When to Accept a Job Offer? Search with Job Heterogeneity

704 Macroeconomics
Topic 3

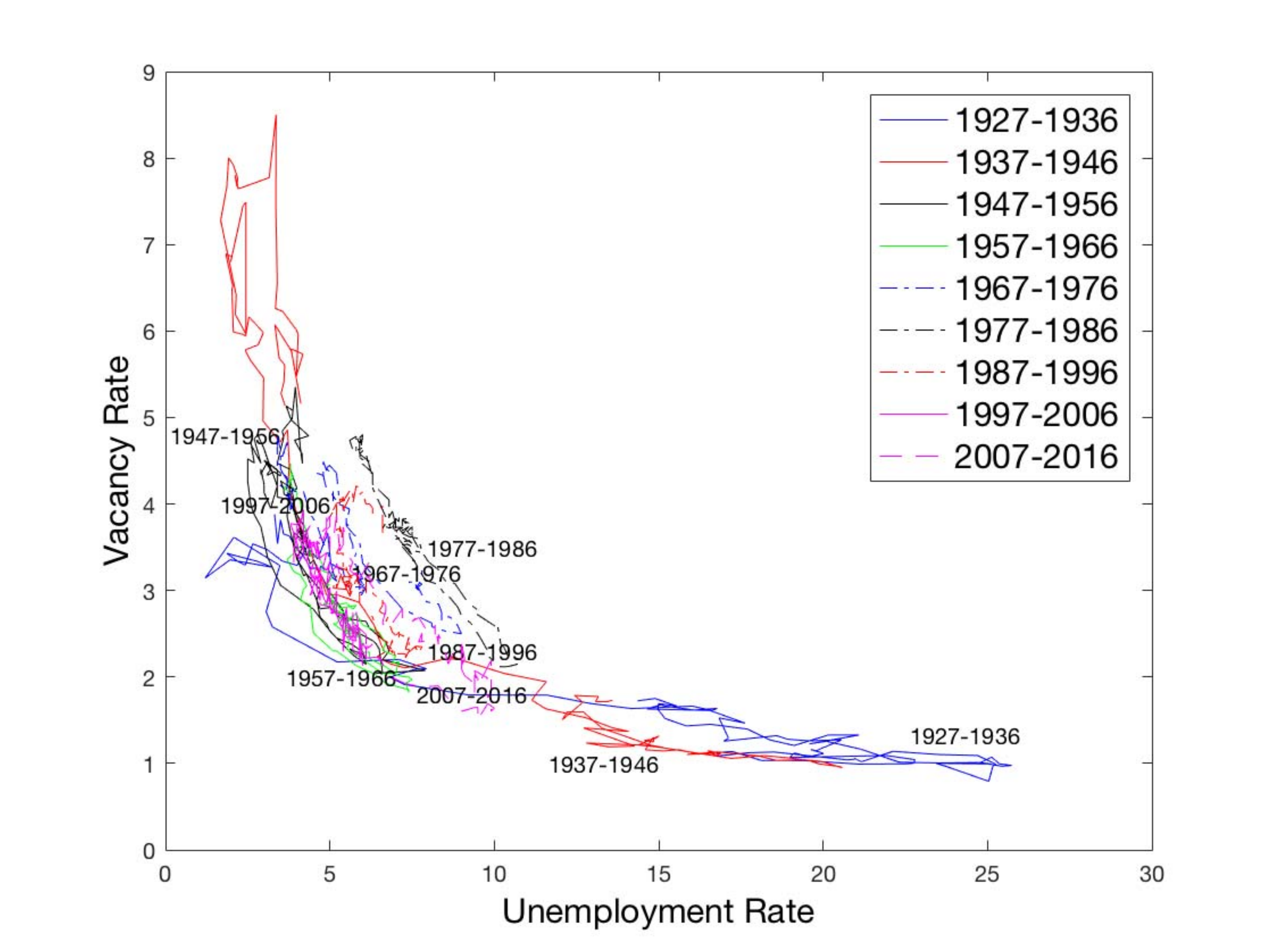
Masao Fukui

2025 Spring

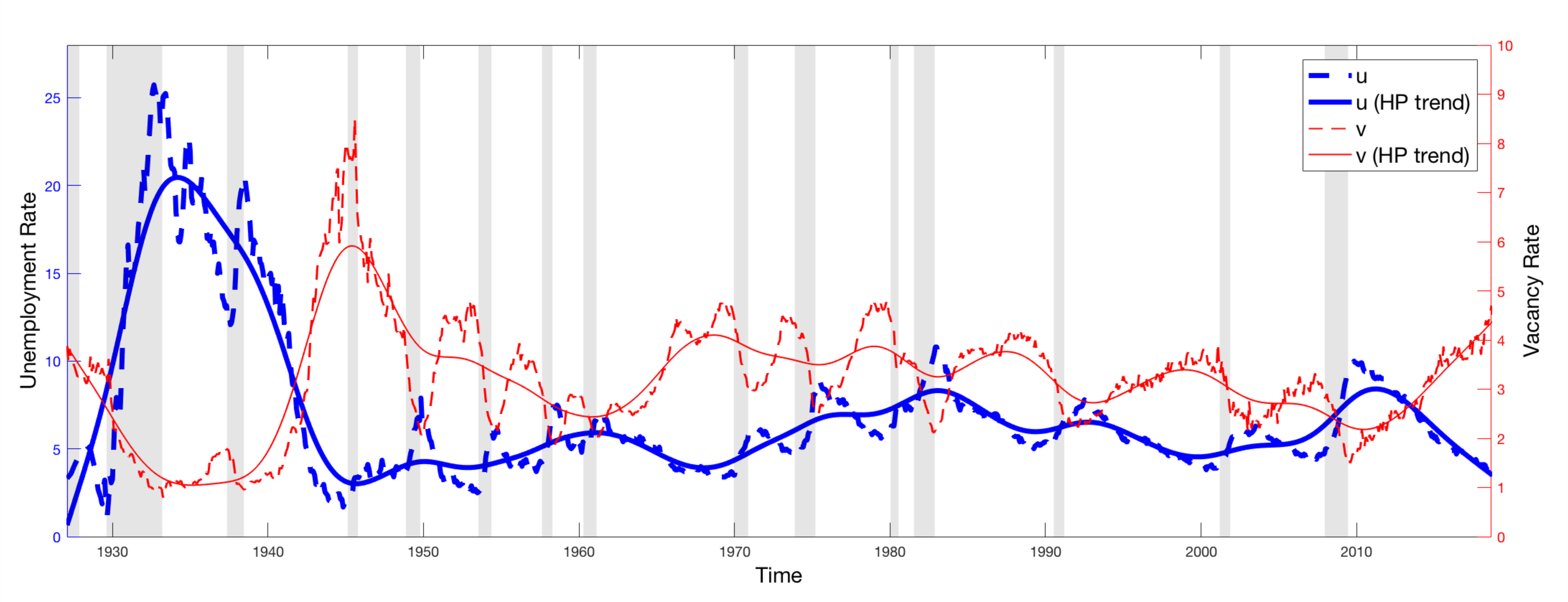
Search and Matching in the Long-Run

- Previous lecture focused on short-run labor market dynamics
- Now shift our focus to long-run
- Is DMP a good model for long-run labor market dynamics?

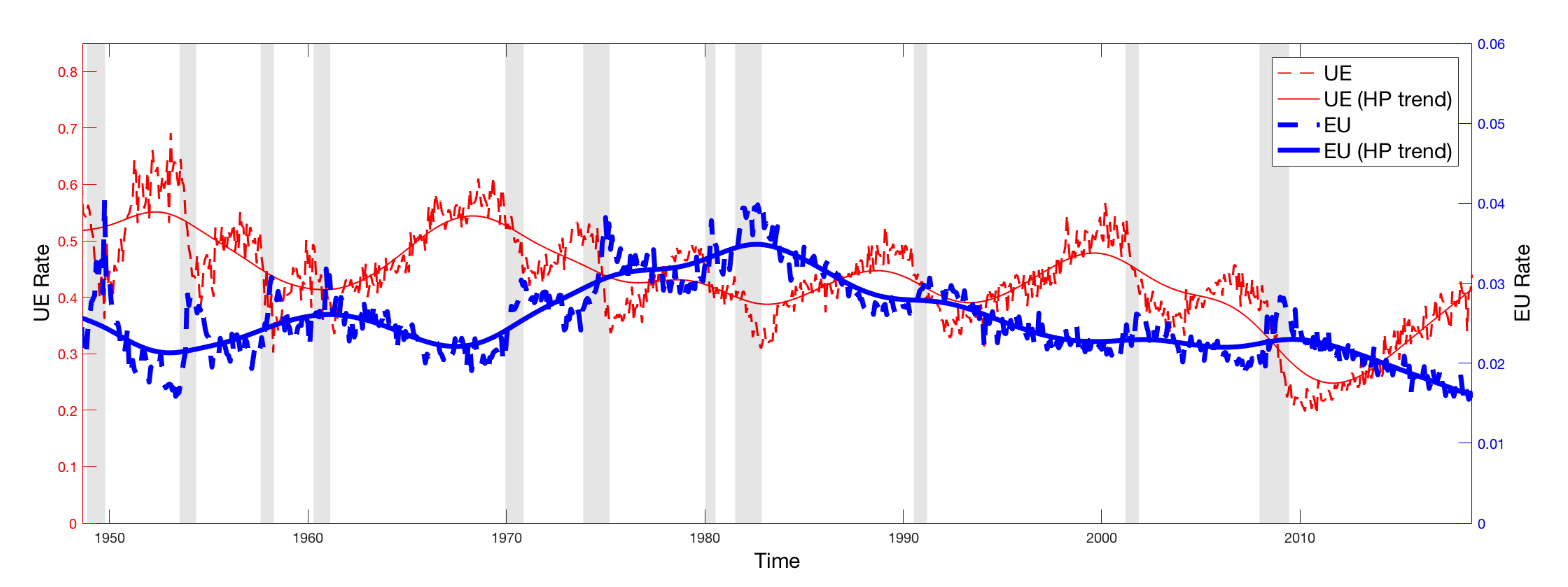
Beveridge Curve in the Long-run



Unemployment and Vacancy Rate



UE and EU Rates



“Puzzle” of DMP Paradigm

- Suppose a matching function is $A_t M(u_t, v_t)$
- We have seen
  1. No secular movement in the Beveridge curve
  $$\frac{u_t}{1-u_t} = \frac{s_t}{A_t f(v_t/u_t)}$$
  2. No secular trend in u_t or v_t
  3. No secular trend in $EU_t = s_t$ (or $UE_t \equiv A_t f(v_t/u_t)$)
- Together, these facts imply there is no improvement in matching technology A_t
- Telephone? Fax? Mobile phone? PC? Internet? Air travel?
  All irrelevant for finding a match? – “Puzzle”

Balanced Growth in Unemployment Rate

- Martellini & Menzio (2020) solve the puzzle with a simple idea
- When it becomes easier to meet, workers…
  1. are more likely to find the job
  2. become pickier because hunting for a better job offer is easier
- Under certain conditions, these two forces exactly offset $\Rightarrow$ no changes in u
- The second force is missing in DMP because jobs are homogenous
- We first introduce job heterogeneity in a partial equilibrium setup
- This model is called McCall's (1970) model of job search

McCall’s Search Model

Environment

- Time: $t = \Delta, 2\Delta, \ldots,$
- Workers are risk neutral with preferences

$$\sum_{t=0}^{\infty} e^{-r\Delta t} c_{\Delta t} \Delta$$

  - $c_t = w$ if employed
  - $c_t = b$ if unemployed
- When unemployed, workers receive a job offer with a probability $1 - e^{-f\Delta}$
- The wage of job-offer is exogenously drawn from $w \sim G(w)$ iid over time
- Workers decide whether to accept or reject the offer (no recall)
- After accepting the offer, the worker loses the job with probability $1 - e^{-s\Delta}$

Bellman Equations

- Value functions:

$$U = b\Delta + e^{-r\Delta}\left[(1 - e^{-f\Delta})\int \max\{E(w), U\}dG(w) + e^{-f\Delta}U\right]$$

$$E(w) = w\Delta + e^{-r\Delta}[e^{-s\Delta}E(w) + (1 - e^{-s\Delta})U]$$

- Take the continuous-time limit $\Delta \to 0$:

$$rU = b + f\int \max\{E(w) - U, 0\}dG(w)$$

$$rE(w) = w + s(U - E(w))$$

Reservation Wage Determination

- Combining the previous two value functions

$$rU = b + f\int \max\{\frac{w + sU}{r + s} - U, 0\}dG(w)$$

- Workers accept the job offer if $w \geq w^R$, and **reservation wage** w^R satisfies

$$\frac{w^R + sU}{r + s} = U$$

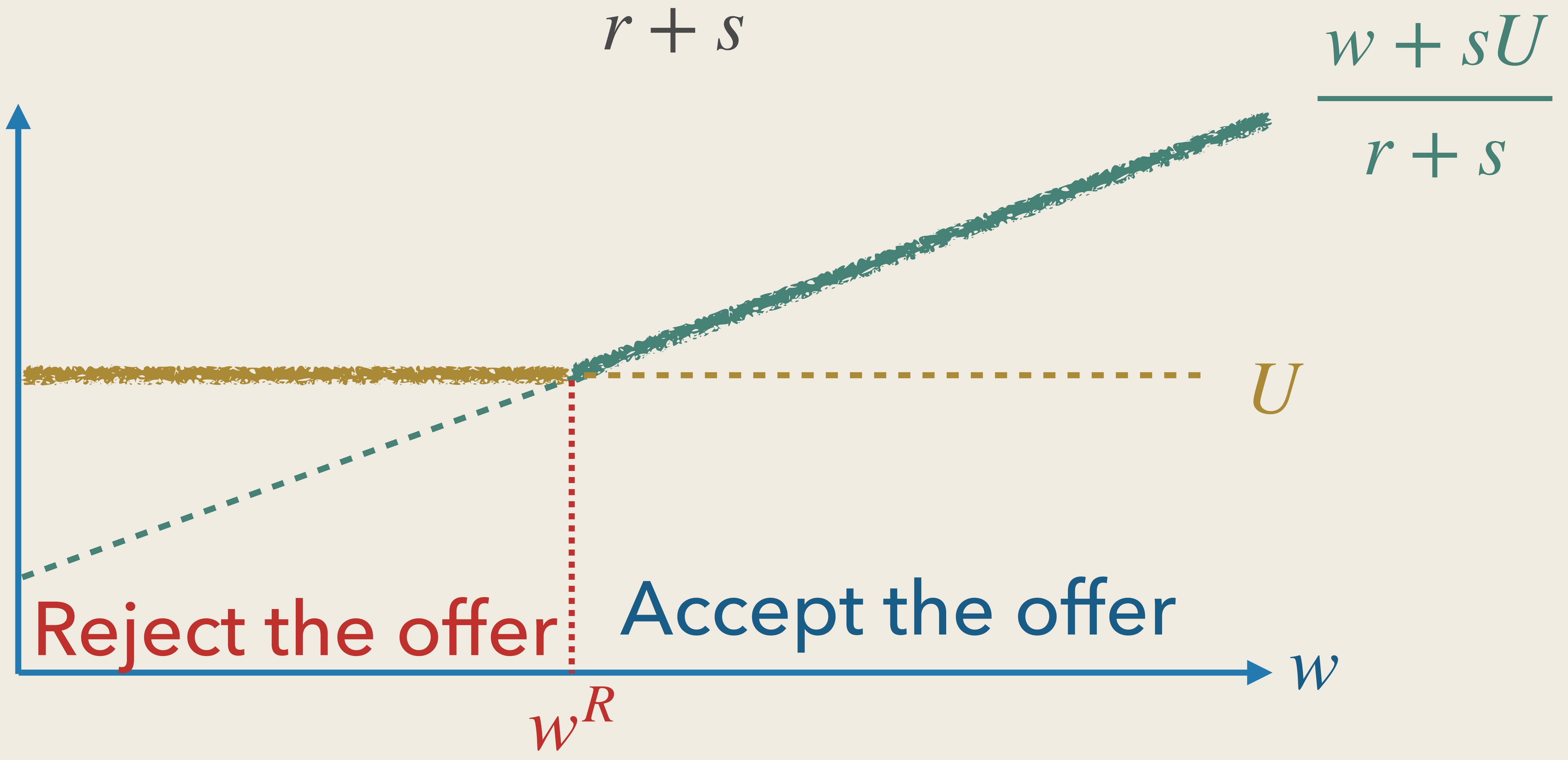
Reservation Wage Determination

- Combining the previous two value functions

$$rU = b + f\int \max\{\frac{w+sU}{r+s} - U, 0\}dG(w)$$

- Workers accept the job offer if $w \geq w^R$, and **reservation wage** w^R satisfies

$$\frac{w^R + sU}{r+s} = U$$



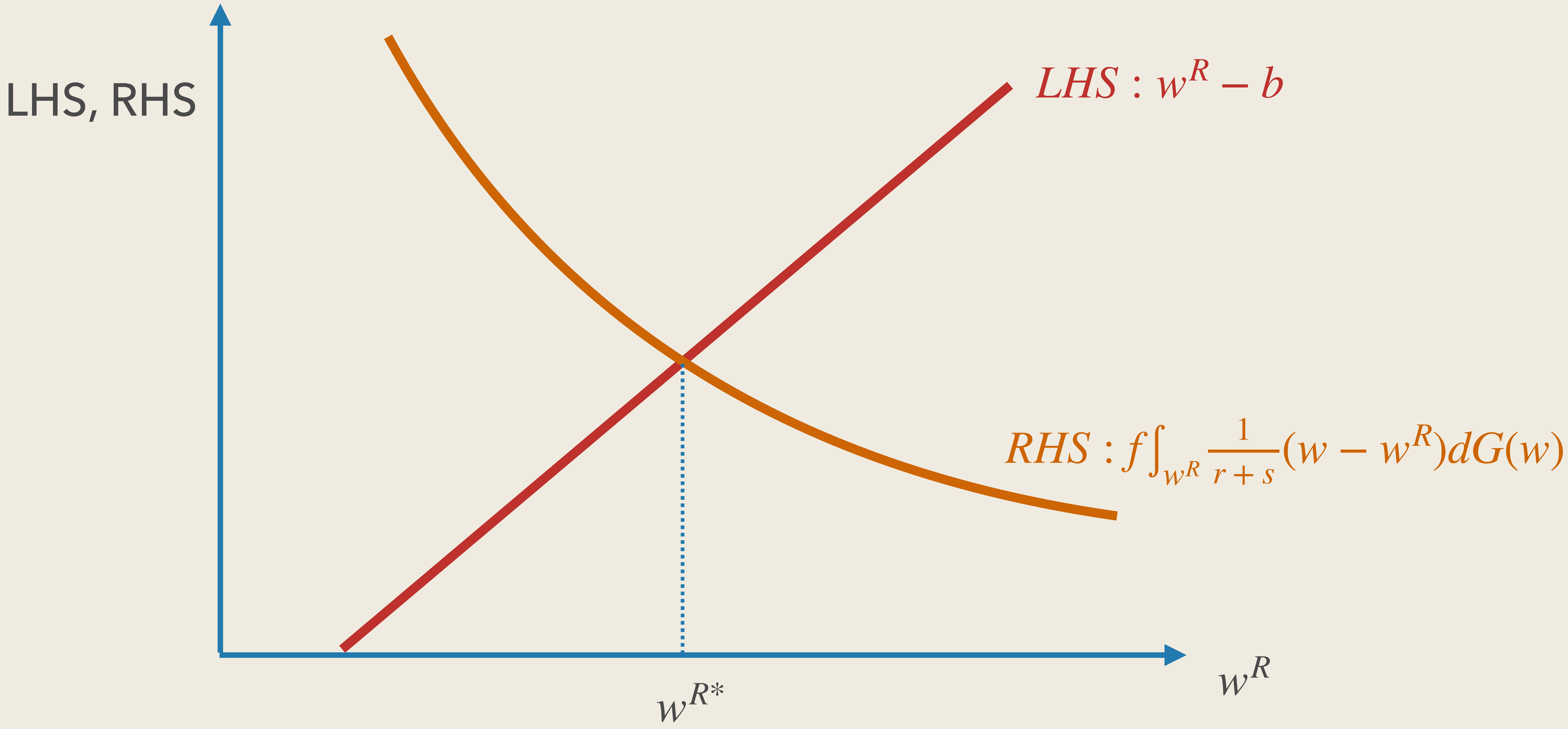
Reservation Wage

- Combining the previous two equations to eliminate U:

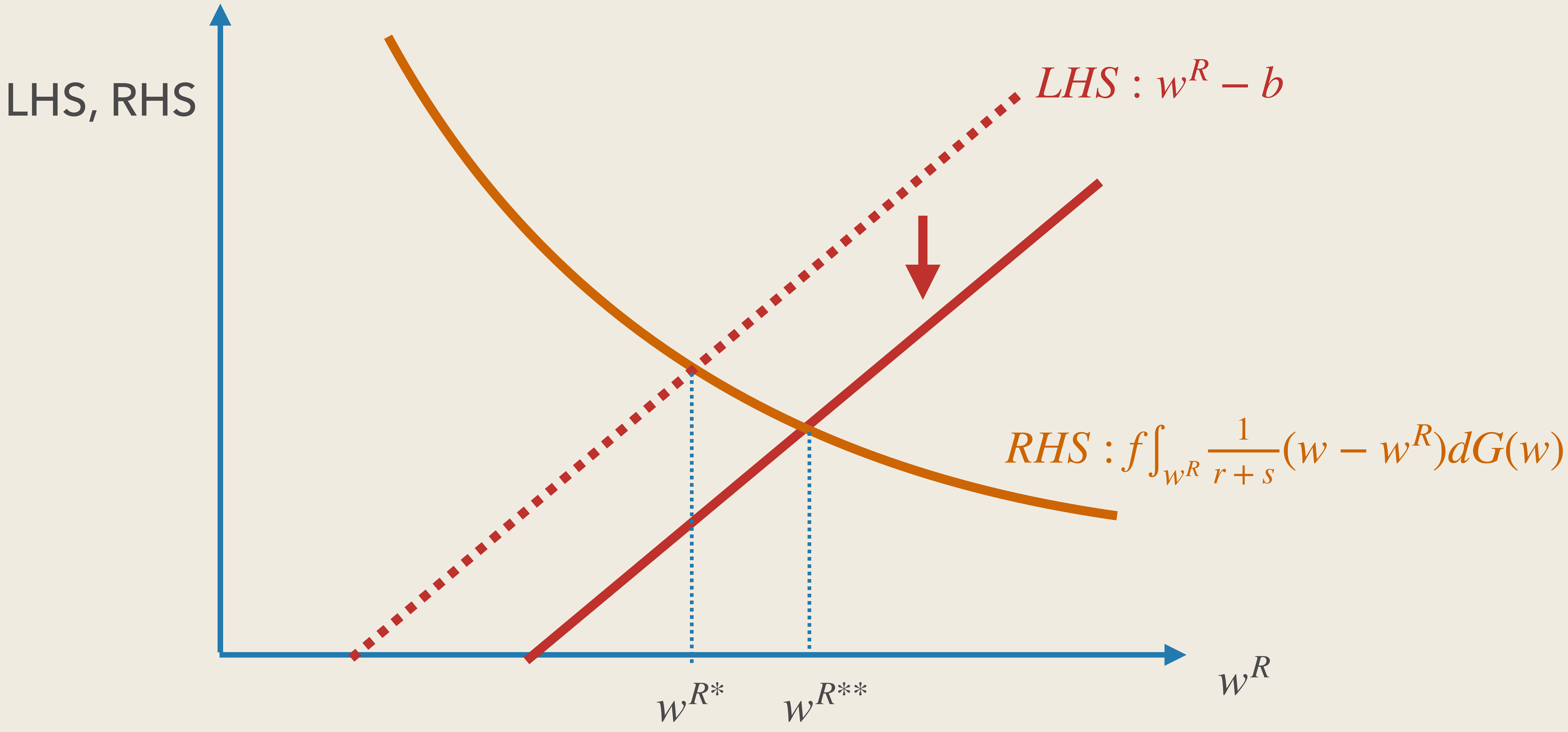
$$w^R - b = f\int_{w^R} \frac{1}{r+s}(w - w^R)dG(w) \tag{1}$$

  - LHS: benefit of accepting a wage offer w^R
  - RHS: cost of accepting an offer w^R = foregoing future better offer
  - At the optimum, two should be equated

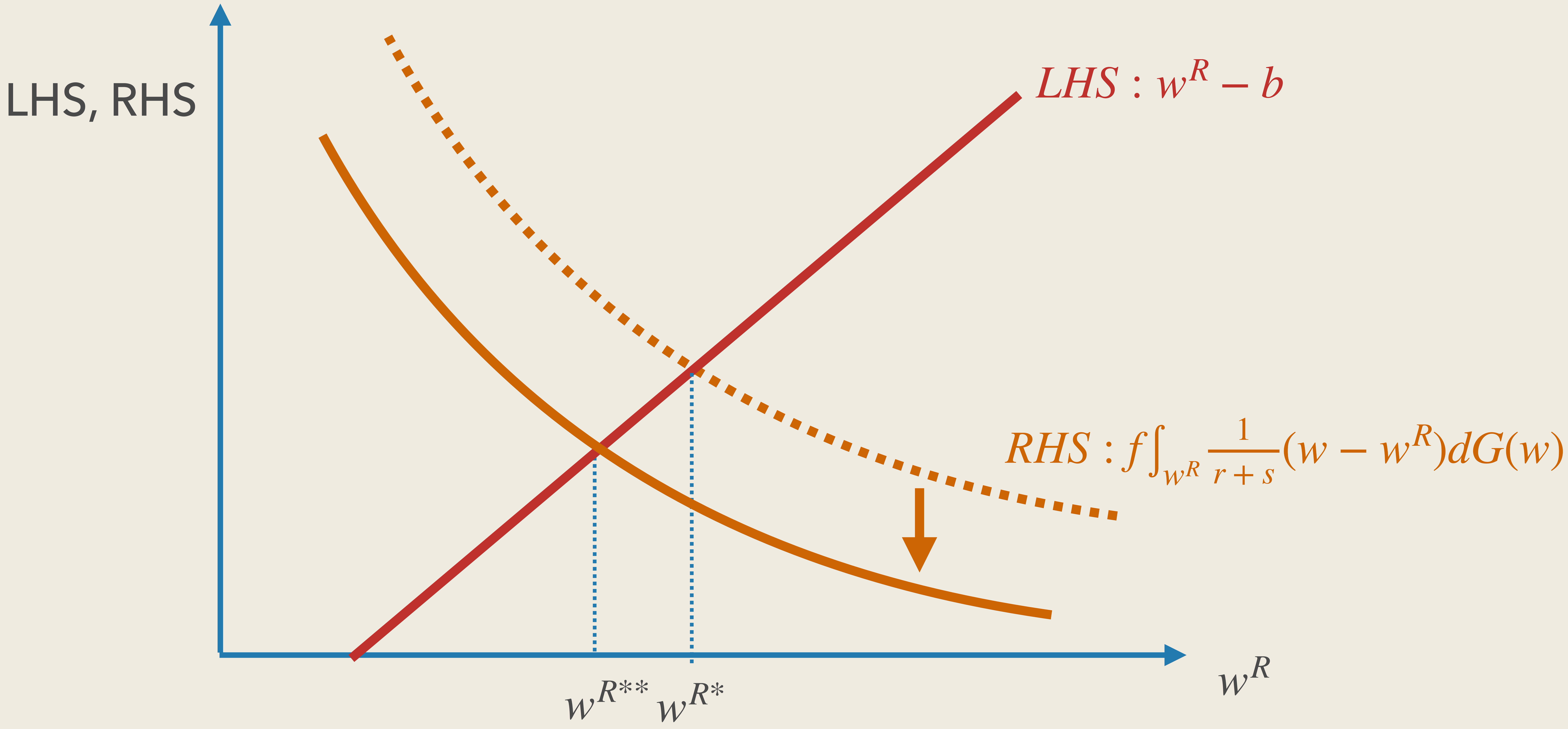
Graphical Illustration



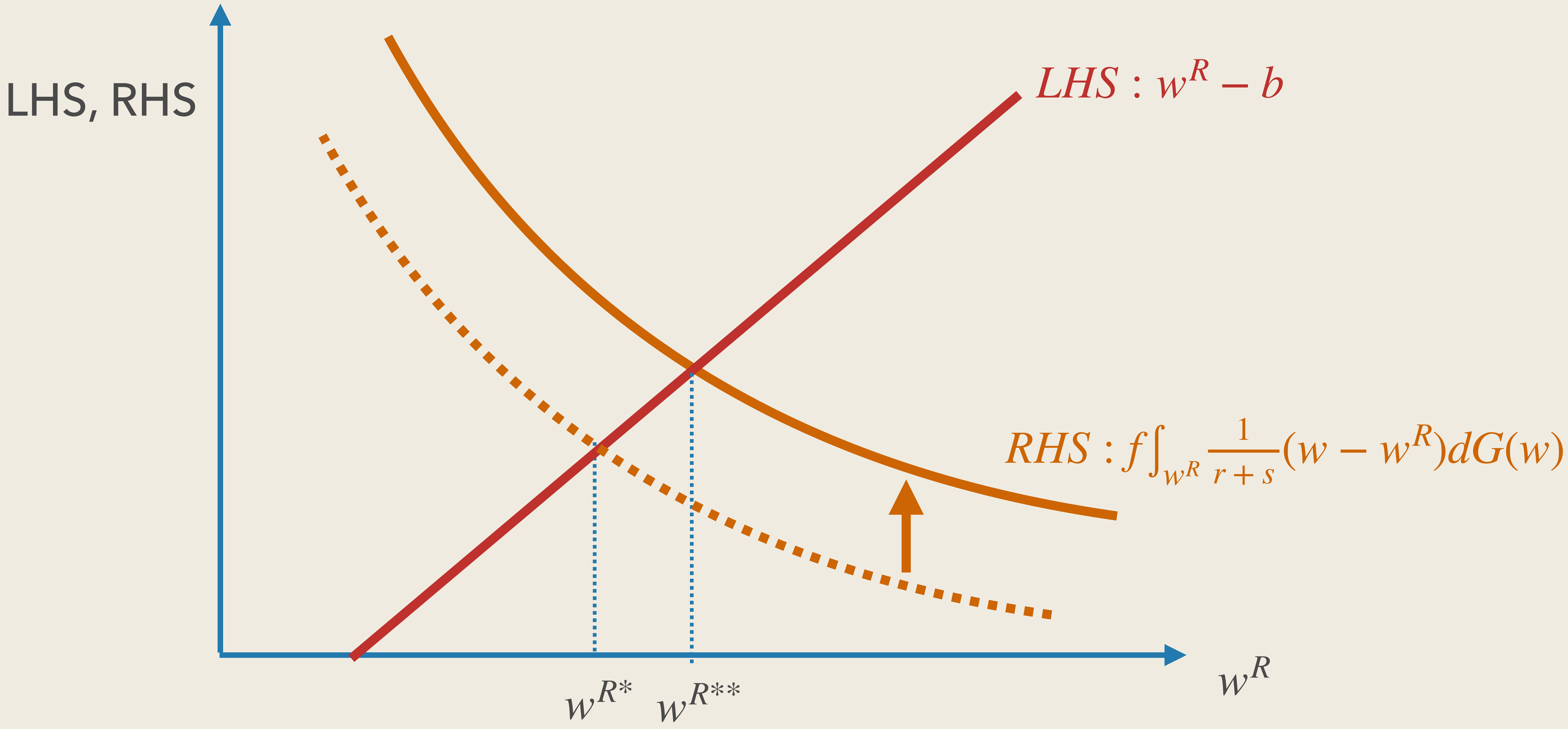
An Increase in b



An Increase in r, s



An Increase in f



UE Rate

- The rate at which workers transition from U to E is

$$UE = f(1 - G(w^R))$$

- What happens if f increases?

$$\frac{d \ln UE}{d \ln f} = 1 - \frac{G'(w^R)w^R}{1 - G(w^R)} \frac{d \ln w^R}{d \ln f}$$

- Under what condition, $\frac{d \ln UE}{d \ln f} = 0$?

Pareto Distribution

- We guess and verify that the following economy features such a property:
  1. Wage distribution follows Pareto distribution,
  $$G(w) = 1 - (w/\underline{w})^{-\alpha}$$
  2. Outside option b is proportional to the average wage in the economy,
  $$\begin{aligned} b &= \bar{b}\mathbb{E}[w \mid w \geq w^R] \\ &= \bar{b}\frac{1}{\alpha - 1}w^R \end{aligned}$$

UE Rate Does not Depend on f

- Plug the conditions 2 into (1),

$$w^R - b = \frac{f}{r+s}\frac{1}{\alpha-1}\underline{w}^{\alpha}\left(w^R\right)^{1-\alpha}$$

- Solving for w^R:

$$w^R = \left[\frac{f}{(r+s)(1-\bar{b}\alpha/(\alpha-1))}\frac{1}{\alpha-1}\right]^{1/\alpha}\underline{w}$$

- The UE rate is

$$UE = (\alpha-1)(r+s)(1-\bar{b}\alpha/(\alpha-1))$$

Main Result

If (i) wage distribution follows Pareto with tail parameter α; and (ii) UI benefit, b, is proportional to the average wage in the economy, an increase in job-finding rate, f,

1. has no effect on the UE rate, $\dfrac{d \ln UE}{d \ln f} = 0$
2. increases the average wage: $\dfrac{d \ln \mathbb{E}[w \,|\, w \geq w^R]}{d \ln f} = \dfrac{1}{\alpha} > 0$

- If it becomes easier to meet, workers become pickier.
  - This offsets the direct effect, leaving no effect on the unemployment rate
  - … yet workers find a better match and the average wage increases

McCall + DMP

DMP with Job Heterogeneity

- Now we endogenize the wage distribution and job-finding rate f
- Firm produces z unit of output per worker, where z is match quality and $z \sim G(z)$
- Assume match quality follows Pareto distribution, $G(z) = 1 - (z/\underline{z})^{-\alpha}$
- Firm posts vacancy at cost $c = \bar{c}\bar{z}$ where $\bar{z}$ is the average output in the economy
- Unemployed workers receive UI benefits of $b = \bar{b}\bar{z}$
- When v and u meet, draw match quality z, and decide whether to form the match
- Wages are set according to Nash bargaining with worker bargaining power γ
- The matching function is CRS and is given by $AM(u, v)$

Steady State Equilibrium

- The firm's value of filled job with match quality z satisfies

$$(r+s)J(z) = z - w(z) + sV$$

- The employed worker's value

$$(r+s)E(z) = w(z) + sU$$

- The unemployed worker's value

$$rU = b + Af(\theta)\int \max\{E(z) - U, 0\}dG(z)$$

- The value of vacancy is

$$rV = -c + Aq(\theta)\int \max\{J(z) - V, 0\}dG(z)$$

- Free entry: $V = 0$

Reservation Match Quality

- Define $S(z) \equiv E(z) + J(z) - U - V$. Then

$$S(z) = \frac{z}{r+s} - \frac{r}{r+s}U$$

- Nash bargaining implies

$$E(z) = U + \gamma S(z)$$

$$rU = b + Af(\theta)\int \max\{\gamma S(z), 0\}dG(z)$$

- Therefore the reservation match quality z^R satisfies $S(z^R) = 0$ or

$$z^R = rU$$

Steady State (θ, z^R)

- Steady-state equilibrium (z^R, θ) solves

$$z^R - b = \gamma A f(\theta) \int_{z^R} \frac{1}{r+s}(z - z^R) dG(z)$$

$$\beta(1-\gamma) A q(\theta) \int_{z^R}^{\infty} \frac{1}{r+s}(z - z^R) dG(z) = c$$

- Using $b = \bar{b}\mathbb{E}[z \,|\, z \geq z^R]$ and $c = \bar{c}\mathbb{E}[z \,|\, z \geq z^R]$, and $G(z) = 1 - (z/\underline{z})^{-\alpha}$

$$\boxed{\theta = \beta \frac{1-\gamma}{\gamma} \frac{(\alpha - 1 - \alpha \bar{b})}{\alpha \bar{c}} \qquad z^R = \left[\frac{\gamma A f(\theta)}{(r+s)(1 - \bar{b}\alpha/(\alpha-1))} \frac{1}{\alpha - 1} \right]^{1/\alpha} \underline{z}}$$

$$\rightarrow$$

$$\boxed{UE = A f(\theta)(1 - G(z^R)) \qquad \frac{d \ln UE}{d \ln A} = 1 - \underbrace{\frac{d \ln f(\theta)}{d \ln A}}_{=0} + \underbrace{\frac{G'(z^R) z^R}{1 - G(z^R)}}_{-\alpha} \underbrace{\frac{d \ln z^R}{d \ln A}}_{1/\alpha}}$$

Balanced Growth in the Labor Market

If (i) match quality distribution follows Pareto with tail parameter α; and (ii) UI benefit, b, and vacancy cost, c, are proportional to the average output in the economy, an increase in matching technology, A,

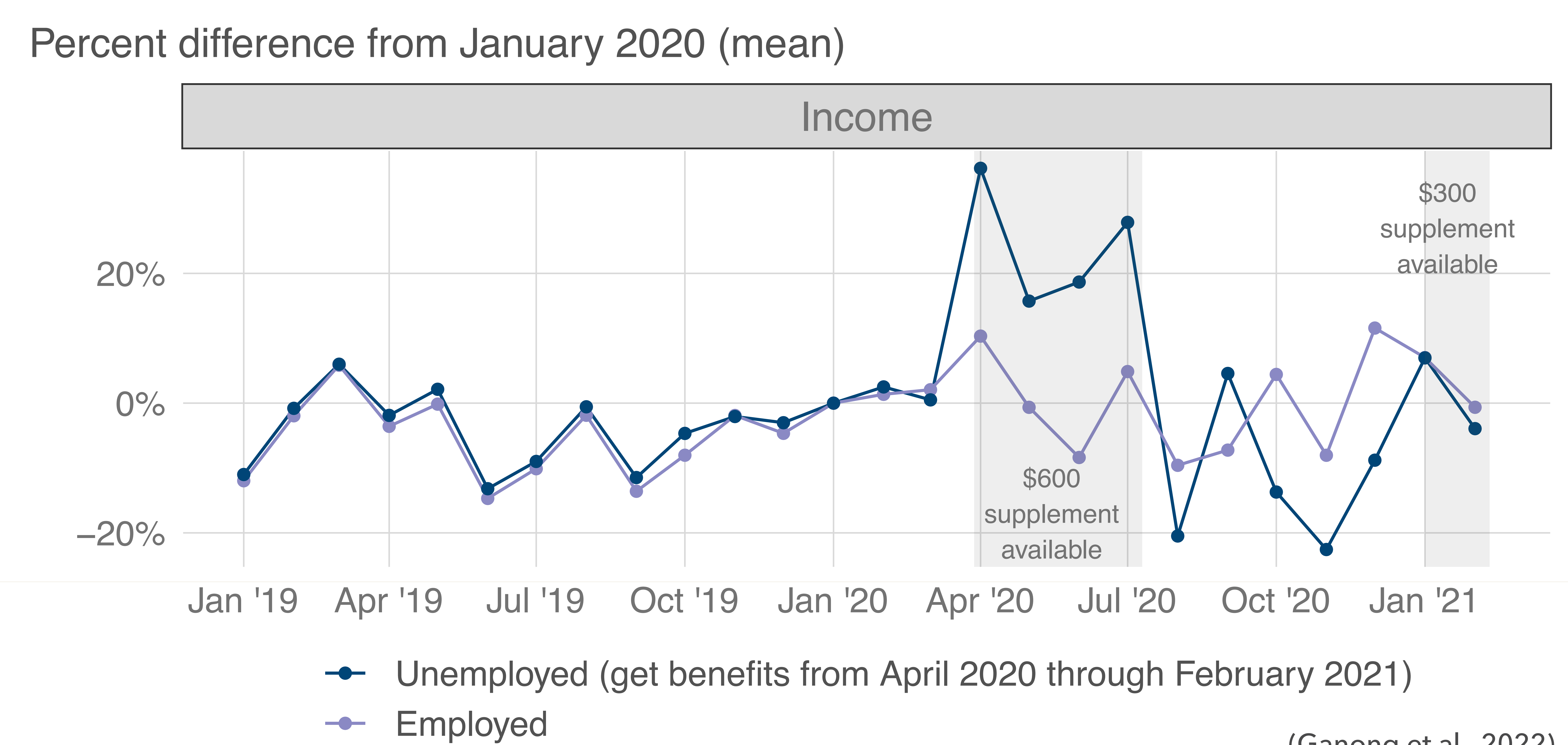
1. has no effect on (u, v, θ, UE)
2. increases the output in the economy: $\dfrac{d \ln Y}{d \ln A} = \dfrac{1}{\alpha} > 0$

- An improvement in matching technology does not show up in the labor market
- Yet, it increases output in the economy $\Rightarrow$ source of economic growth

Mirco Consequences of Increasing UI Benefits

Ganong, Greig, Noel, Sullivan, and Vavra (2022)

UI Benefit Expansions



(Ganong et al., 2022)

Micro Effect of UI Benefit

- What is the micro consequence of UI benefit expansion?
  - micro: individual worker's response to an increase in UI
  - macro: economy-wide response to an increase in UI
- Let us go back to McCall's model, where we hold f and $G(w)$ fixed
- How does the increase in UI benefit affect the $UE \equiv f(1 - G(w^R))$ rate?

$$\frac{dUE}{d\ln b} = -fG'(w^R)w^R\frac{d\ln w^R}{d\ln b} < 0$$

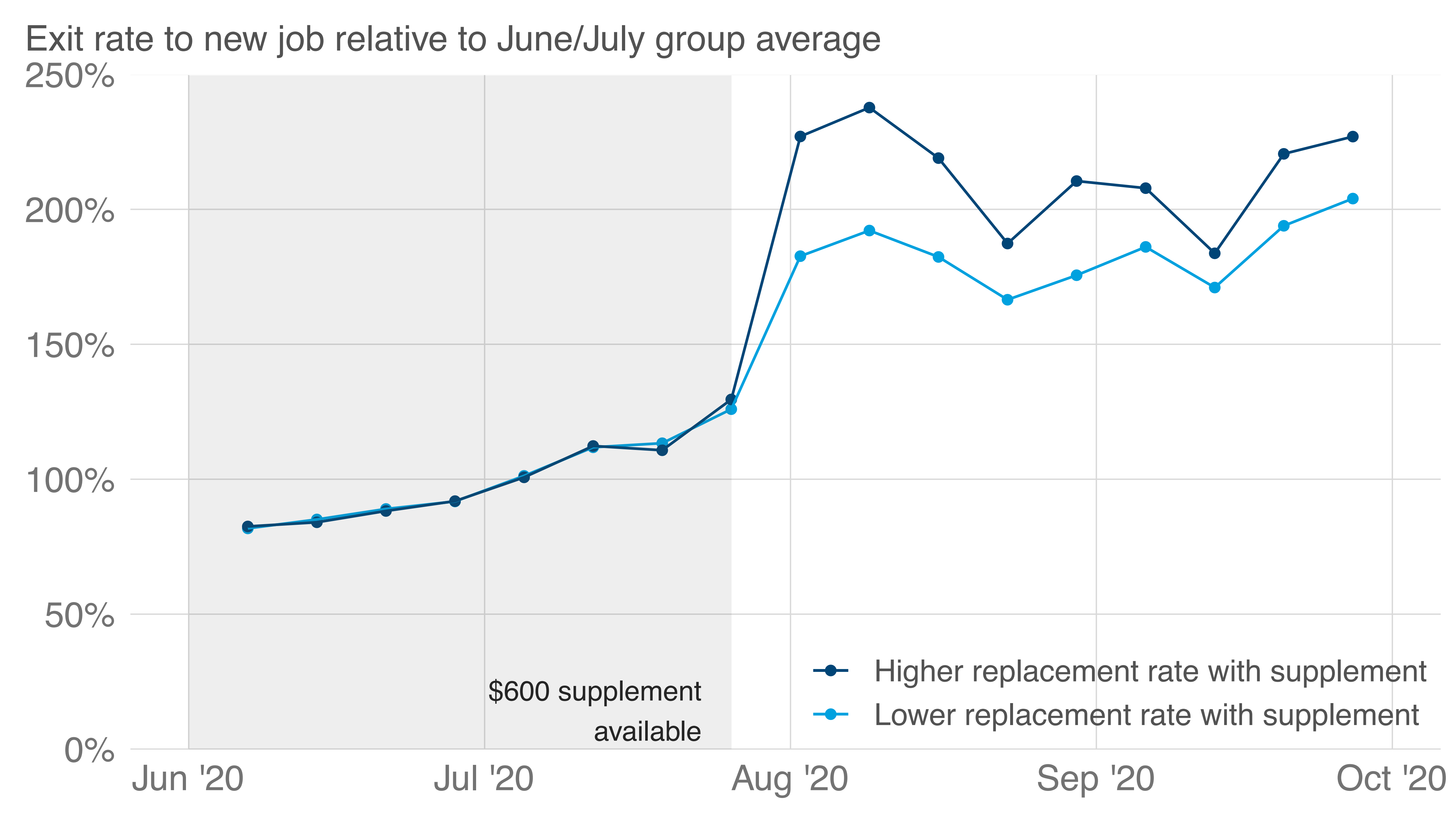
- How large was it?

Identification Stragety

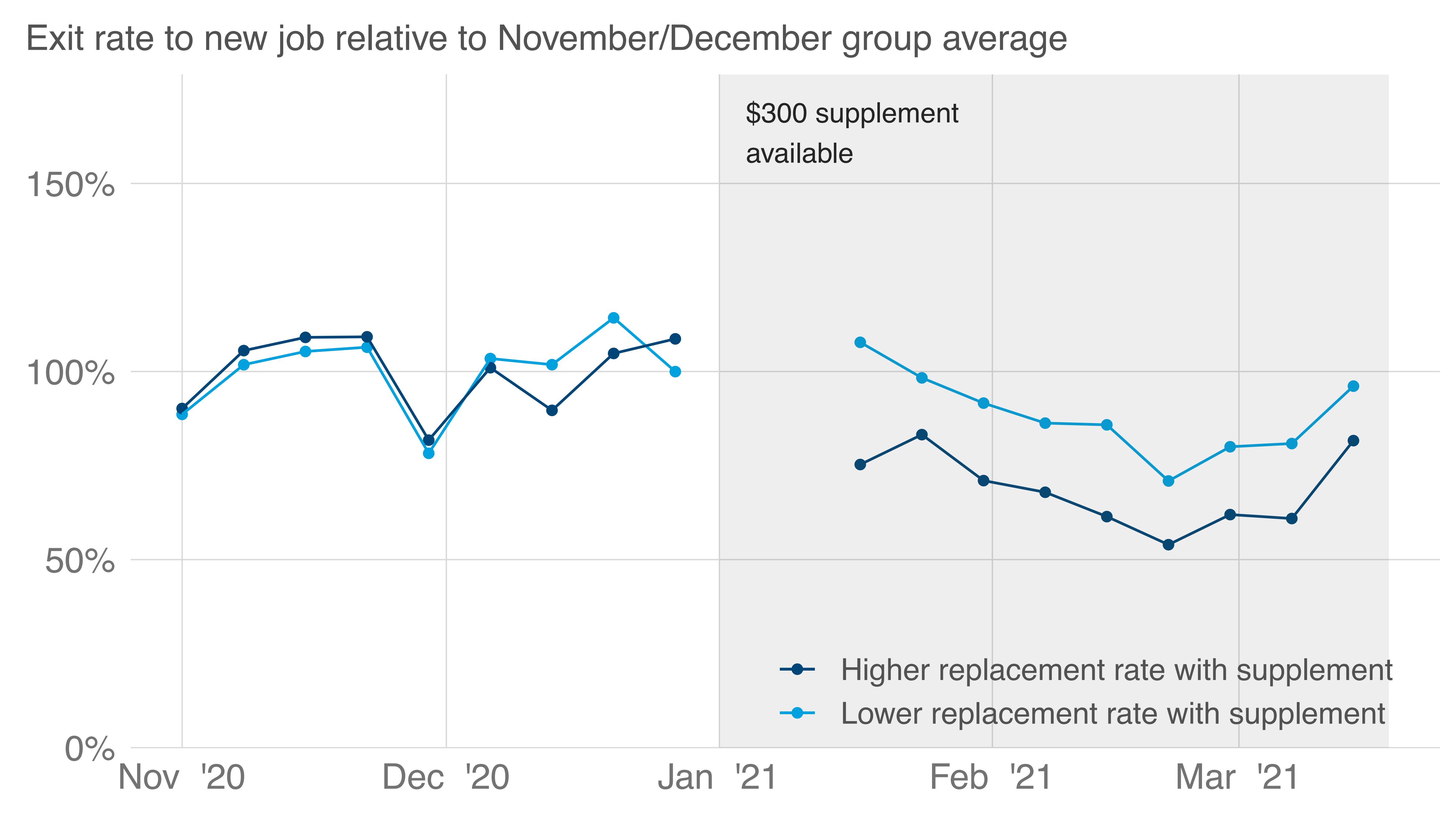
- Suppose a worker i's UI is $\bar{b}_i$ in a normal time.
- Adding \$600/\$300 result in differential proportional increase in UI:

$$\frac{\bar{b}_i + \$600(\text{or } \$300)}{\bar{b}_i}$$

(a) Expiration of \$600

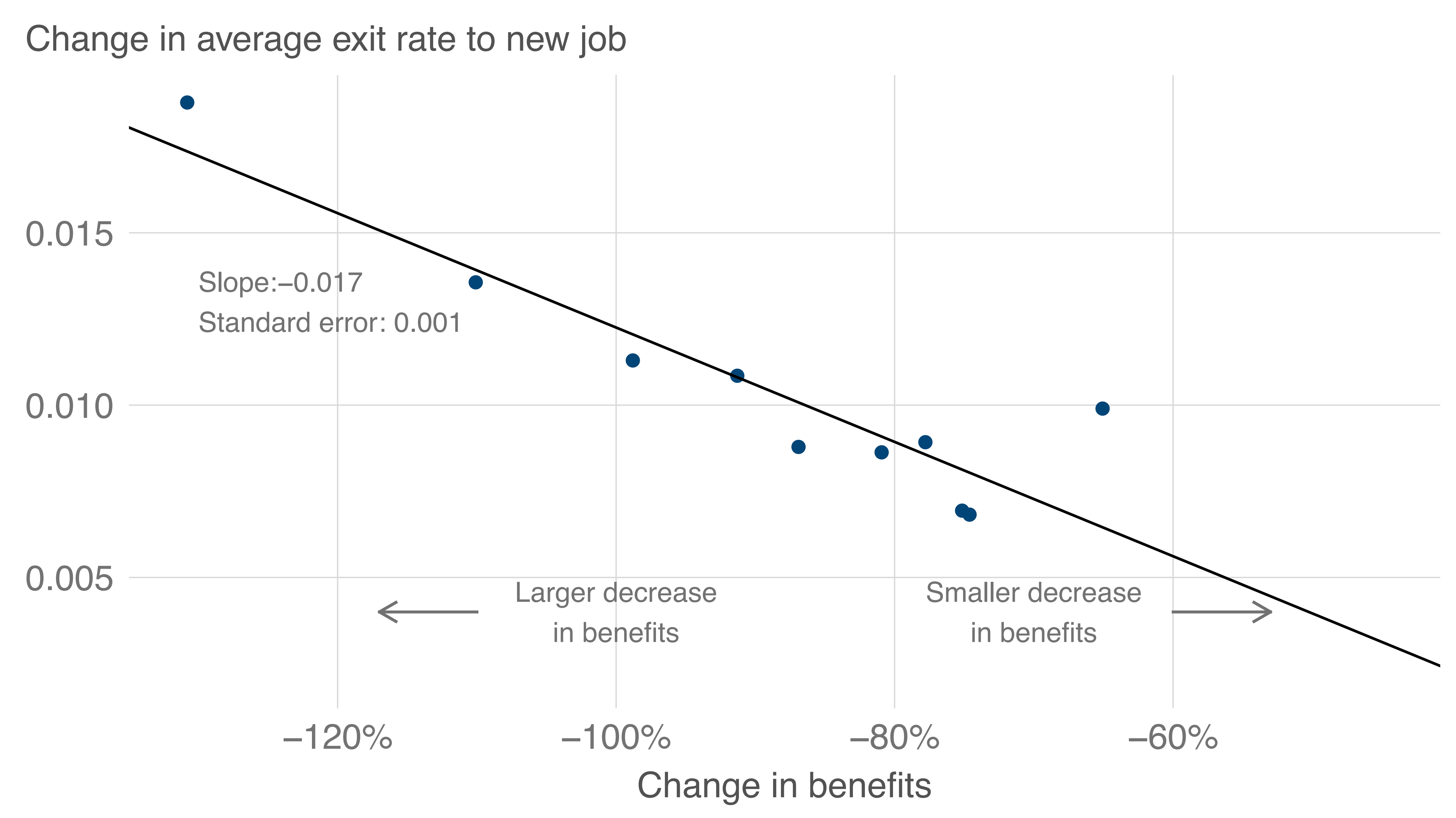


(b) Onset of \$300

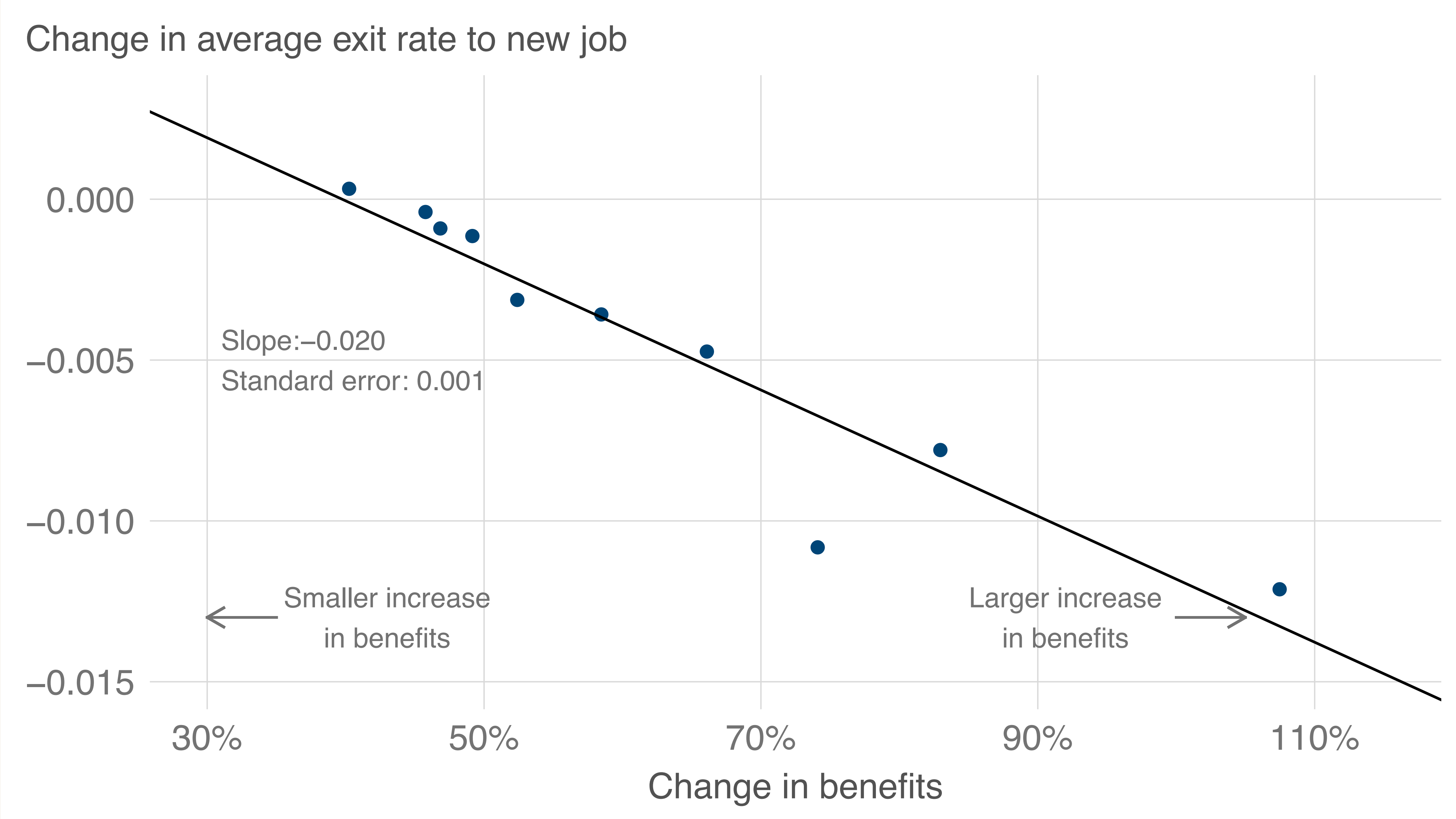


Small Micro Effects

(a) Expiration of \$600



(b) Onset of \$300



- Result: $\dfrac{dUE}{d \ln b} \approx -0.02$
- 1% increase in UI benefit decreases the job-finding rate by 0.02 percentage points
- \$600 UI Benefit expansion decreased employment by 0.6% – small effect

Macro Consequences of Increasing UI Benefits

Chodorow-Reich, Coglianese, and Karabarbounius (2019)

Macro Effect

- Does the previous result imply the **macro** impact of UI expansion is small as well?
  - At the macro level, f changes
- Not necessarily. Now consider McCall + DMP with exogenous (b, c)

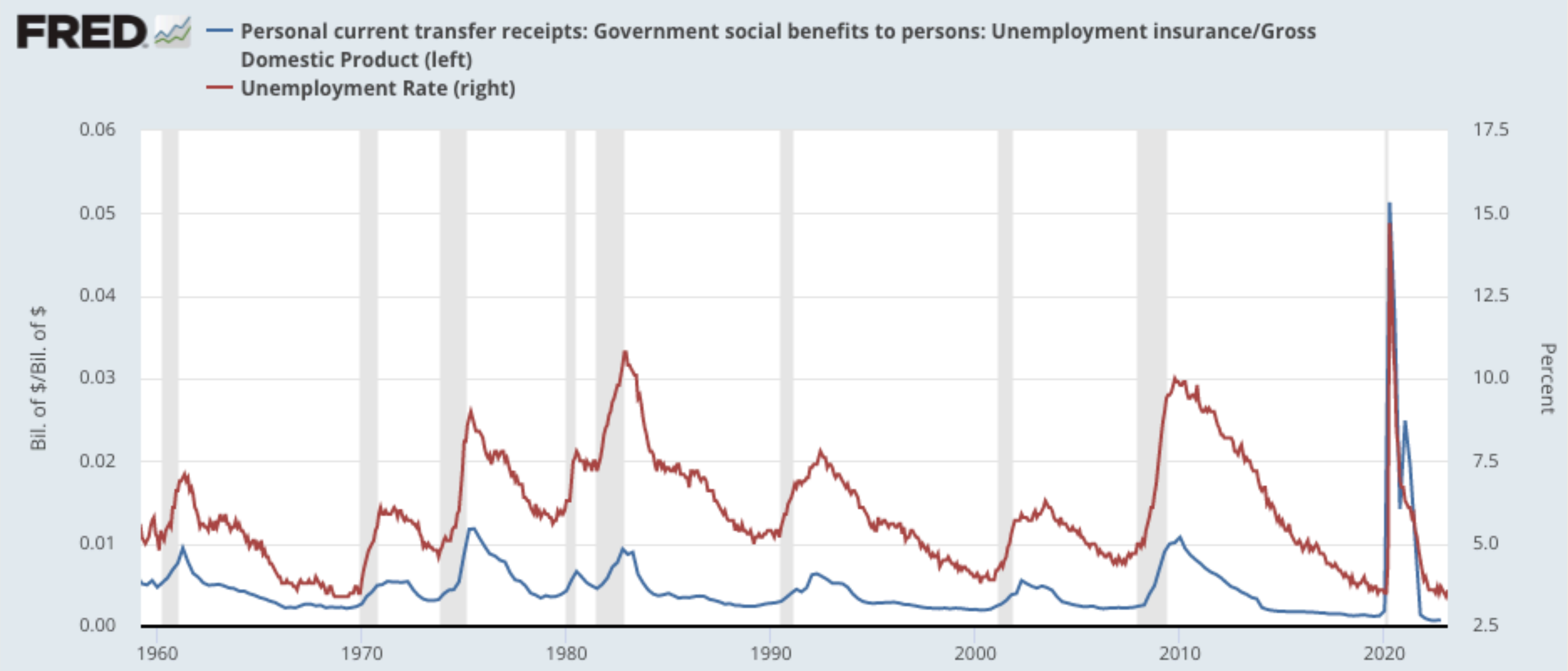
$$\frac{dUE}{d\ln b} = \underbrace{-f(\theta)G'(w^R)w^R\frac{d\ln w^R}{d\ln b}}_{\text{micro (-)}} + \underbrace{(1 - G(w^R))f'(\theta)\theta\frac{d\ln\theta}{d\ln b}}_{\text{macro (-)}}$$

- How large is the macro effect? – Much harder question to answer empirically
- Suppose we run time-series regression:

$$y_t = \alpha + \beta \ln b_t + \epsilon_t$$

What are the problems?

UI Benefit and Unemployment Rate



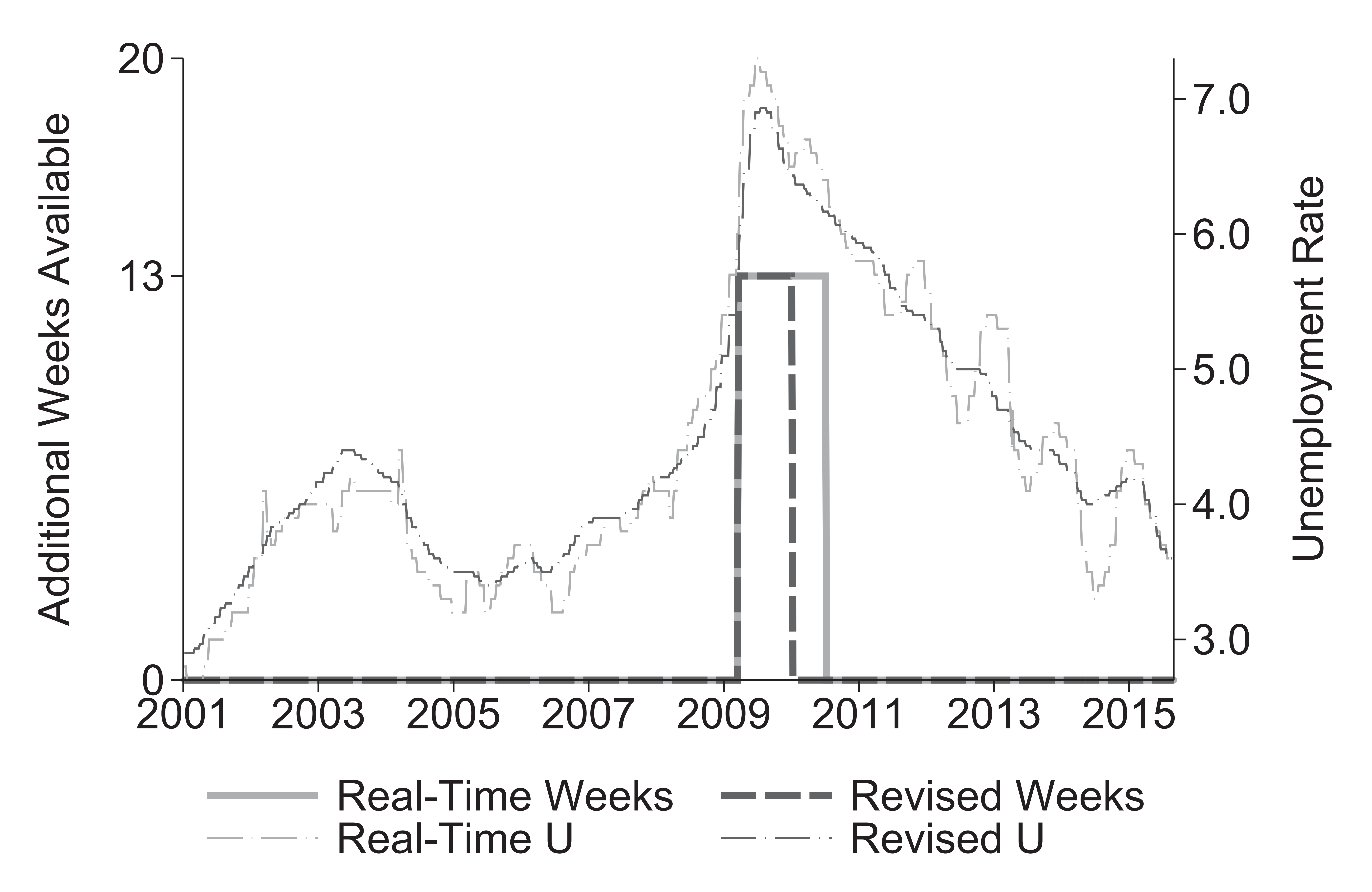
Measurement Error Approach

APRIL 2013 EXAMPLE

		Louisiana	Wisconsin
Real-time data	Unemployment rate (moving average)	5.9%	6.9%
	duration of benefit extensions	14 weeks	28 weeks
Revised data	Unemployment rate (moving average)	6.9%	6.9%
	duration of benefit extensions	28 weeks	28 weeks
	UI error	−14 weeks	0 weeks

- The duration of UI benefits is determined through real-time estimates of unemp. rate
- Contain measurement errors with revision later on
- Measurement error plausibly orthogonal to underlying economic fundamentals

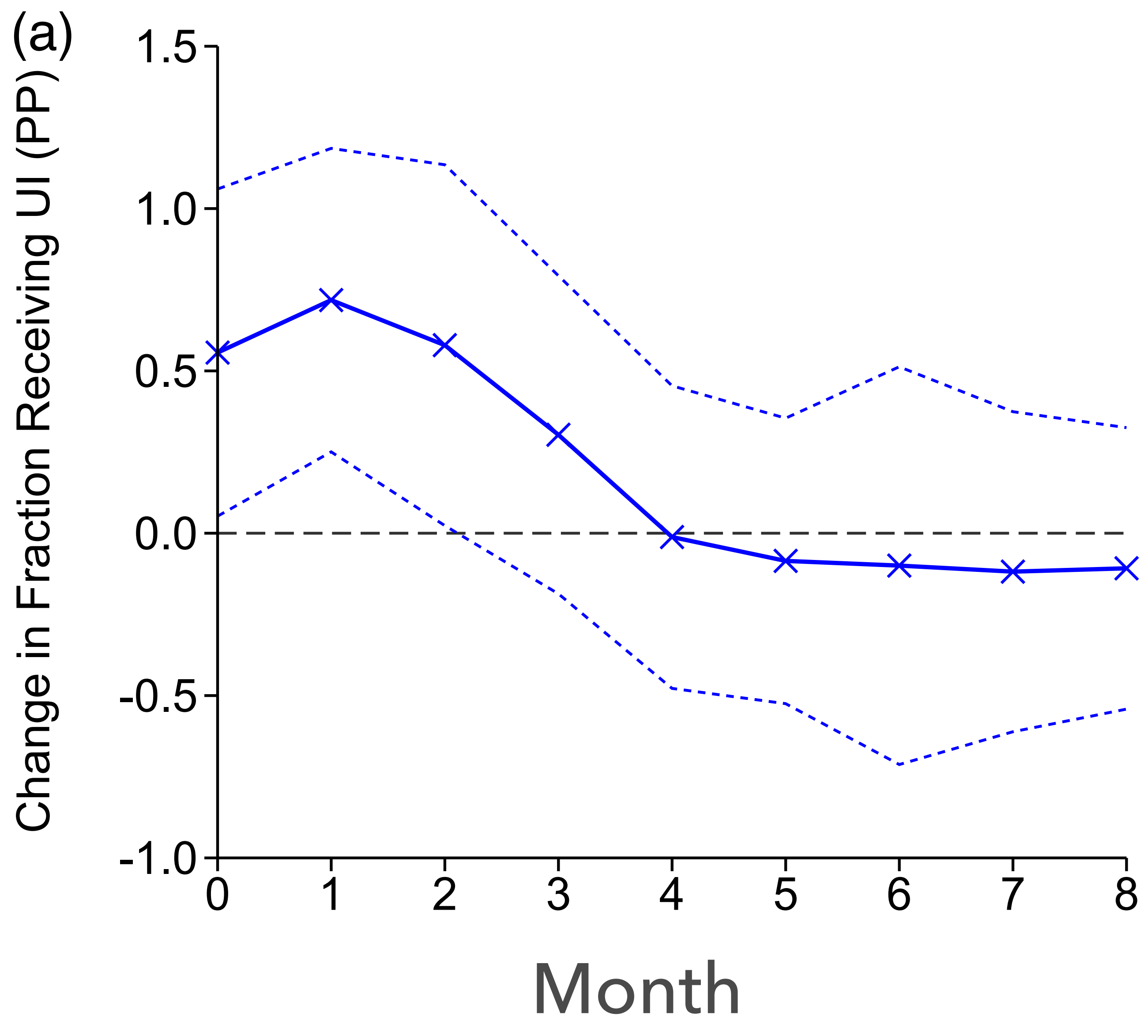
Extended Benefits and Unemployment in Vermont



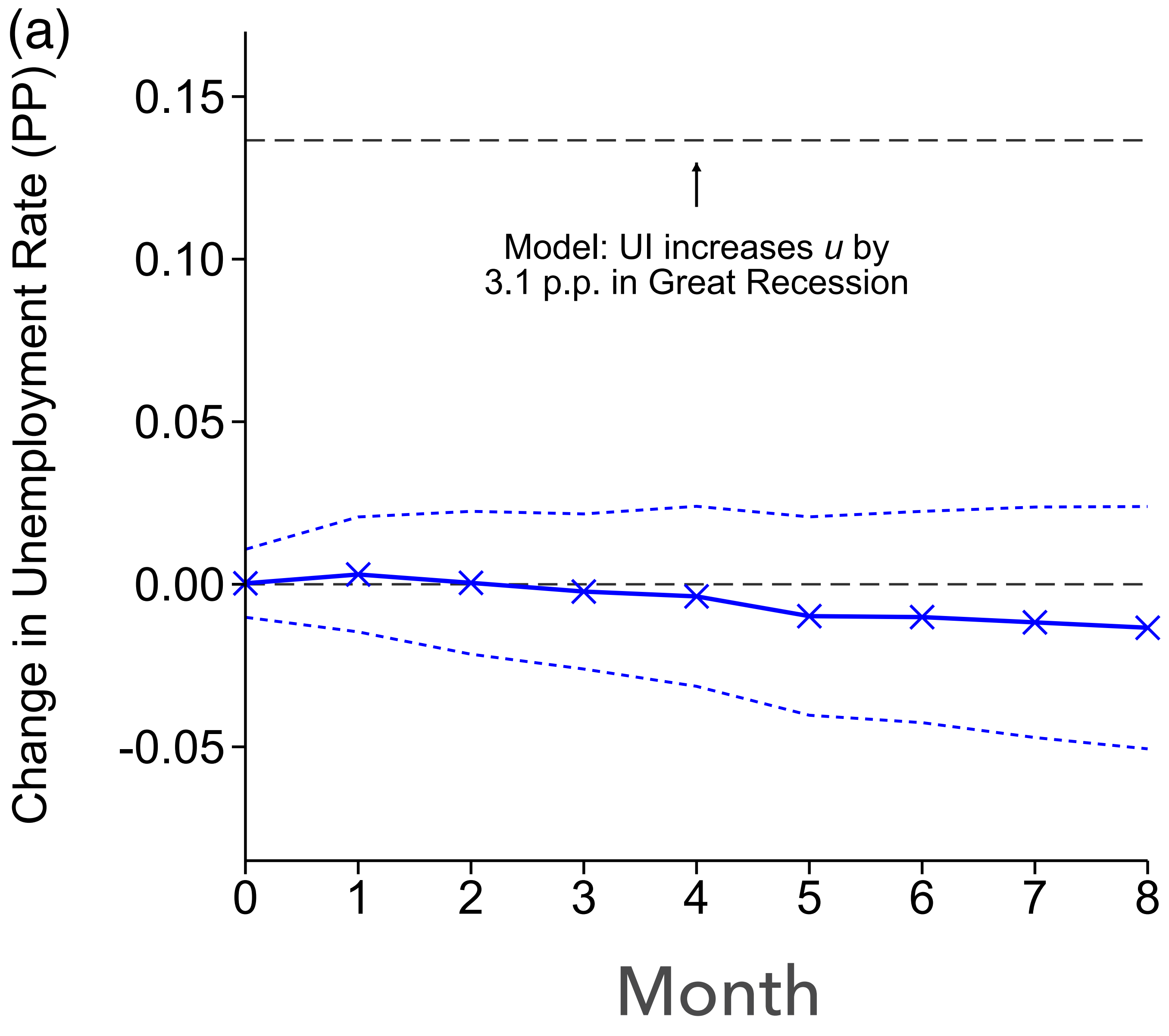
Small Macro Effect

$$y_{t+h} - y_{t-1} = \beta_h \times (\text{UI Benefit Increase from Measurement Error}) + \gamma' \mathbf{X}_{t,h} + \epsilon_{t,h}$$

Fraction Receiving UI



Unemployment Rate



Vacancies