DMP Model

704 Macroeconomics II
Topic 2

Masao Fukui

2025 Spring

DMP Model

- The last lecture took vacancy, v_t, as given
- We now model firm's optimal choice of vacancy creation
- Known as Diamond-Mortensen-Pissarides (DMP) model or the search model
- Firms and workers are optimizing given search & matching technology
- "Equilibrium" model of unemployment

Preferences

- Discrete time, $t = 0,1,\ldots,\infty$
- Unit measure of ex-ante identical risk-neutral workers, discount factor $\beta < 1$:

$$\sum_{t=0}^{\infty} \beta^t c_t$$

  - Equivalently can be hand-to-mouth
  - Risk-neutraly shuts down Euler equation: $\beta R = 1$
- Employed workers receive wage income, w_t
- Unemployed workers receive unemployment benefits/utility from leisure, b

Technology

- A firm (job) uses one worker to produce $z_t > b$ units of output
- z_t is stochastic and follows some Markov process
- Alternatively can have CRS firms that employ many workers

Technology

- Firms can post a vacancy per unit cost c every period
  - A pool of infinitely many potential firms in the background (free entry)
- Assume the number of matches is given by CRS matching function

$$M(u_t, v_t)$$

Let $\theta_t \equiv v_t / u_t$ be the market tightness

  - Probability of unemployed finding a job in current period:

$$M(u_t, v_t)/u_t = M(1, \theta_t) \equiv f(\theta_t) = f_t, \quad f'(\theta) > 0$$

  - Probability that a vacant firm finds a worker

$$M(u_t, v_t)/v_t = M(1/\theta_t, 1) \equiv q(\theta_t) = q_t \quad q'(\theta) < 0$$

- For now, the probability that a job terminates s is exogenous

Value Functions

- Value of being unemployed:

$$U_t = b + \beta \mathbb{E}_t[f_t E_{t+1} + (1 - f_t) U_{t+1}] \tag{1}$$

- Value of being employed:

$$E_t = w_t + \beta \mathbb{E}_t[(1 - s) E_{t+1} + s U_{t+1}] \tag{2}$$

- Value of a filled job:

$$J_t = z_t - w_t + \beta \mathbb{E}_t[(1 - s) J_{t+1} + s V_{t+1}] \tag{3}$$

- Value of a vacant job:

$$V_t = -c + \beta \mathbb{E}_t[q_t J_{t+1} + (1 - q_t) V_{t+1}] \tag{4}$$

Free Entry

- Free entry implies $V_t = 0$:

$$c = \beta \mathbb{E}_t[q_t J_{t+1}] \tag{5}$$

  - LHS: cost of creating a vacancy
  - RHS: benefit of creating a vacancy
  - $RHS > LHS$ $(V_t > 0) \Rightarrow$ more firms enter $(v_t \uparrow) \Rightarrow$ congest the market $q_t = q(v_t/u_t) \downarrow$

- The value of filling a position is

$$J_t = z_t - w_t + \beta \mathbb{E}_t[(1-s)J_{t+1} + sV_{t+1}]$$
$$= \mathbb{E}_t \sum_{n=t}^{\infty} (\beta(1-s))^{n-t}[z_n - w_n]$$

which is present discounted value of profits from a match.

- Given wages $\{w_t\}_t$, (5) pins down θ_t. But how is w_t pinned down?

Joint Match Surplus

- Useful to introduce a notion of joint match surplus: $S_t = E_t + J_t - U_t - V_t$
  - Gains from trade between workers and firms
  - Wage determines how workers and firms split the pie S_t
- The joint match surplus recursively solves (use (1)-(3) and $V_t = 0$):

$$S_t = z_t - b + \beta \mathbb{E}_t[(1 - s)S_{t+1} - f_t(E_{t+1} - U_{t+1})] \tag{6}$$

  - $f_t(E_{t+1} - U_{t+1})$: the opportunity cost of continuing the employment relationship

Wage Determination

Non Competitive Labor Market

- Unlike competitive labor market, there is no unique way to pin down wages
- Workers and firms face a **bilateral monopoly** when they meet
  - If firm walks away, worker loses wage this period and must search again
  - If worker walks away, firm loses profits this period and must search again
- Any wages $\{w_t\}$ that satisfy $E_t > U_t$ and $J_t > 0$ constitute an equilibrium
  - Workers do not have incentive to quit
  - Firms do not have incentive to fire
  - There can be continuum of wages

Nash Bargaining

- Assume wages are negotiated period-by-period (no commtiment)
- We first focus on the most standard wage-setting protocol: Nash bargaining
- Wage such that surplus is split between worker and firm with shares γ, $1-\gamma$:

$$w_t = \arg\max_w (E_t(w) - U_t)^\gamma J_t(w)^{1-\gamma}$$

  - γ is called bargaining power of workers
  - Microfoundation through alternative offer game a la Rubinstein (1980)
- Using the first-order conditions,

$$E_t(w) - U_t = \gamma S_t, \quad J_t(w) = (1-\gamma) S_t$$

Solving for Wages

- Substituting $E_t - U_t = \gamma S_t$ into (6),

$$S_t = z_t - b + \beta \mathbb{E}_t[(1-s)S_{t+1} - f_t \gamma S_{t+1}] \tag{7}$$

- Taking the difference between (1) and (2),

$$\underbrace{E_t - U_t}_{\gamma S_t} = w_t - b + \beta \mathbb{E}_t[(1-s-f_t)\underbrace{(E_{t+1} - U_{t+1})}_{\gamma S_{t+1}}] \tag{8}$$

- Multiply (8) by $1/\gamma$ and subtract it from (7):

$$z_t - b - \frac{1}{\gamma}(w_t - b) + \frac{1-\gamma}{\gamma}\beta \mathbb{E}_t f_t \gamma S_{t+1} = 0$$

- Note that (5) with $J_{t+1} = (1-\gamma)S_{t+1}$ implies $\beta \mathbb{E}_t f_t (1-\gamma) S_{t+1} = \theta_t c$

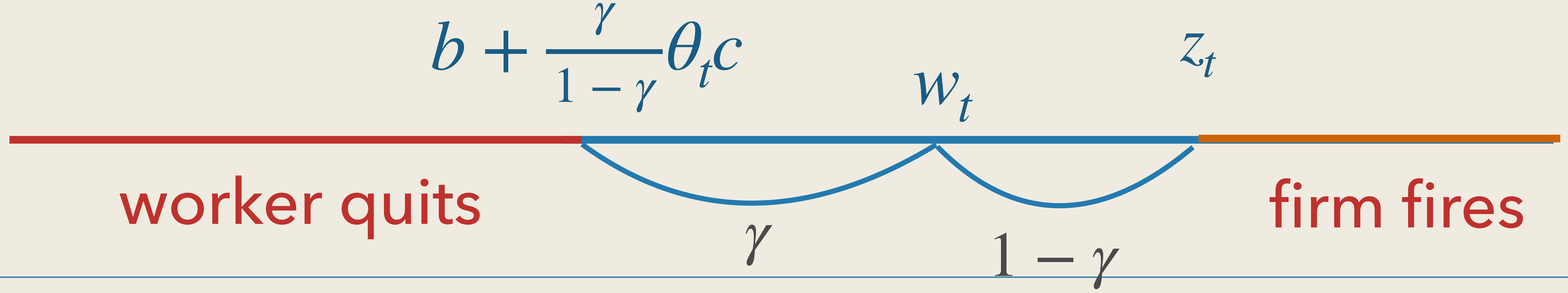
Wage

- Solving for w_t,

$$w_t = \gamma z_t + (1-\gamma)\left[b + \frac{\gamma}{1-\gamma}\theta_t c\right]$$

- Weighted average between
  - Firm's flow output z_t
  - Worker's flow outside option b + opportunity cost of not searching $\frac{\gamma}{1-\gamma}\theta_t c$
    - $\theta_t c = v_t c / u_t$: the total expected firm profits per unemployed
    - $\gamma/(1-\gamma)$: relative bargaining power of workers

$\Rightarrow$ Workers are expected to obtain $\frac{\gamma}{1-\gamma}\theta_t c$ from searching if they quit



Equilibrium Conditions

The decentralized equilibrium $\{\theta_t, w_t, u_t\}$ solve

- Free-entry:

$$c = \beta q(\theta_t)\mathbb{E}\sum_{n=t}^{\infty}(\beta(1-s))^{n-t}[z_{n+1} - w_{n+1}]$$

- Wage determination:

$$w_t = (1-\gamma)b + \gamma(z_t + \theta_t c)$$

- The stock-flow equation:

$$u_{t+1} - u_t = s(1-u_t) - f(\theta_t)u_t$$

Steady State Equilibrium

Steady State Eqm Characterization

- From now on, focus on the steady-state equilibrium: $z_t = z$ for all t.

- The equilibrium $\{\theta, w, u\}$ solve

$$c = \beta q(\theta) \frac{1}{1-\beta(1-s)}[z - w] \qquad \text{(Free entry)}$$

$$w = (1-\gamma)b + \gamma(z + \theta c) \qquad \text{(wage)}$$

$$0 = s(1-u) - f(\theta)u \qquad \text{(Beveridge curve)}$$

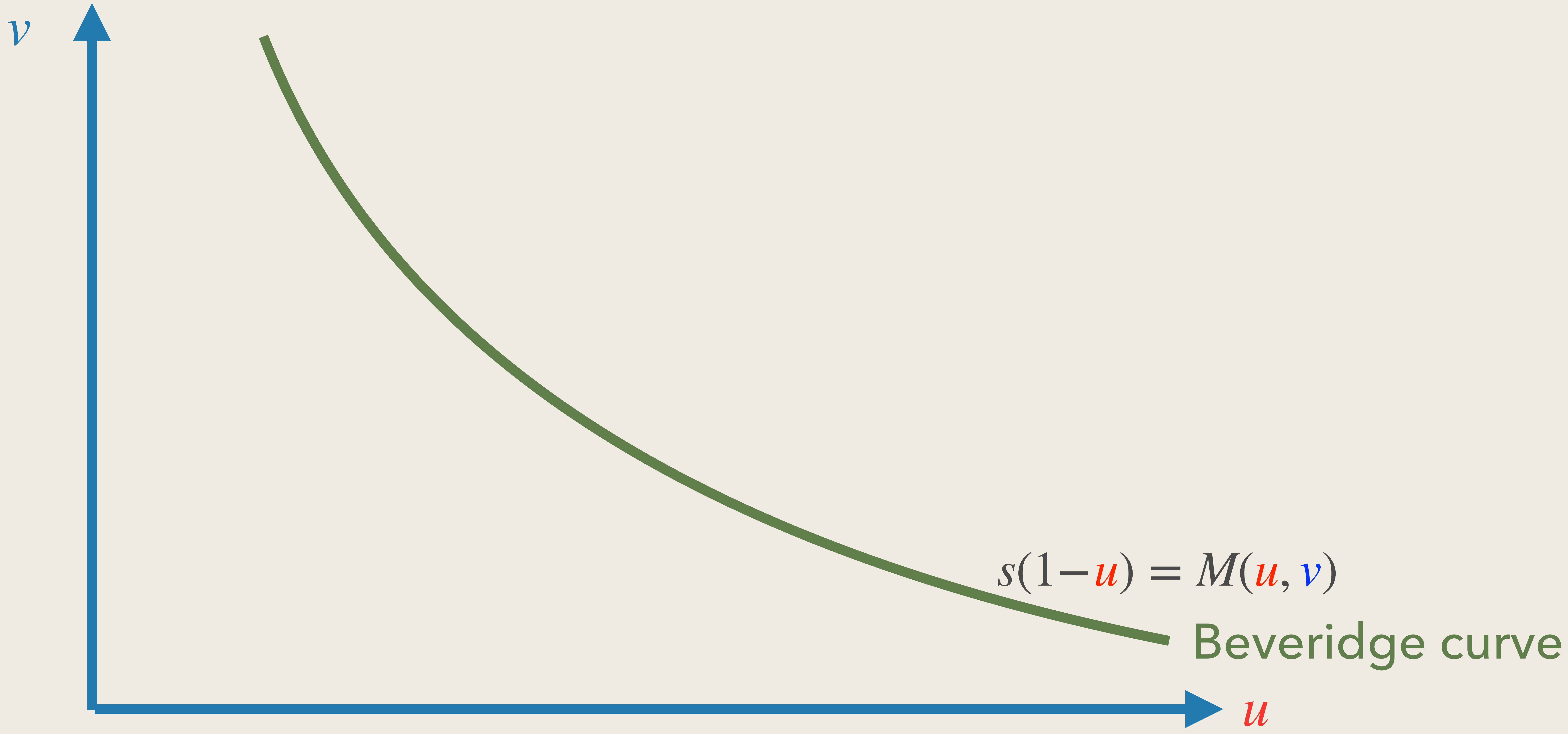
- Combining (Free-entry) and (wage),

$$c = \beta q(\theta) \frac{1}{1-\beta(1-s)}[(1-\gamma)(z-b) - \gamma\theta c]$$

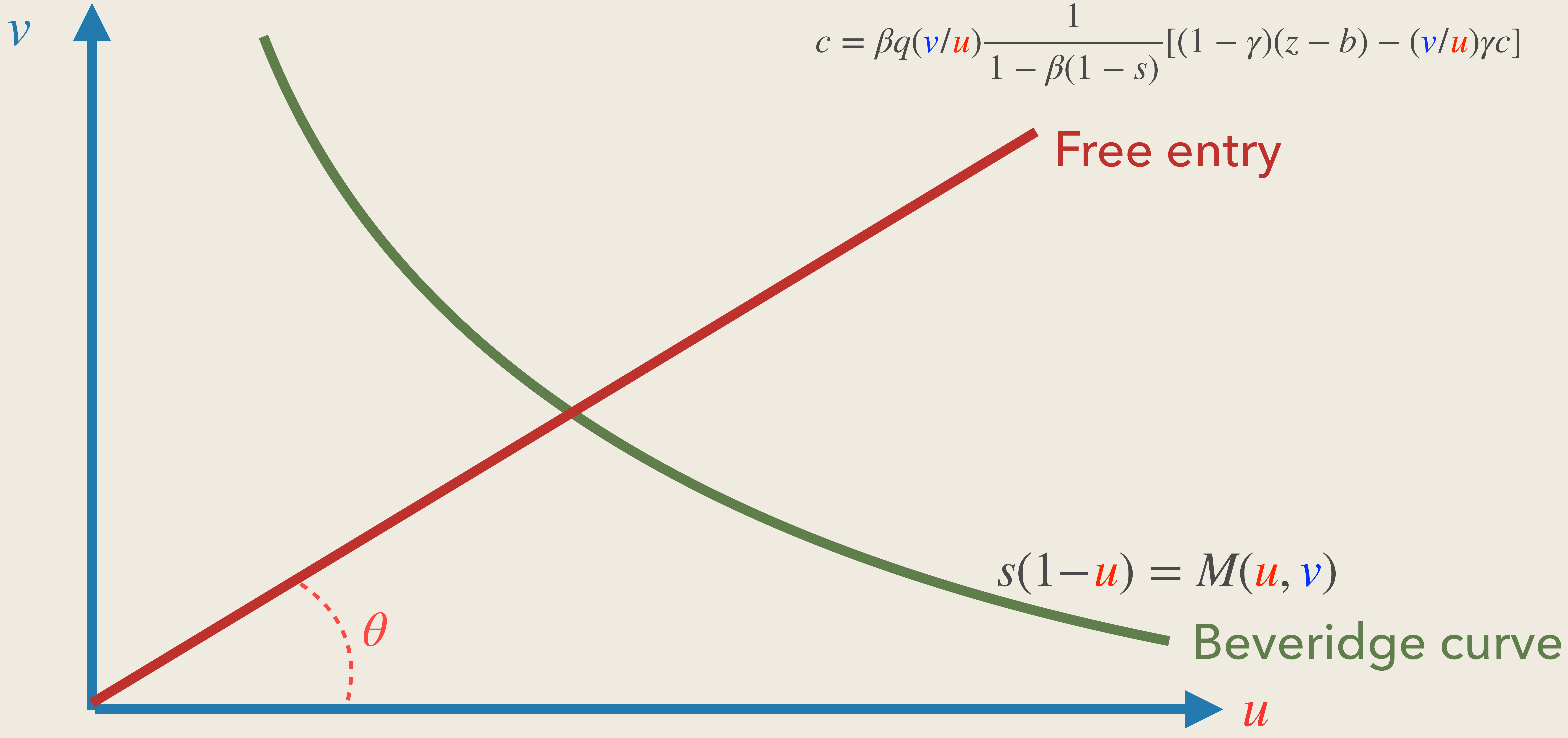
  This pins down $\theta = v/u$.

- (Beveridge curve) pins down u given θ

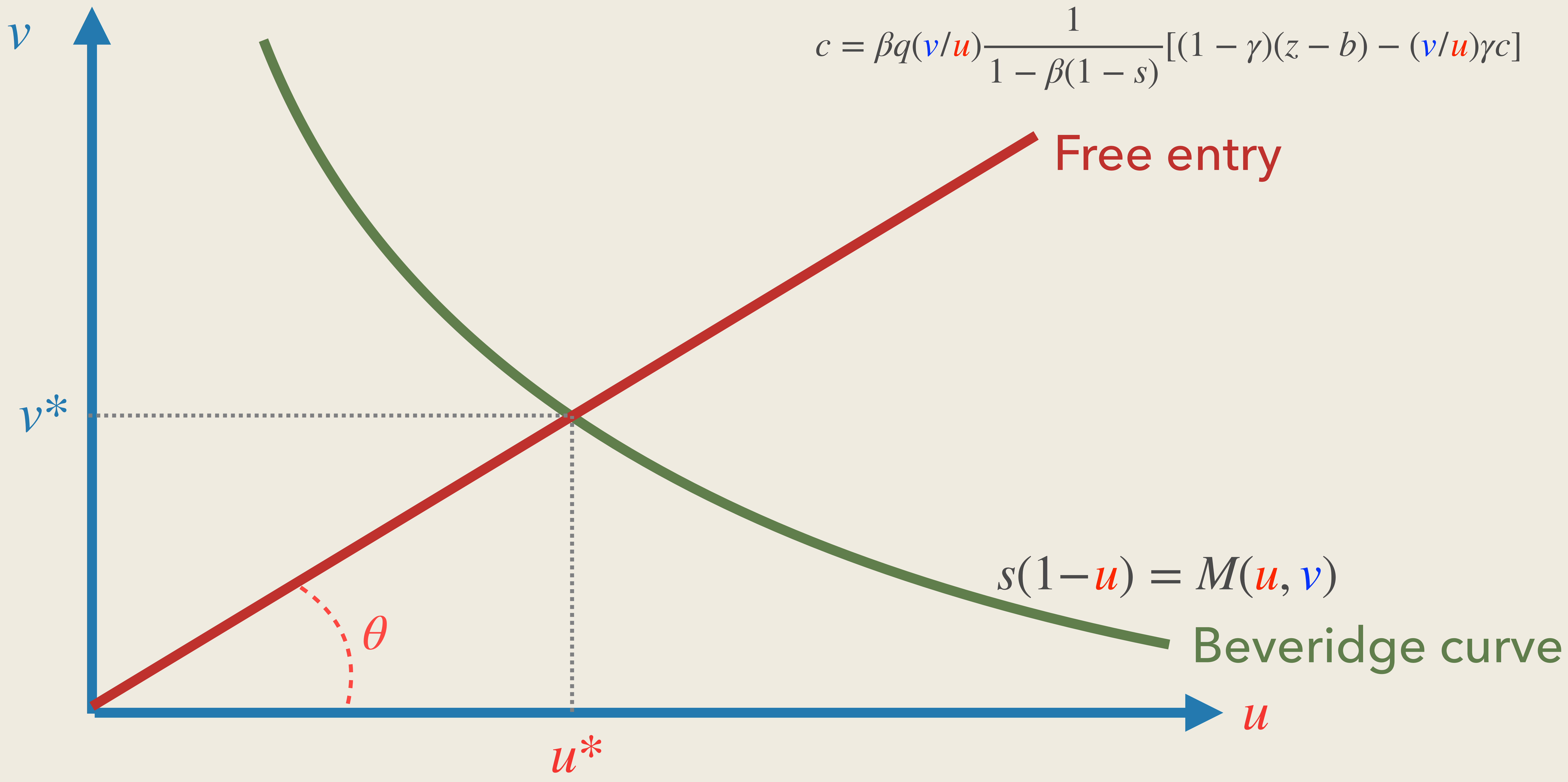
Graphical Characterization



Graphical Characterization



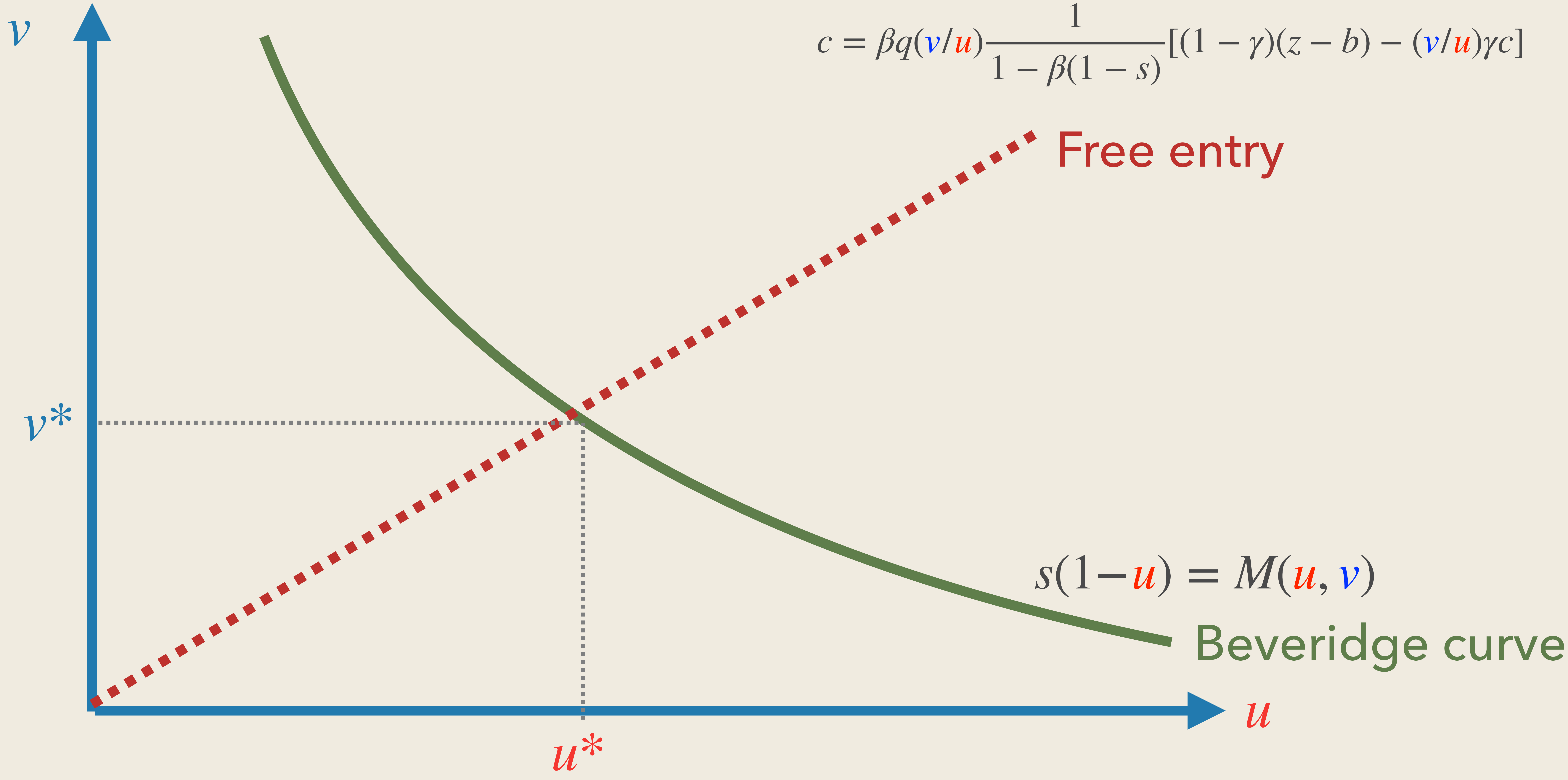
Graphical Characterization



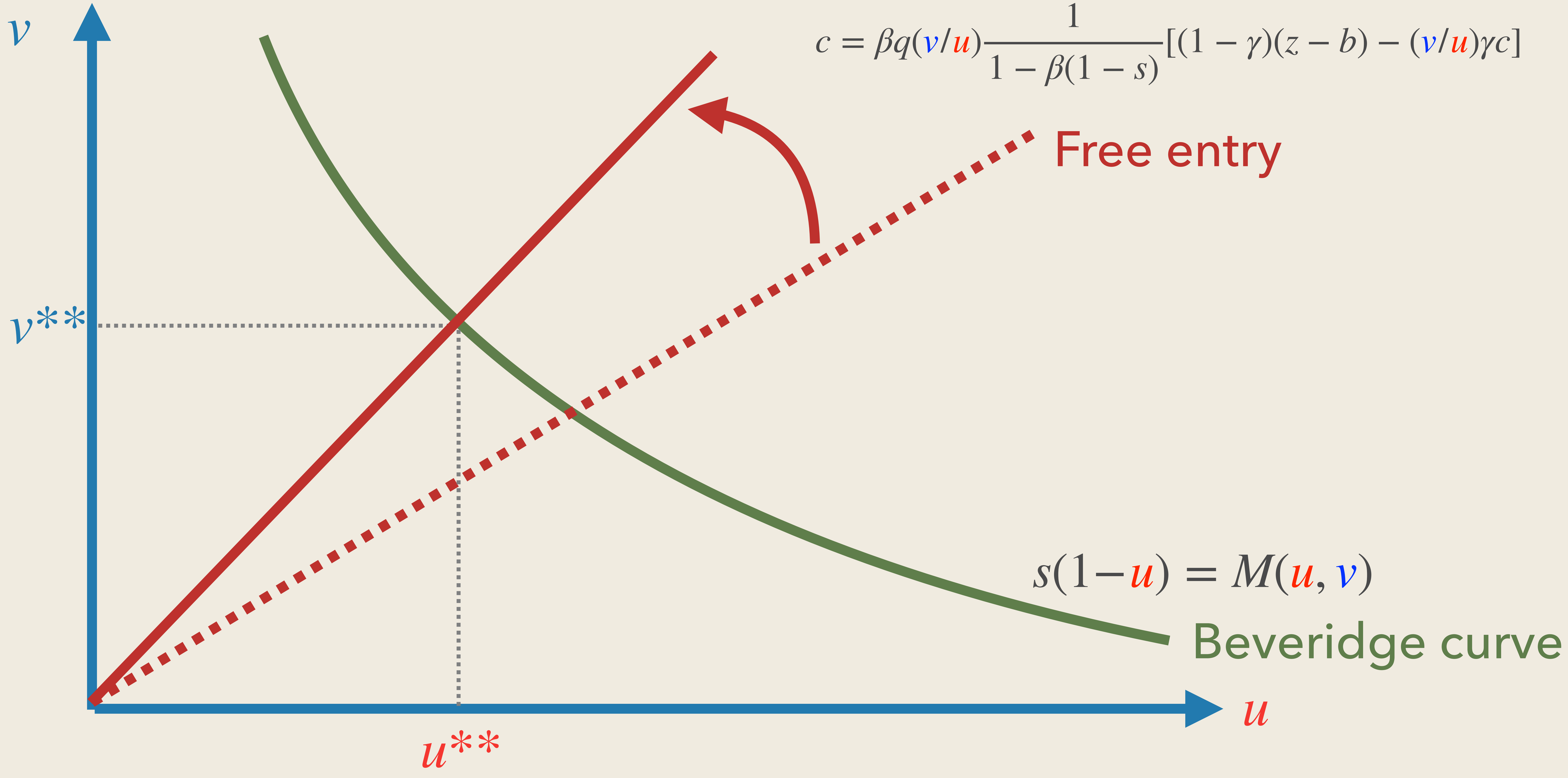
Comparative Statics

An Increase in z

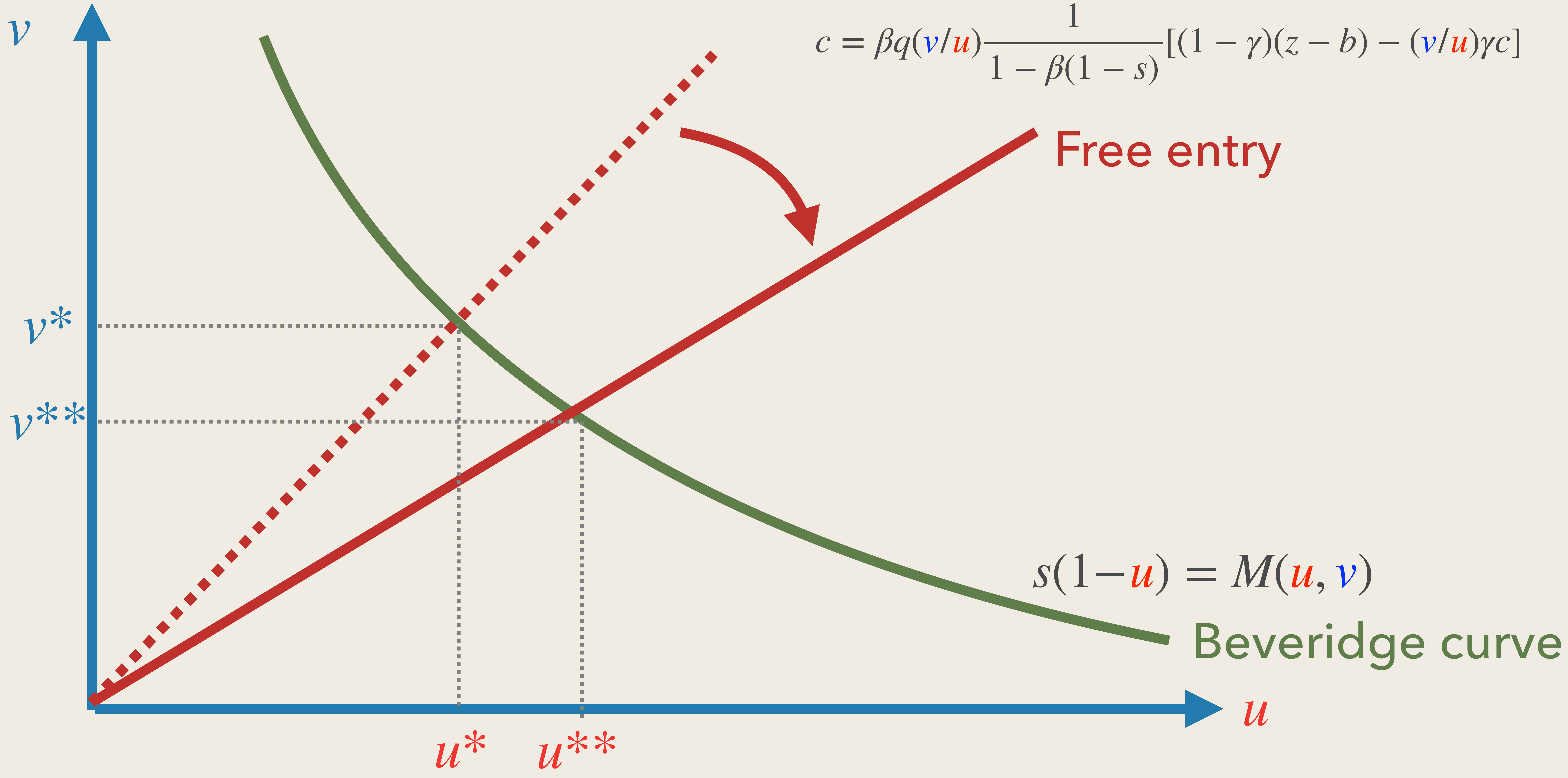
$$c = \beta q(v/u)\frac{1}{1-\beta(1-s)}[(1-\gamma)(z-b)-(v/u)\gamma c]$$



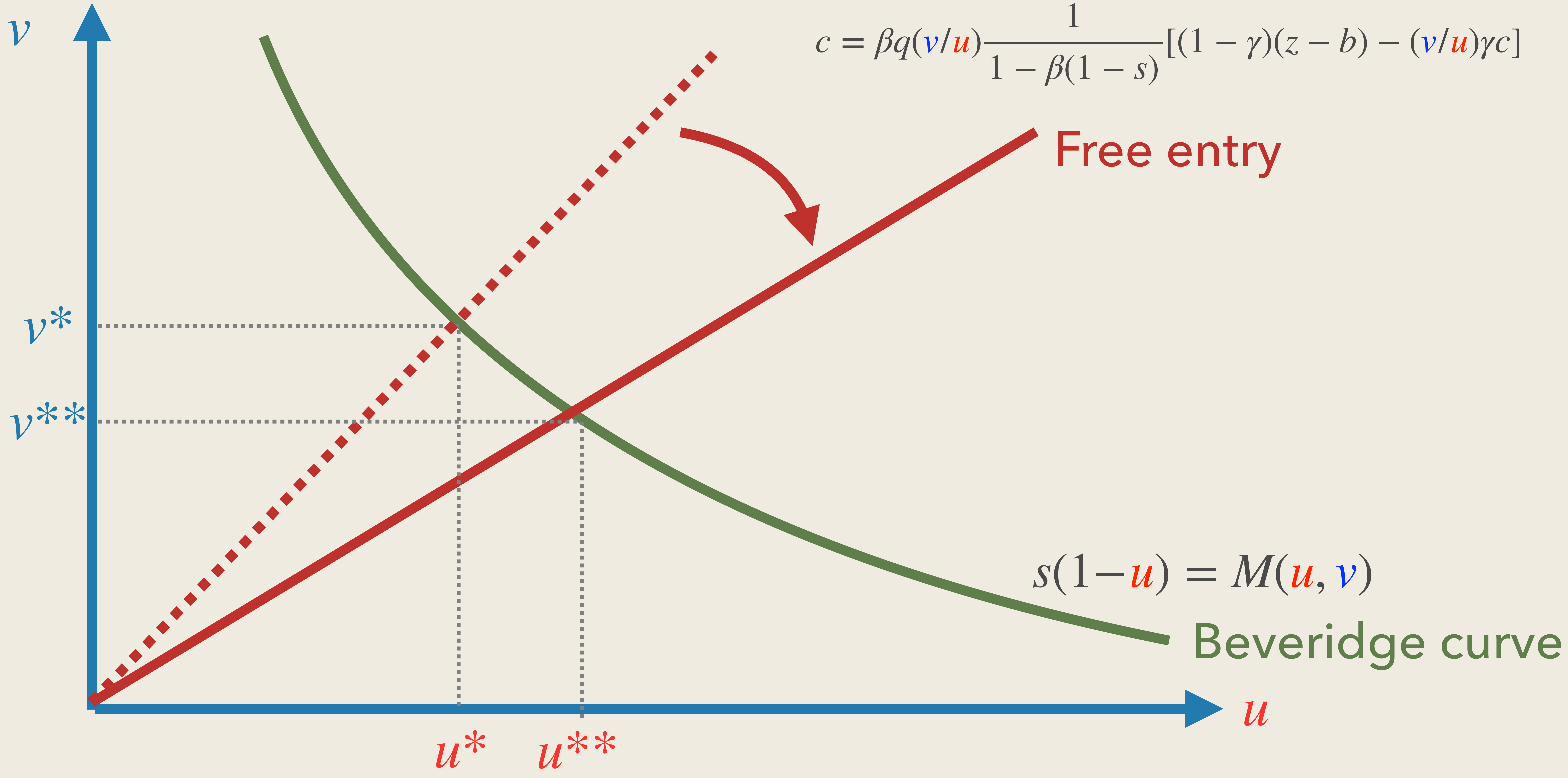
An Increase in z



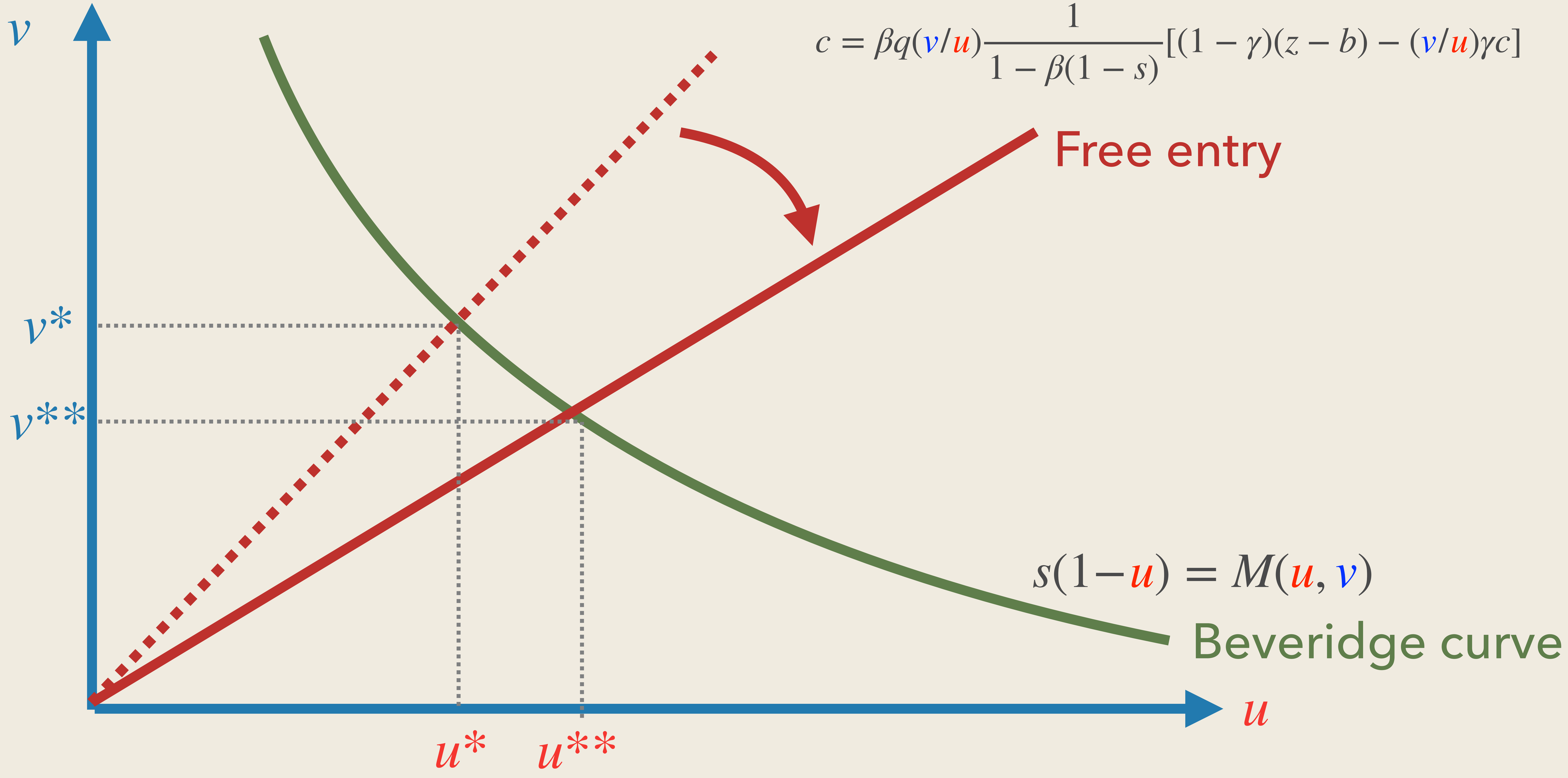
An Increase in b



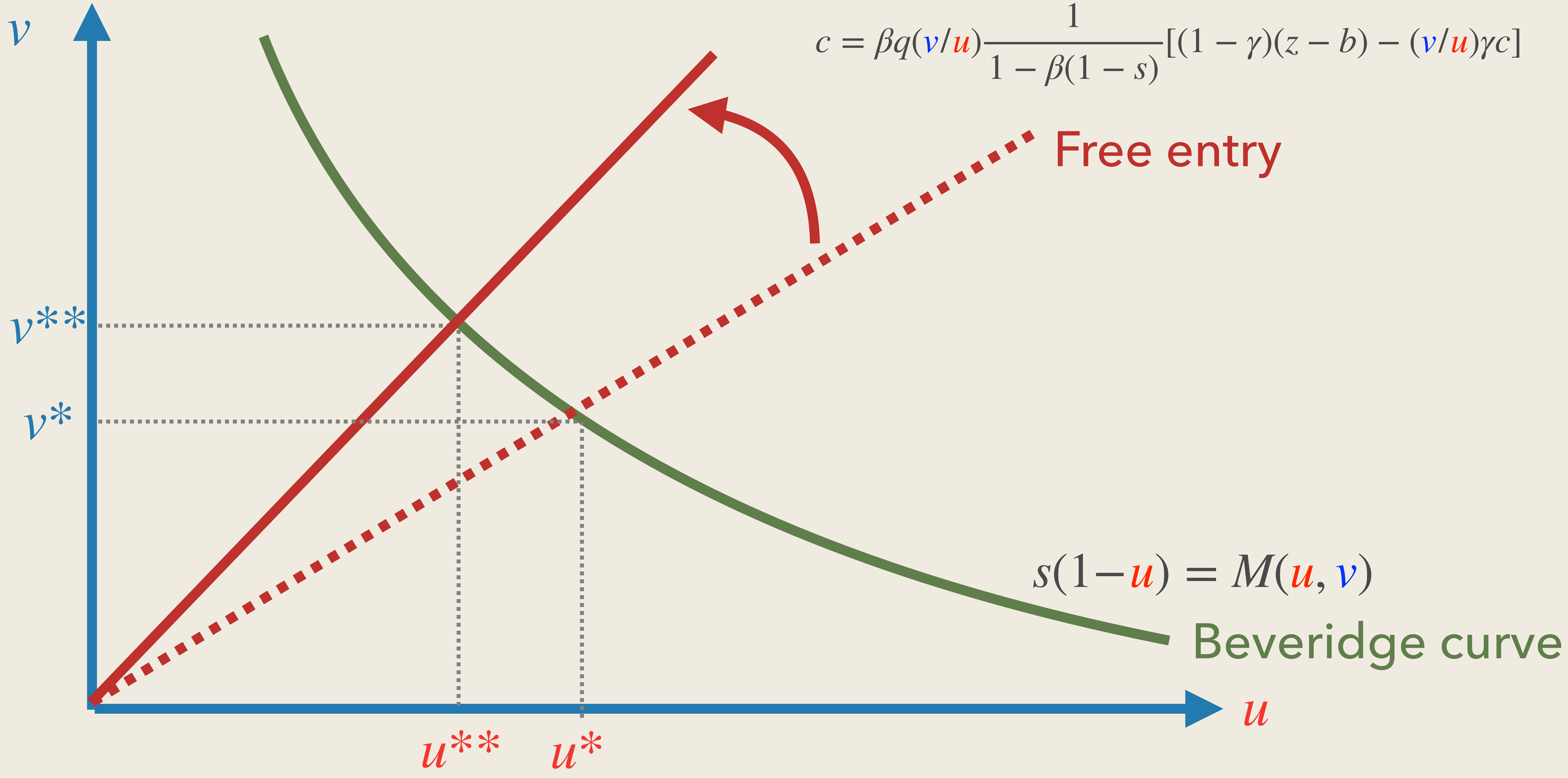
An Increase in γ



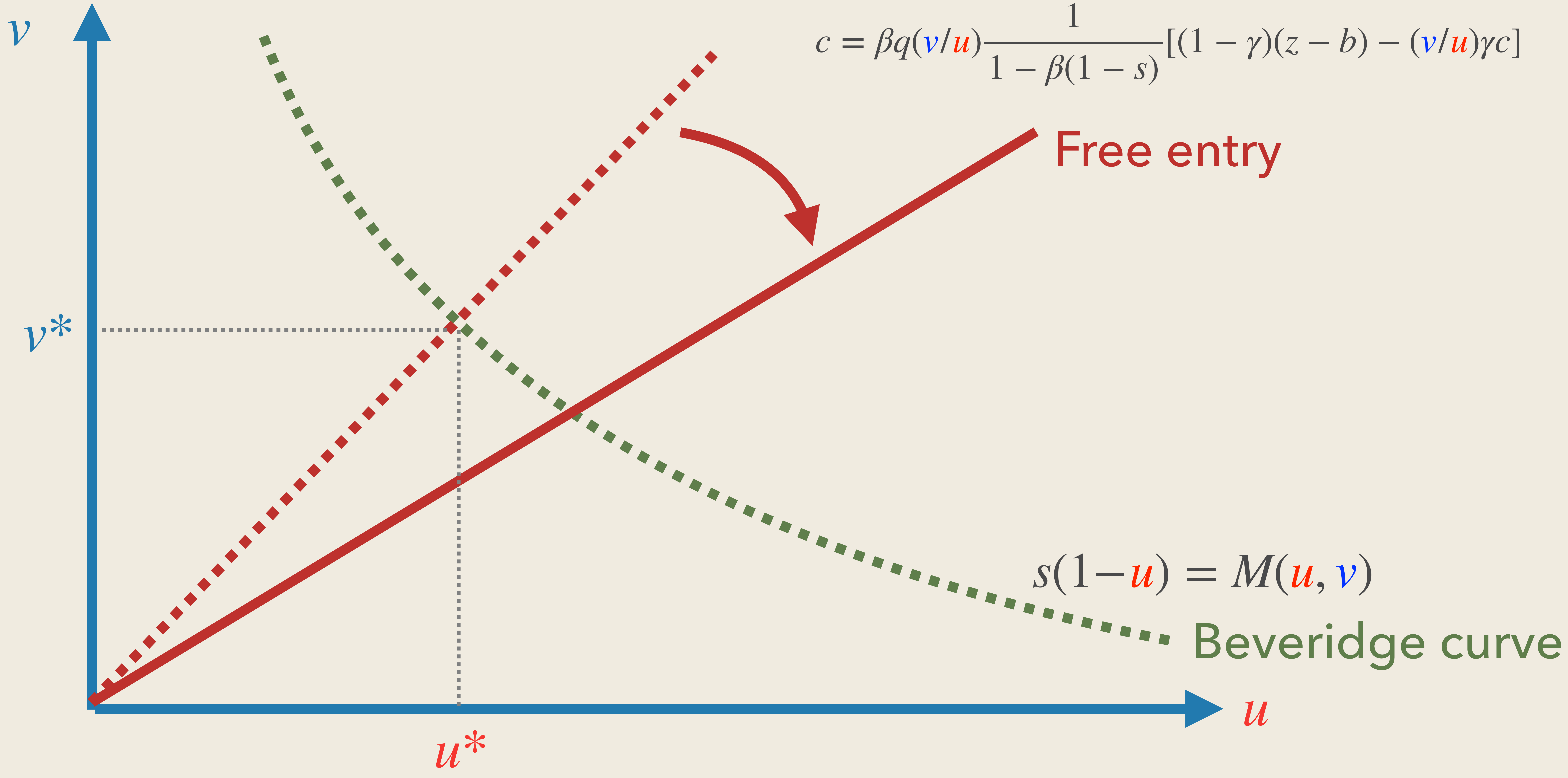
An Increase in c



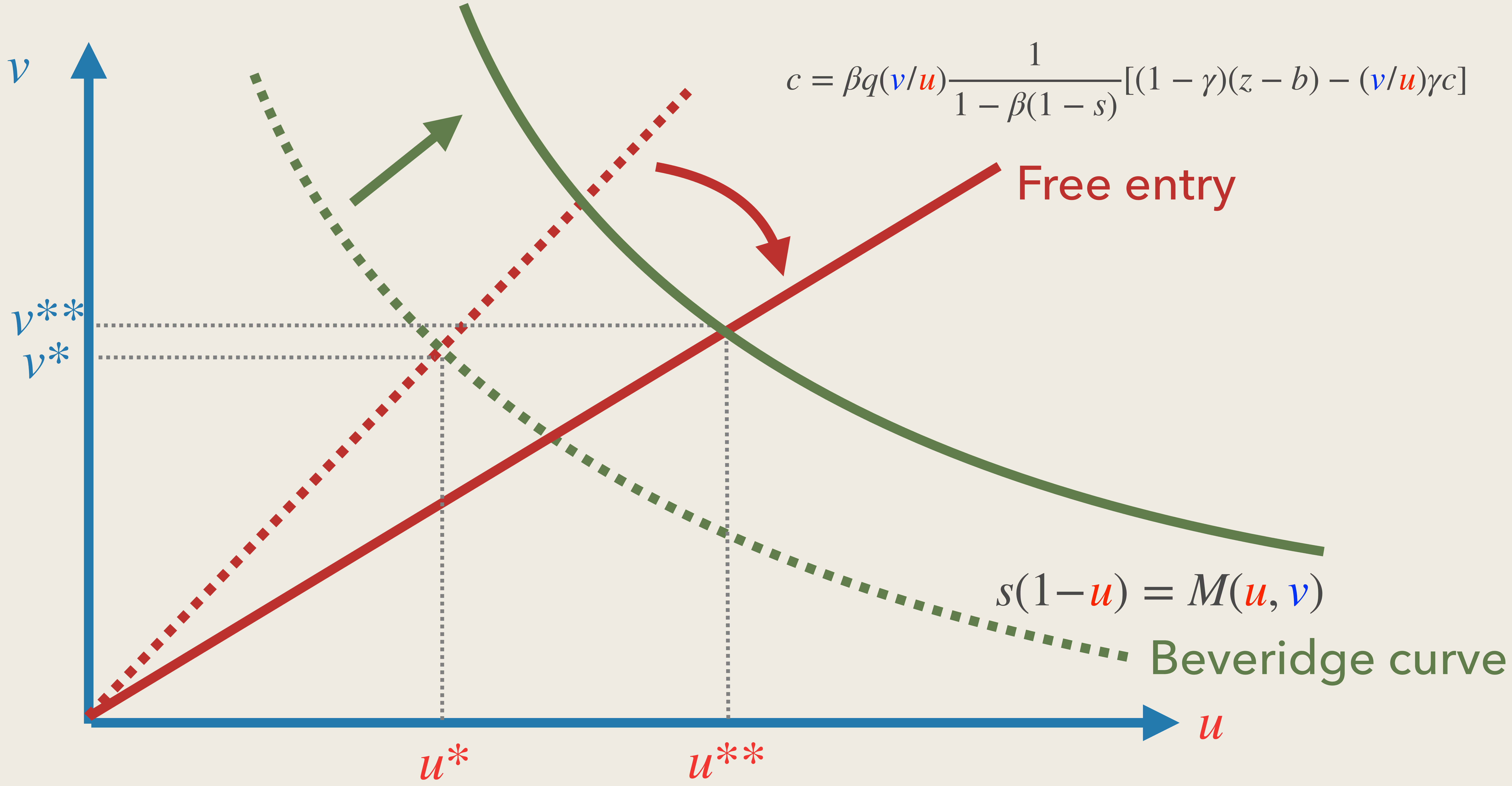
An Increase in β



An Increase in s



An Increase in s



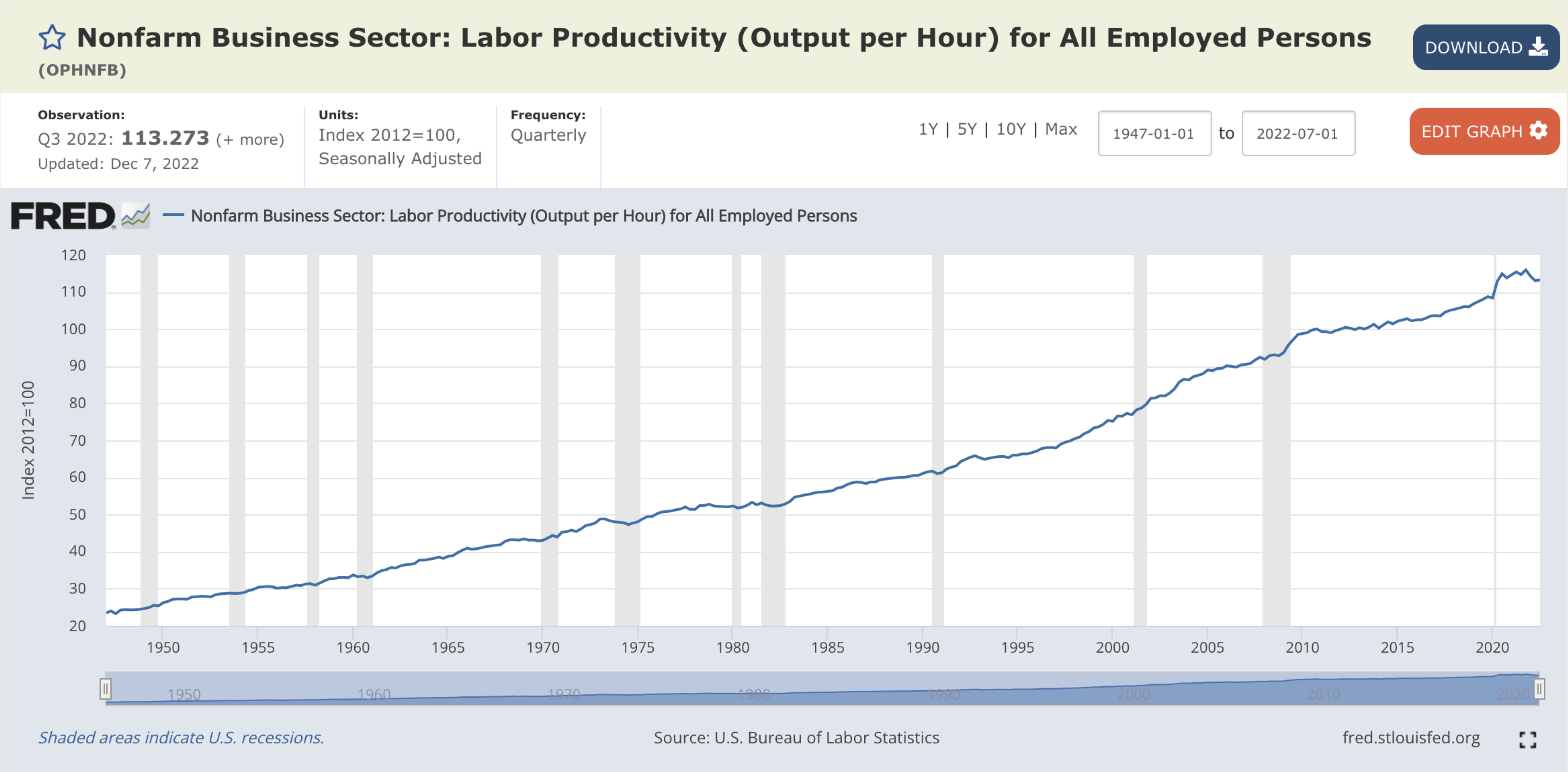
Unemployment Volatility Puzzle (Shimer Puzzle)

Does DMP Model Explain Unemployment Volaitliy?

- Can the DMP model explain unemployment fluctuations quantitatively?
  - Surprisingly, no one asked this question until Shimer (2005)
- Shimer (2005) argued that the model performs terribly. Let us replicate it.
- Calibration (monthly frequency):
  - Matching function: $M(v, u) = \bar{m} v^{1-\alpha} u^{\alpha}$ and set $\alpha = 0.75$ based on lecture 1
  - Following Shimer (2005), we set $\gamma = \alpha$.
  - Job separation rate is set to $s = 2\,\%$ based on the historical average
  - Discount rate is set at 4% annually, so $\beta = 0.96^{1/12}$
  - Following Shimer (2005), set $b = 0.4$ to replicate the UI replacement rate
  - Set c so that the steady-state unemployment rate is 5%

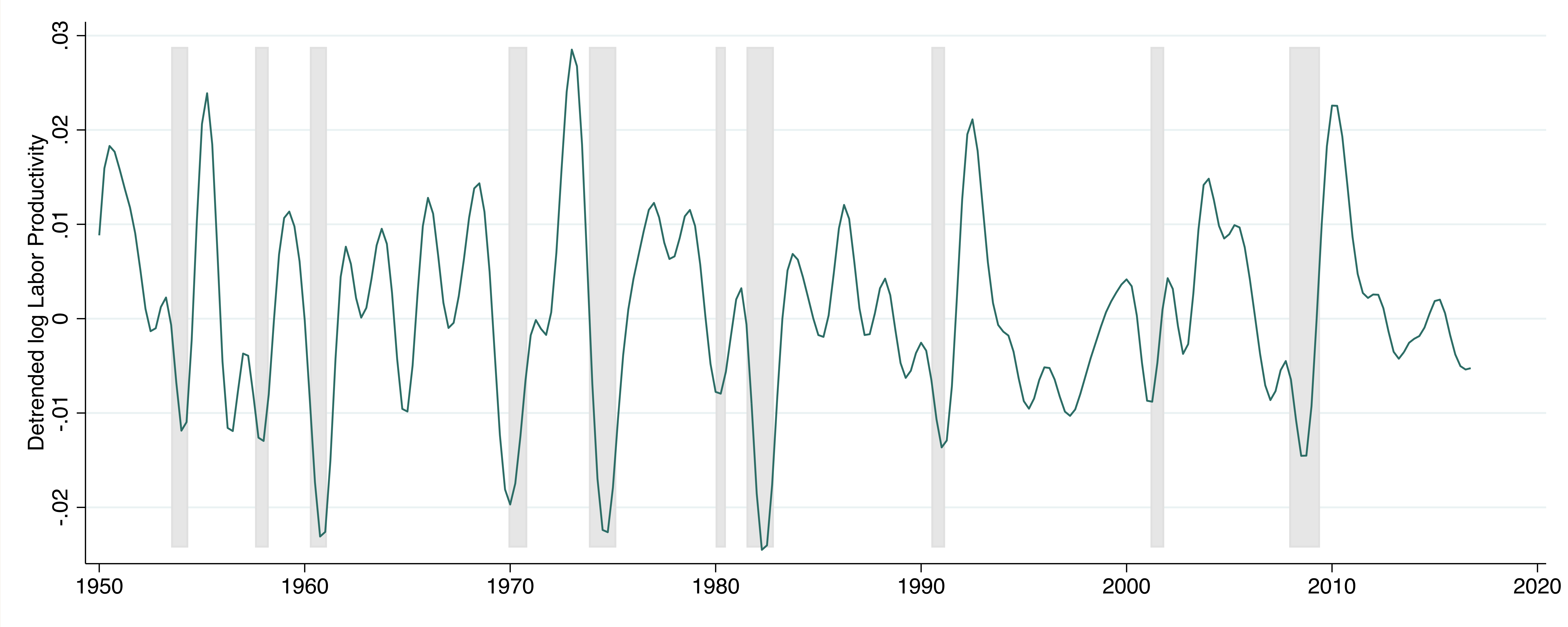
What is z?

- We measure z as the labor productivity (output per hour)



What is z?

- Take log and detrend using Baxter and King (1999) bandpass filter:
- Assume $\ln z$ follows AR(1), $\ln z_t = \rho_z \ln z_{t-1} + \epsilon_t^z$, and we estimate $\rho_z \approx 0.97$

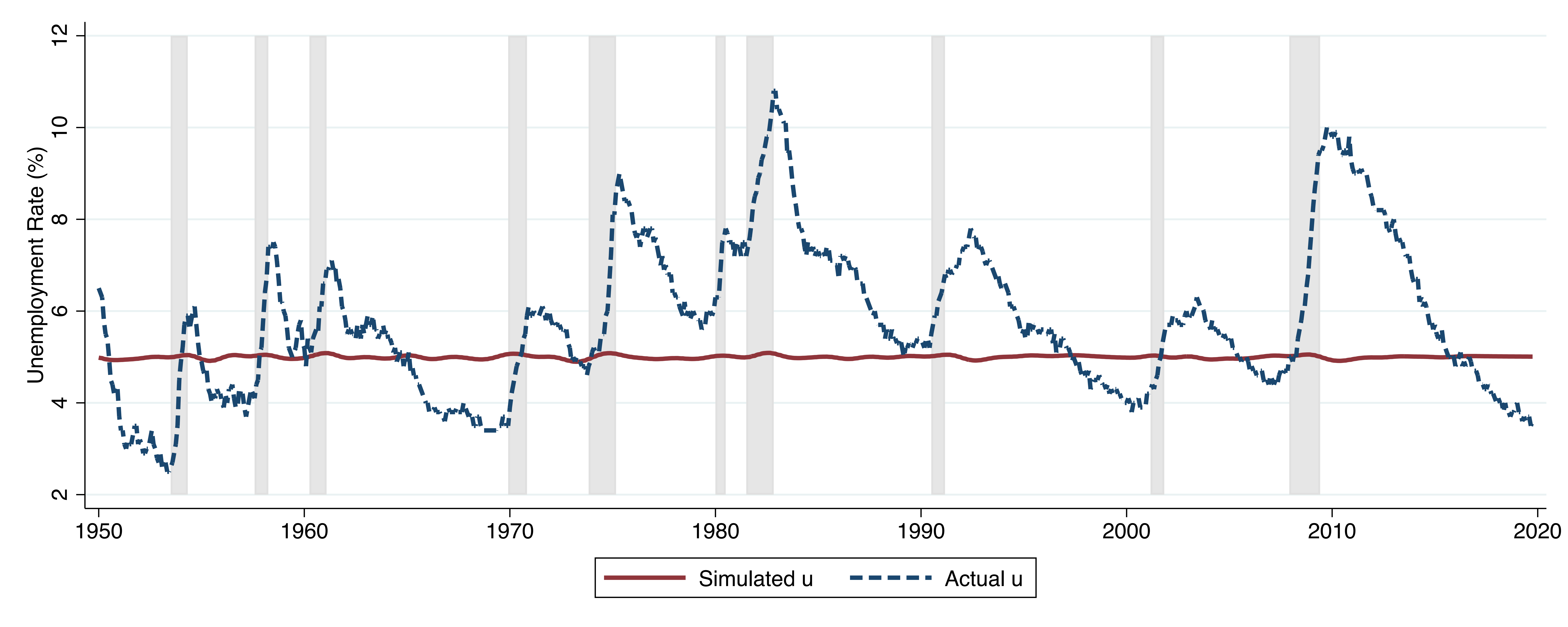


Simulated vs. Actual Unemployment

- Now feed the realized value of $\{z_t\}$ and simulate…

Simulated vs. Actual Unemployment

- Now feed the realized value of $\{z_t\}$ and simulate…



Shimer Puzzle

- The model's unemployment volatility nowhere close to the data
- What is going on?
- To understand, let us focus on comparative statistics w.r.t. z in the steady state
  - Estimated process for z quite persistent
  - Transitions are fast in DMP model (especially so when calibrated to the US data)
  - $\Rightarrow$ Steady-state comparisons provide a good approximation

Unpacking Shimer Puzzle

- Let $1/(1+r) \equiv \beta$. Combining

$$d\ln u = -\underbrace{(1-u)\frac{r+\gamma f(\theta)}{\alpha(r+s)+\gamma f(\theta)}}_{\equiv A}\underbrace{(1-\alpha)\frac{z}{z-b}}_{\equiv B}d\ln z$$

  - Any reasonable calibration implies s, r small relative to $\gamma f(\theta)$ $\Rightarrow A \approx 1$
  - Our calibration implies $(1-\alpha) = 0.25$ and $z/(z-b) = 1.66 \Rightarrow B \approx 0.42$

- Therefore,

$$d\ln u \approx -0.42 \times d\ln z$$

- In the data, std$(\ln u) \approx$ 0.28 and std$(\ln z) \approx 0.009 \Rightarrow |d\ln u| \approx 31 \times |d\ln z|$

Taking Stock

- A huge disappointing failure
- We built an equilibrium model of unemployment
  but it generates less than 2% of volatility in unemployment compared to the data
- This "Shimer puzzle" spurred subsequent research
- We will cover how we might be able to solve the puzzle

Solutions to Unemployment Volatility Puzzles

1. Hagedorn-Manovskii (2008)

$$d \ln u = -\underbrace{(1-u)\frac{r+\gamma f(\theta)}{\alpha(r+s)+\gamma f(\theta)}}_{\approx 1}(1-\alpha)\frac{z}{z-b}d\ln z$$

- The first attack to Shimer puzzle is Hagedorn-Manovskii (2008)
- They argue b closed to z is the reasonable calibration
  - Vacancy cost c is small in the data
  - Therefore profits must be small in order to match observed θ. (Recall: $c = \beta q(\theta)\frac{1}{1-\beta(1-s)}[(1-\gamma)(z-b)-\gamma\theta c]$)
- With $b \approx 0.96$, they solved the puzzle.
  - Mathematically, this is because $z/(z-b)$ term above is high
  - What is the economics?

Intuition

- Firms care about PDV of $\pi_t \equiv z_t - w_t$
- Under Nash bargaining, $w_t = (1-\gamma)b + \gamma(z_t + \theta_t c)$
- When z drops by dz, wage drops by $dw = \gamma dz$
- Profit drops by $d\pi = dz - dw = (1-\gamma)dz$
- A proportional drop in profits is

$$\frac{d\pi}{\pi} = \frac{(1-\gamma)}{(1-\gamma)(z-b) - \gamma\theta c} dz$$

This is larger when steady state profit is small (i.e., $z - b$ is small)

Chodorow-Reich & Karabarbounius (2016)

- But the previous argument critically relies on the fact b is invariant to business cycle
- Chodorow-Reich & Karabarbounius (C-K) asks: is it reasonable to assume constant b?
- How should we think about b?

$$b = \text{(UI benefit)} + \text{(Value of Leisure)}$$

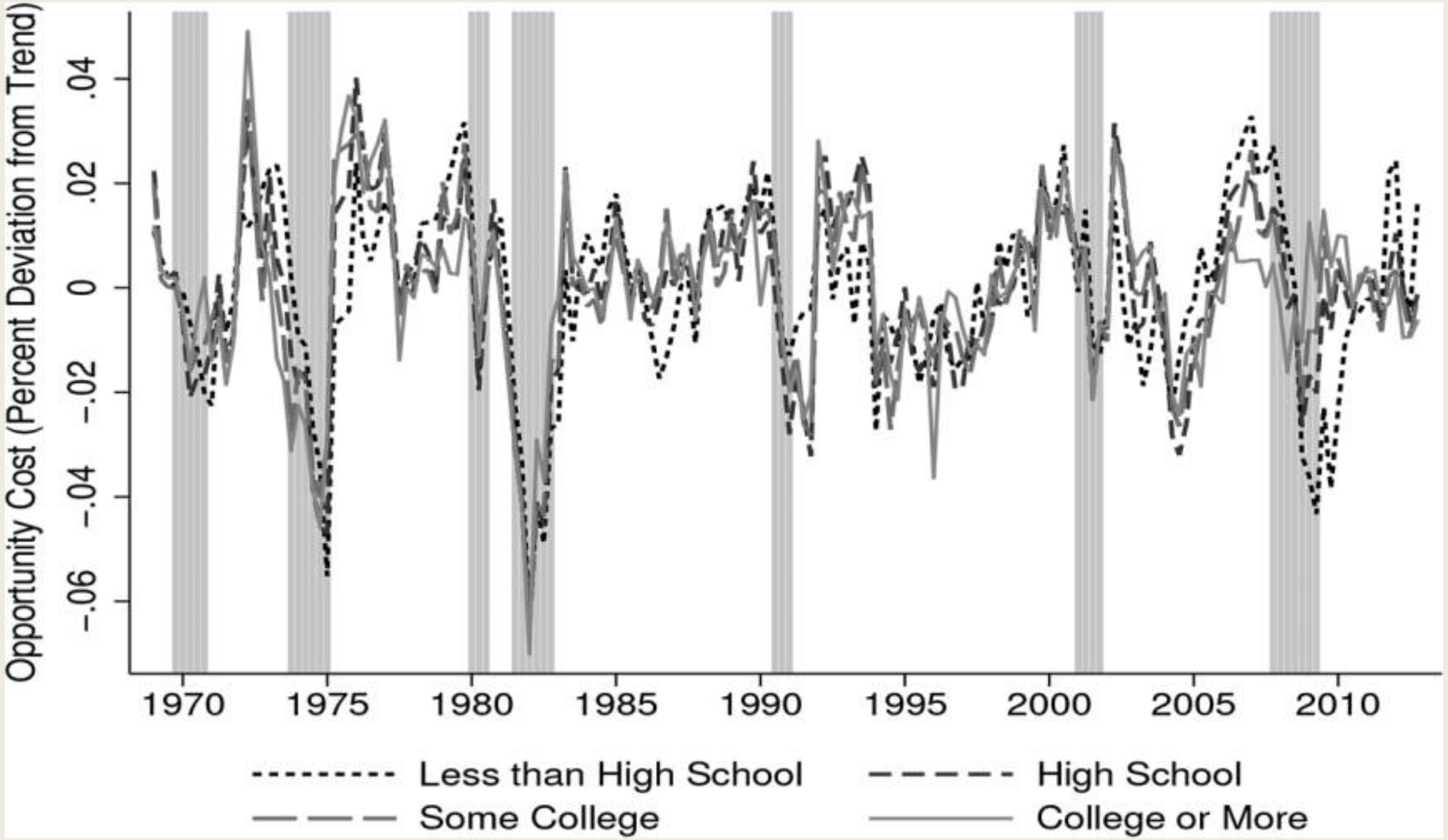
- The value of leisure corresponds to MRS between leisure and consumption

$$MRS_{cl} = \frac{U_l(c,l)}{U_c(c,l)}$$

which was constant in DMP because of linear utility

- More standard assumpion on $U(c,l)$ implies:
  - In recession, c is lower $\Rightarrow MRS_{cl}$ is lower because U_c is higher
  - In recession, l is higher $\Rightarrow MRS_{cl}$ is lower because U_l is lower

Pro-cyclical Opportunity Cost of Unemployment



- C-K measure b as well as they could
- Find b_t is strongly pro-cyclical and $d \ln b_t / d \ln z_t \approx 1$

Implications of Procyclical b

- What does it imply for Shimer puzzle?
- Now replace b with b_t and assume $b_t = \bar{b} z_t$
- When z drops by dz, wage drops now by $dw = ((1-\gamma)\bar{b} + \gamma)dz$ (recall $w_t = (1-\gamma)b + \gamma(z_t + \theta_t c)$)
- Profit drops by $d\pi = dz - dw = (1-\gamma)(1-\bar{b})dz$
- A proportional drop in profits is

$$\frac{d\pi}{\pi} = \frac{(1-\gamma)(1-\bar{b})}{(1-\gamma)z(1-\bar{b}) - \gamma\theta c} dz$$

- Now higher $\bar{b}$ no longer helps. It lowers both the denominator and numerator.

Puzzle Gets Worse

- More formally, the unemployment response to z is

$$d \ln u = -\underbrace{(1-u)\frac{r+s+f(\theta)\gamma}{(r+s)\,\alpha+f(\theta)\gamma}}_{\approx 1}(1-\alpha)d\ln z$$

  - $|d \ln u / d \ln z| \approx (1-\alpha) < 1$
  - Impossible to solve Shimer puzzle irrespective of the value of $\bar{b}$

- In recession, workers are desperate to get a job (lower b)

- Lower workers' outside option $\Rightarrow$ w goes down as much as $z \Rightarrow$ profits little affected

2. Wage Rigidity

- The second attack by Hall (2005) focuses on wage setting
- Why should we stick to Nash Bargaining?
  – As we discussed, ***any*** wage that satisfies individual rationality is an equilibrium
- Suppose wages are fully rigid, $w_t = \bar{w} \in [b, z_t + \theta_t c]$.

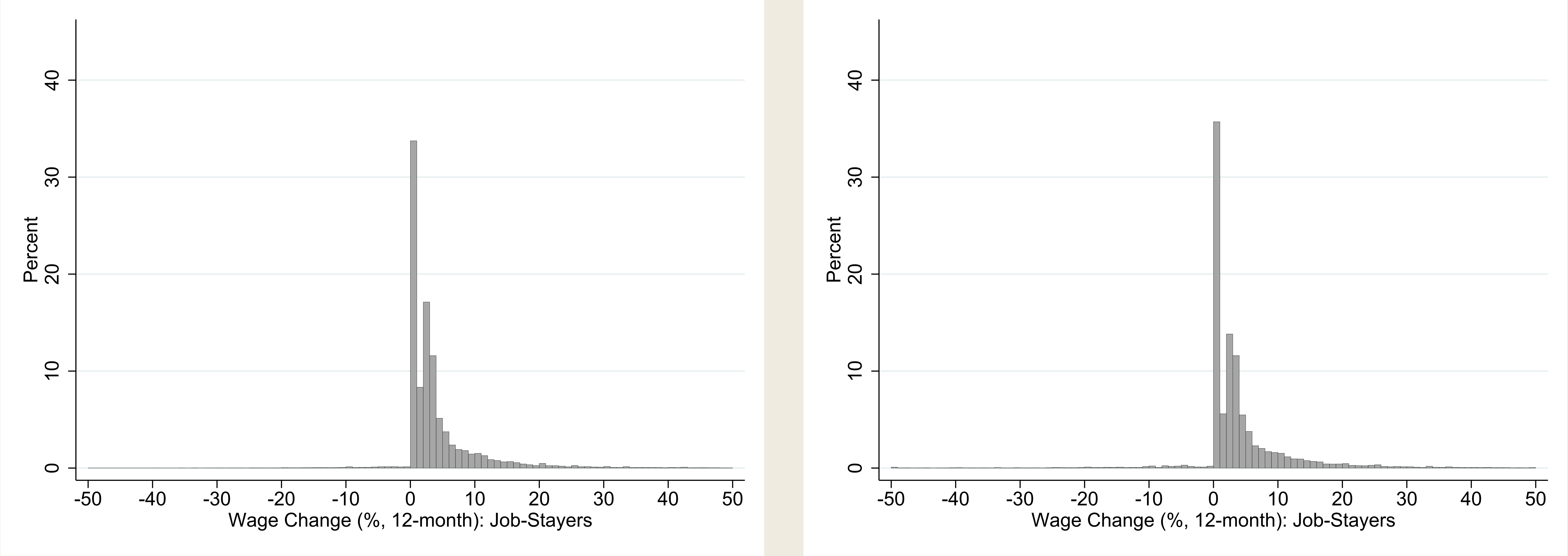
$$\frac{d \ln u}{d \ln z} = -(1-u)\frac{1-\alpha}{\alpha}\frac{z}{z-\bar{w}}$$

- Hall (2005) sets $\bar{w} = 0.96$ and $\alpha = 0.23$, so that $d \ln u / d \ln z \approx -80$
- Note that wage rigidity is not sufficient to solve the puzzle. Need high $z/(z-w)$.
- Immune to Chodorow-Reich & Karabarbounis critique.

Wages of Job-Stayers are Downwardly Rigid

- Ultimately, whether wages are rigid or not is an empirical question
- Wages of job-stayers are downwardly rigid. Is this what we should measure?

Figure 2: 12-Month Nominal Base Wage Change Distribution, Job-Stayers



Panel A: Hourly Workers   Panel B: Salaried Workers

Source: Grisby, Hurst & Yildirmaz (2020)

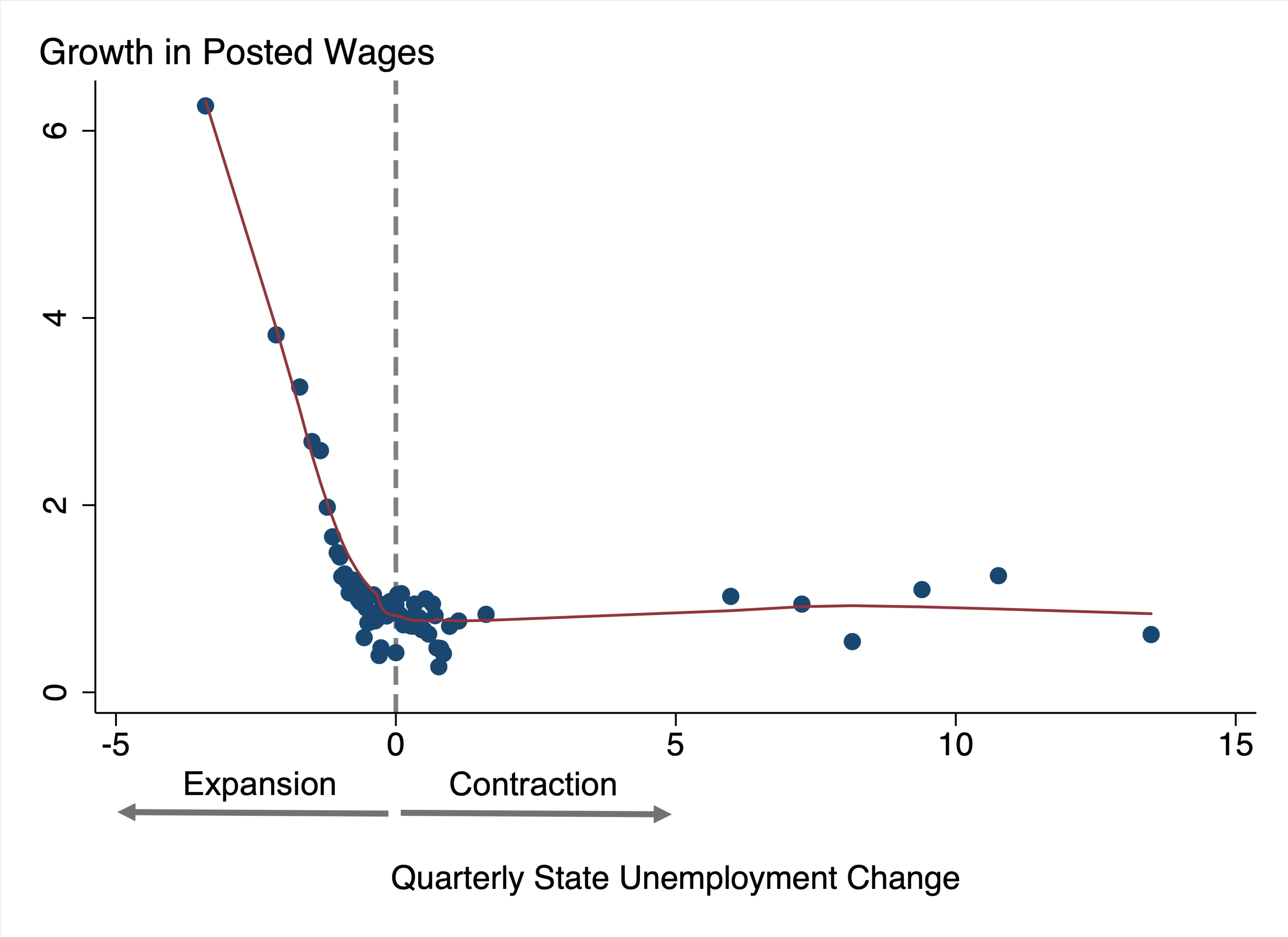
Measuring Wage Rigidity

Two issues:

1. What matters is the (PDV of) wages for *new hires*:
   - wage of workers hired before t irrelevant for firms' incentive to create new job
   - what matters is how much firms need to pay for workers newly hiring at time t
2. Difficult to measure rigidity in new hire wages due to compositional differences:
   - naive idea is to compare workers hired in booms and recessions
   - workers/jobs in recessions and booms might be very different
   - not an apple-to-apple comparison

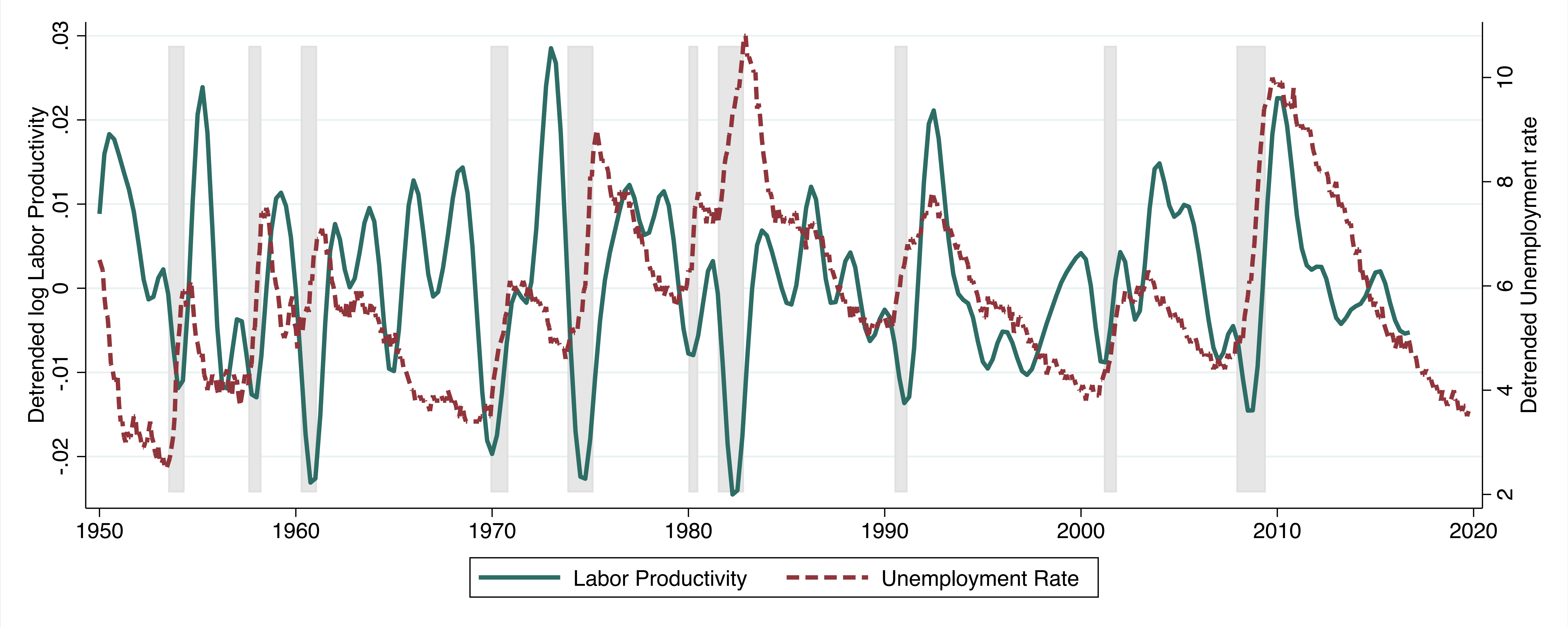
Hazell and Taska (2022)

- Posted wages in online job vacancies are rigid downward



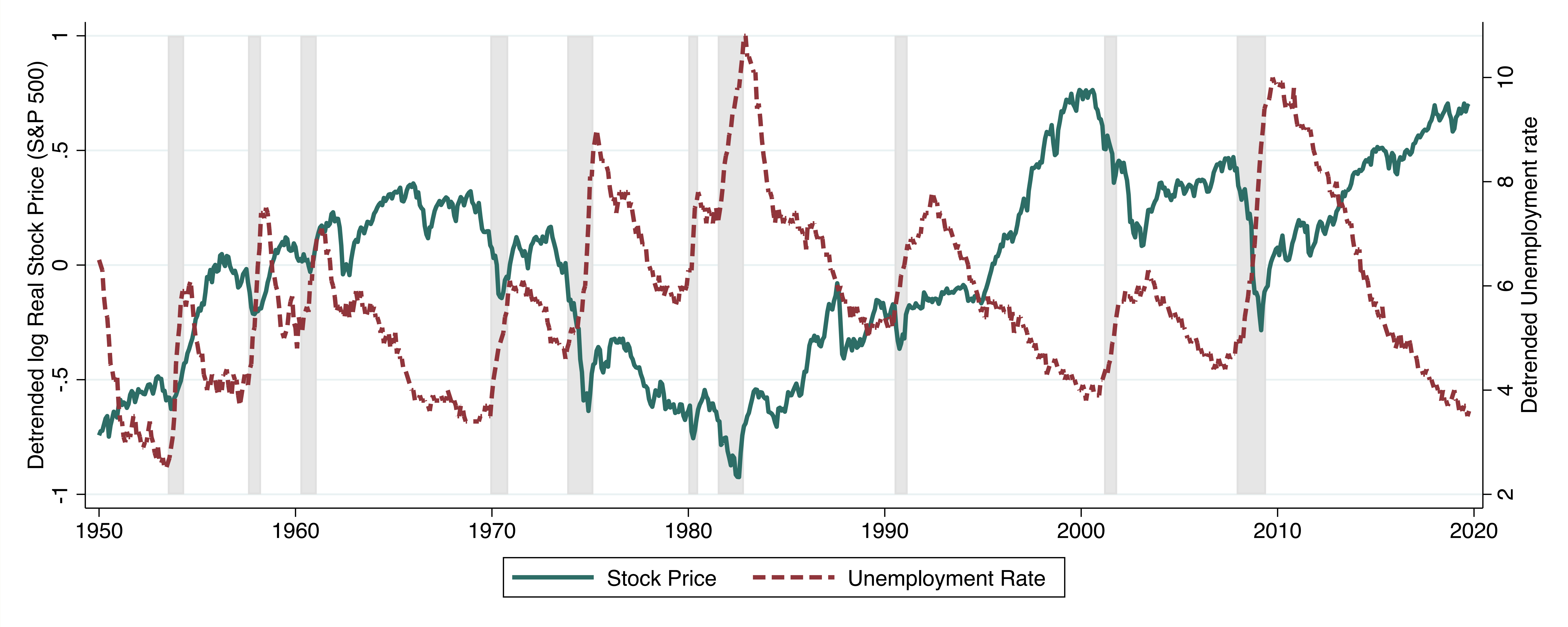
3. Discount Rate Shock

- The third attack by Hall (2017) & Kehoe et al. (2022) focus on the nature of the shock
- In the end, labor productivity is not correlated with unemployment (even wrong sign)



Asset Prices and Unemployment

- Stock prices feature strong negative correlation with unemployment



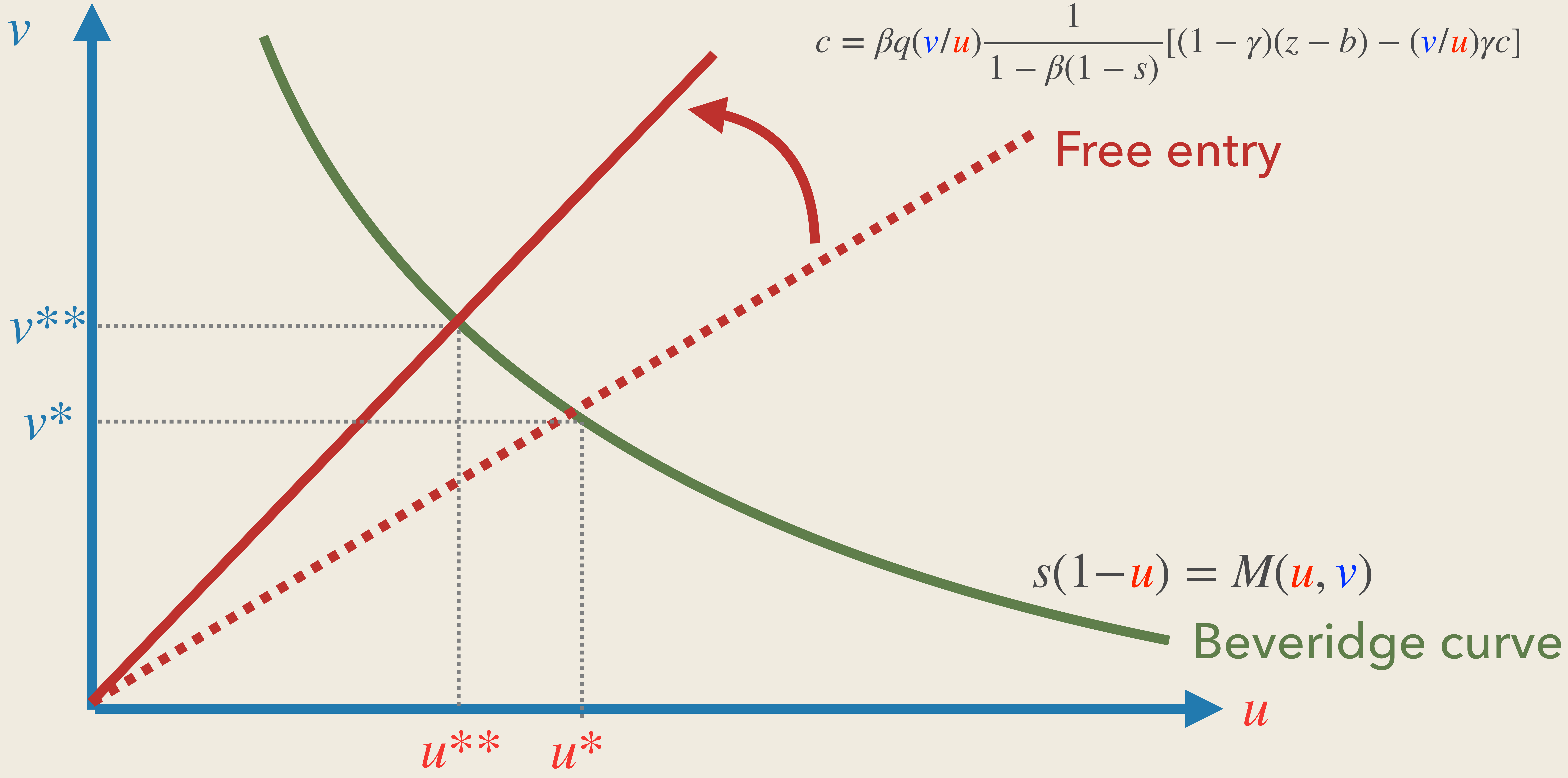
Financial Dicounts

- Asset pricing equation:

$$P_t = \sum_{s=0}^{\infty} \frac{1}{(1+r)^s} d_{t+s}$$

  - P_t fluctuates massively due to fluctuations in r, not d.
- What happens to unemployment if $\beta \equiv 1/(1+r)$ changes?

Recap: An Increase in β



Quantification

- Kehoe et al. (2022) argued this effect is quantitatively small in DMP
- Why? In DMP, creating a job is a short-term investment (expected duration 5 years)
- Once we introduce on-the-job human capital accumulation…
  - Job creation becomes a long-term investment
  - When investment is long-term, its return is more sensitive to discounting
- They argued this solves the Shimer puzzle

4. Separation Shock

- So far, we have not considered time variation in the separation rate, s_t
- Shimer (2005) argued that fluctuations in separation cannot be important
- Why?
  Let's take the baseline DMP model and simulate the impulse response to s shock.

IRF to Separation Shock

Counterfactual Vacancy Response

- Highly counterfactual response of vacancy
- Recall that Beveridge curve in the data tells us $corr(u, v) < 0$
- Separation shock implies $corr(u, v) > 0$
- Separation shock cannot be a major driver of unemployment fluctuations

- Coles & Kellishomi (2018) argue this relies on counterfactual free-entry assumption
  - Infinitely many firms are waiting to create a vacancy

Inelastic Vacancy Creation

- Instead of free-entry, suppose new vacancy creation is inelastic
- Consider the extreme: a fixed number of vacancies, ω, can be created every period
- The stock of vacancy in the economy evolves

$$v_{t+1} - v_t = \omega - q(\theta_t)v_t$$

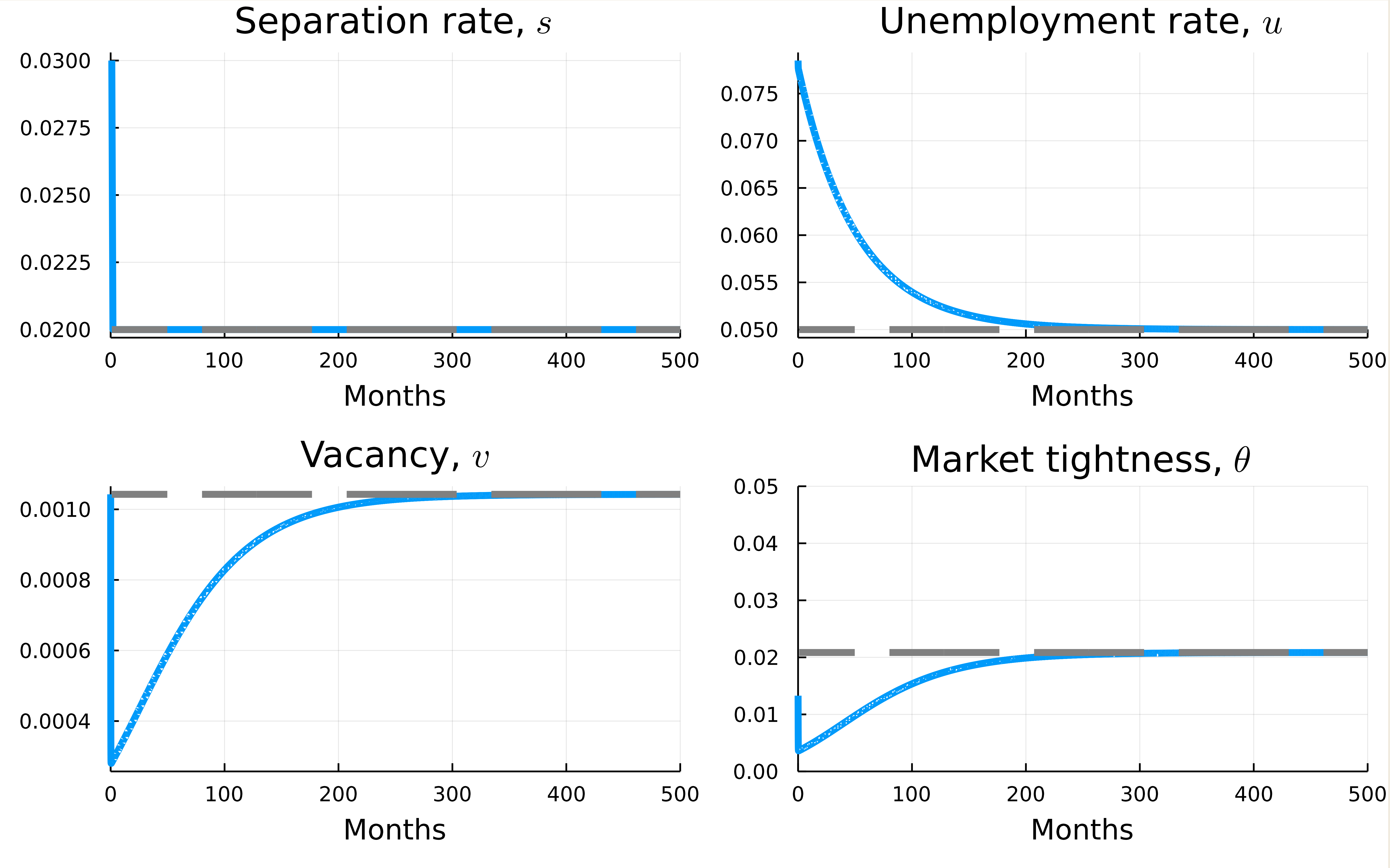
- The law of motion of unemployment is fully characterized by

$$u_{t+1} - u_t = s_t(1 - u_t) - f(\theta_t)u_t$$

$$v_{t+1} - v_t = \omega - q(\theta_t)v_t$$

$$\theta_t = v_t / u_t$$

IRF to Separation Shock with Inelastic Vacacancy



More General Case

- Of course, the previous example is extreme
- Coles-Kellishomi (2018) considers a model in-between:

$$c \times \omega^{1/\xi} = \beta q(\theta_t)\mathbb{E}\sum_{n=t}^{\infty}(\beta(1-s))^{n-t}[z_{n+1} - w_{n+1}]$$

  - When $\xi = \infty$, we have DMP
  - When $\xi \to 0$, we have the previous example

IRF to Separation Shock in Coles-Kellishomi

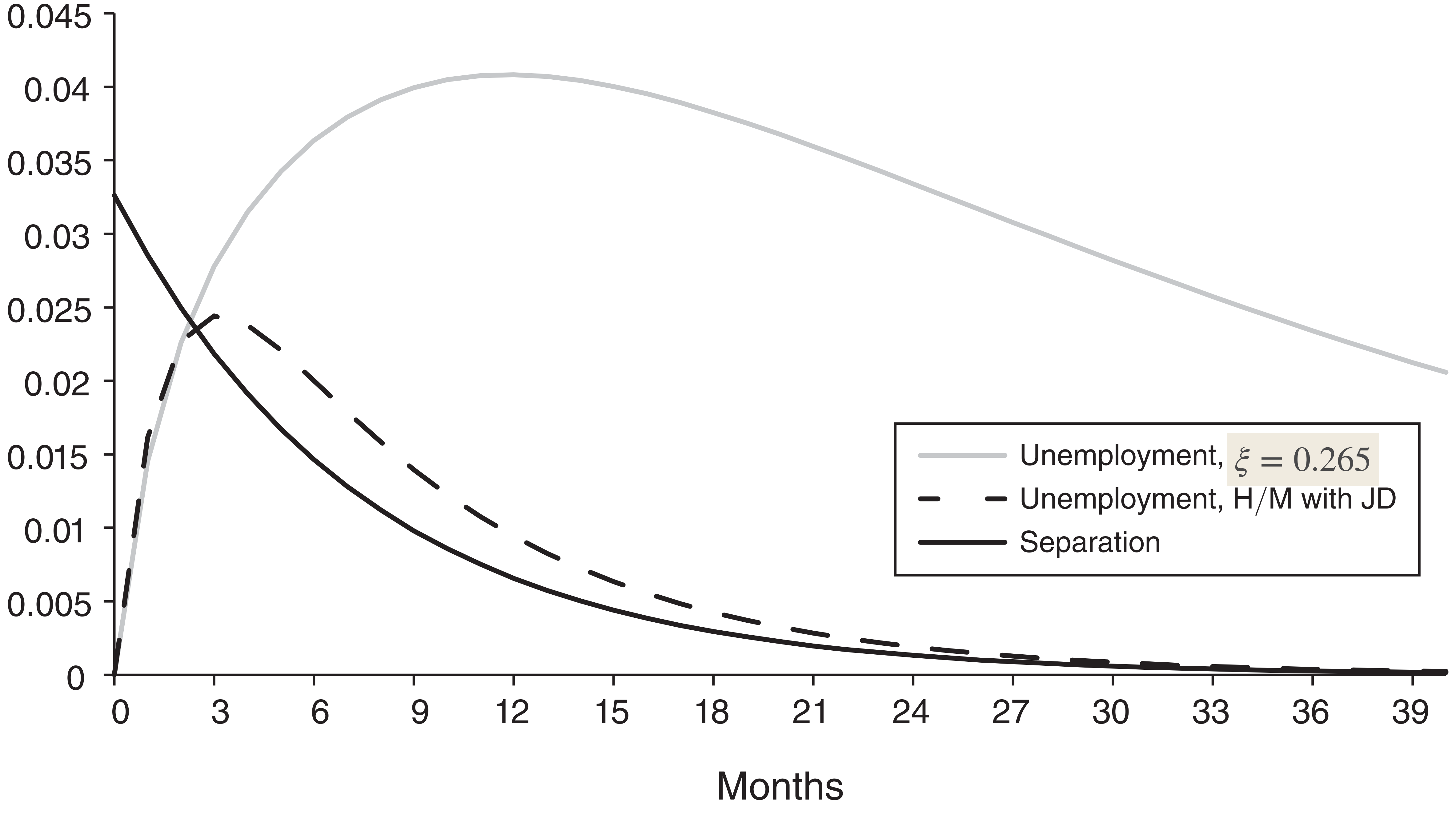


FIGURE 2. IMPULSE RESPONSE OF UNEMPLOYMENT TO A SEPARATION SHOCK

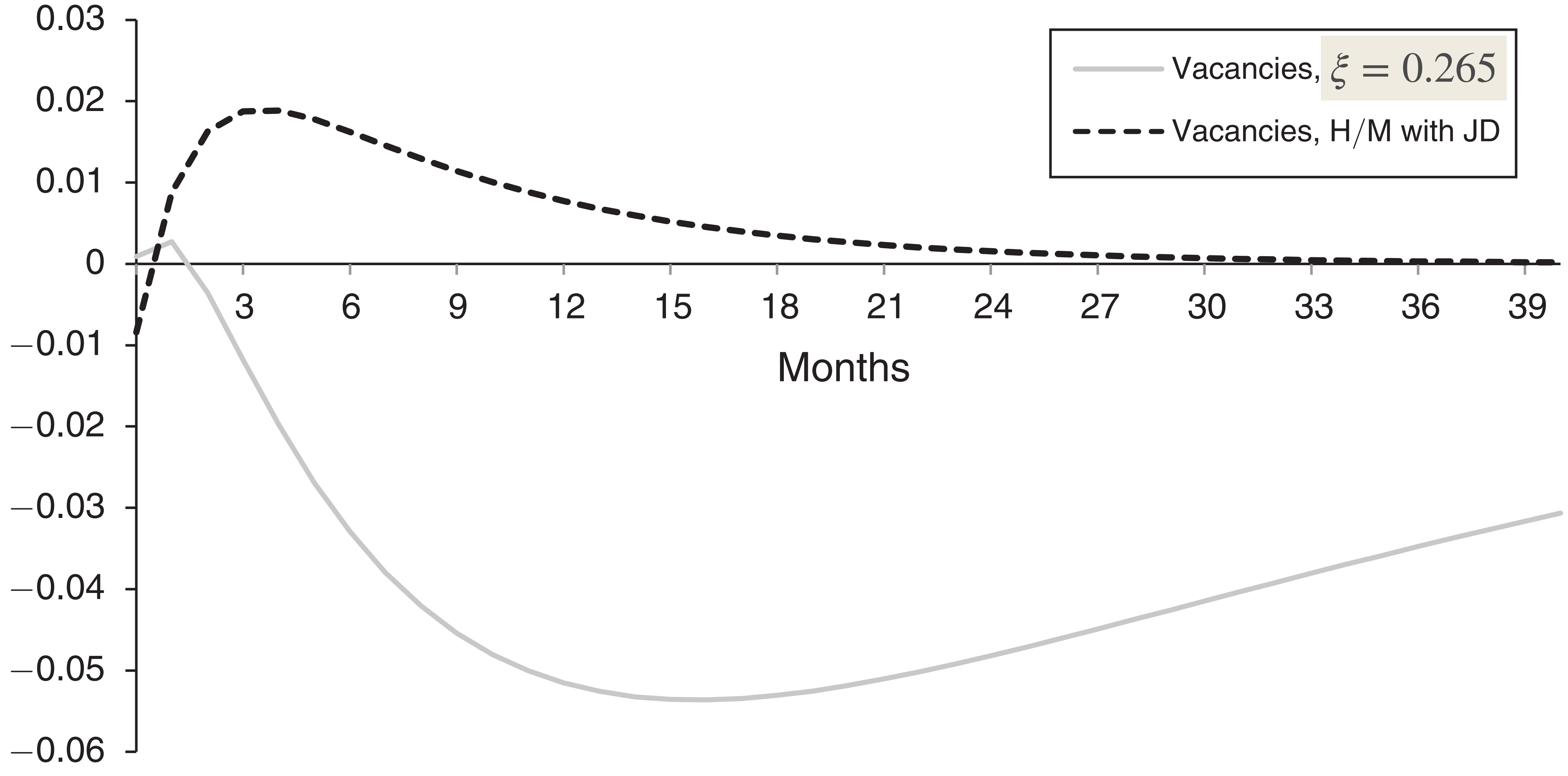


FIGURE 3. IMPULSE RESPONSE OF VACANCIES TO A SEPARATION SHOCK

Hiring Looks Inelastic in the Data

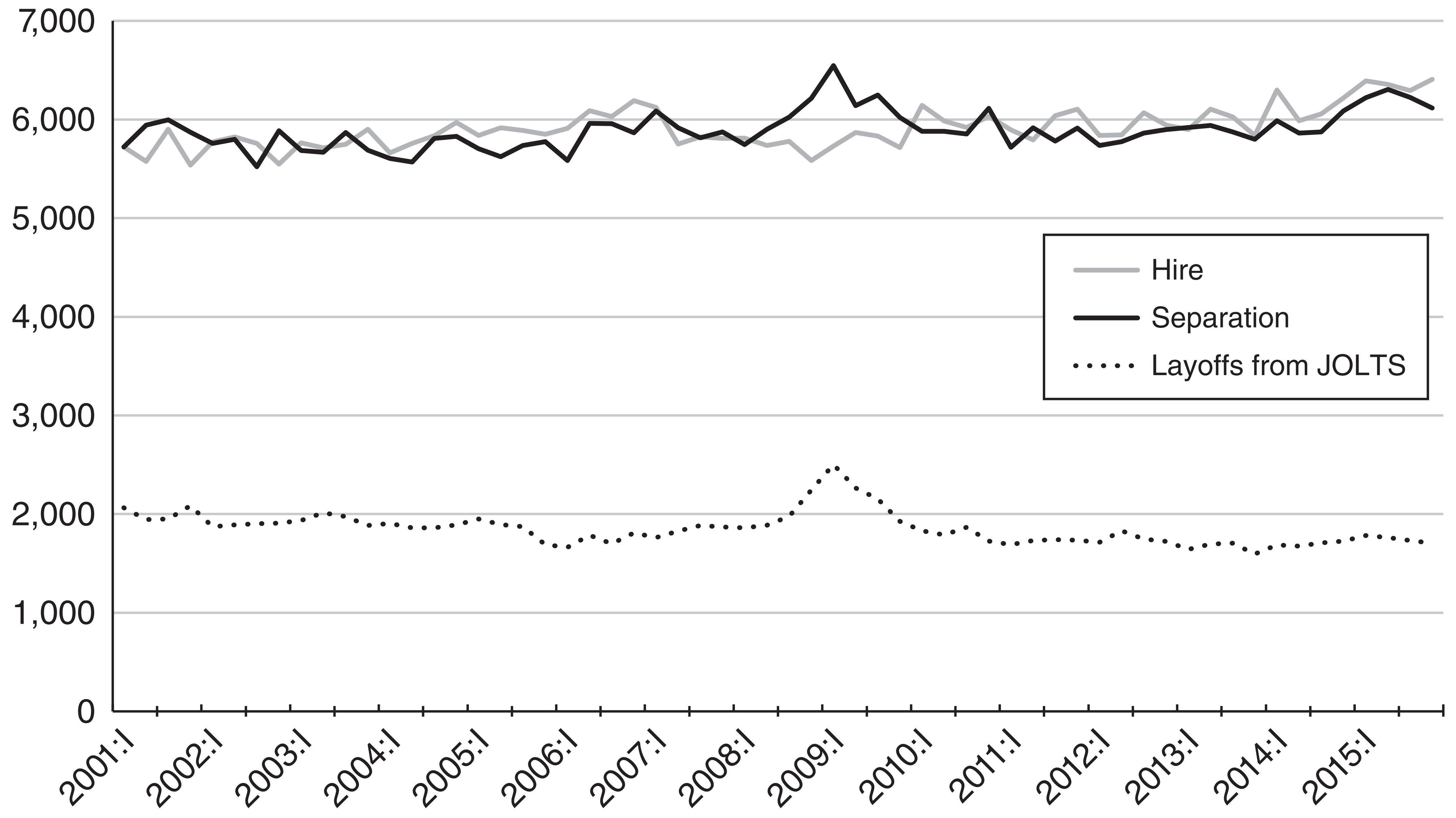


FIGURE 4. US JOB TURNOVER (2000–2015)

Notes: Hires measure is the flow of workers from unemployment and nonlabor force to employment. Job separations is the flow of workers from employment to unemployment and nonlabor force (all in thousands).

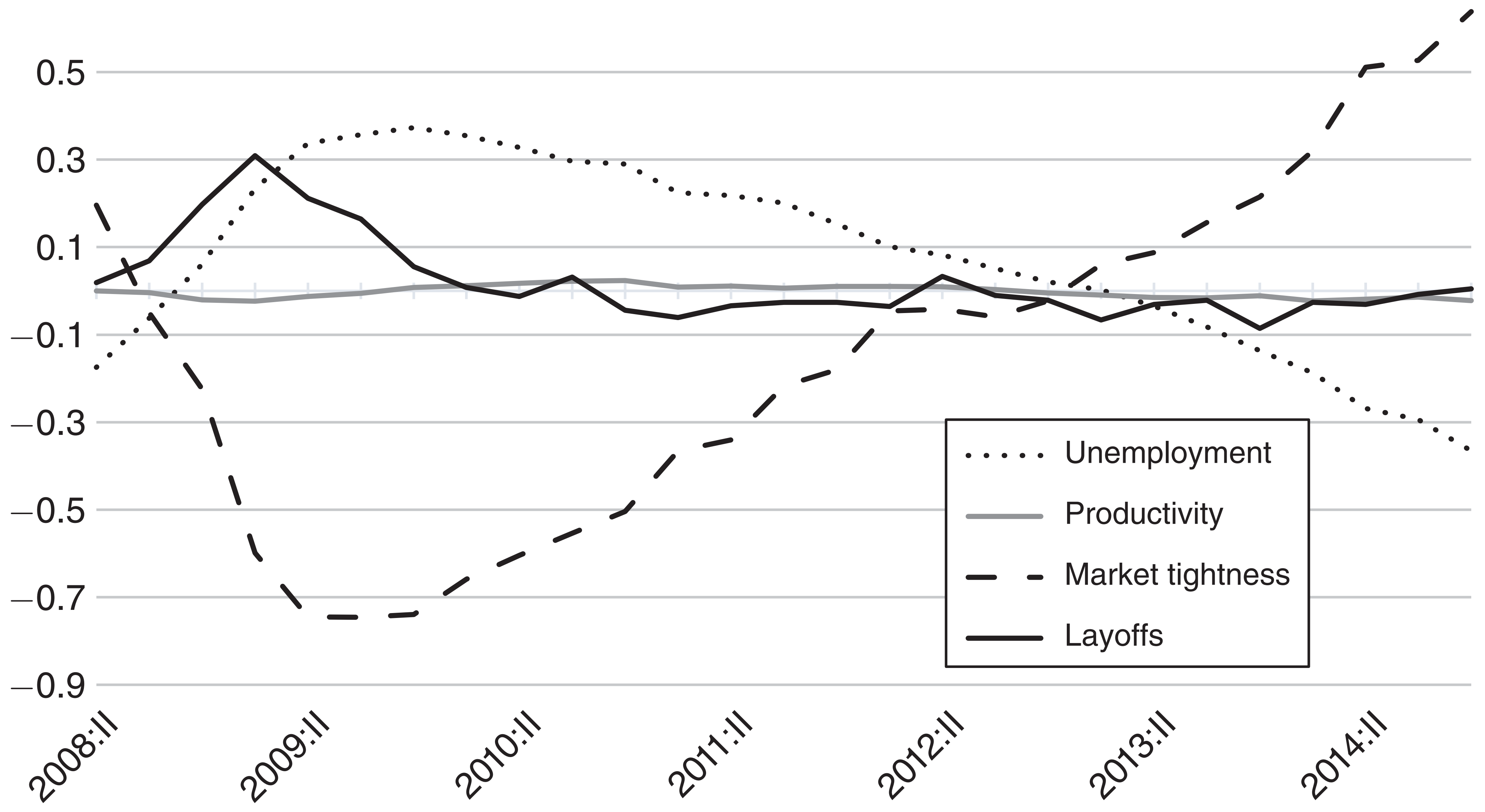


FIGURE 5. US LABOR MARKET INDICATORS (2008–2015)

Notes: Series are quarterly deviations from HP trends ($\lambda = 10^5$). Productivity is the Bureau of Labor Statistics (BLS) output per worker from Major Sector Productivity and Costs; unemployment is BLS constructs from CPS; vacancies used in market tightness is job openings from JOLTS; and layoffs are also from JOLTS (nonfarm business).

- But what is "separation shock"?
- We endogenize separation in the problem set

Summary

$$c = \beta q(\theta_t)\mathbb{E}\sum_{n=t}^{\infty}(\beta(1-s))^{n-t}[z_{n+1} - w_{n+1}]$$

- DMP model where vacancy creation is endogenous
- But it fails terribly in explaining unemployment fluctuations ("Shimer puzzle")
- Broadly, three attacks to Shimer puzzle:
  1. Make profits volatile
  2. Make discount rates volatile
  3. Abandon free-entry (and consider separation shock)

Summary

$$c = \beta q(\theta_t)\mathbb{E}\sum_{n=t}^{\infty}(\beta(1-s))^{n-t}[z_{n+1} - w_{n+1}]$$

1

- DMP model where vacancy creation is endogenous
- But it fails terribly in explaining unemployment fluctuations ("Shimer puzzle")
- Broadly, three attacks to Shimer puzzle:
  1. Make profits volatile
  2. Make discount rates volatile
  3. Abandon free-entry (and consider separation shock)

Summary

$$c = \beta q(\theta_t)\mathbb{E}\sum_{n=t}^{\infty}(\beta(1-s))^{n-1}[z_{n+1} - w_{n+1}]$$

2 1

- DMP model where vacancy creation is endogenous
- But it fails terribly in explaining unemployment fluctuations ("Shimer puzzle")
- Broadly, three attacks to Shimer puzzle:
  1. Make profits volatile
  2. Make discount rates volatile
  3. Abandon free-entry (and consider separation shock)

Summary

$$c = \beta q(\theta_t)\mathbb{E}\sum_{n=t}^{\infty}(\beta(1-s))^{n-t}[z_{n+1} - w_{n+1}]$$

- DMP model where vacancy creation is endogenous
- But it fails terribly in explaining unemployment fluctuations ("Shimer puzzle")
- Broadly, three attacks to Shimer puzzle:
  1. Make profits volatile
  2. Make discount rates volatile
  3. Abandon free-entry (and consider separation shock)