Efficiency in DMP Model

704 Macroeconomics II
Topic 4

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Is Unemployment Efficient?

- We built a model of unemployment and studied the positive implications
- Today, we focus on the normative implications
- Is the equilibrium efficient? Is unemployment too high or too low?

Constrained Efficient Allocation

Constrained Efficiency

- Consider the canonical DMP model as in lecture 2
- We want to study how a benevolent planner would allocate resources
- If the planner could get rid of search frictions, would do so
  - neither interesting nor realistic
- Instead, we treat search friction as part of technology
- Use the concept of **constrained efficiency**:
  Planner's problem taking search friction as given

Planning Problem

$$\max_{\{C_t, v_t, u_{t+1}\}} \sum_{t=0}^{\infty} \beta^t C_t$$

s.t. $$C_t = z_t(1 - u_t) + bu_t - cv_t$$

$$u_{t+1} - u_t = s(1 - u_t) - f(v_t/u_t)u_t, \quad u_0 \text{ given}$$

- Here, b is treated as home production
- With linear preferences, maximizing consumption = maximizing output
  - Transfers immaterial: everyone has the same marginal utility of consumption
- The last constraint captures "constrained" efficiency
  - Without it, $u_t = v_t = 0$ iff $z_t > b$ (again, neither interesting nor realistic)

Reducing the Constraints

- Planner's problem simplifies to a standard dynamic optimization:

$$\max_{\{v_t, u_{t+1}\}} \sum_{t=0}^{\infty} \beta^t [z_t(1-u_t) + bu_t - cv_t]$$

s.t. $\quad u_{t+1} - u_t = s(1-u_t) - M(v_t, u_t), \quad u_0$ given

- Can solve using
  - Lagrangian method
  - Dynamic programming

Recursive Formulation

- The Bellman equation is

$$\Omega(u, z) = \max_{u', v} z(1 - u) + bu - cv + \beta \mathbb{E} \Omega(u', z')$$

$$\text{s.t.} \quad u' - u = s(1 - u) - M(v, u)$$

- FOC:

$$c = -\beta \frac{\partial M(u, v)}{\partial v} \mathbb{E} \left[\frac{\partial \Omega(u', z')}{\partial u'} \right]$$

- Envelope:

$$\frac{\partial \Omega(u, z)}{\partial u} = -z + b + \beta \left(1 - s - \frac{\partial M(u, v)}{\partial u} \right) \mathbb{E} \left[\frac{\partial \Omega(u', z')}{\partial u'} \right]$$

Algebra

- We rewrite

$$\frac{\partial M(u,v)}{\partial u} = \underbrace{\frac{\partial \ln M(u,v)}{\partial \ln u}}_{\equiv \alpha} \underbrace{\frac{M(u,v)}{u}}_{\equiv f(\theta)}$$
$$= \alpha f(\theta)$$

- Under CRS matching function, $M = (\partial_u M)u + (\partial_v M)v$, so

$$\frac{\partial M(u,v)}{\partial v} = \frac{1}{v}M(u,v) - \frac{\partial M(u,v)}{\partial u} u \frac{1}{v}$$
$$= \frac{1}{v}M(u,v) - \underbrace{\frac{\partial \ln M(u,v)}{\partial \ln u}}_{\equiv \alpha} \frac{M(u,v)}{u} u \frac{1}{v}$$
$$= q(\theta)(1-\alpha)$$

Planner's Solution vs. Equilibrium

- Defining the planner's surplus from a job as $S_t^{SP} \equiv -\partial_u \Omega(u_t, z_t)$
- The planner's solution $\{S_t^{SP}, \theta_t^{SP}\}$ solves

$$S_t^{SP} = z_t - b + \beta(1 - s - \alpha_t f(\theta_t^{SP}))\mathbb{E}S_{t+1}^{SP}$$

$$c = (1 - \alpha_t)\beta q(\theta_t^{SP})\mathbb{E}_t S_{t+1}^{SP}$$

- Recall in the decentralized equilibrium, $\{S_t^{DE}, \theta_t^{DE}\}$ solves

$$S_t^{DE} = z_t - b + \beta(1 - s - \gamma f(\theta_t^{DE}))\mathbb{E}S_{t+1}^{DE}$$

$$c = (1 - \gamma)\beta q(\theta_t^{DE})\mathbb{E}_t S_{t+1}^{DE}$$

- Planner and eqm share the same stock-flow equation: $u_{t+1} - u_t = -f(\theta_t)u + s(1-u)$
- Find the difference?

Hosios Condition

Hosios (1990) Condition

Decentralized equilibrium is constrained efficient if and only if $\alpha_t = \gamma$

- Under Cobb-Douglas, $M(u, v) = \bar{m}u^{\alpha}v^{1-\alpha}$, efficiency is achieved when $\alpha = \gamma$
- Holds only in a knife-edge case
- To understand, it is useful to break down into two margins
  1. Investment margin:
     Is vacancy creation incentive efficient given the value of matches?
  2. Valuation margin:
     Given market tightness, are the matches valued correctly?

Investment Margin

$$c = (1 - \alpha_t)\beta q(\theta_t^{SP})\mathbb{E}_t S_{t+1}^{SP} \qquad c = (1 - \gamma)\beta q(\theta_t^{DE})\mathbb{E}_t S_{t+1}^{DE}$$

- If matches are valued correctly ($S_{t+1}^{SP} = S_{t+1}^{DE}$), is market tightness θ_t efficient?
- When a firm creates a vacancy, it creates a social surplus of $\frac{\partial M(u,v)}{\partial v} S = (1 - \alpha) q(\theta) S$
  - Less than $q(\theta)S$ because it lowers the meeting prob. of other firms
  - Impose negative externality $\Rightarrow$ force toward too many vacancy creations
- Firm's private incentive to create a job is $(1 - \gamma)q(\theta)S$
  - Less than $q(\theta)S$ because workers capture part of rents (hold-up problem)
  - Firms cannot capture full surplus $\Rightarrow$ force toward too little vacancy creations
- When $1 - \gamma = 1 - \alpha$, these two forces exactly cancel

Valuation Margin

$$S_t^{SP} = z_t - b + \beta(1 - s - \alpha_t f(\theta_t^{SP}))\mathbb{E}S_{t+1}^{SP}$$

$$S_t^{DE} = z_t - b + \beta(1 - s - \gamma f(\theta_t^{DE}))\mathbb{E}S_{t+1}^{DE}$$

- If market tightness is the same ($\theta_t^{SP} = \theta_t^{DE}$), is the valuation of the job S_t efficient?
- When the match separates, it creates a social surplus of $\frac{\partial M(u,v)}{\partial u}S = \alpha_t f(\theta)S$
  - Lower than $f(\theta)S$ because it congests the market
- When the match separates, it creates a private surplus of $\gamma f(\theta)S$
  - Lower than $f(\theta)S$ because workers can only get a fraction of surplus
- When $\alpha_t = \gamma$, private and social valuation are aligned

Magic of Hosios Condition

- Despite there being **two** sources of inefficiency, **one** condition ensures efficiency
- This is magical to me

Unemployment Too High or Too Low?

- Focus on the steady state.
- Then

$$c = \beta q(\theta^{DE}) \frac{z - b}{1 - \beta(1 - s - \gamma f(\theta^{DE}))} \quad \text{vs.} \quad c = \beta q(\theta^{SP}) \frac{z - b}{1 - \beta(1 - s - \alpha f(\theta^{SP}))}$$

- One can show

$$\gamma < \alpha \Leftrightarrow \theta^{DE} > \theta^{SP} \Leftrightarrow u^{DE} < u^{SP}$$

- No clear empirical guidance on the choice of γ and α
- Often suggested: $\gamma \ll \alpha$, which means unemployment is too low!
- Can restore efficiency with income tax or tax on vacancy cretion

Random vs. Directed Search

- There is an alternative way of modeling search friction: **directed search**
  - Pioneered by Moen (1997), and popularized by Menzio and Shi (2011)
- Firms post wages and workers direct what jobs to search (can apply only one)
- In this class of models, the equilibrium is always efficient
- Reality is clearly a mix of random vs. directed search
- How much are searches in the real world directed? How can we tell from the data?
  - See Lentz, Maibom, and Moen (2024) for a recent attempt