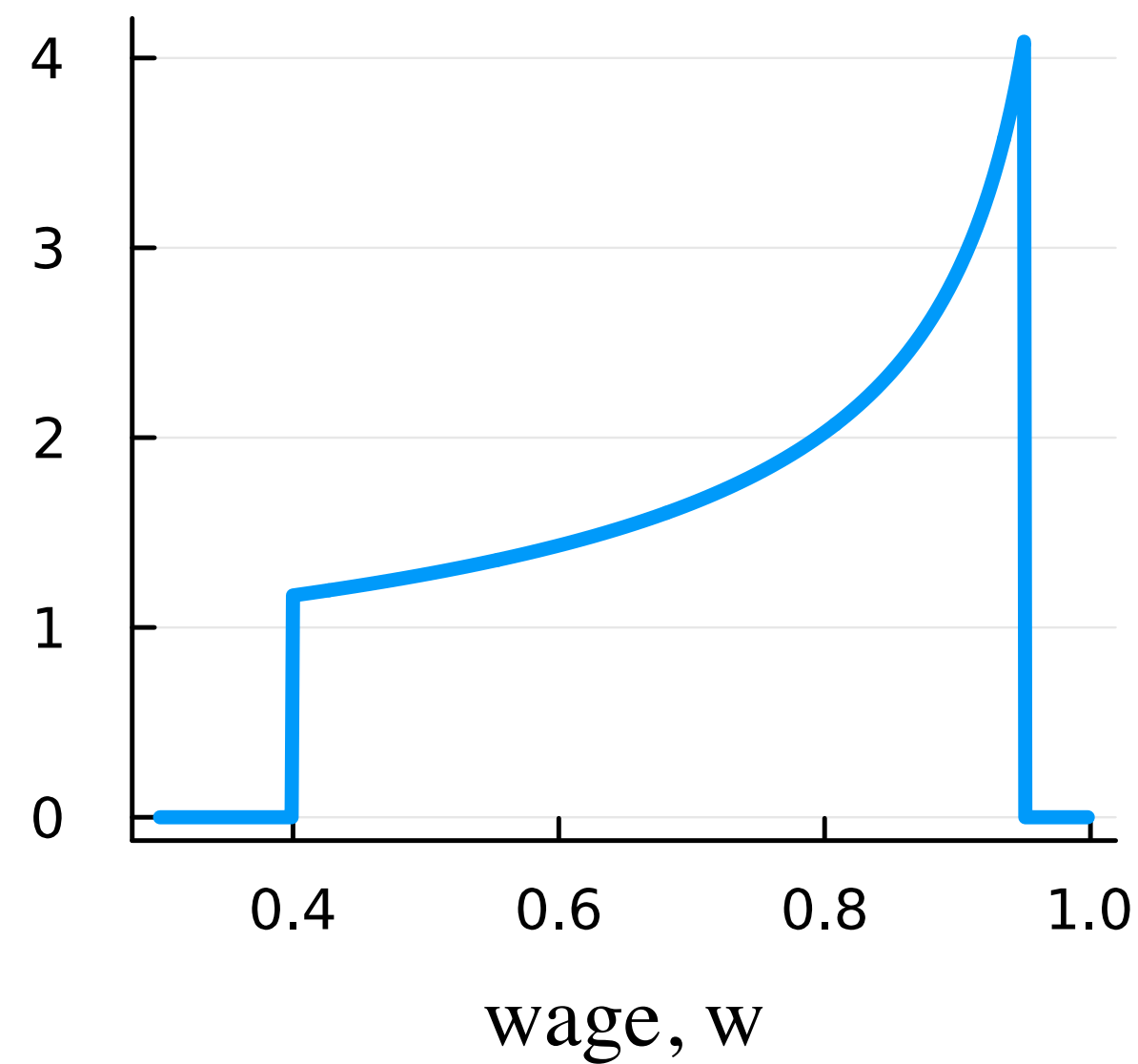
$$G(w) = (1 + s/f^E) \left(1 - \sqrt{\frac{(z - w)}{(z - w^R)}} \right) \quad \text{for } w \in [w^R, \bar{w}], \quad \text{where } G(\bar{w}) = 1$$

- Plug back to the definition of $H(w)$:

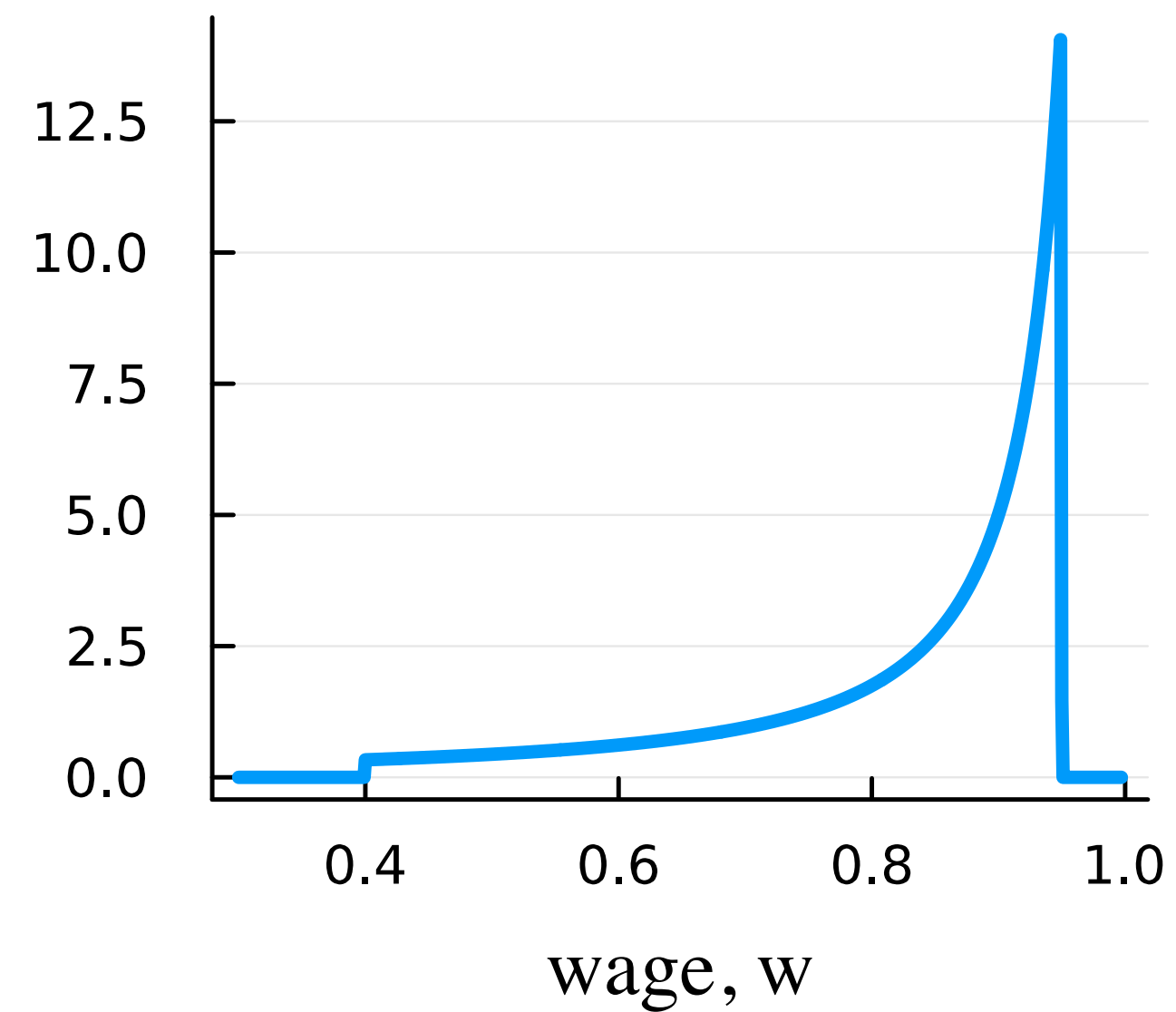
$$H(w) = \frac{s}{f^E} \left(\sqrt{\frac{(z - w^R)}{(z - w)}} - 1 \right)$$

Numerical Example

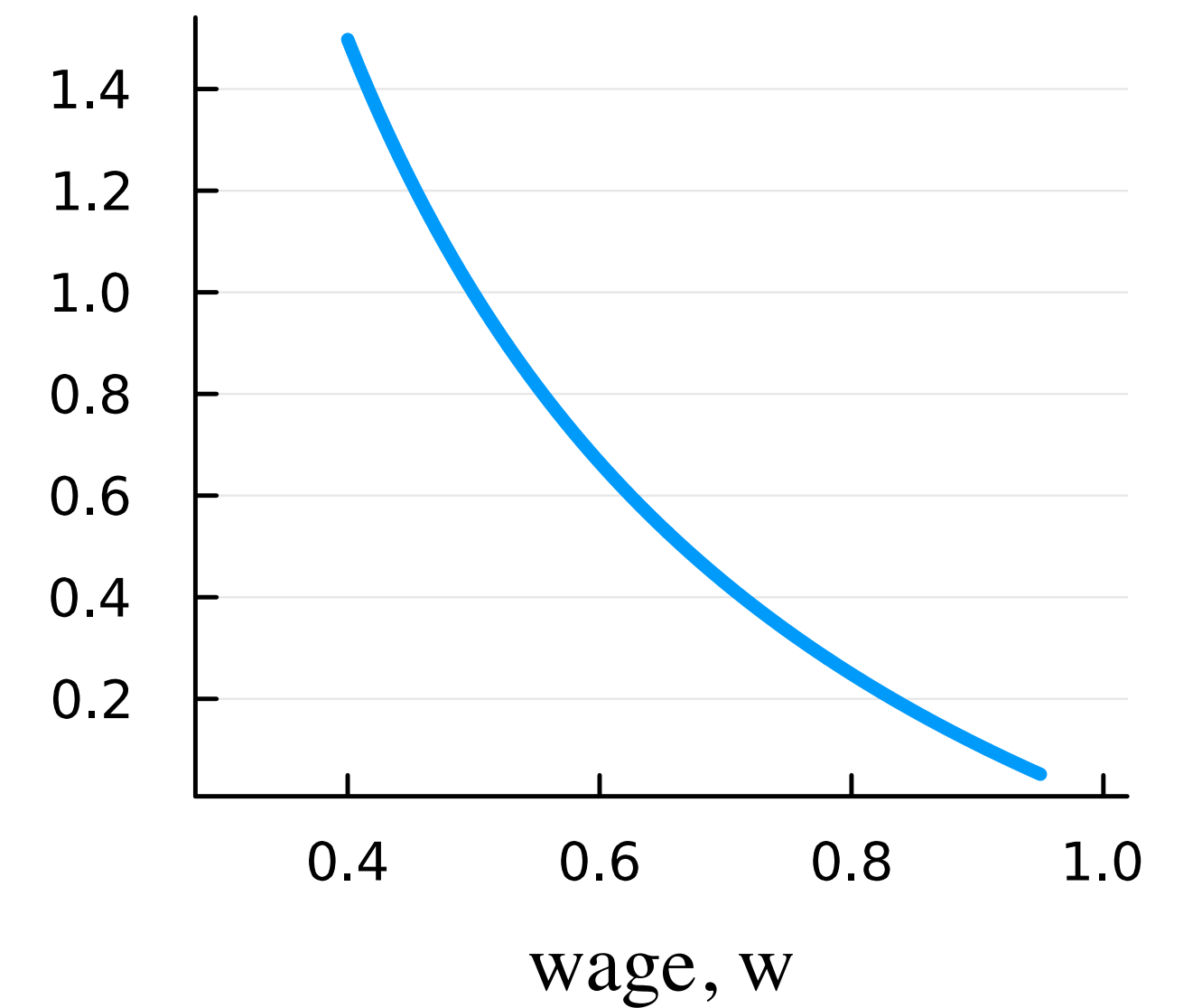
Wage Offer Distribution, $g(w)$



Wage Distribution, $h(w)$

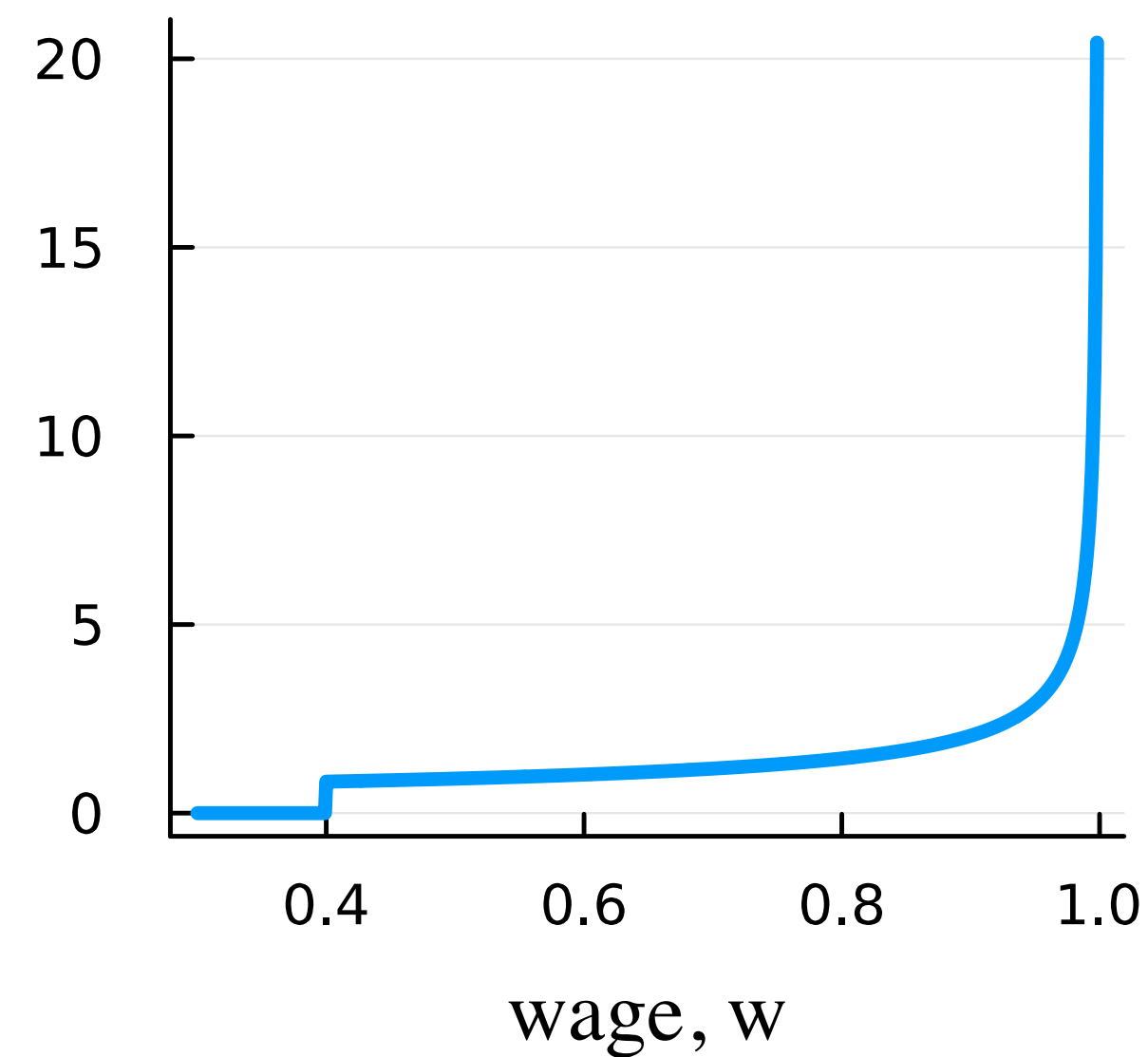


Wage Markdown $(z - w)/w$

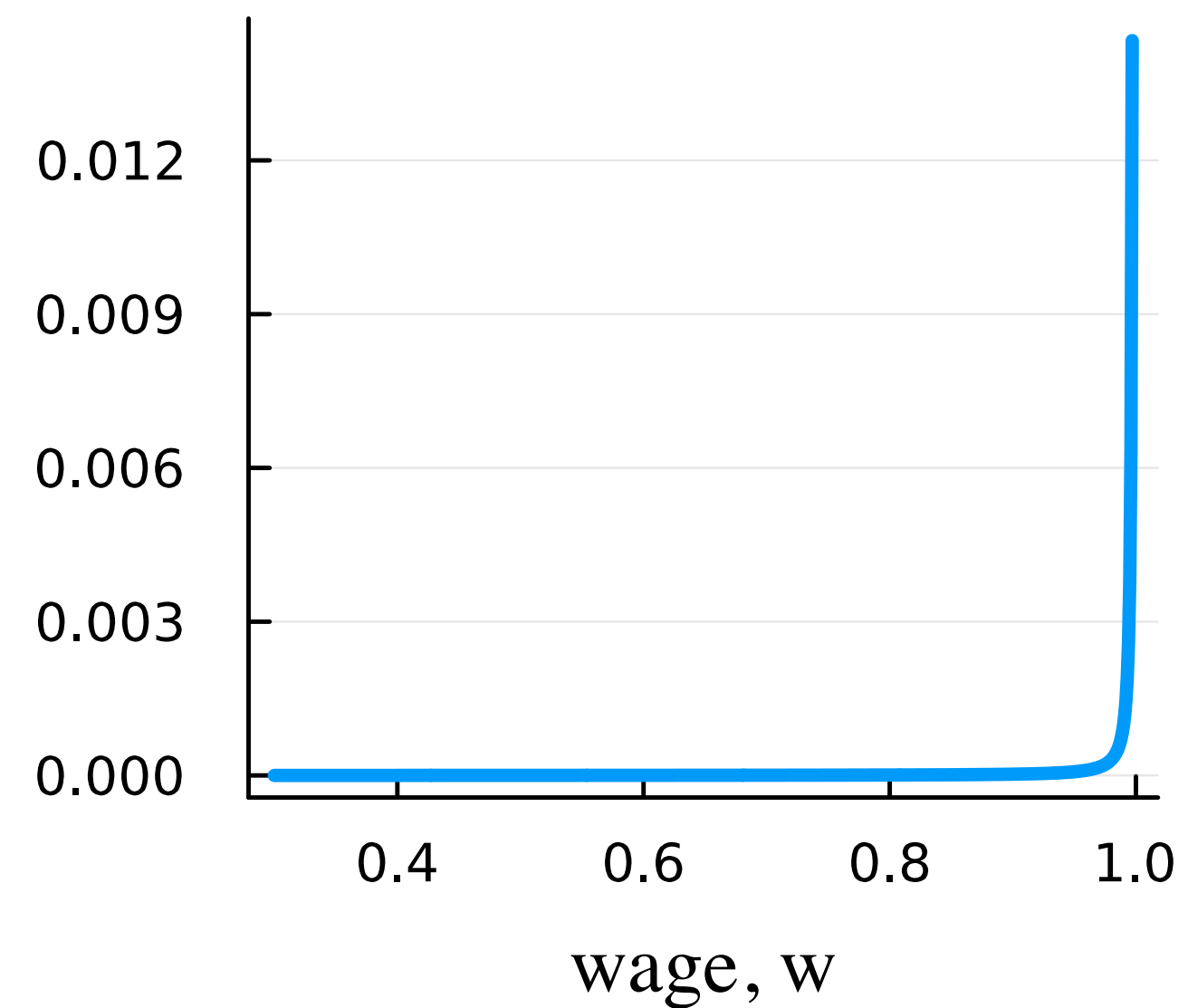


Numerical Example ($f^E \rightarrow \infty$)

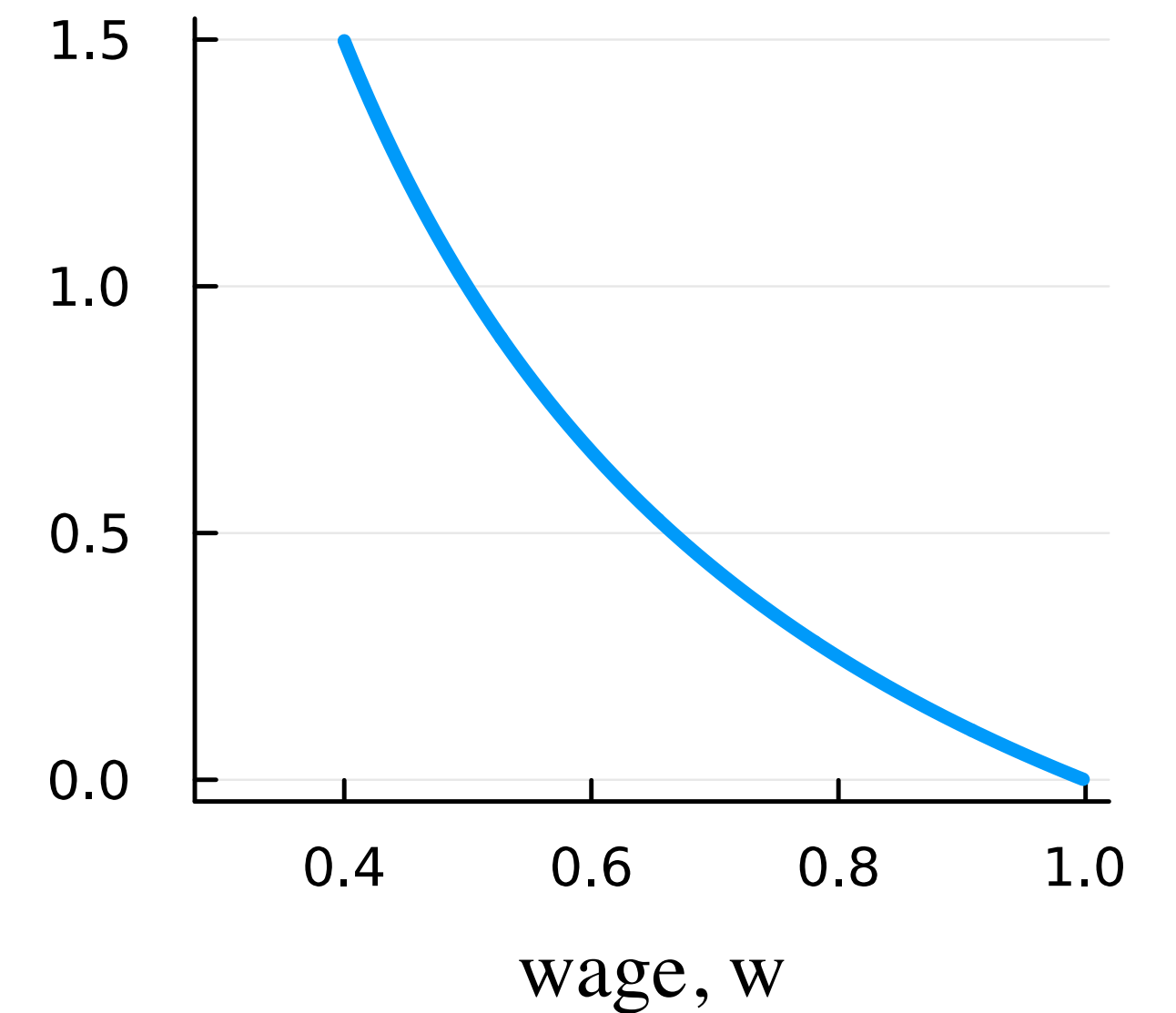
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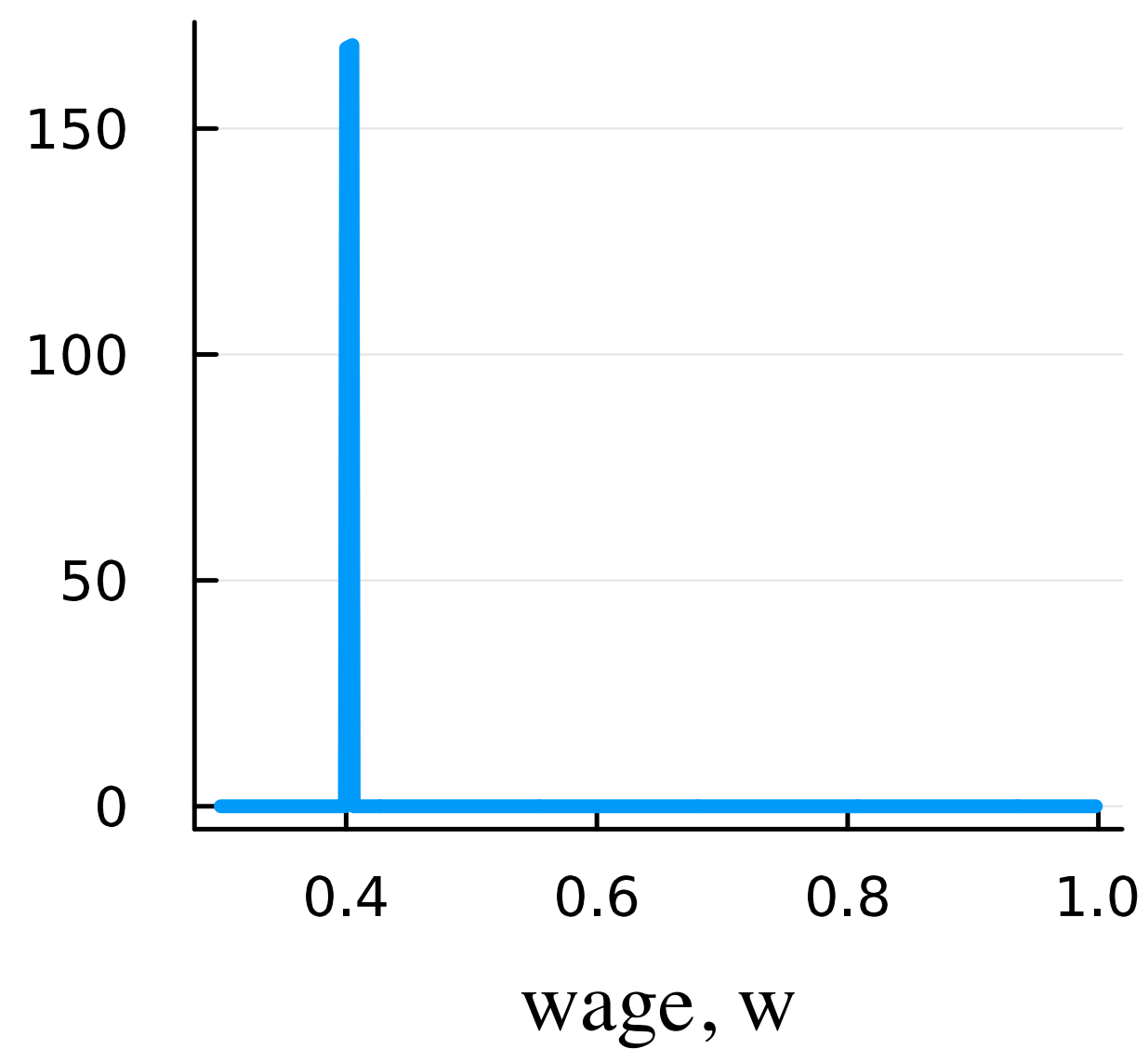


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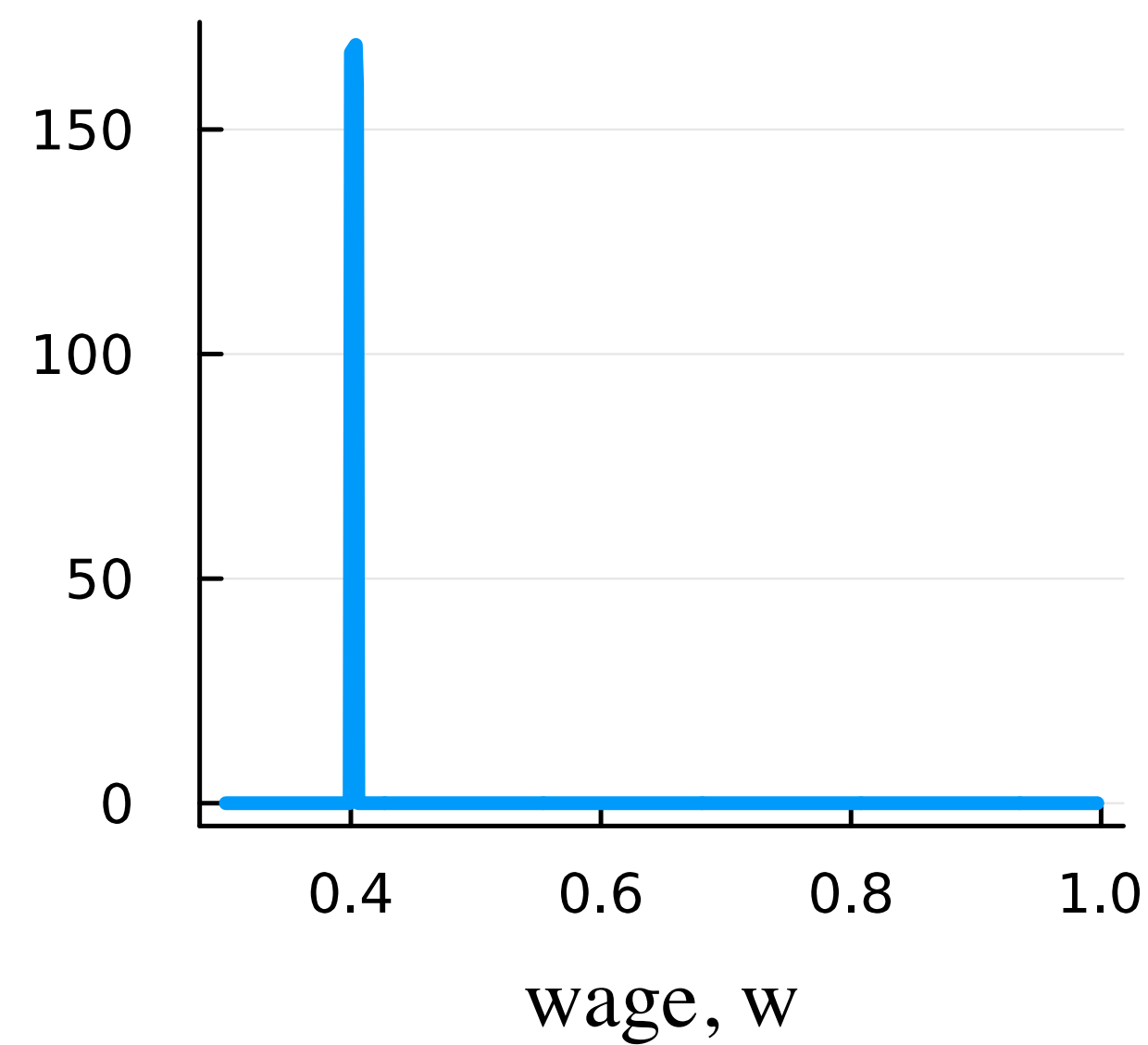


Numerical Example ($f^E \rightarrow 0$)

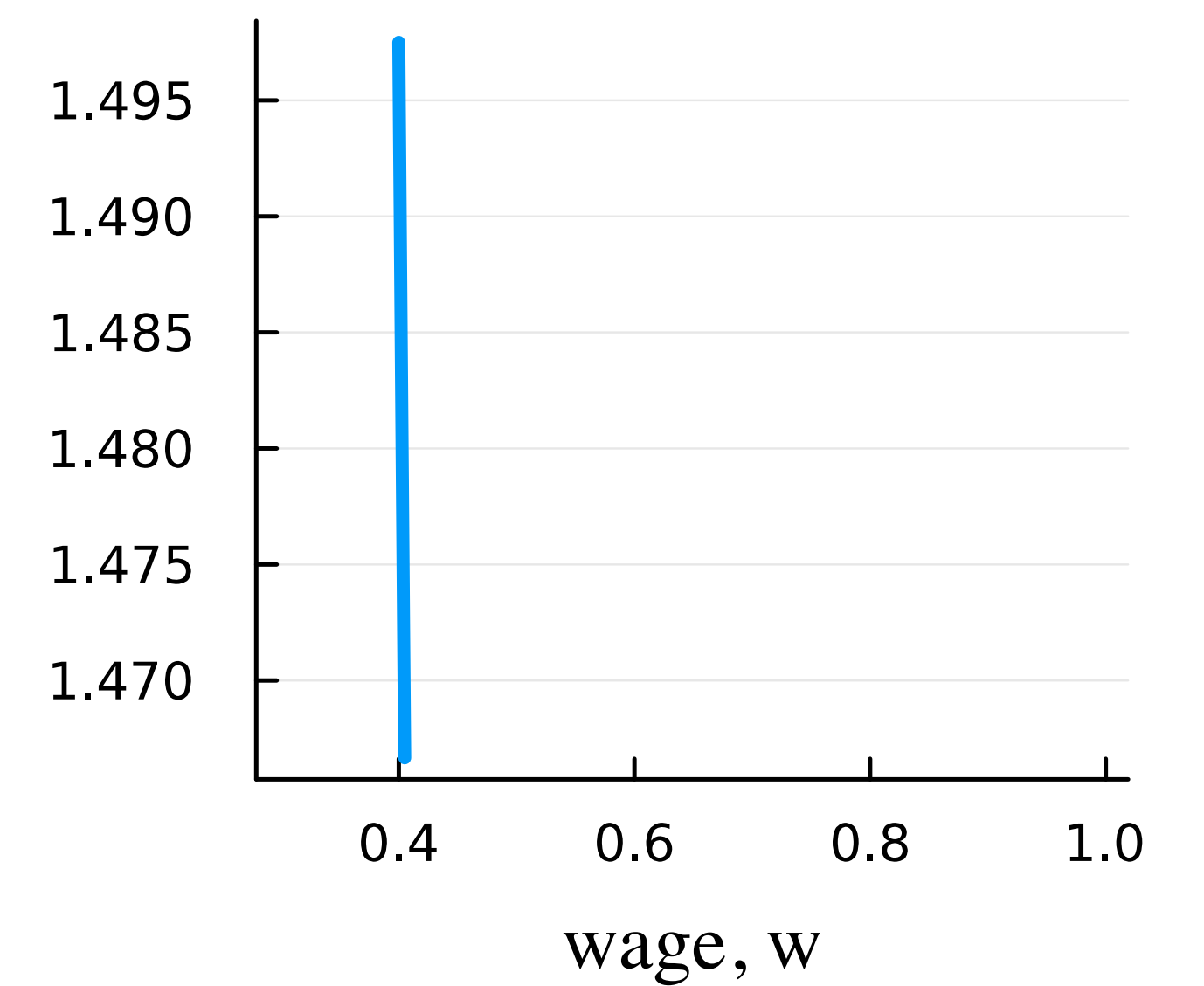
Wage Offer Distribution, $g(w)$



Wage Distribution, $h(w)$



Wage Markdown $(z - w)/w$



Heterogenous Firms

Heterogenous Firm Setup

- Now suppose the firm's productivity distribution is continuous and given by $J_0(z)$

- Firms with $z \leq w^R$ cannot make profits so inactive. Let $J(z) \equiv \frac{J_0(z) - J_0(w^R)}{1 - J_0(w^R)}$

- Firms with $z \geq w^R$ solve

$$\max_w (z - w)l(w) \quad (\star)$$

- The first-order condition is ($\epsilon_w \equiv l'(w)w/l(w)$)

$$\underbrace{\frac{z - w}{w}}_{\text{wage markdown}} = \frac{1}{\underbrace{\epsilon_w}_{\text{inv. LS elasticity}}} \Rightarrow (z - w) \frac{2f^E G'(w)}{(s + f^E(1 - G(w)))} = 1$$

- At this point, what do we know about $G(w)$? – Nothing... big fixed point problem!

Rank-Preserving Property

- Realize that (★) is strictly supermodular in (z, w)
 $\Rightarrow w(z)$ is strictly increasing in z

- Then we know a lot about $G(w)$

$$G(w(z)) = J(z)$$

$$\Rightarrow G'(w(z))w'(z) = J'(z)$$

- It is this **rank-preserving property** that makes all the job-ladder models tractable
 - More productive firms poach from less productive firms

Solving for Wage Function

- Combine with FOCs to obtain an ODE that characterizes the equilibrium wage

$$w'(z) = (z - w(z)) \frac{2f^E J'(z)}{s + f^E(1 - J(z))}$$

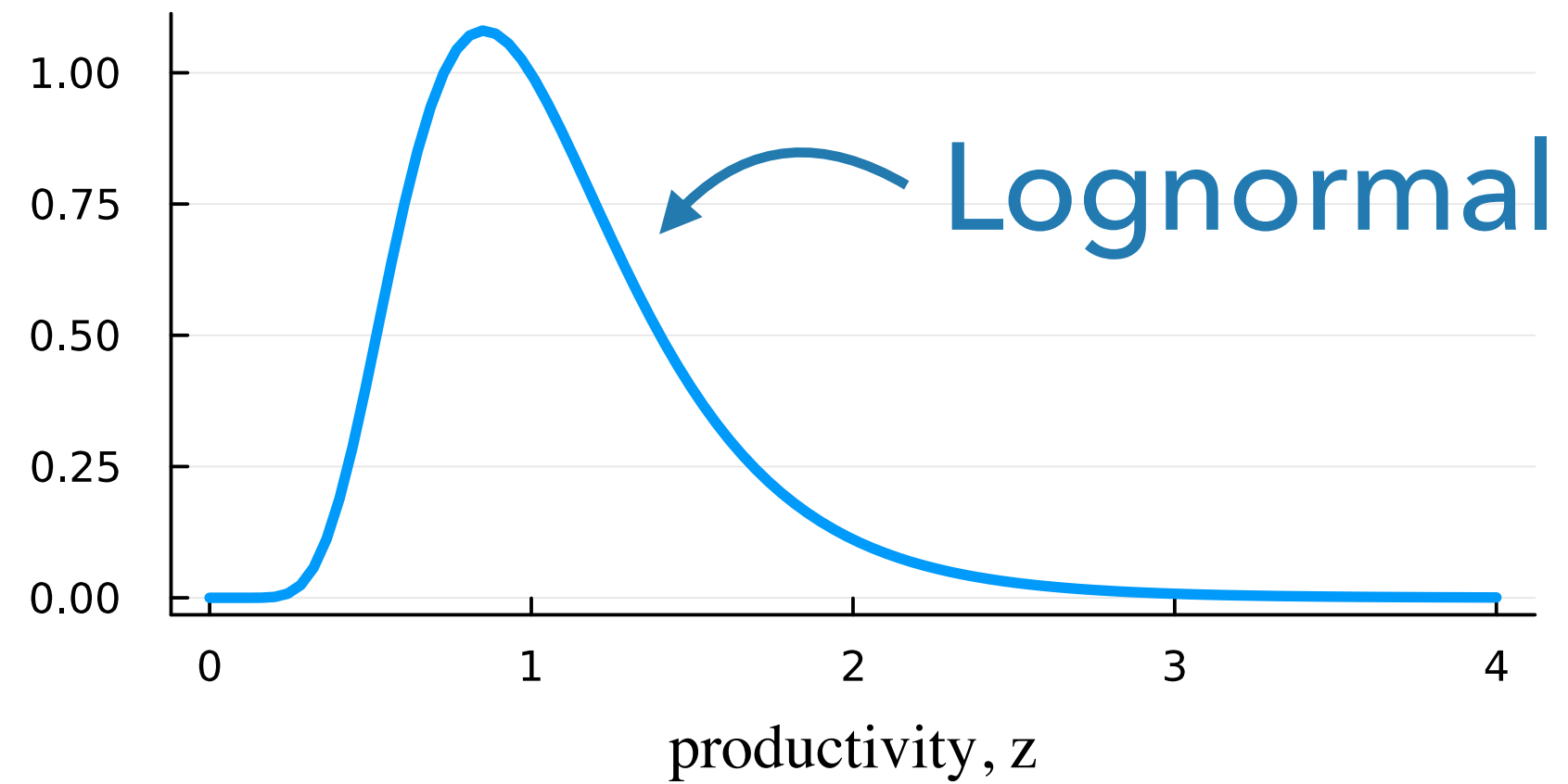
- Solving the ODE with boundary condition $w(w^R) = w^R$

$$w(z) = z - \int_{w^R}^z \frac{(s + f^E(1 - J(\tilde{z})))^2}{(s + f^E(1 - J(\tilde{z})))^2} d\tilde{z}$$

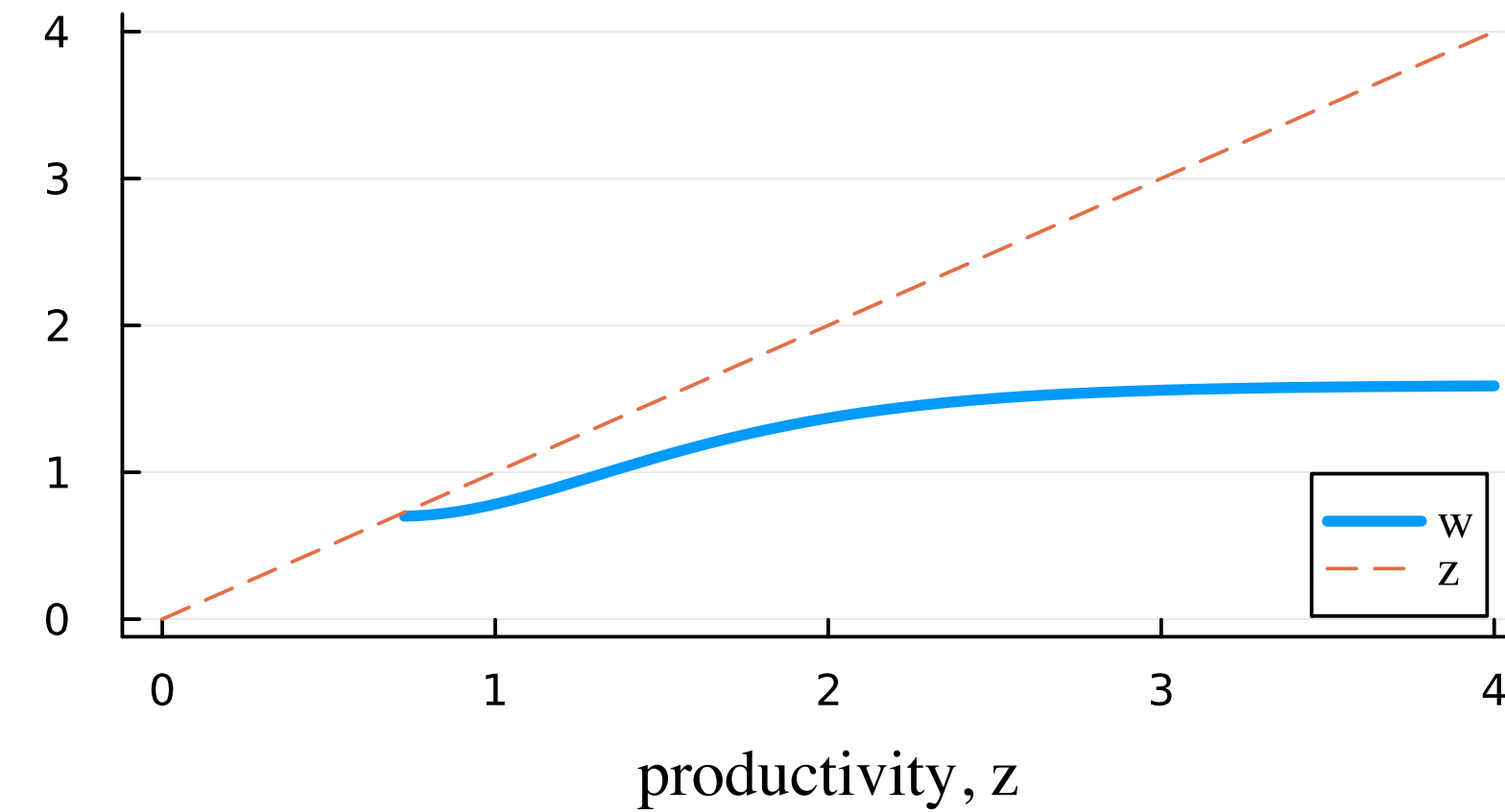
- One can check the second-order condition is also satisfied

Numerical Example

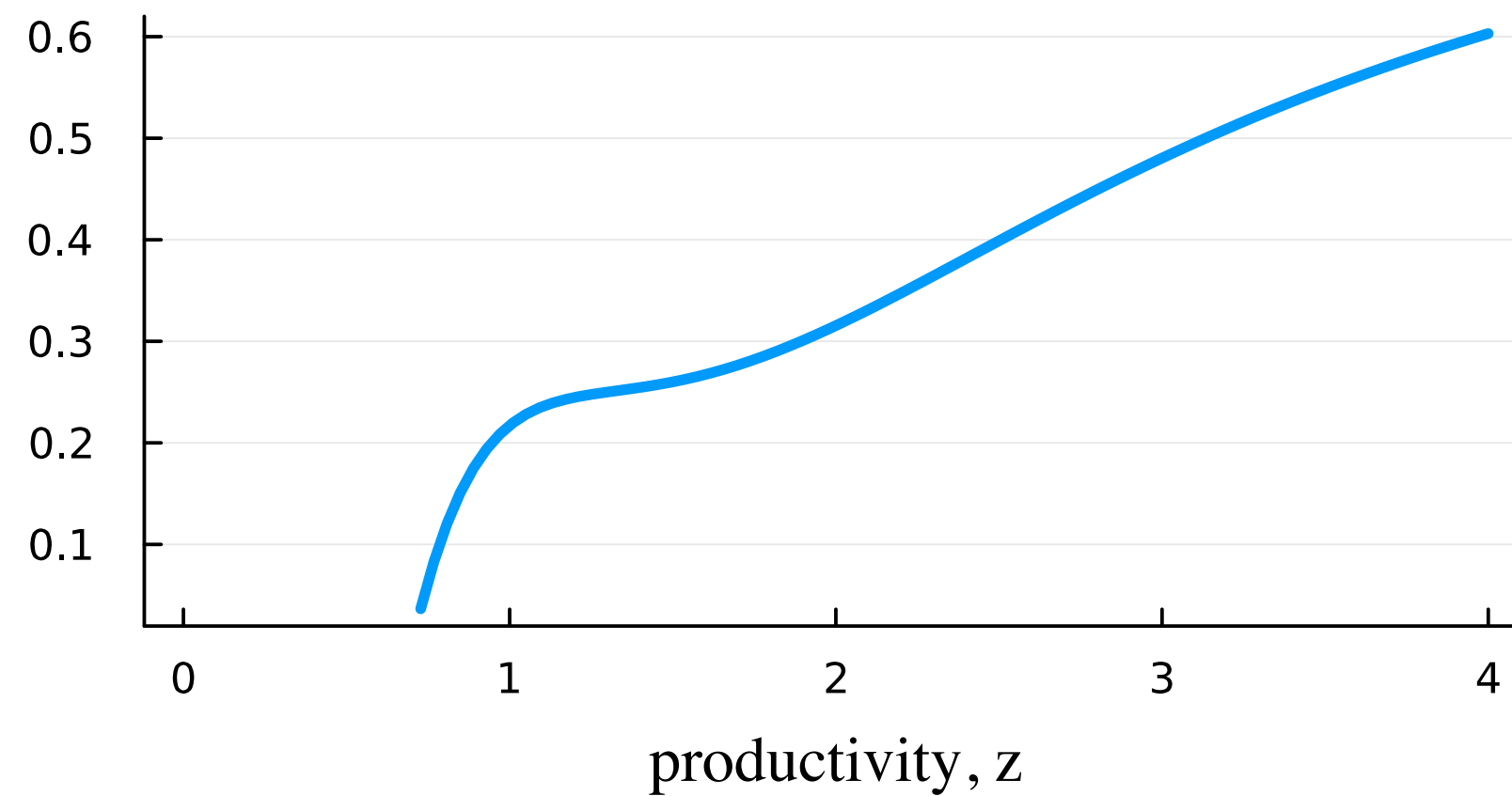
Productivity Distribution, $J'(z)$



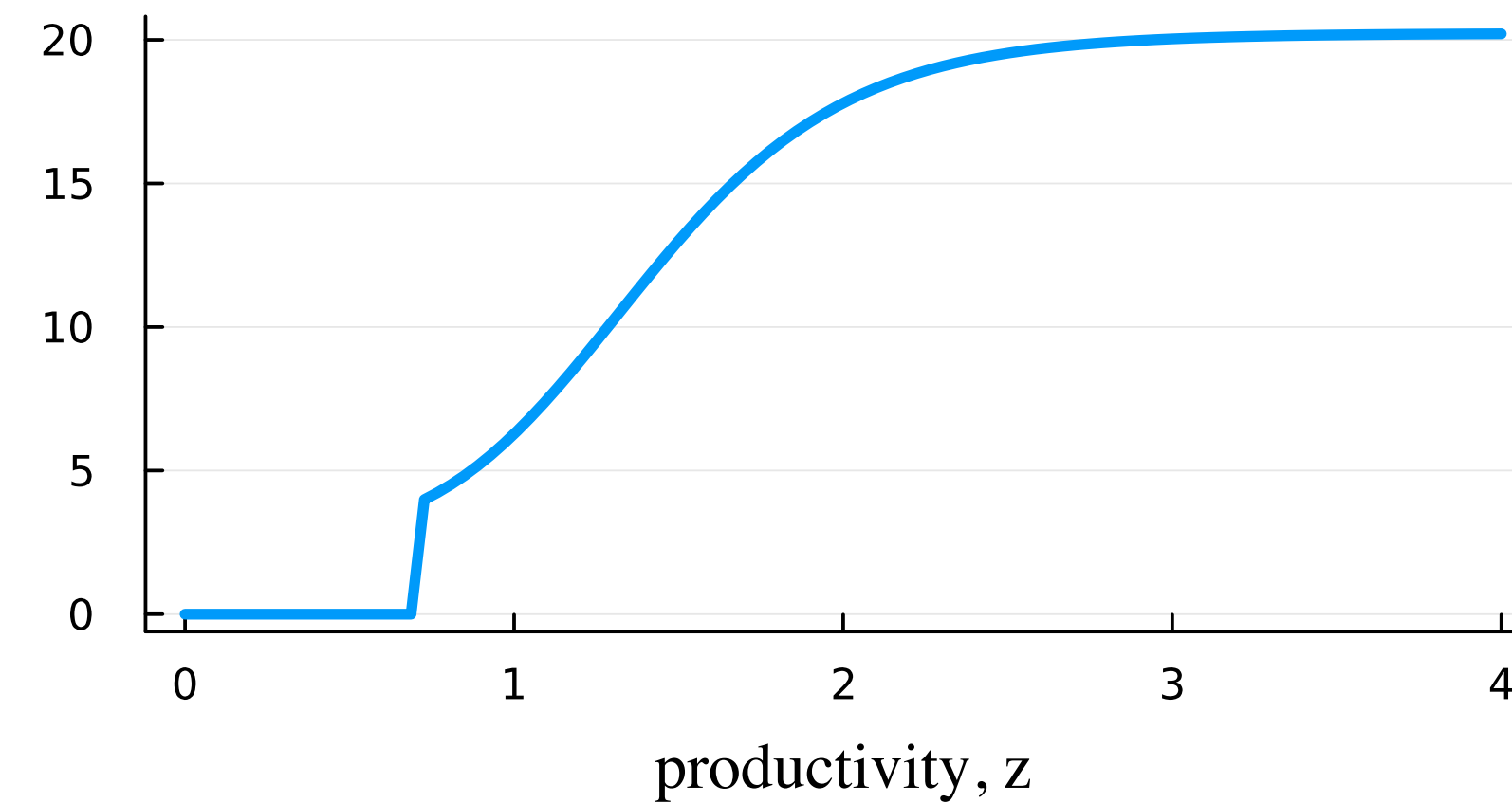
Wage, $w(z)$



Wage Markdown, $(z - w)/z$



Firm Size, $l(w(z))$



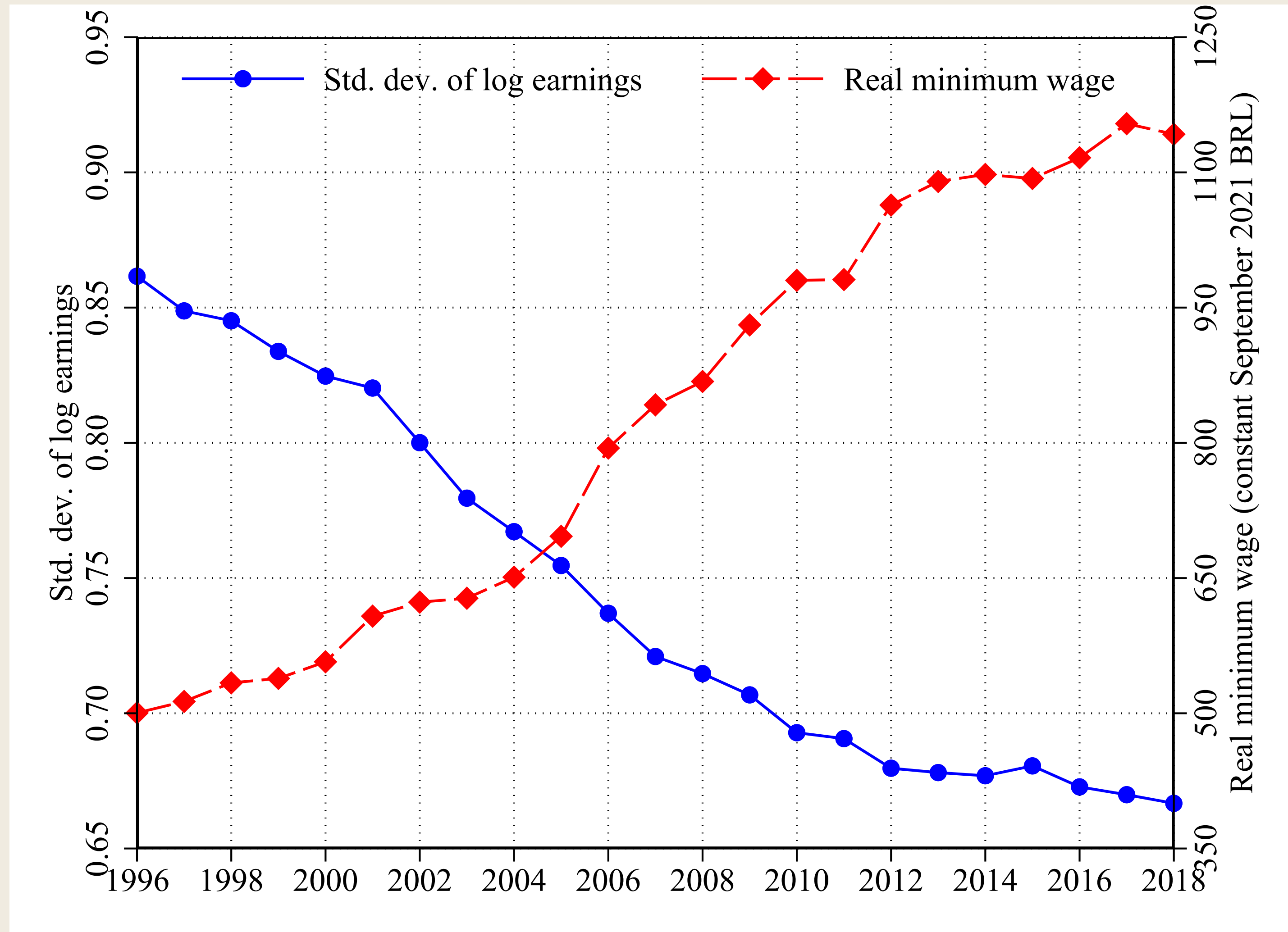
Wage Distribution



Spillover Effect of Minimum Wage

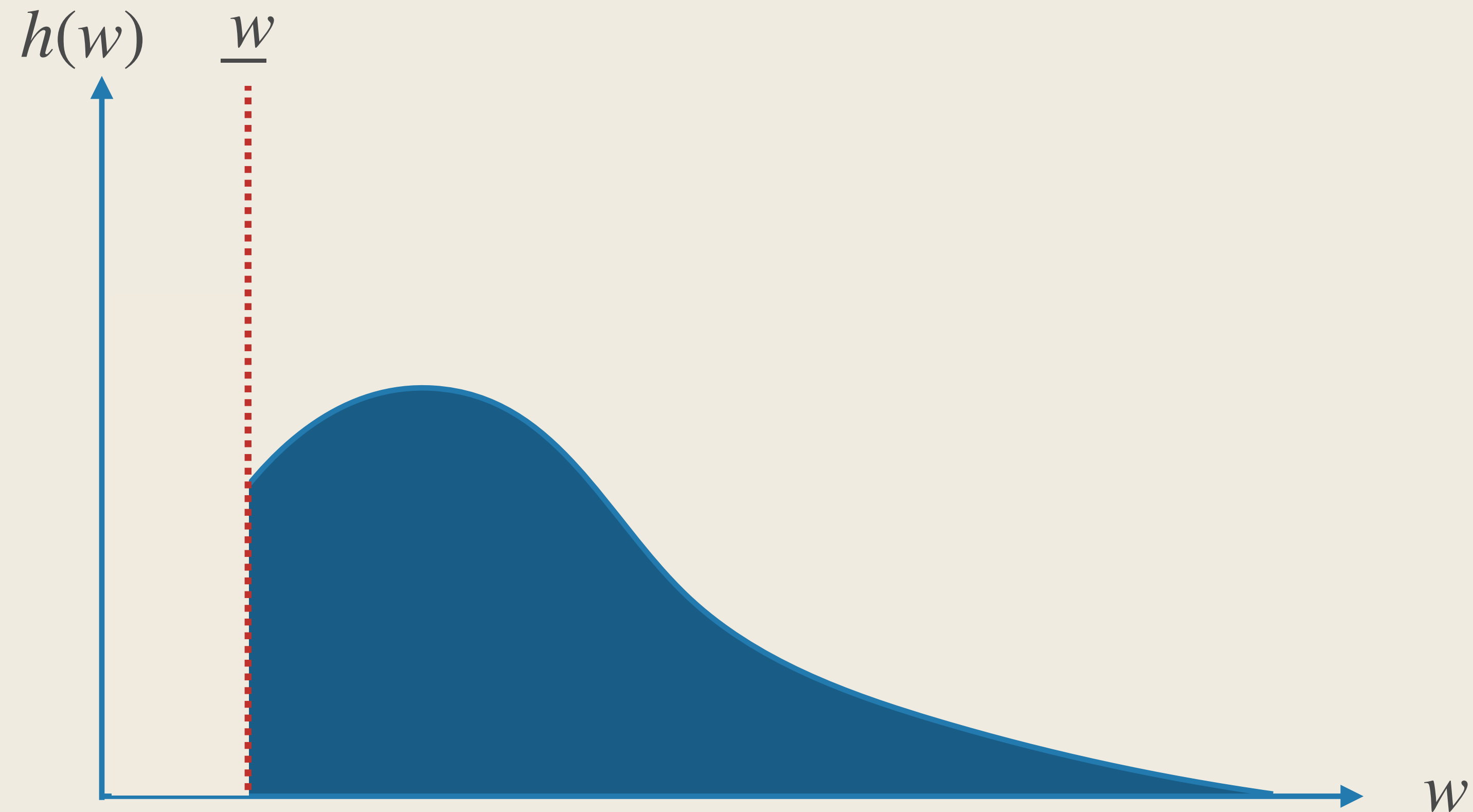
– Engbom and Moser (2021)

Earnings Inequality and Minimum Wage in Brazil



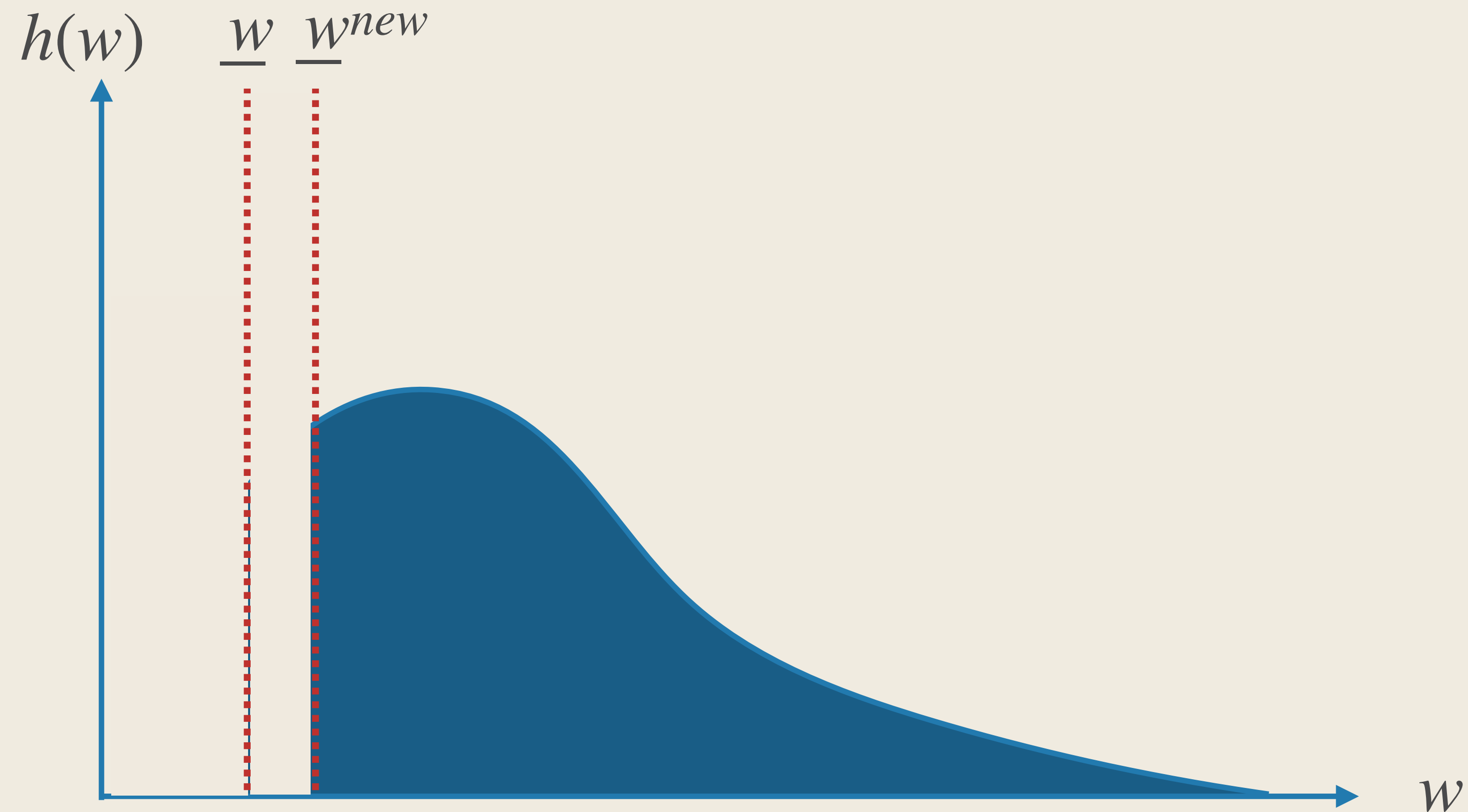
Minimum Wage Spillover

- Interpret \underline{w} as the minimum wage
- Suppose we raise \underline{w} . What would happen to the wage distribution?



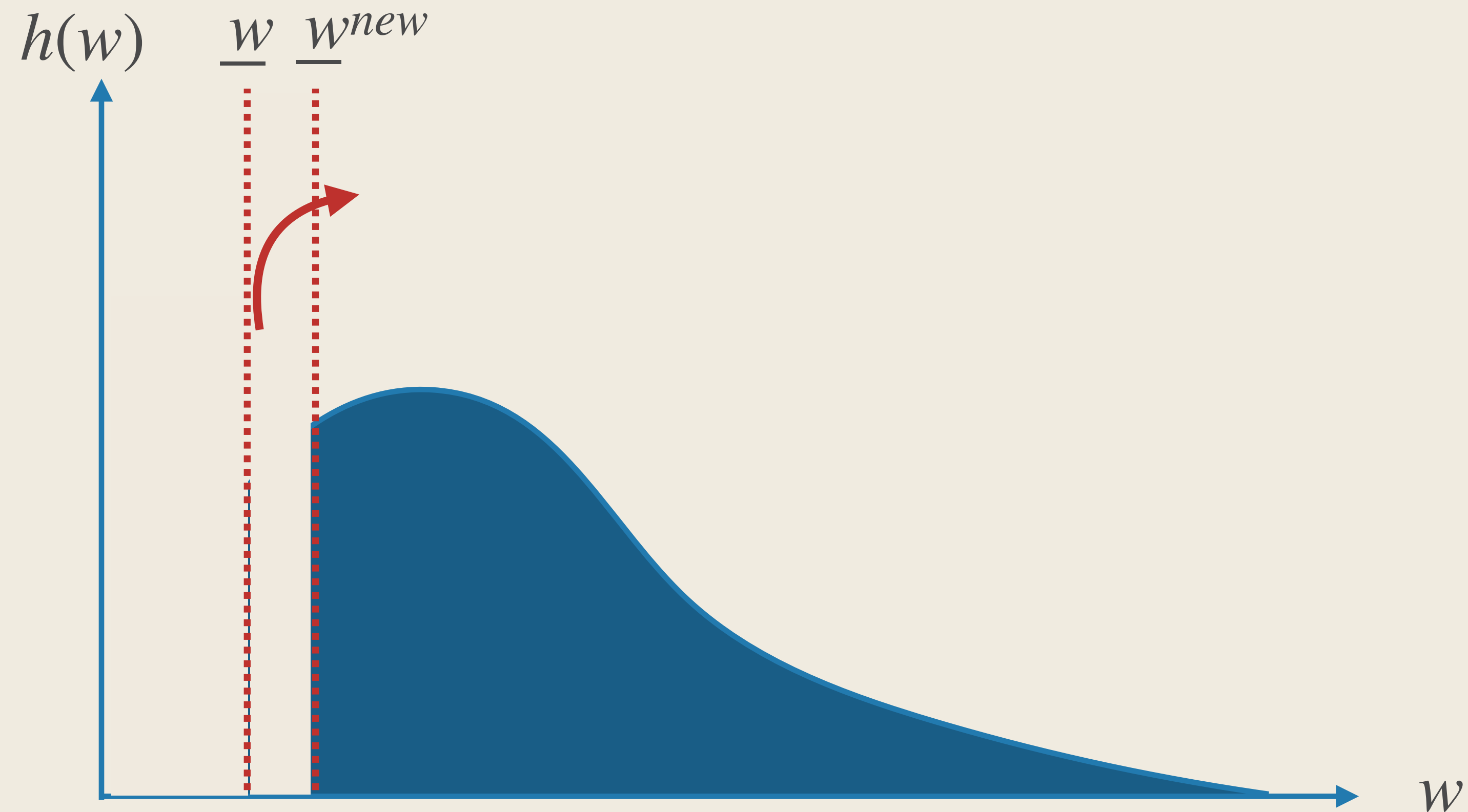
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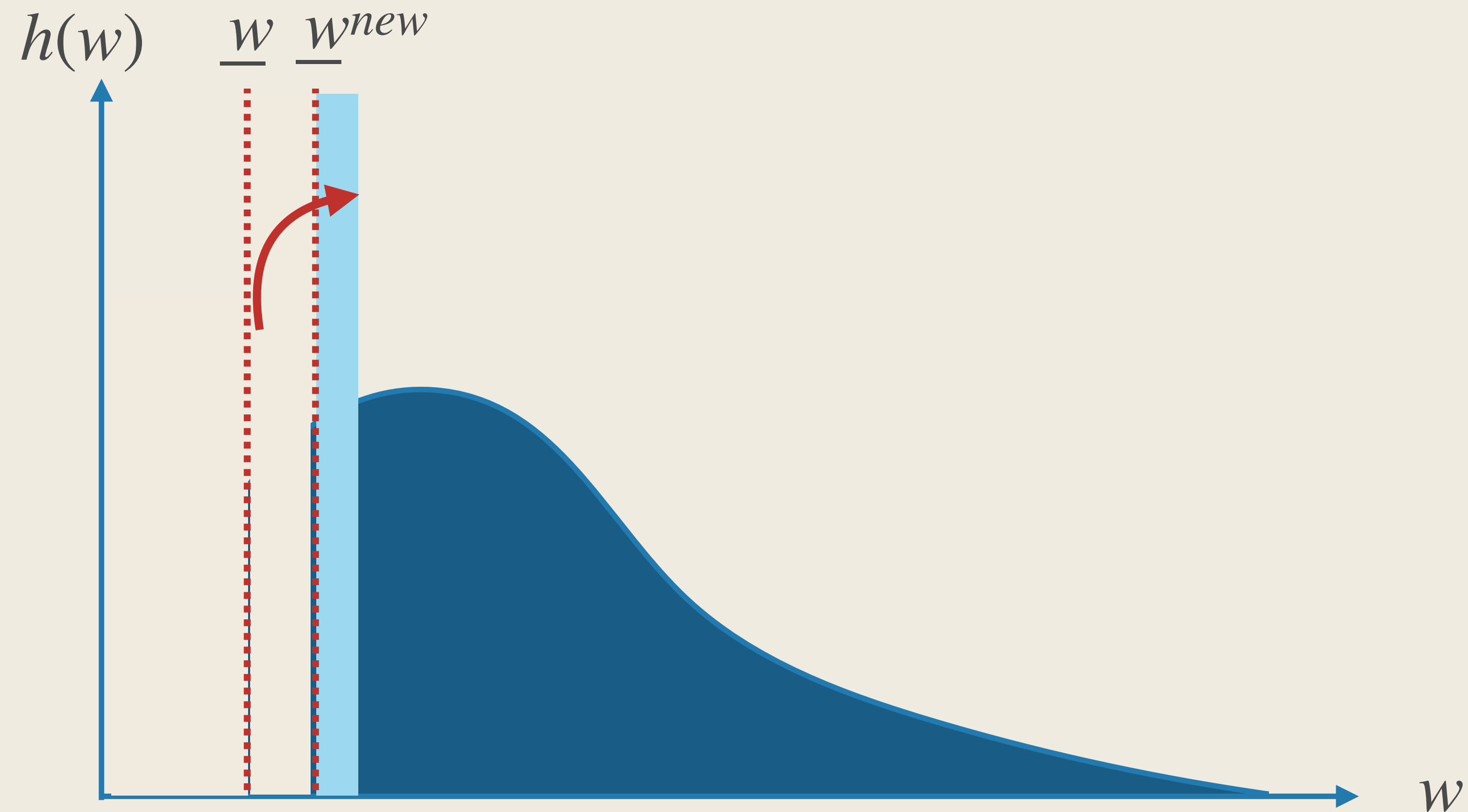
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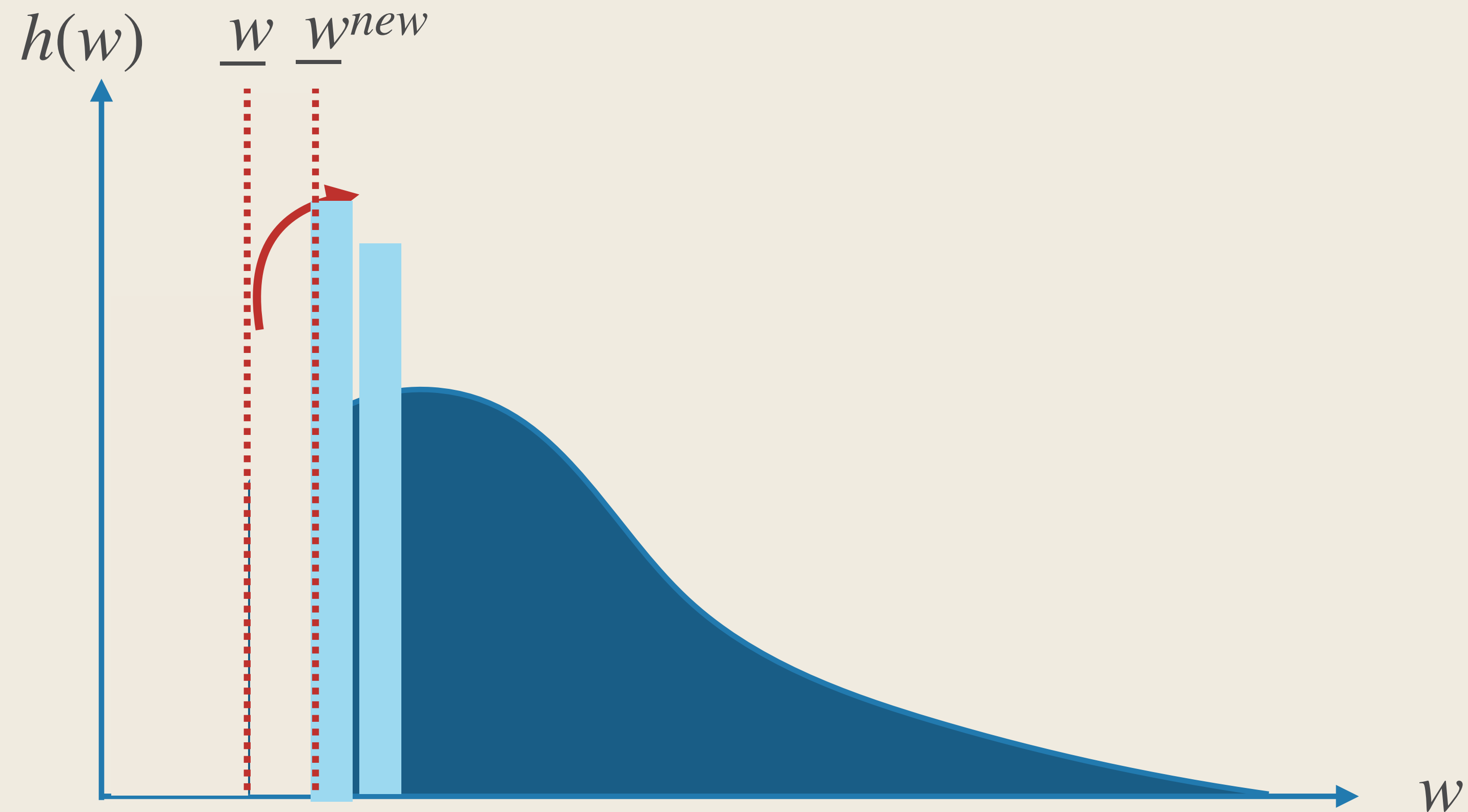
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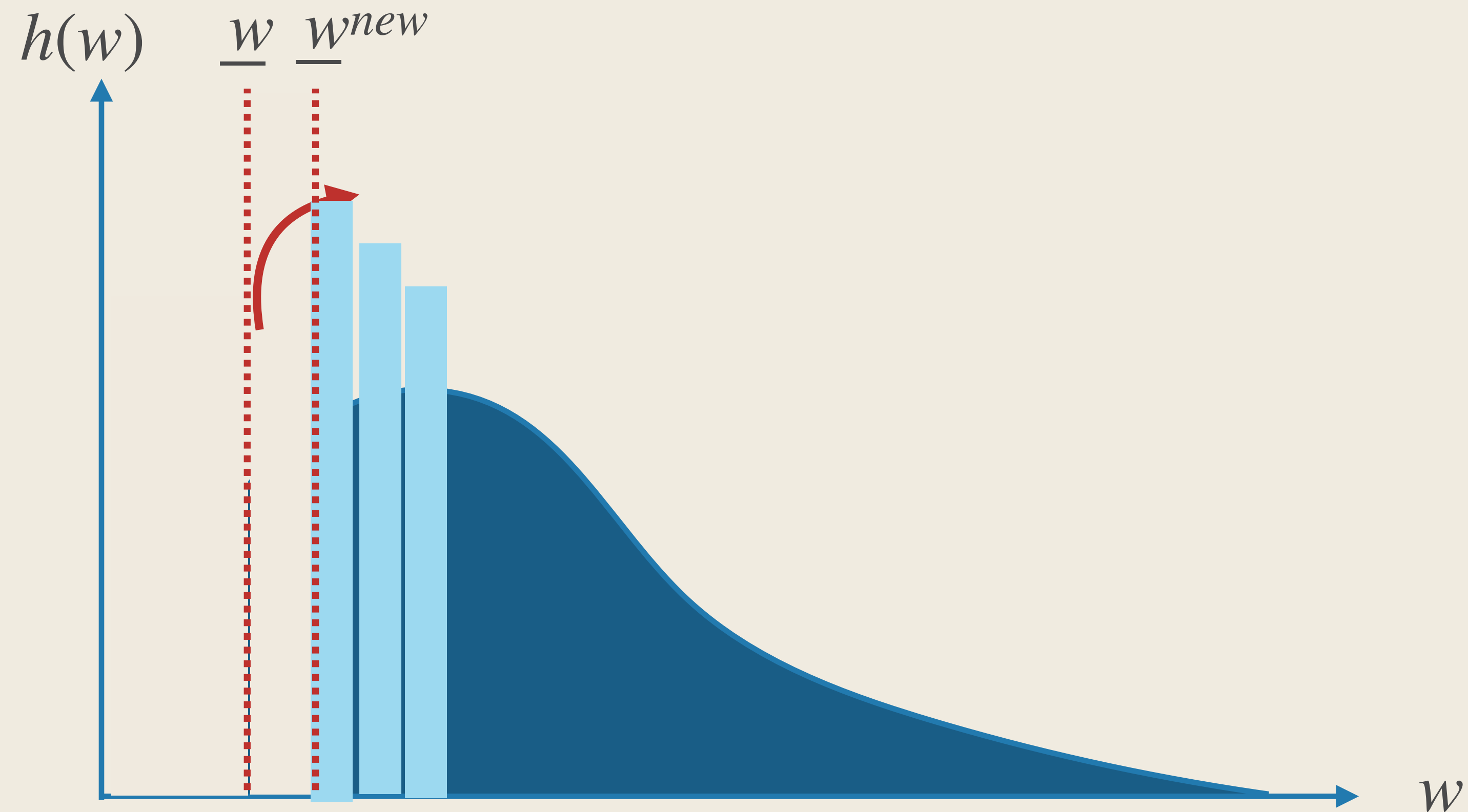
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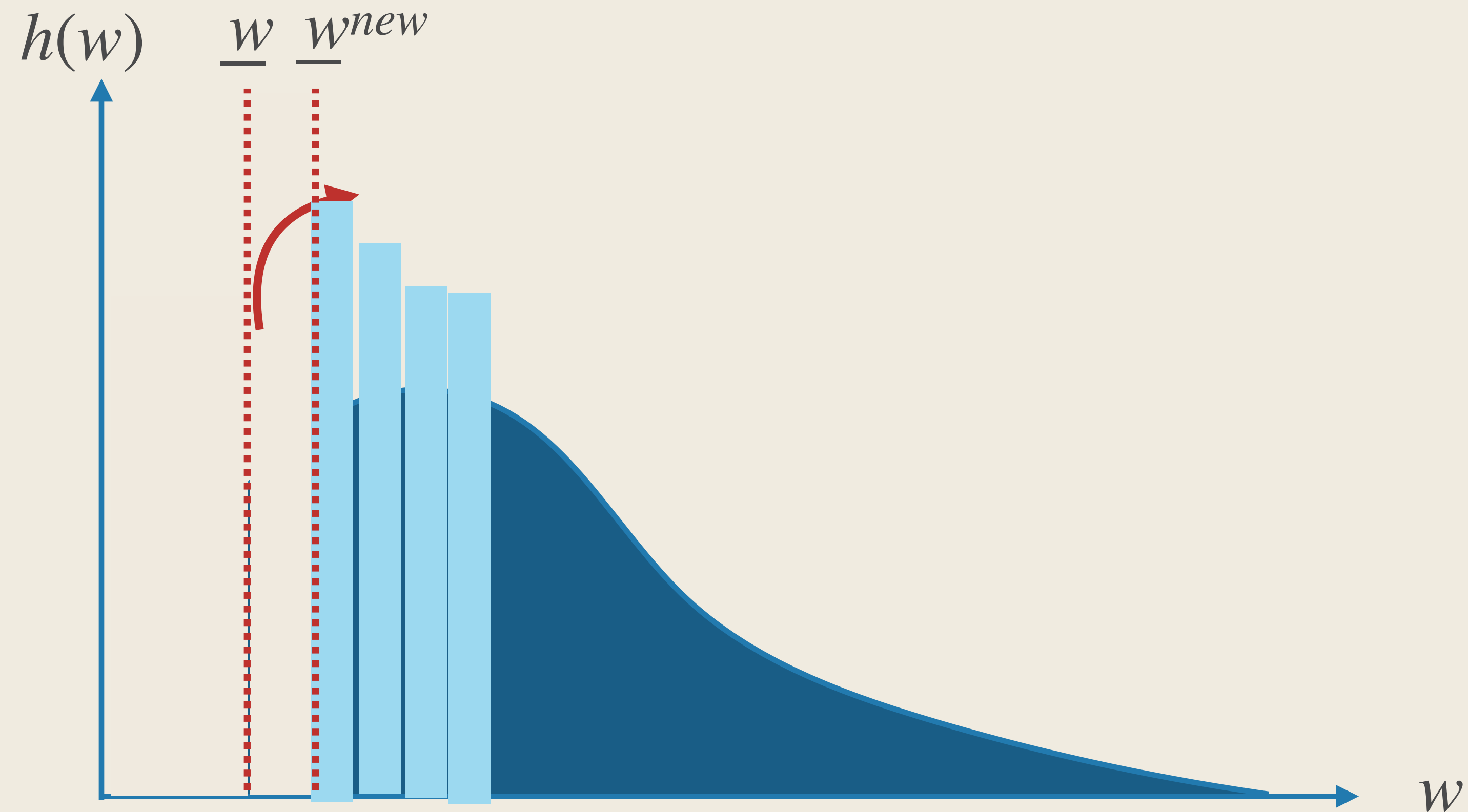
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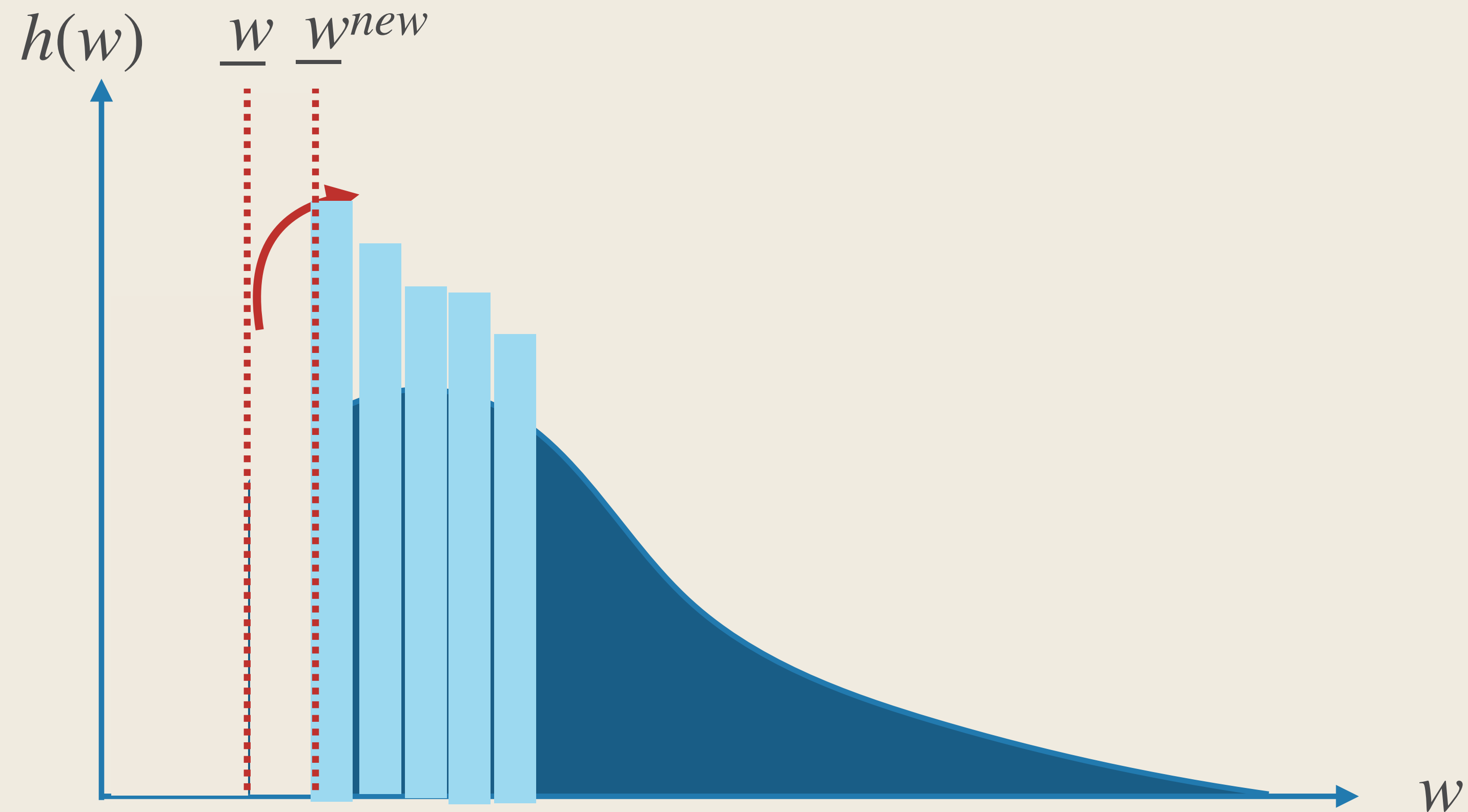
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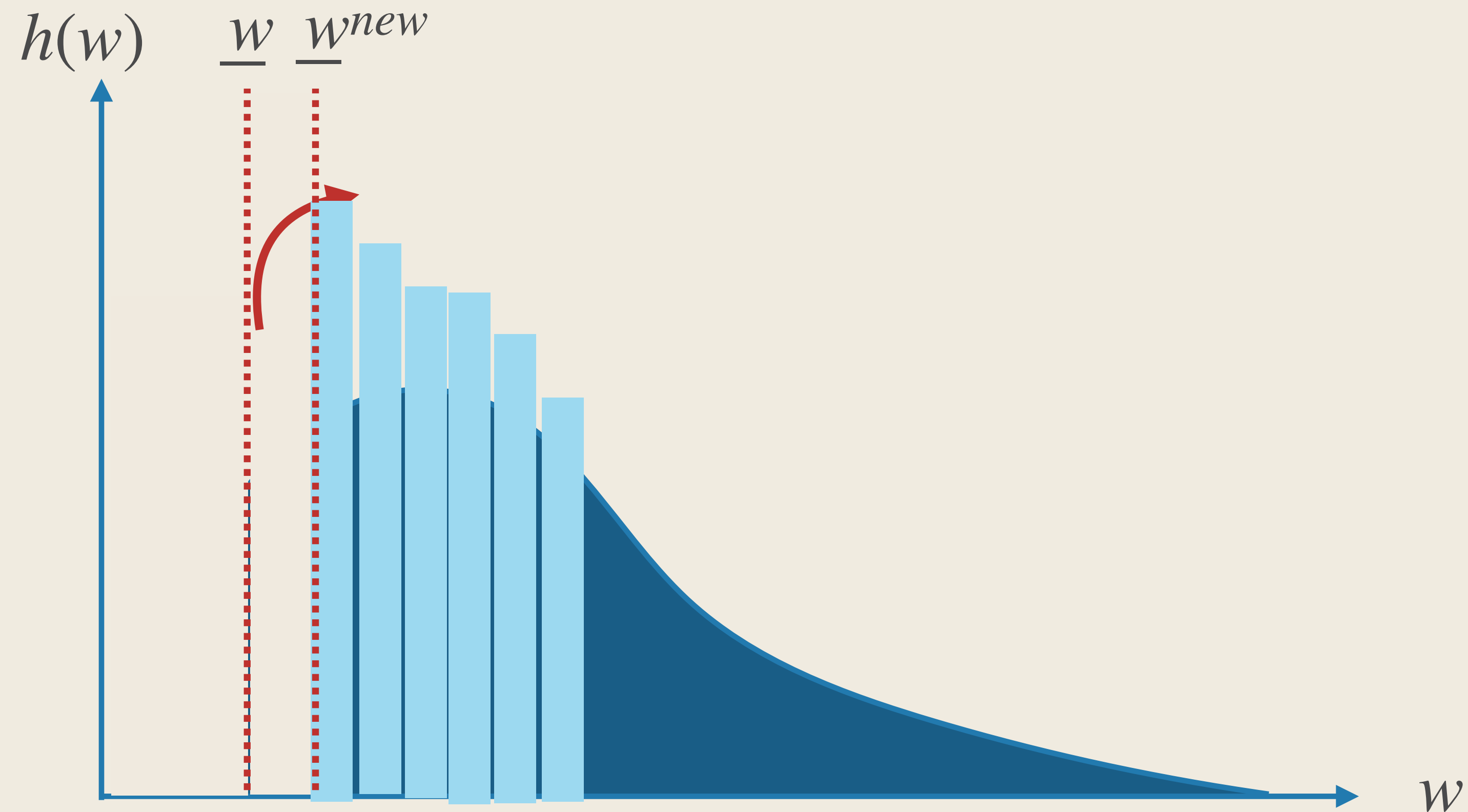
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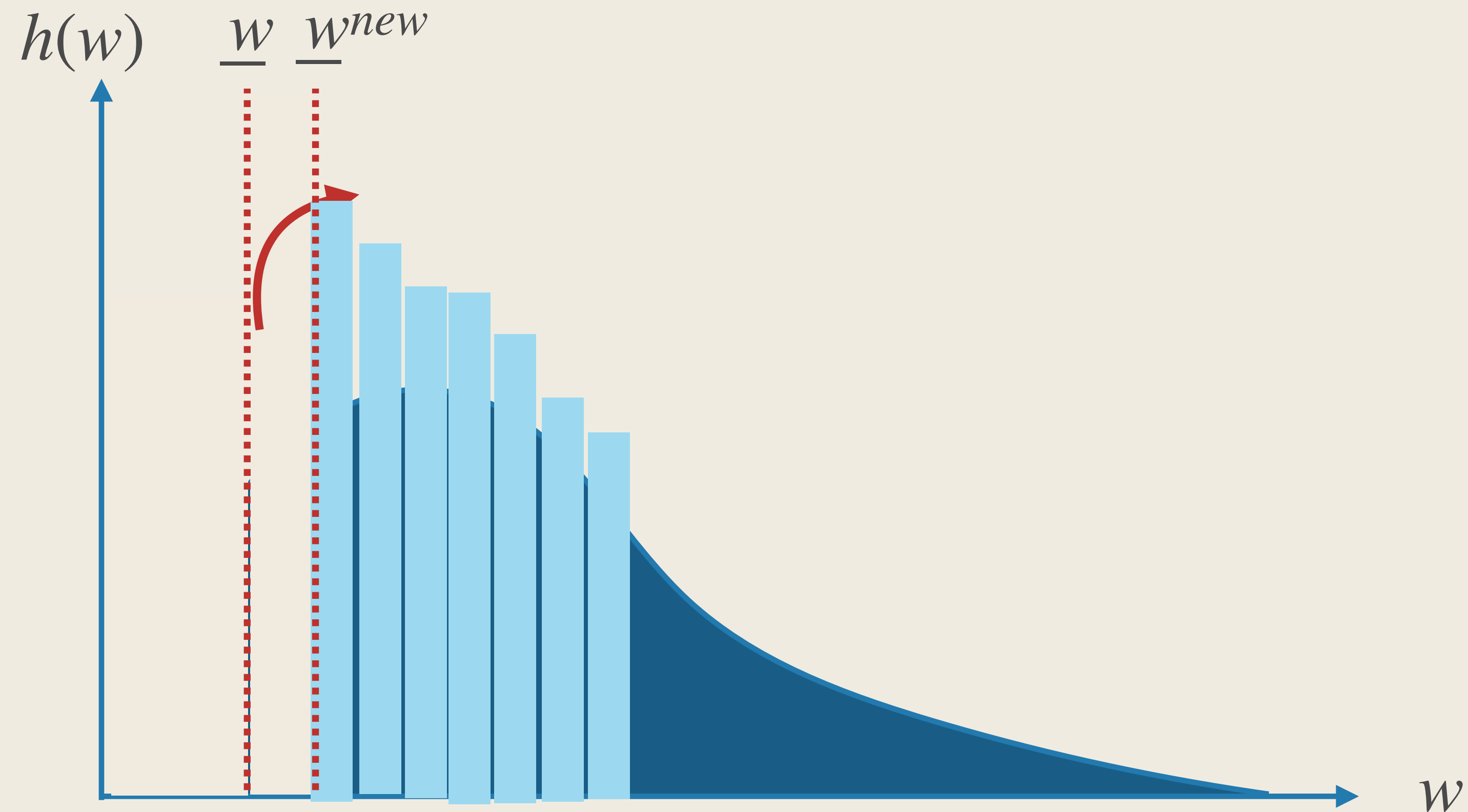
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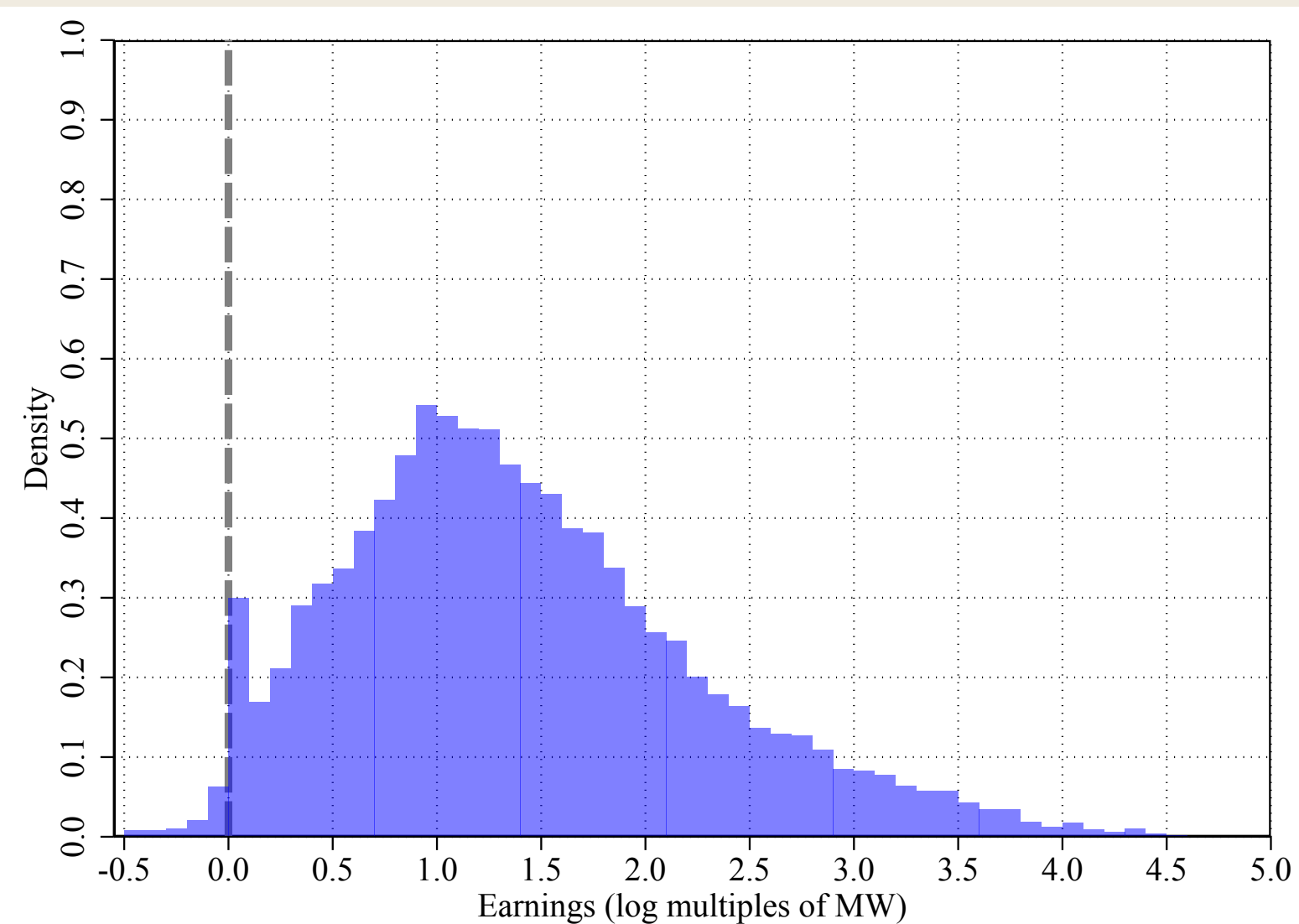
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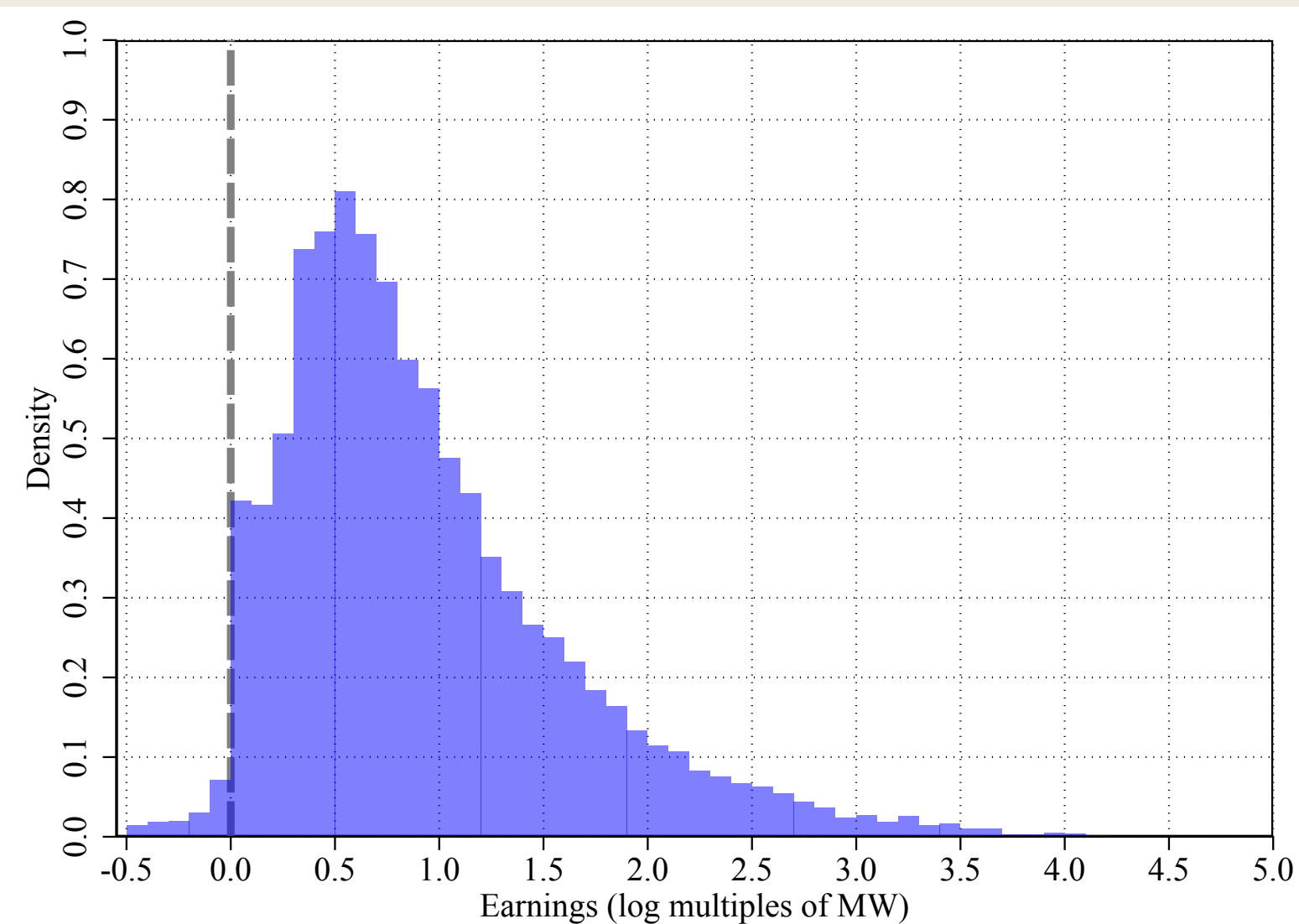


Falling Inequality at the Bottom

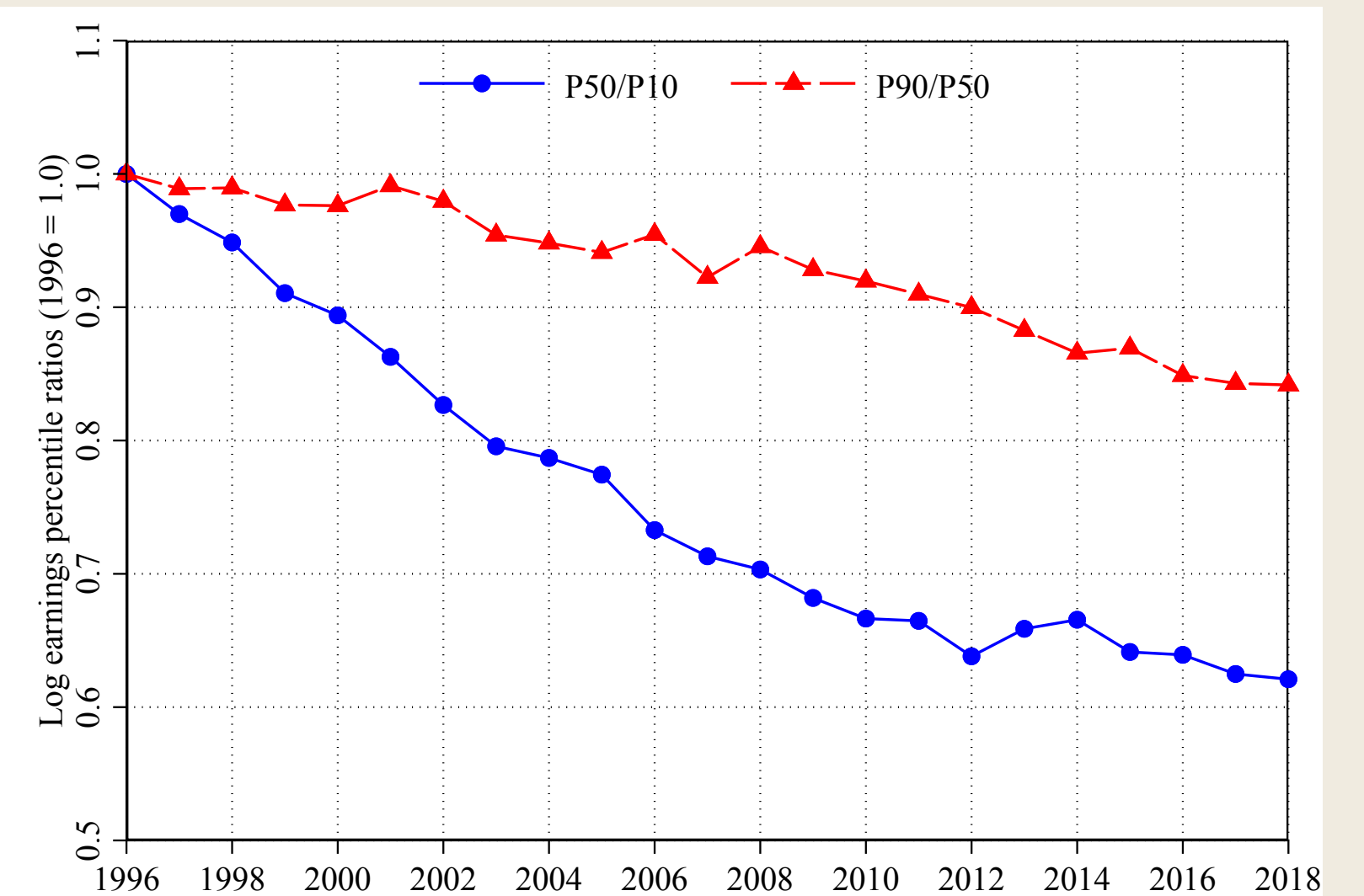
A. Histogram of log wages, 1996



B. Histogram of log wages, 2018



C. Percentile ratios, 1996–2018

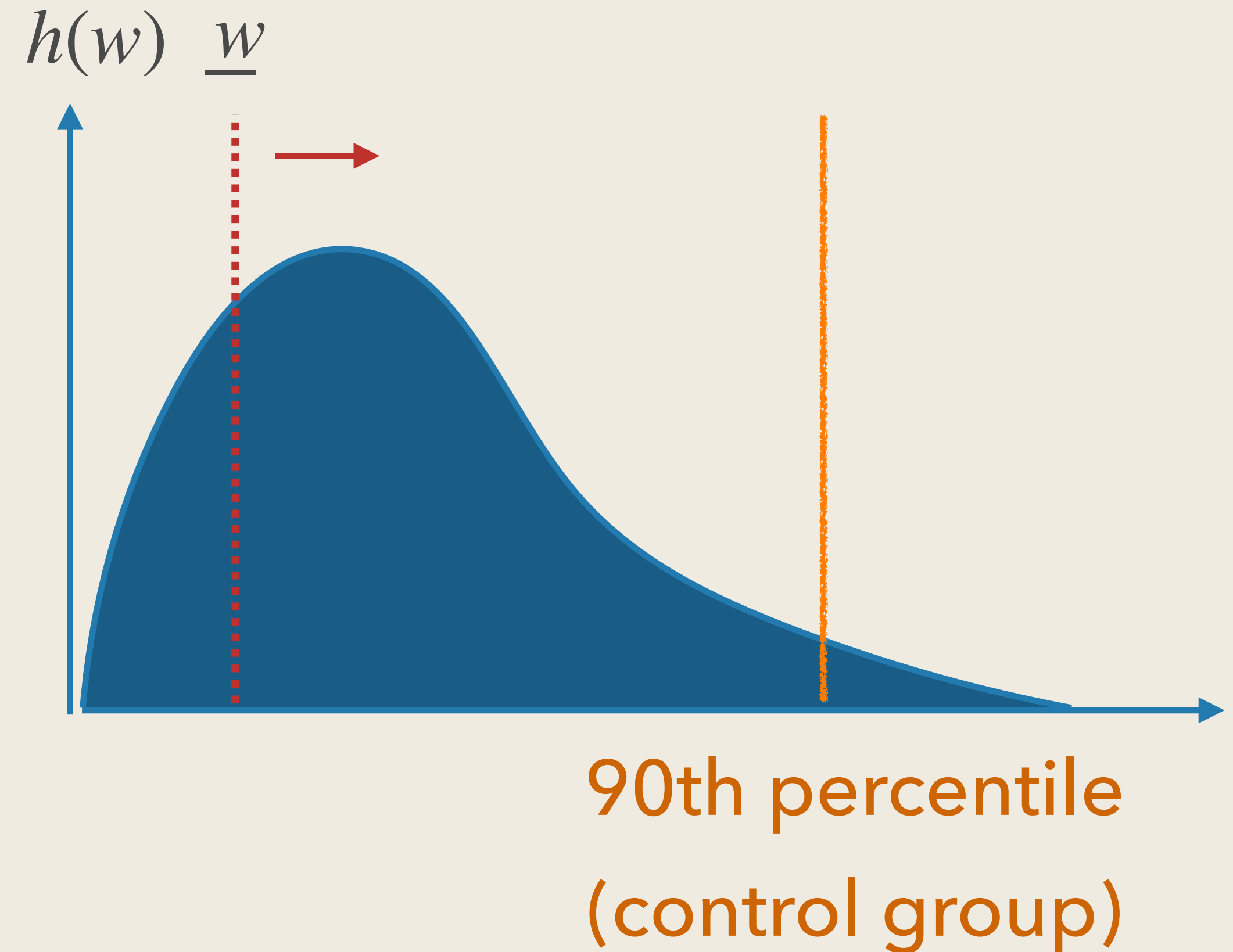
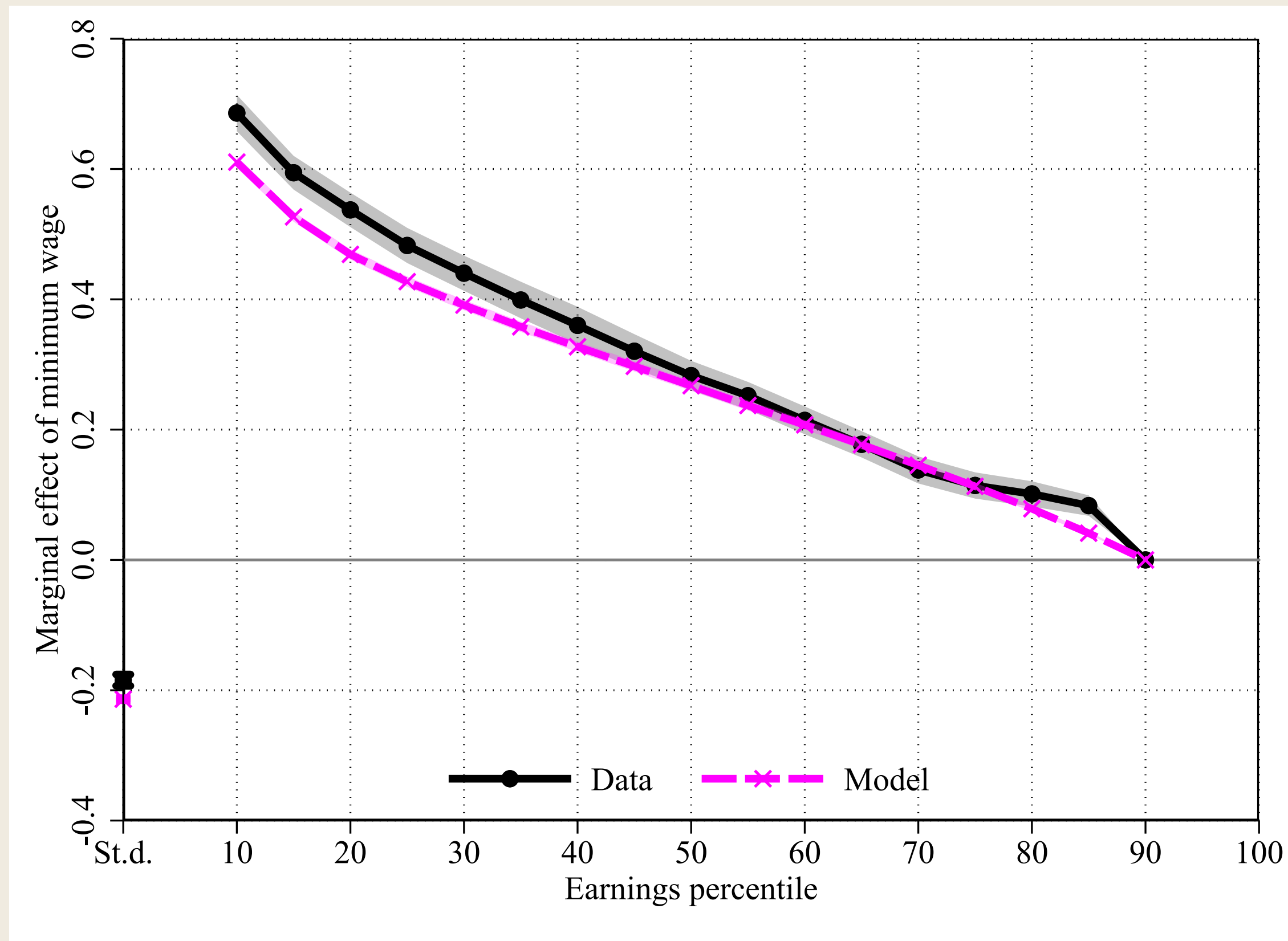


Notes: Panels A and B show histograms of log wages in multiples of the current minimum wage based on 60 equispaced bins for population of male workers aged 18–54 for 1996 and 2018, respectively. Panel C plots lower- and upper-tail wage inequality, as measured by the P50/P10 and the P90/P50 log wage percentile ratios between 1996 and 2018, normalized to 1.0 in 1996. Source: RAIS, 1996–2018.

Data versus Model

$$\ln w_{st}^p - \ln w_{st}^{90} = \beta^p [\ln w_{st}^{\min} - \ln w_{st}^{90}] + \gamma_s^p + \delta_s^p \times t + \epsilon_{st}^p$$

B. Relative to P90



Impact of Minimum Wage on Inequality

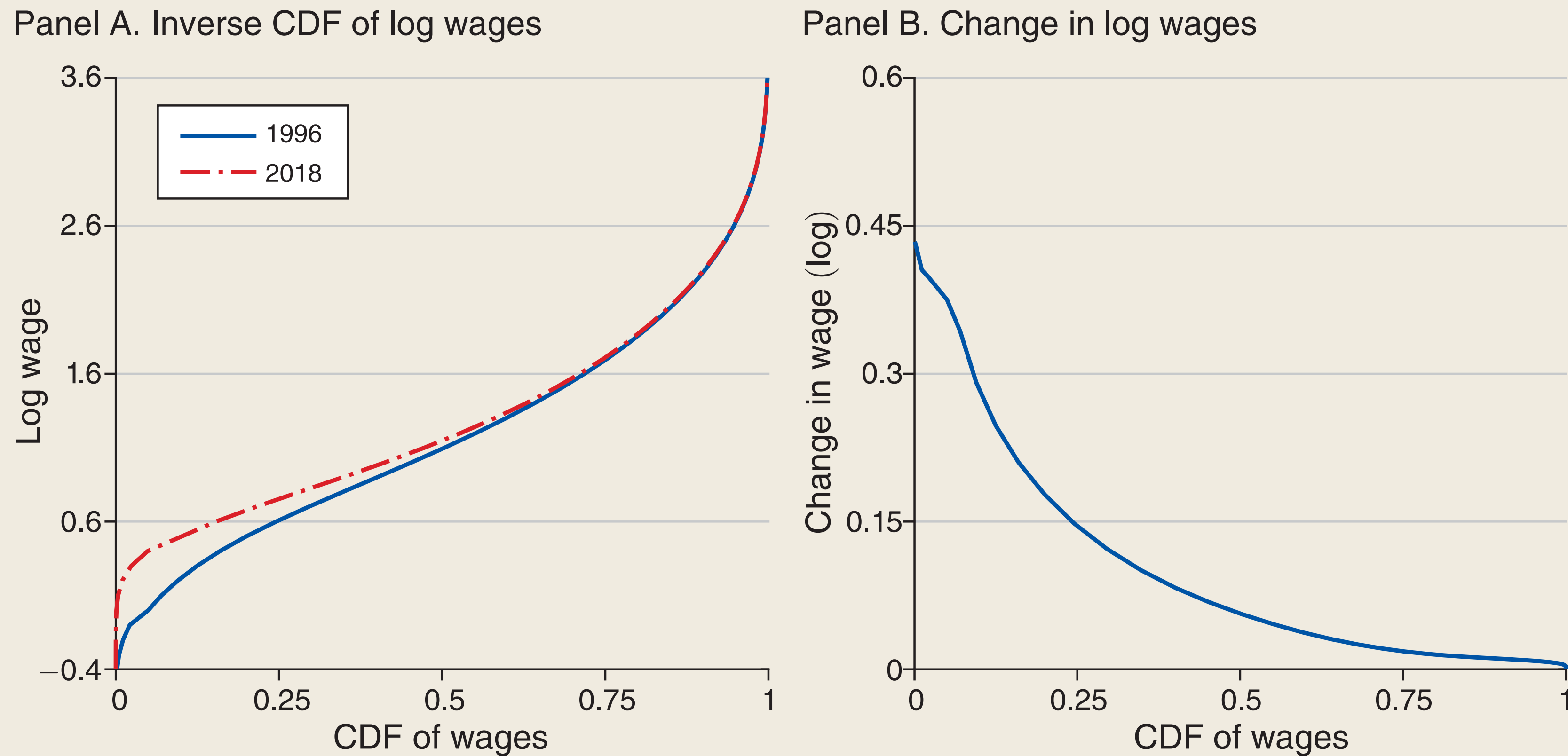


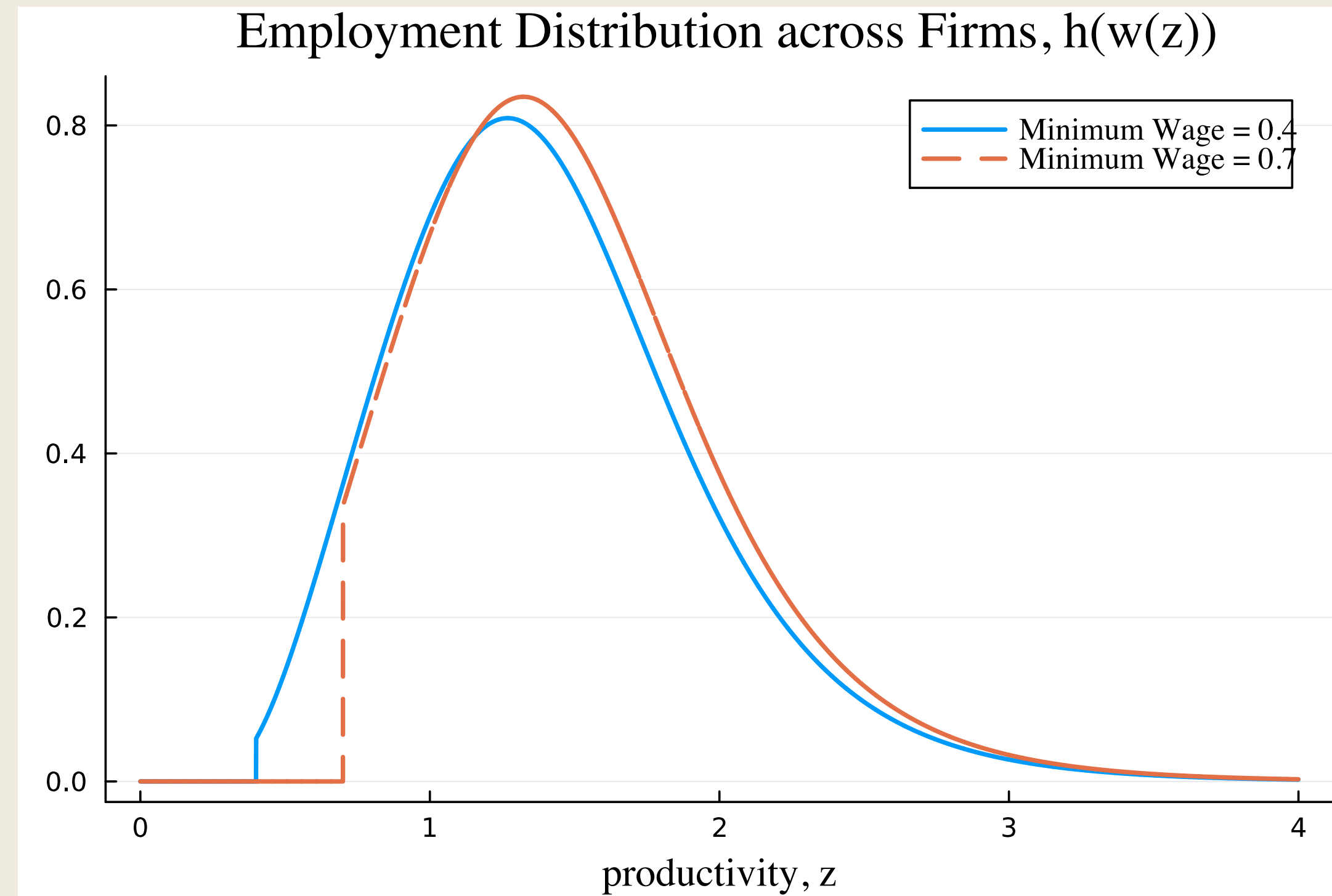
FIGURE 9. IMPACT OF THE MINIMUM WAGE THROUGHOUT THE WAGE DISTRIBUTION IN THE MODEL

- Increases in MW account for 45% of the reduction in $\text{Var}(\ln w)$ over 1996-2018

Reallocation Effect of Minimum Wage

– **Dutsman, Linder, Schönberg, Umkehrer, and Berge (2022)**

Reallocation Effect of Minimum Wage



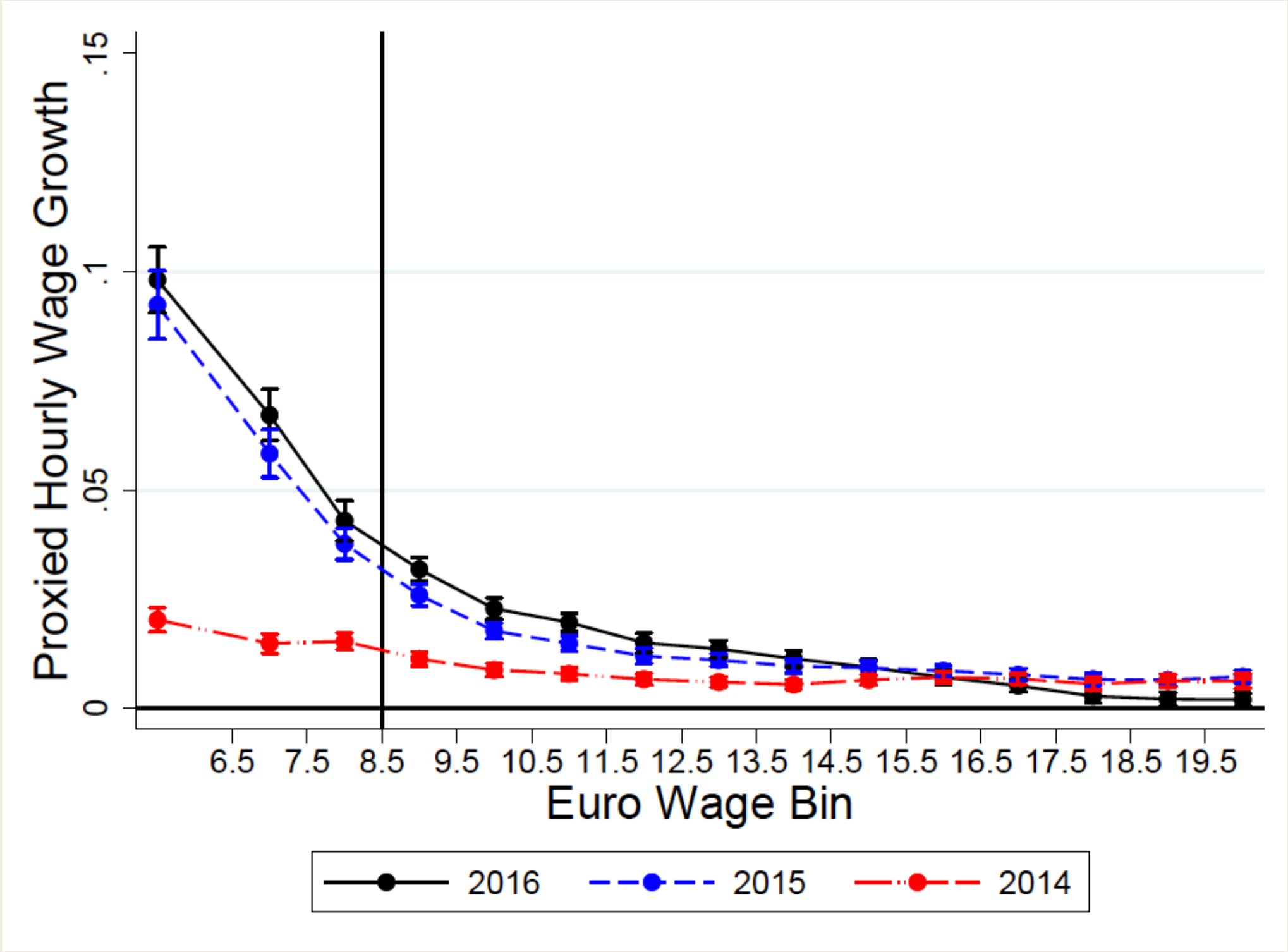
- # of matches: $M(u + \zeta(1 - u), v(1 - J_0(\underline{w})))$
- $\underline{w} \uparrow$ implies $q \uparrow$ for firms with $z \geq \underline{w}$
- The effects will be further amplified once we endogenize vacancy creation, v

Minimum Wage Policy in Germany

- Jan 2015: first introduction of minimum wage, €8.5
 - 15% of workers earn below €8.5 in 2014
- Ask:
 1. how are workers earning below €8.5 impacted relative to those above?
 2. how are regions with more binding workers impacted relative to those with less?

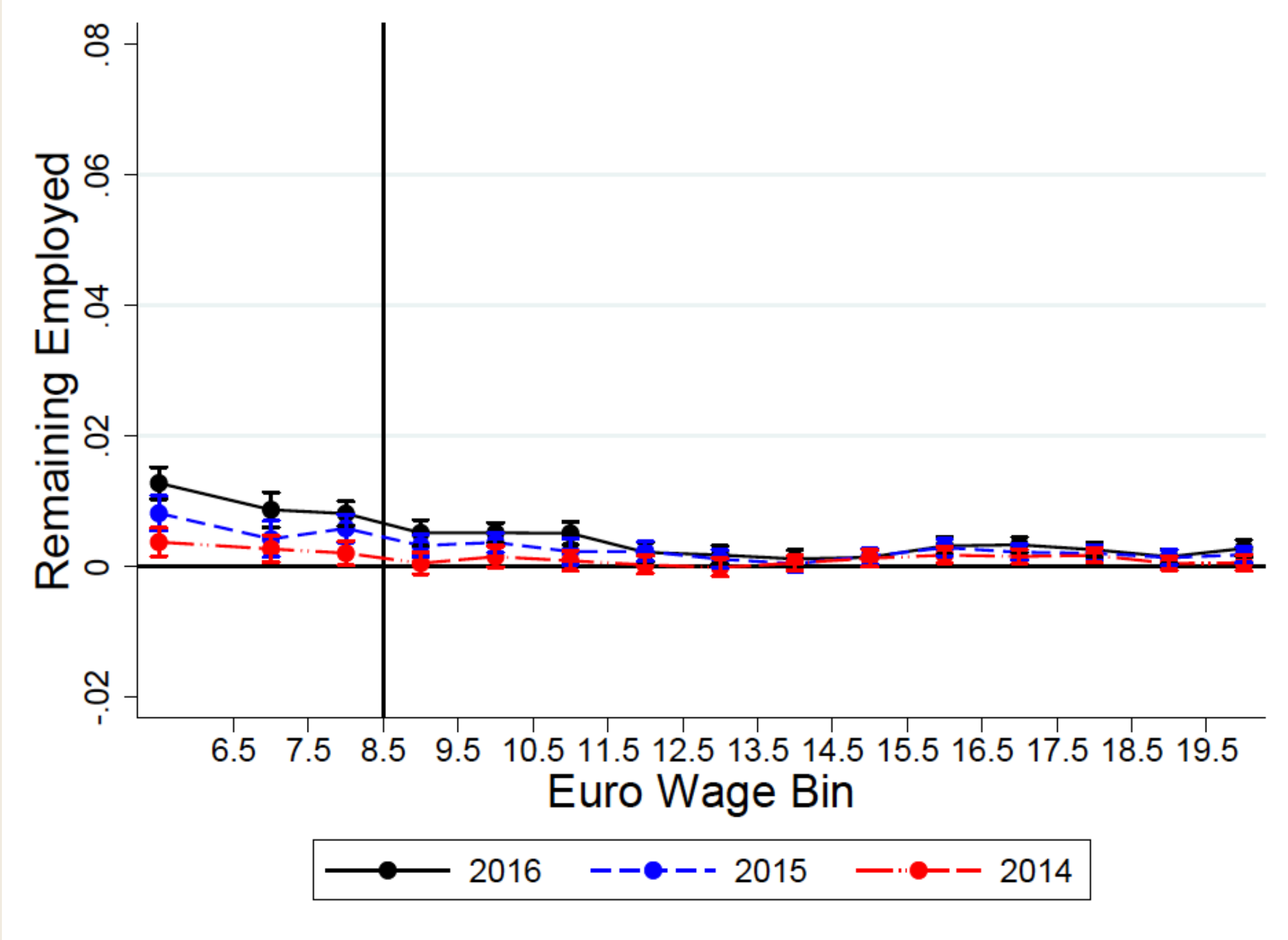
Wage and Employment Effect

Wage



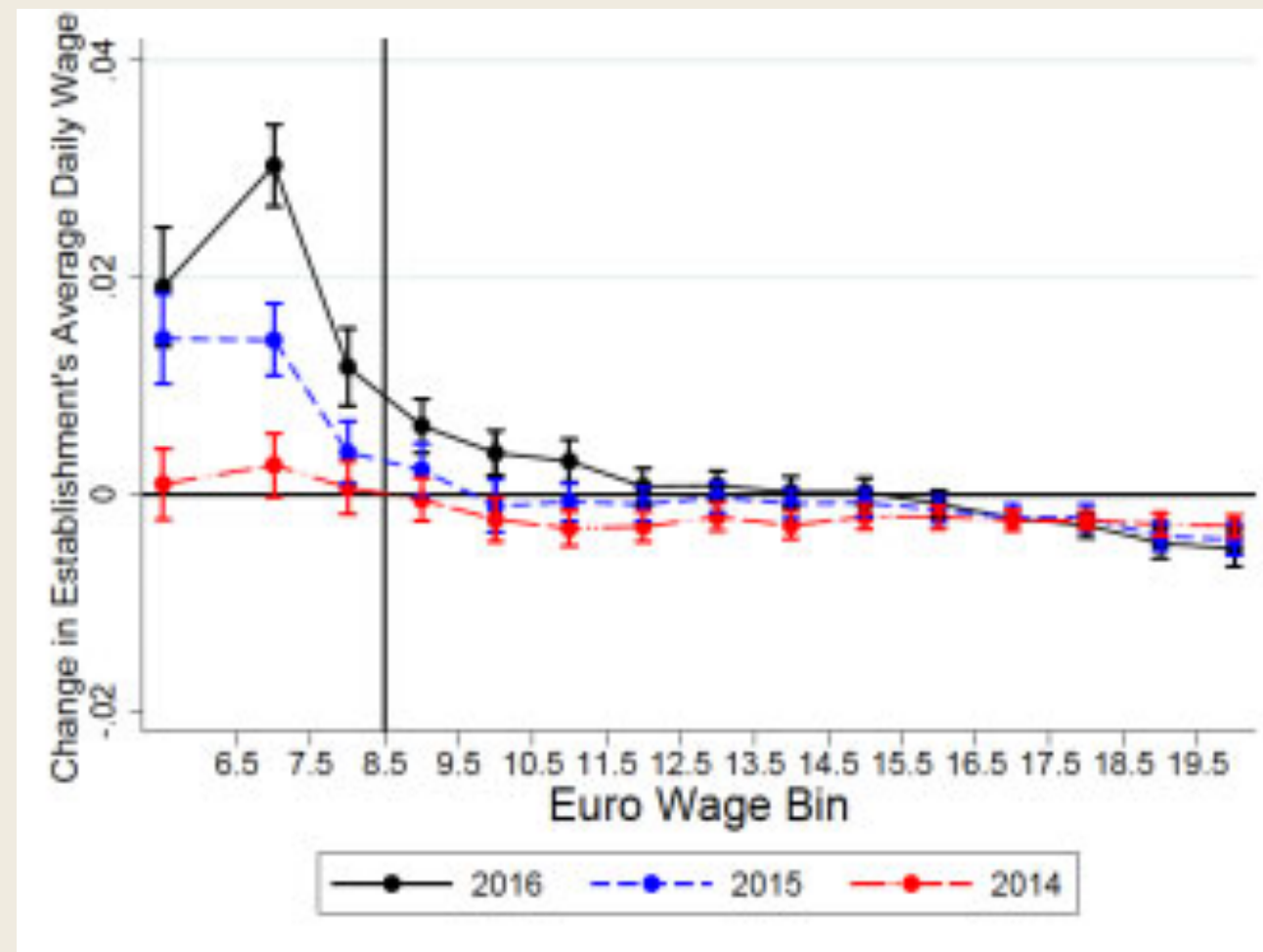
(B) Two-Year Hourly Wage Growth by Initial Wage Bin, relative to 2011 vs 2013

Employment

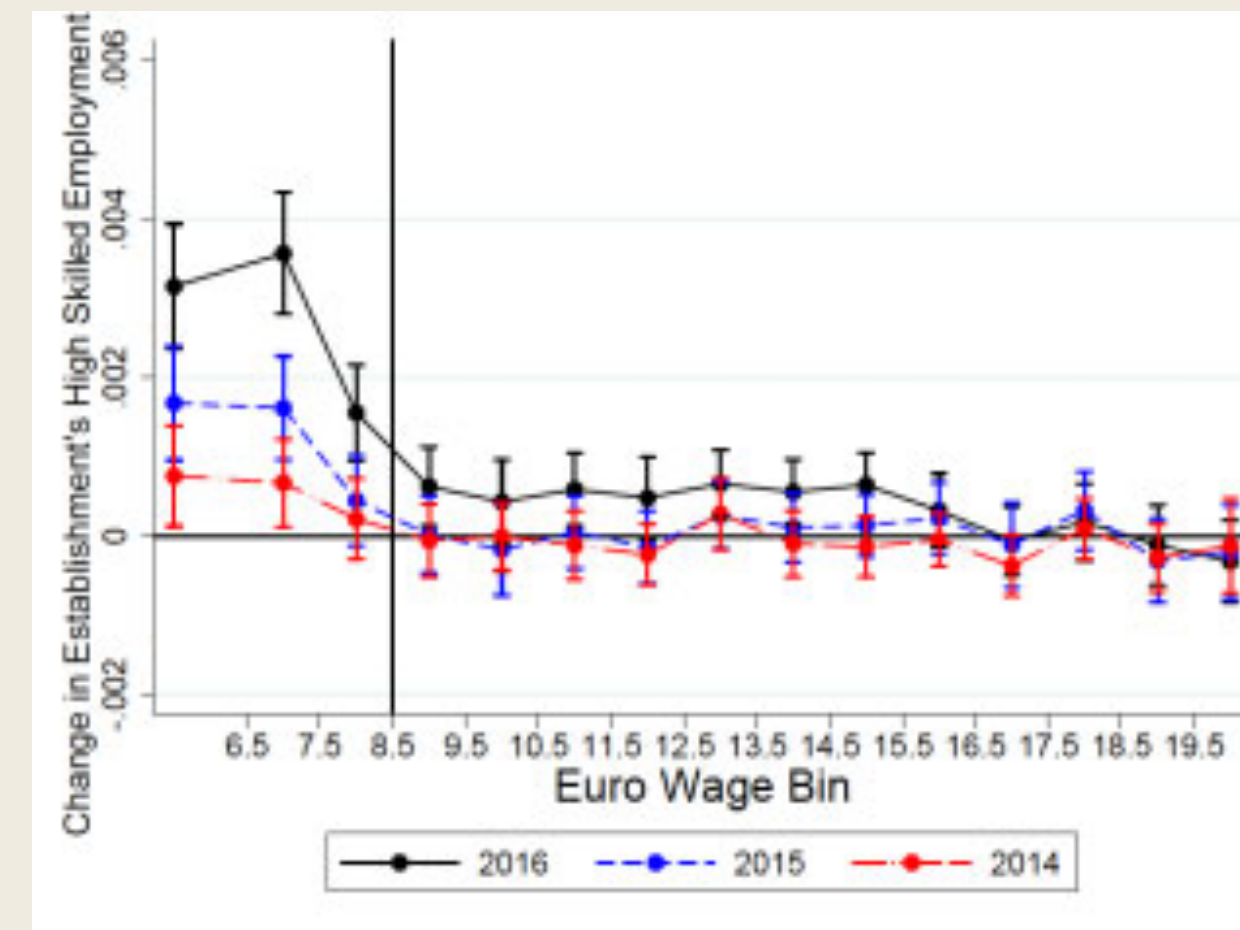


(B) Employment Probability in Year t by Initial Wage Bin, relative to 2011 versus 2013

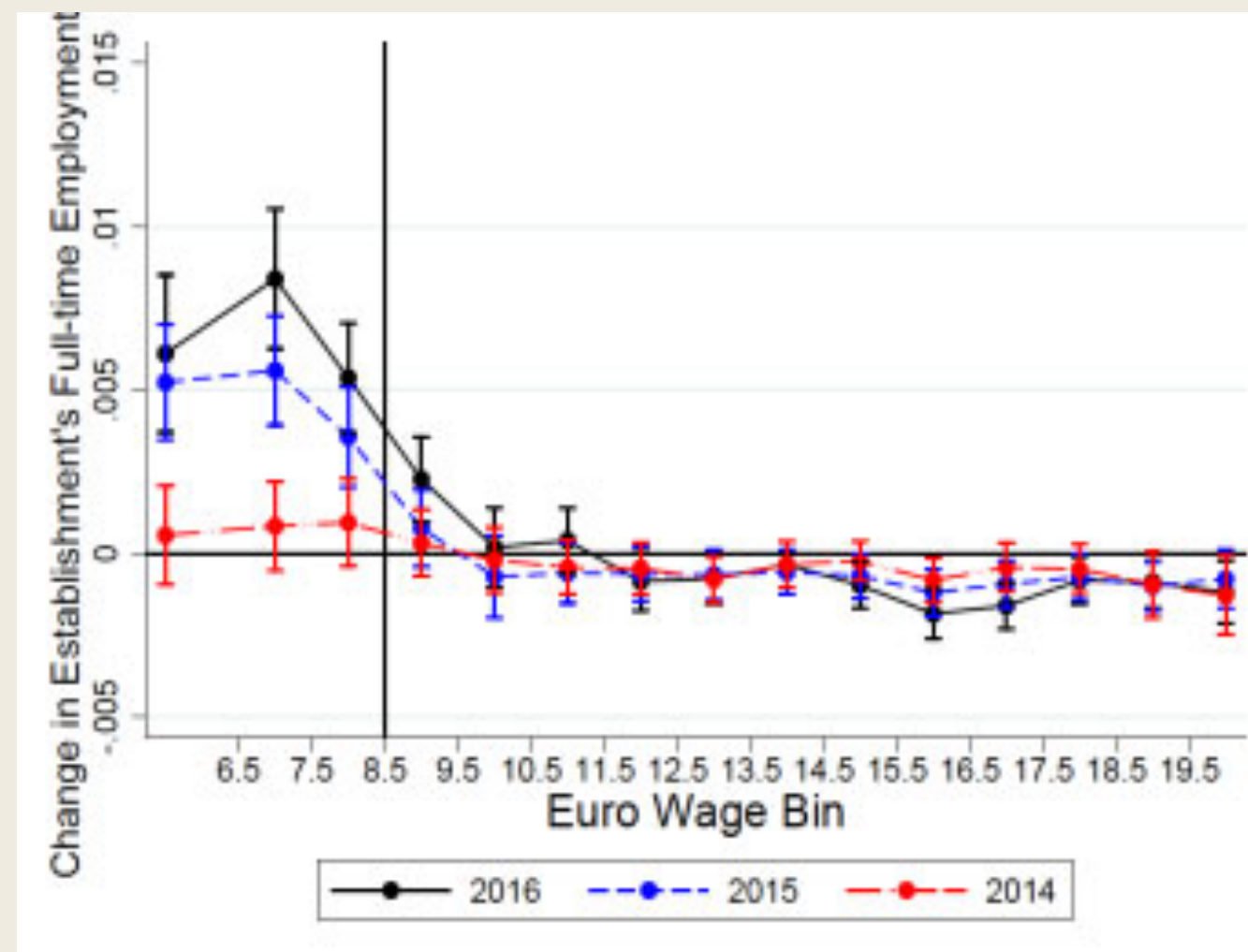
Reallocation Toward "Good" Firms



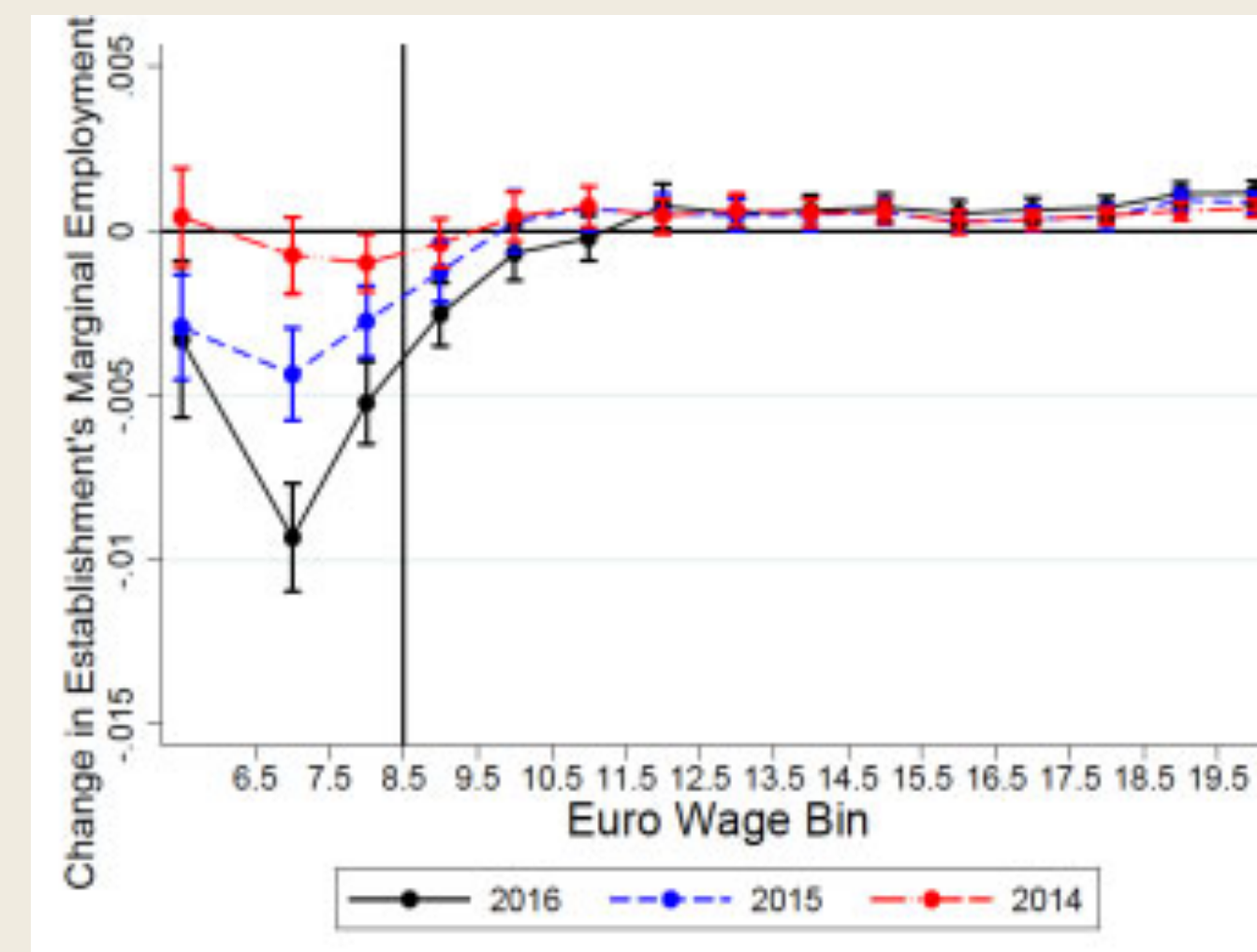
(A) Establishment's Average Daily Wage



(B) Establishment's High-Skilled Employment Share



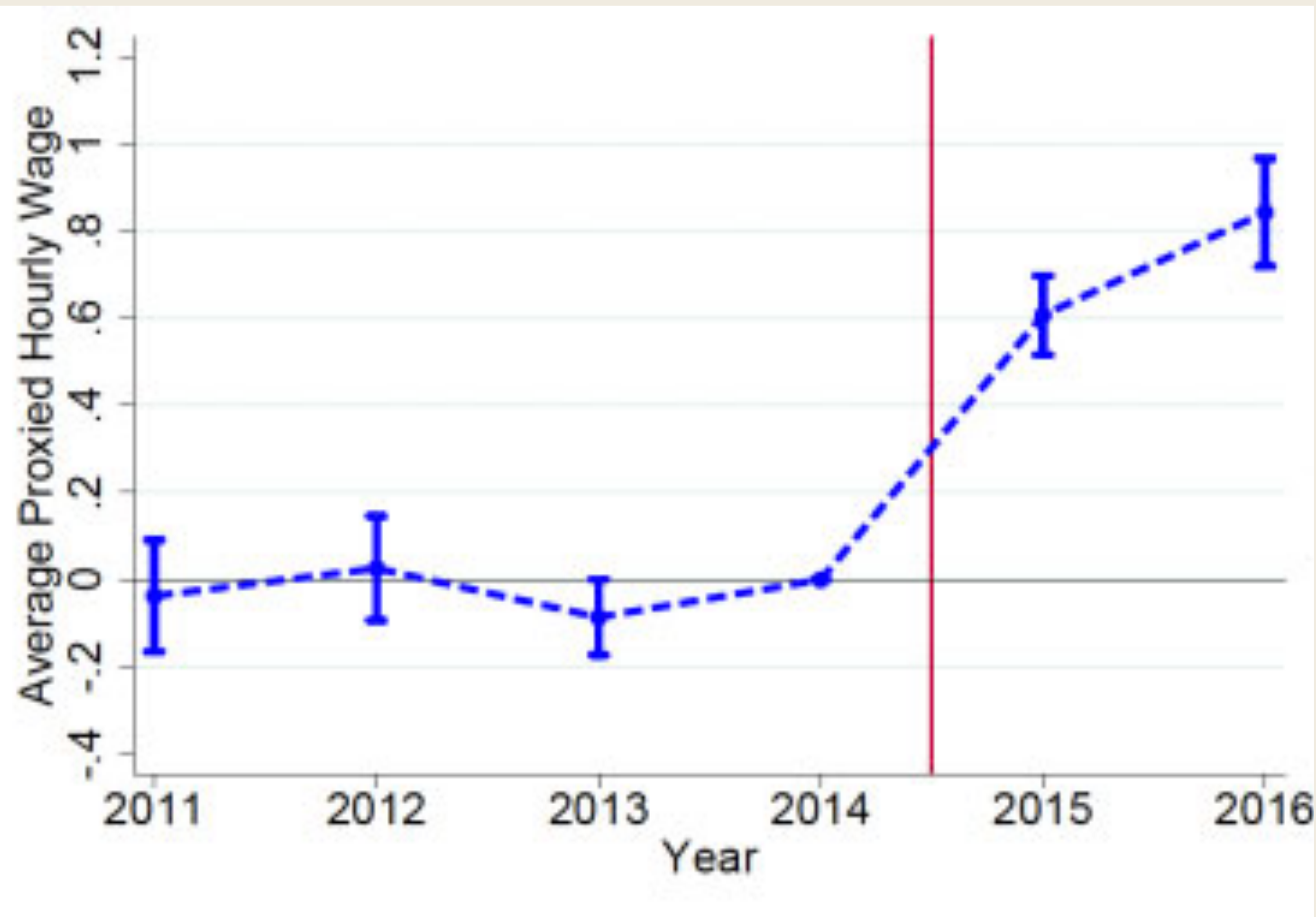
(C) Establishment's Full-Time Employment Share



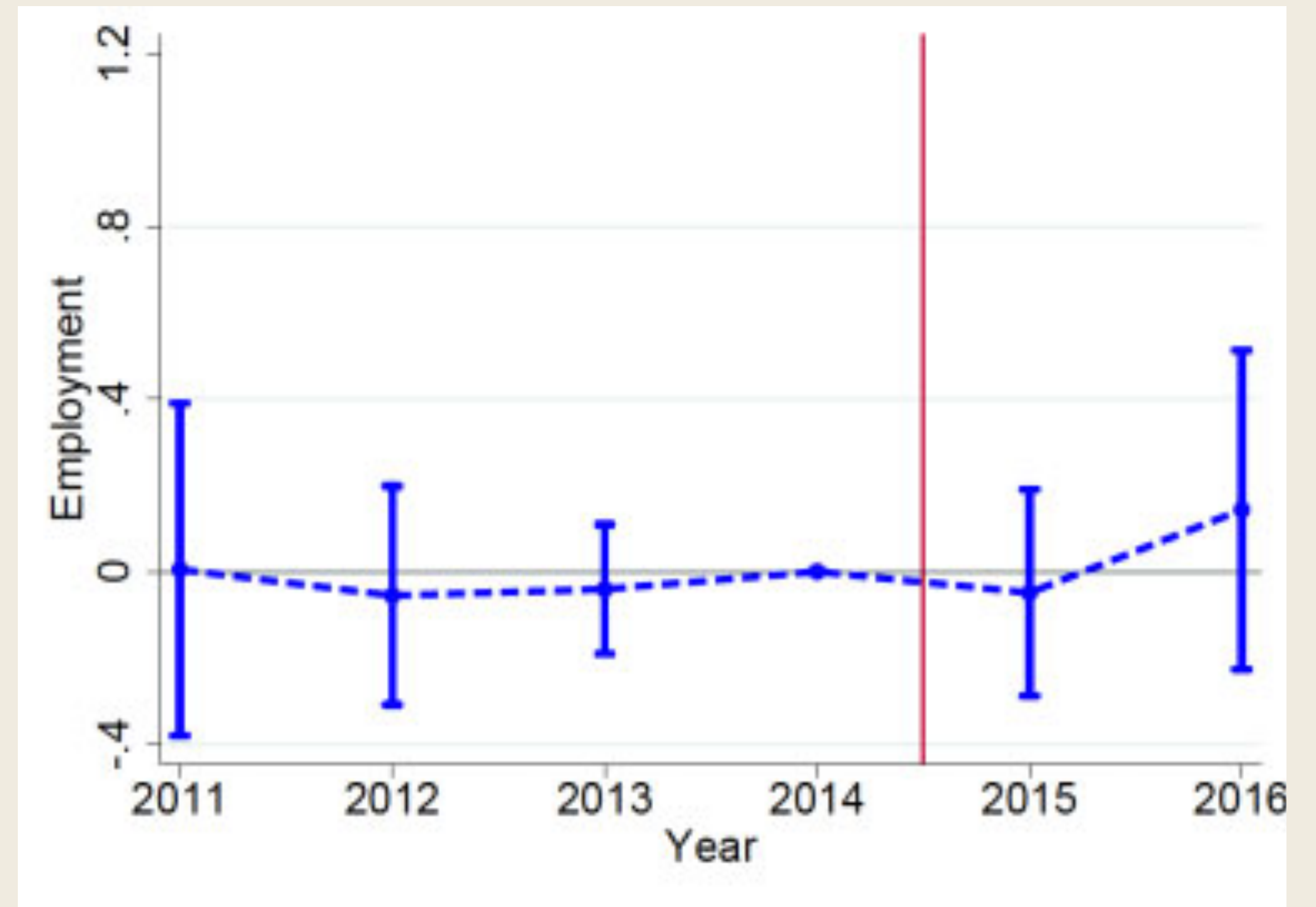
(D) Establishment's Marginal Employment Share

Wage and Employment Effect at the Region Level

$$y_{m,h} - y_{m,2014} = \beta_h \times (\text{Fraction binding in 2014 in market } m) + \gamma'_h \mathbf{X}_m + \epsilon_{m,h}$$

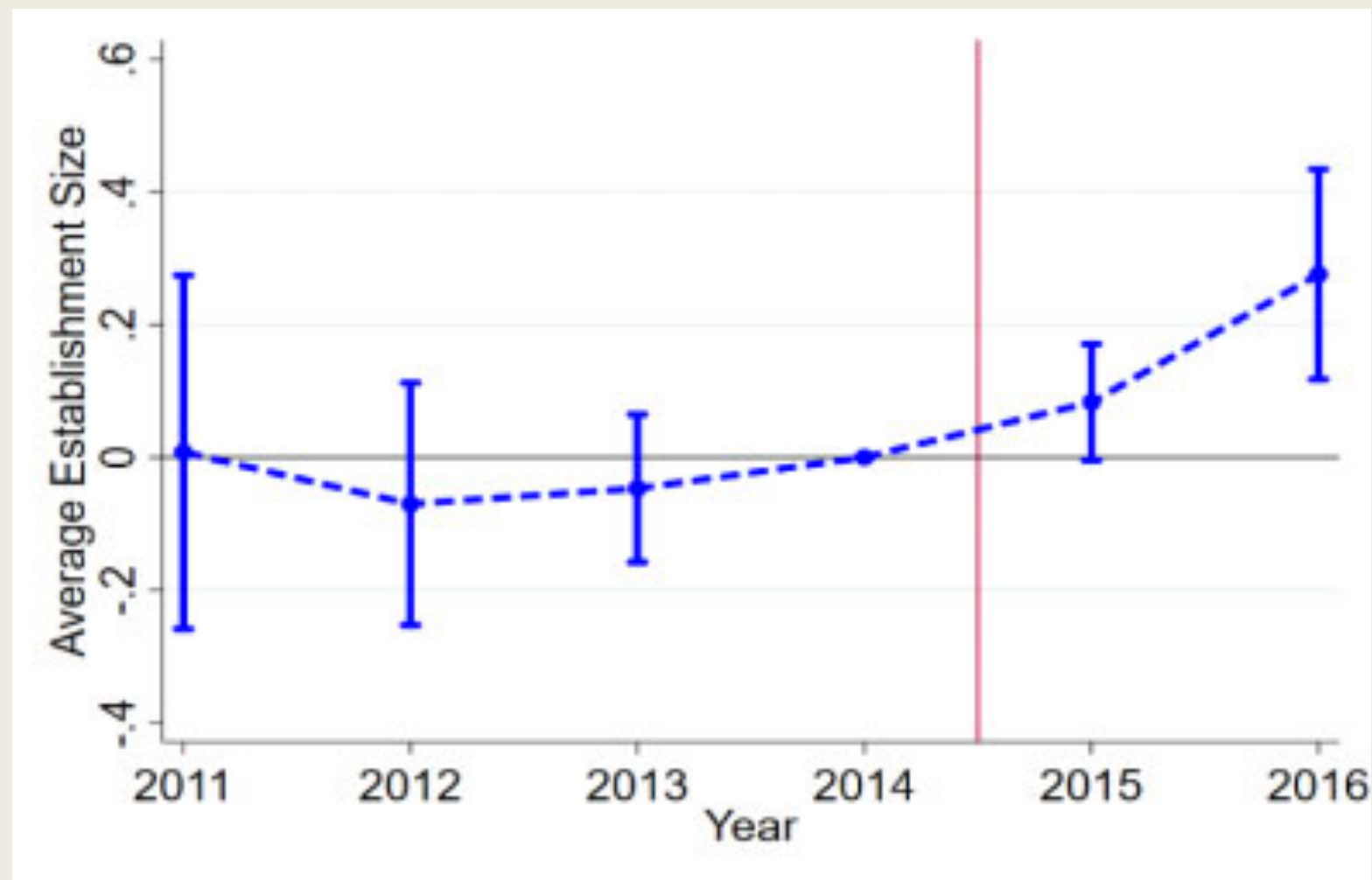


(B) Detrended Proxied Hourly Wages

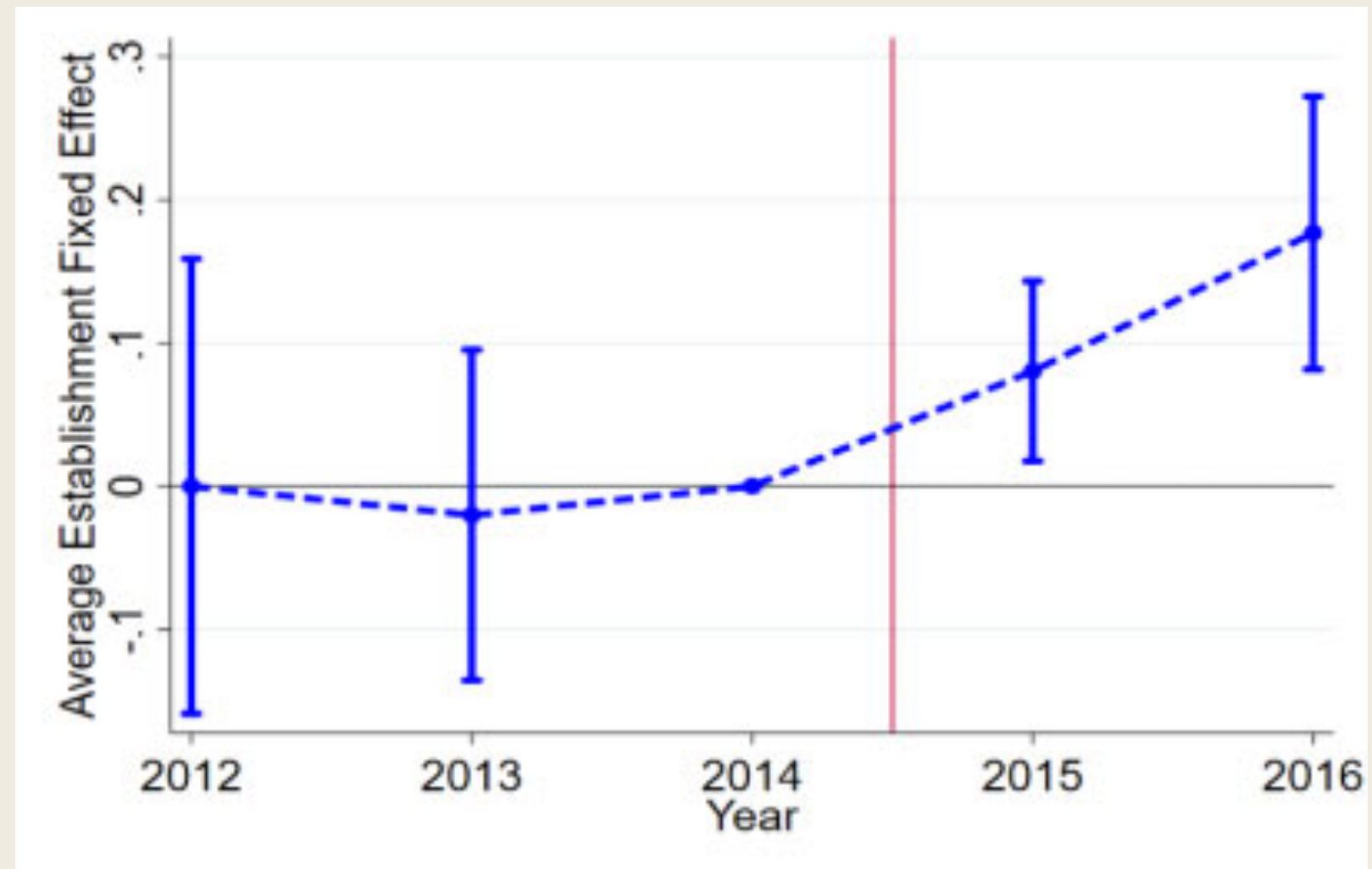


(D) Detrended Employment

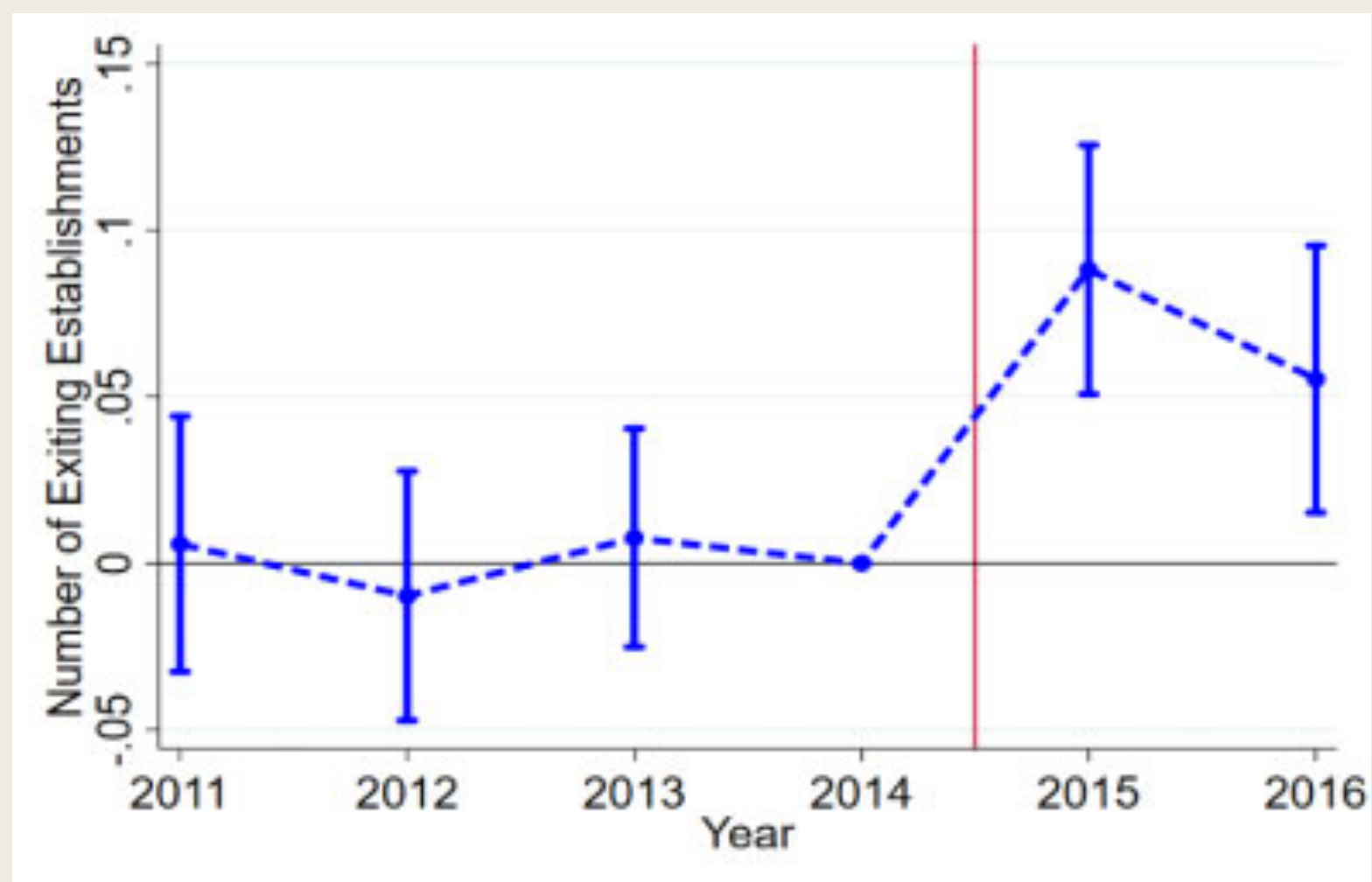
Reallocation at Regional Level



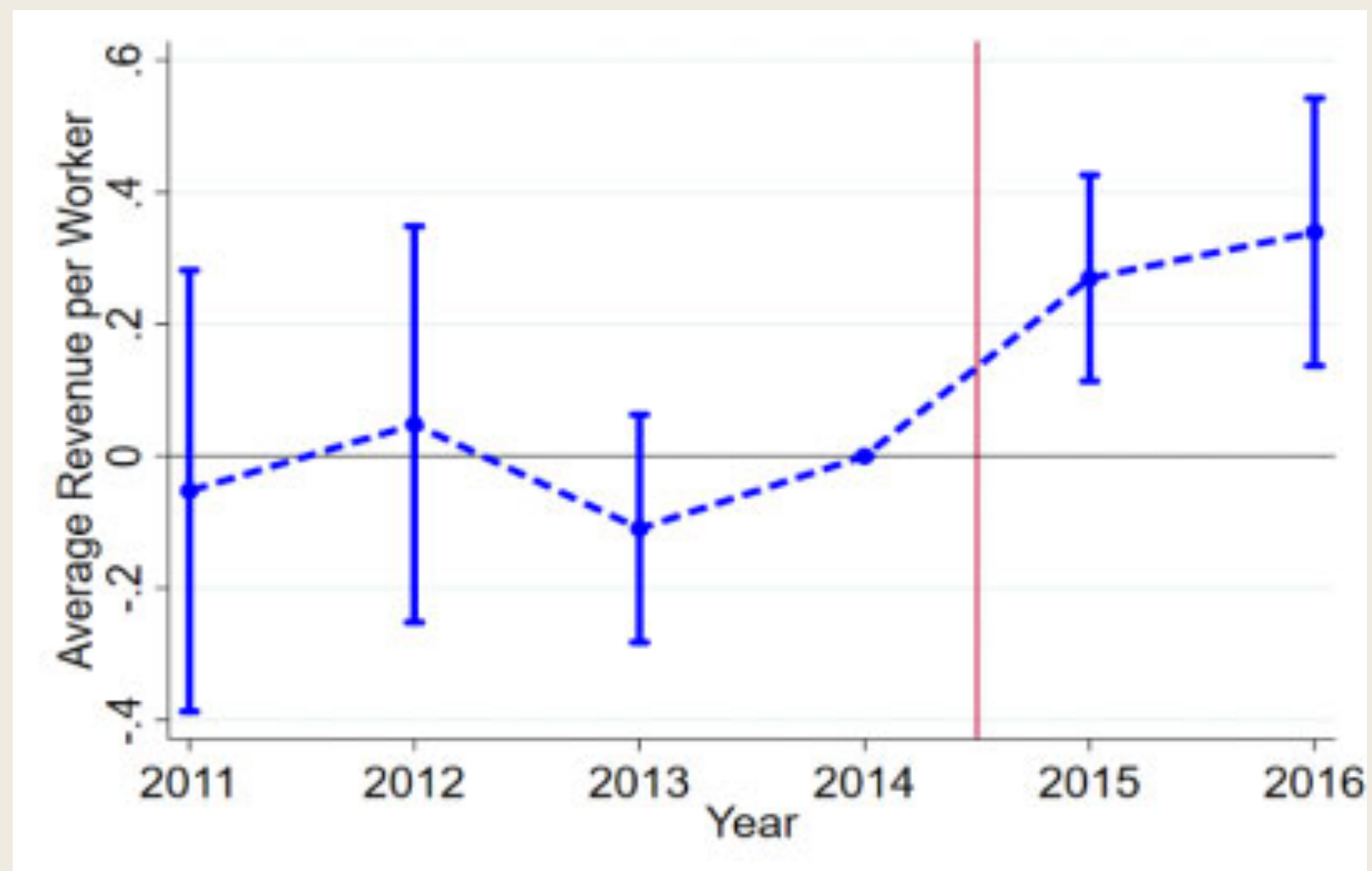
(C) Average Establishment Size



(D) Average AKM Establishment Fixed Effects



(E) Exit of Small Businesses (1-2 Employees)



(F) Revenue per Worker

Taking Stock

- Burdett-Mortensen model with wage-posting instead of bargaining
 - Tractable framework with many empirical predictions
- However, we have restricted the contract space significantly
 - firms offer a single wage to all workers
 - why not wage-tenure contracts?
 - why not counteroffer?
- Active research going on how firms set wages