

---

# **Supply-Side View of Financial Frictions:**

## **Borrowing Constraints and Misallocation**

704 Macroeconomics II  
Topic 6

Masao Fukui

---

# **Do Financial Frictions Matter in the Short-Run?**



# How the Great Depression Started

*Complete and Closing N. Y. Stock Exchange Prices On Pages 37-40*

Complete Wire Reports of UNITED PRESS, the Greatest World-Wide News Service

**The Pittsburgh Press**

STOCKS EXTRA  
Complete Markets

FORTY-EIGHT PAGES      WEATHER—RAIN.      PITTSBURGH, PA., TUESDAY, OCTOBER 29, 1929      IN TWO SECTIONS—SECTION ONE      THREE CENTS

**HUGE LOSSES IN WALL STREET:  
SALES SET ALL-TIME RECORD**

**200 Escape Seventh Ave. Hotel Fire**

**RACING  
TODAY'S RESULTS**

**BIG DECLINE  
SLOWED UP BY  
LATE RALLY**

*Playful Monk Makes  
Good Escape From  
Fraternity Boys.*

**RESCUE GUEST  
FROM SMOKE**

**BLAZE IN HOTEL**

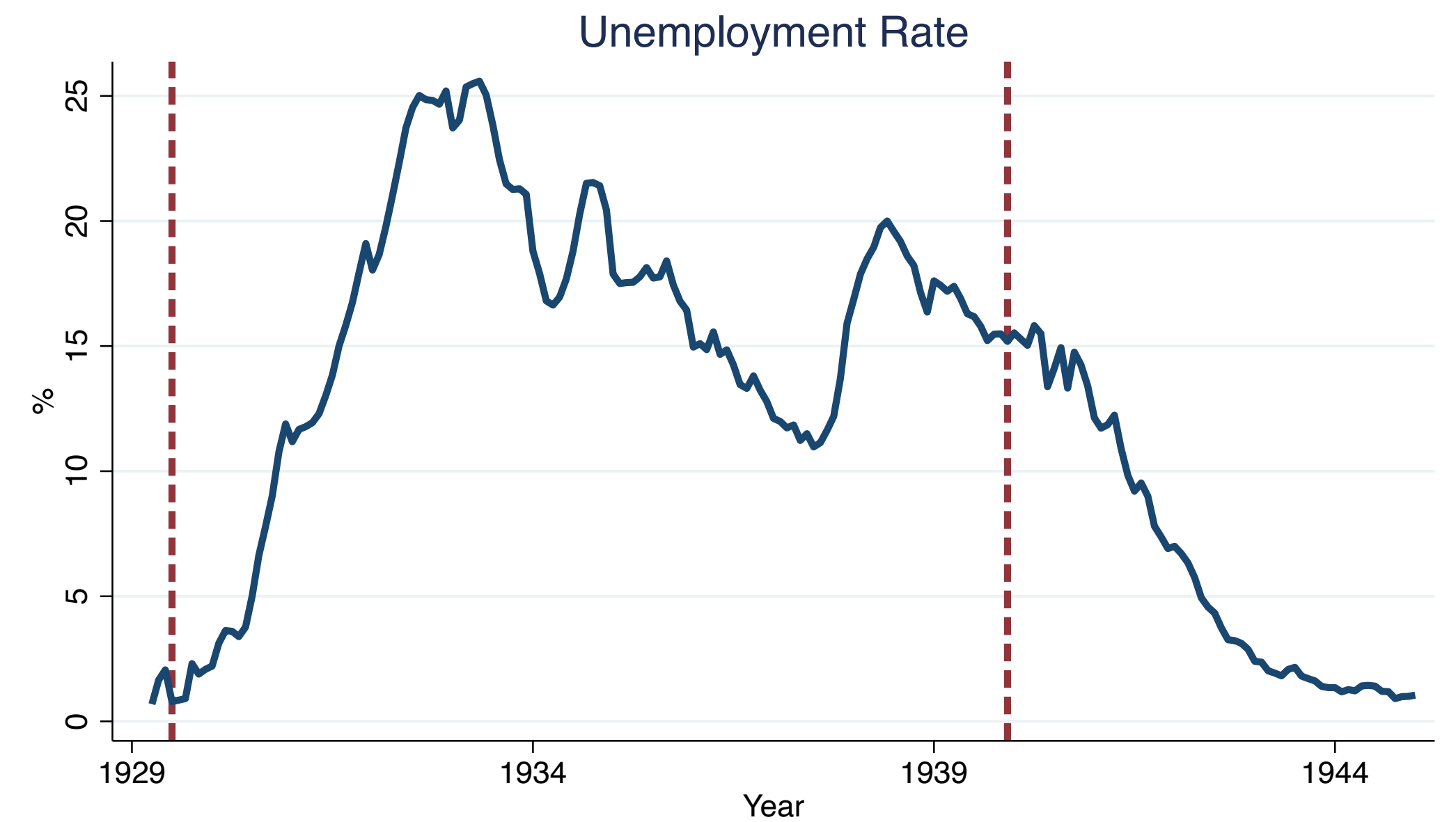
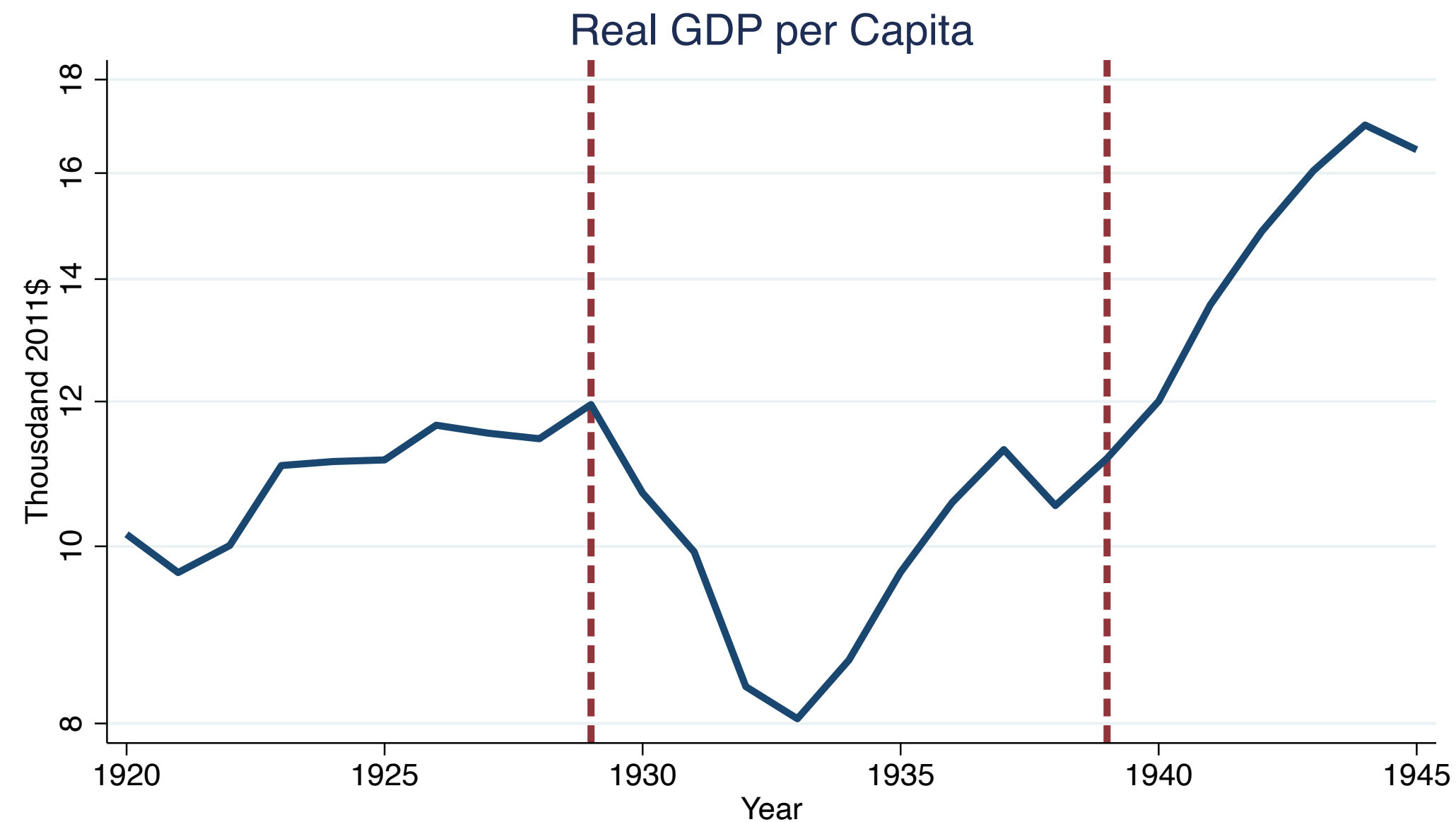
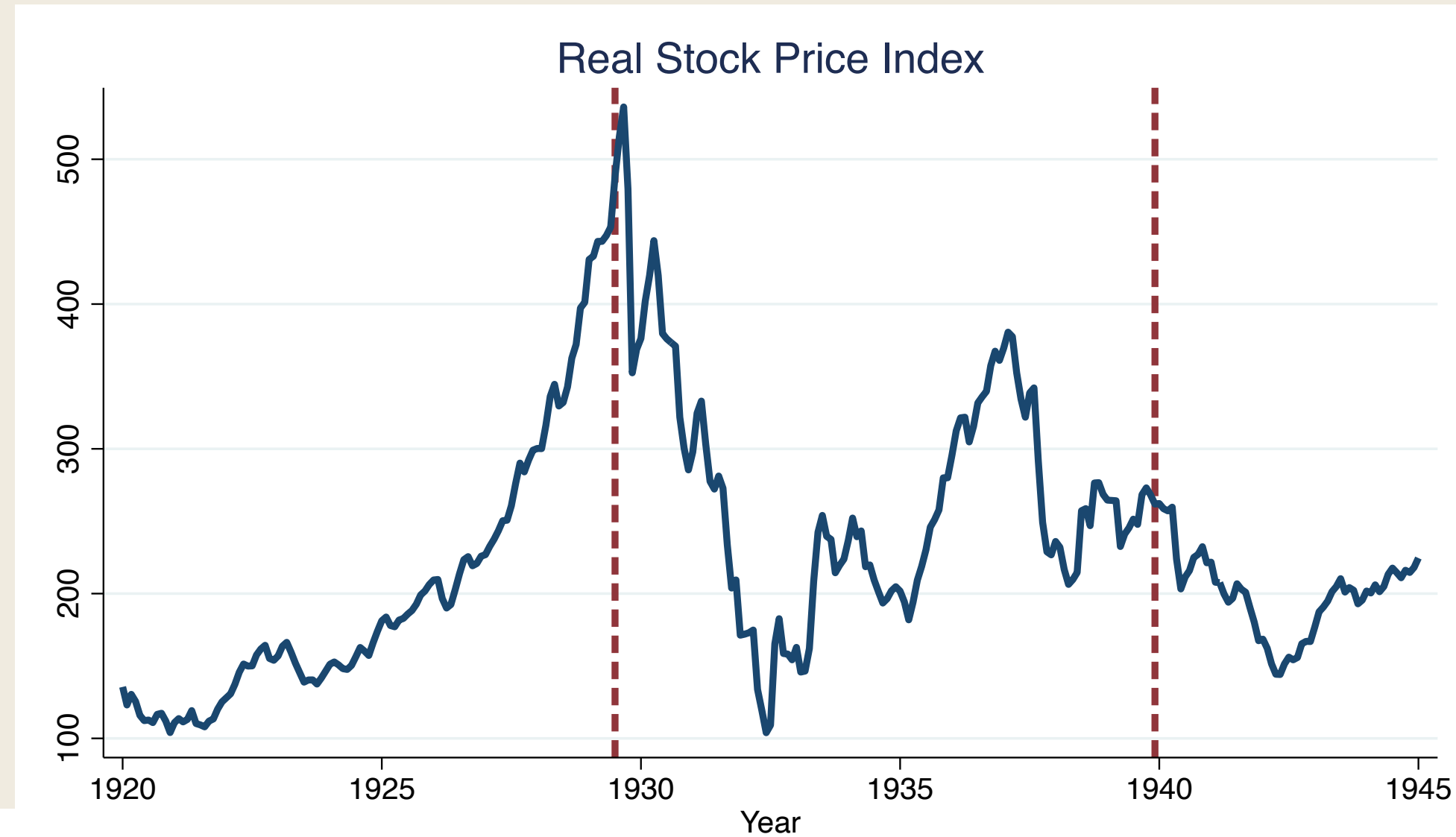
**UNDERGROUND  
TRAFFIC PLAN**

*MONKEYSHINES look the*

LAUREL.			
First race, \$1,500 purse, 2-year-olds, 4-mile.			
Moine	110	Inzelone	22.50, 9.70, 3.90
Sam Fakes	110	Collettili	8.20, 2.90
Monter	110	Leishman	2.20
Time—1:12 4-5. Also ran—Handman, Kings Crier, Guthrie, House Knight, Timon, Black Cloud.			
Second race, \$1,500, claiming, 3-year-olds, 1 mile.			
Donna Tina	105	Serie	5.20, 3.70, 3.30
Loumoro	113	Abel	14.50, 8.20
Lion Healed	110	F. Mann	8.20



# Great Depression





# How the Great Recession Started

## THE WALL STREET JOURNAL.

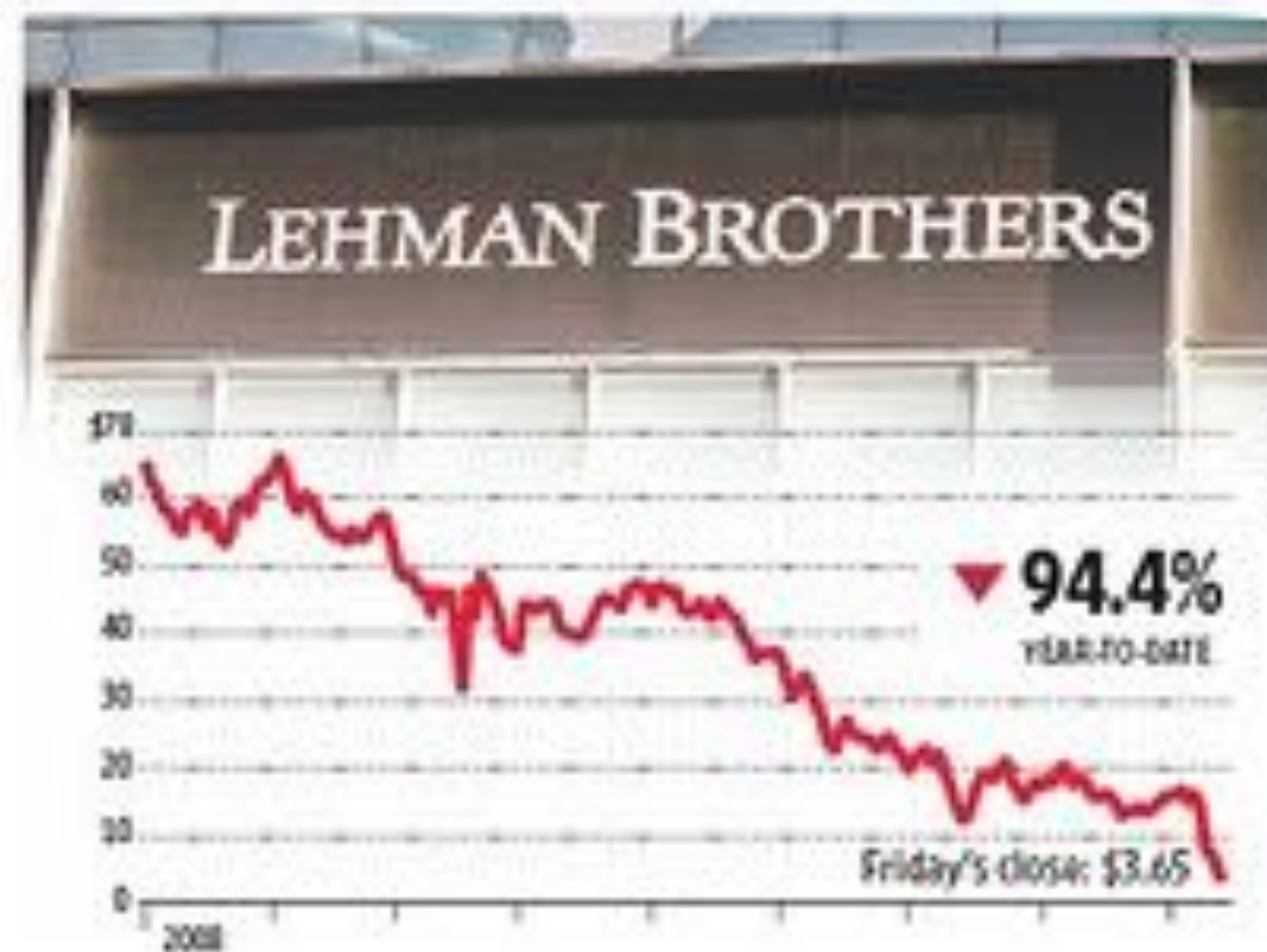
DOW JONES  
A FINANCIAL CORPORATION COMPANY

\*\*\*\*\*

MONDAY, SEPTEMBER 15, 2008 • VOL. CCLII NO. 64

★★★★ \$2.00

Last week: DJIA 11421.99 ▲201.03 1.8% NASDAQ 2261.27 ▲0.2% NIKKEI 12214.76 unch. DJ STOXX 50 2858.68 ▲3.8% 10-YR TREASURY ▼20/32, yield 3.730% OIL \$101.18 ▼\$5.05 EURO \$1.4217 YEN 107.87

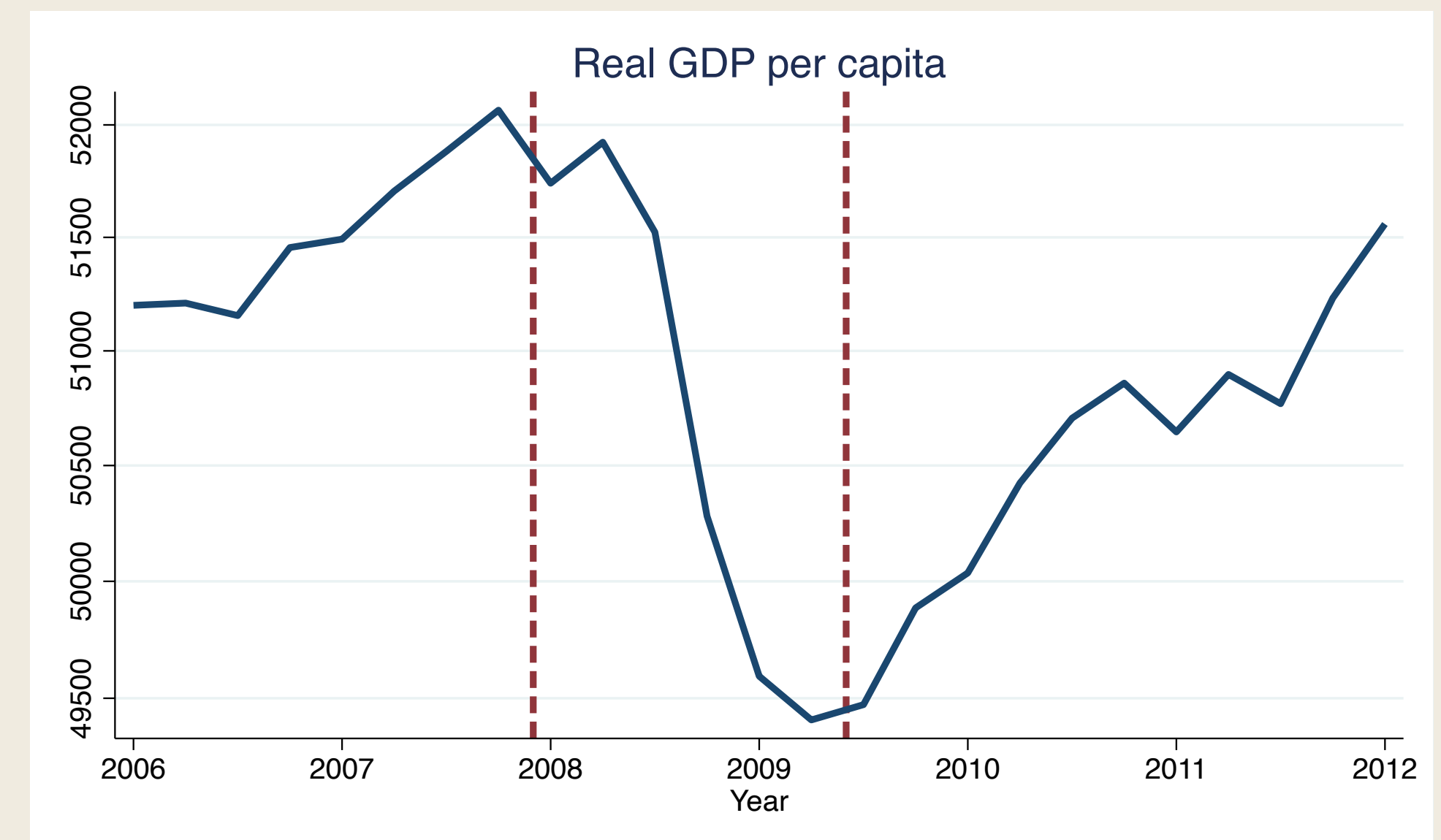
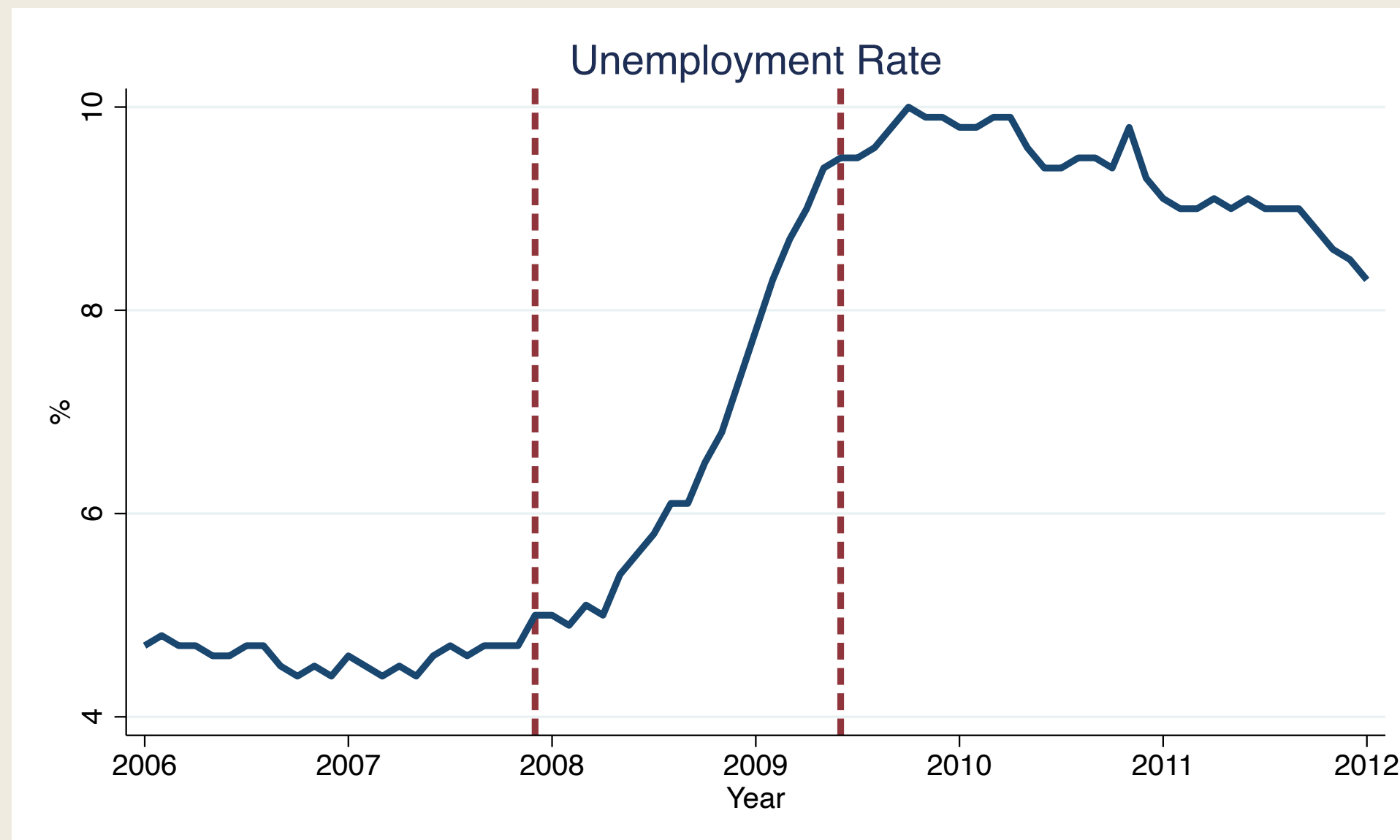
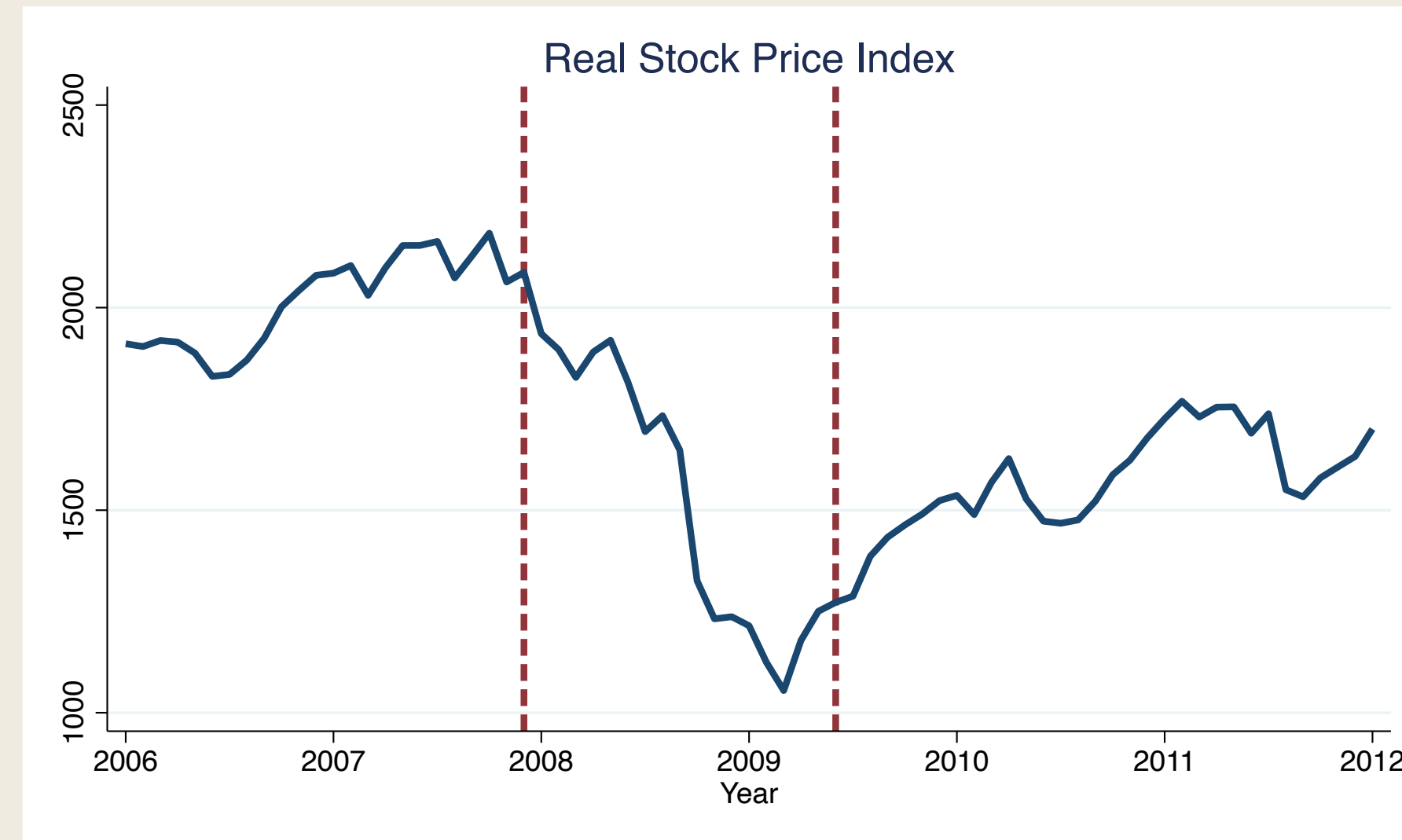


## Crisis on Wall Street as Lehman Totters, Merrill Is Sold, AIG Seeks to Raise Cash

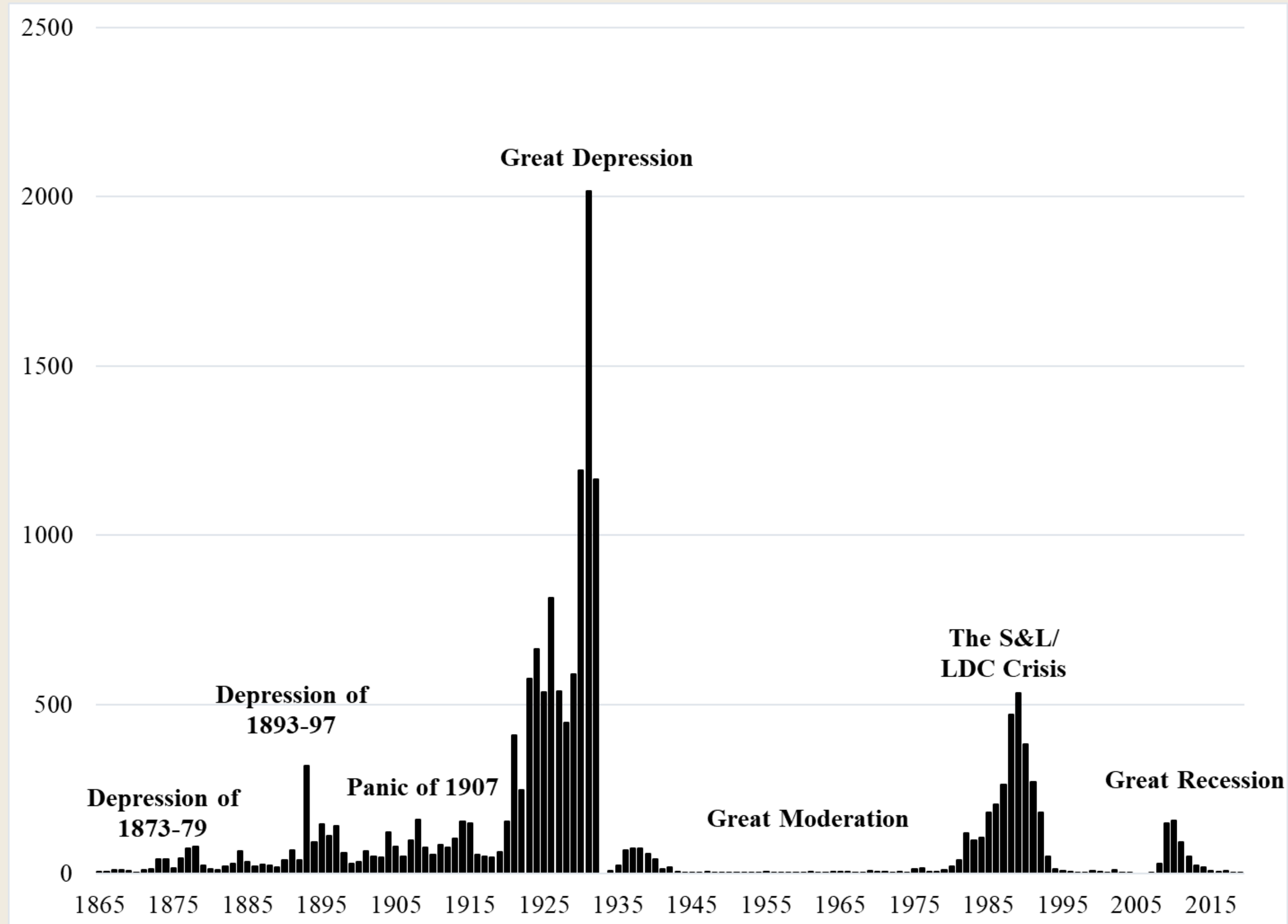
*Fed Will Expand Its Lending Arsenal in a Bid to Calm Markets; Moves Cap a Momentous Weekend for American Finance*



# Great Recession



# Number of Bank Failures



---

# Cause or Consequence?

- Two views on bank failures:
  1. Bank failures are a **consequence** of the Great Depression/Great Recession
  2. Bank failures are the **cause** of the Great Depression/Great Recession
- The first view was dominant after the Great Depression
- In his 1983 paper, Bernanke brought a new perspective and argued for 2



# Bernanke (1983)

$$\Delta Y_t = \alpha + \beta \times \Delta(\text{Bank Health})_t + \gamma' \mathbf{X}_t + \epsilon_t$$

$$\begin{aligned} (3) \quad Y_t = & \frac{.613}{(9.86)} Y_{t-1} - \frac{.159}{(-2.63)} Y_{t-2} + \frac{.332}{(2.92)} (M - M^e)_t + \frac{.113}{(0.99)} (M - M^e)_{t-1} + \frac{.110}{(0.96)} (M - M^e)_{t-2} \\ & + \frac{.156}{(1.38)} (M - M^e)_{t-3} - \frac{.869E-04}{(-4.24)} DBANKS_t - \frac{.406E-04}{(-1.93)} DBANKS_{t-1} \\ & - \frac{.258E-03}{(-1.95)} DFAILS_t - \frac{.325E-03}{(-2.47)} DFAILS_{t-1} \\ & s.e. = .0249 \quad D.W. = 1.99 \quad \text{Sample: 1/21-12/41} \end{aligned}$$

$$\begin{aligned} (4) \quad Y_t = & \frac{.615}{(9.76)} Y_{t-1} - \frac{.131}{(-2.13)} Y_{t-2} + \frac{.455}{(3.99)} (P - P^e)_t + \frac{.231}{(1.97)} (P - P^e)_{t-1} - \frac{.004}{(-0.03)} (P - P^e)_{t-2} \\ & + \frac{.024}{(0.22)} (P - P^e)_{t-3} - \frac{.799E-04}{(-4.03)} DBANKS_t - \frac{.337E-04}{(-1.66)} DBANKS_{t-1} \\ & - \frac{.202E-03}{(-1.52)} DFAILS_t - \frac{.242E-03}{(-1.83)} DFAILS_{t-1} \\ & s.e. = .0246 \quad D.W. = 1.98 \quad \text{Sample: 1/21-2/41} \end{aligned}$$

---

Notes:  $Y_t$  = rate of growth of industrial production (*Federal Reserve Bulletin*), relative to exponential trend.  
 $(M - M^e)_t$  = rate of growth of  $M1$ , nominal and seasonally adjusted (Friedman and Schwartz, Table 4-1), less predicted rate of growth.  
 $(P - P^e)_t$  = rate of growth of wholesale price index (*Federal Reserve Bulletin*), less predicted rate of growth.  
 $DBANKS_t$  = first difference of deposits of failing banks (deflated by wholesale price index).  
 $DFAILS_t$  = first difference of liabilities of failing businesses (deflated by wholesale price index).  
Data are monthly;  $t$ -statistics are shown in parentheses.

---

# (More) Credible Identification

## 1. **Chodorow-Reich (2014)**: Firm-level cross-sectional regression:

$$\Delta Y_i = \beta \times \Delta(\text{Bank Health})_i + \gamma' \mathbf{X}_i + \epsilon_i$$

- $(\text{Bank Health})_i$ : health of banks that the firm  $i$  had a relationship with
- Using data from the US 2007-2009, find  $\beta > 0$

## 2. **Huber (2018)**: County-level cross-sectional regression:

$$\Delta Y_c = \beta \times \Delta(\text{Bank Health})_c + \gamma' \mathbf{X}_c + \epsilon_c$$

- $(\text{Bank Health})_c$ : average health of banks in county  $c$
- Using data from the Germany 2007-2012, find  $\beta > 0$



---

# The Role of Cross-Sectional Identification

A common critique of estimates based on cross-sectional identification in macroeconomics is that they don't answer the right question. While it is true that these estimates don't directly provide estimates of aggregate responses, they often provide a great deal of indirect evidence by helping researchers discriminate between different theoretical views of how the world works.... This “piecemeal” form of inference will, therefore, result in partial identification on the model space.

— Nakamura and Steinsson (2018) “Identification in Macroeconomics”

---

# **Do Financial Frictions Matter in the Long-Run?**

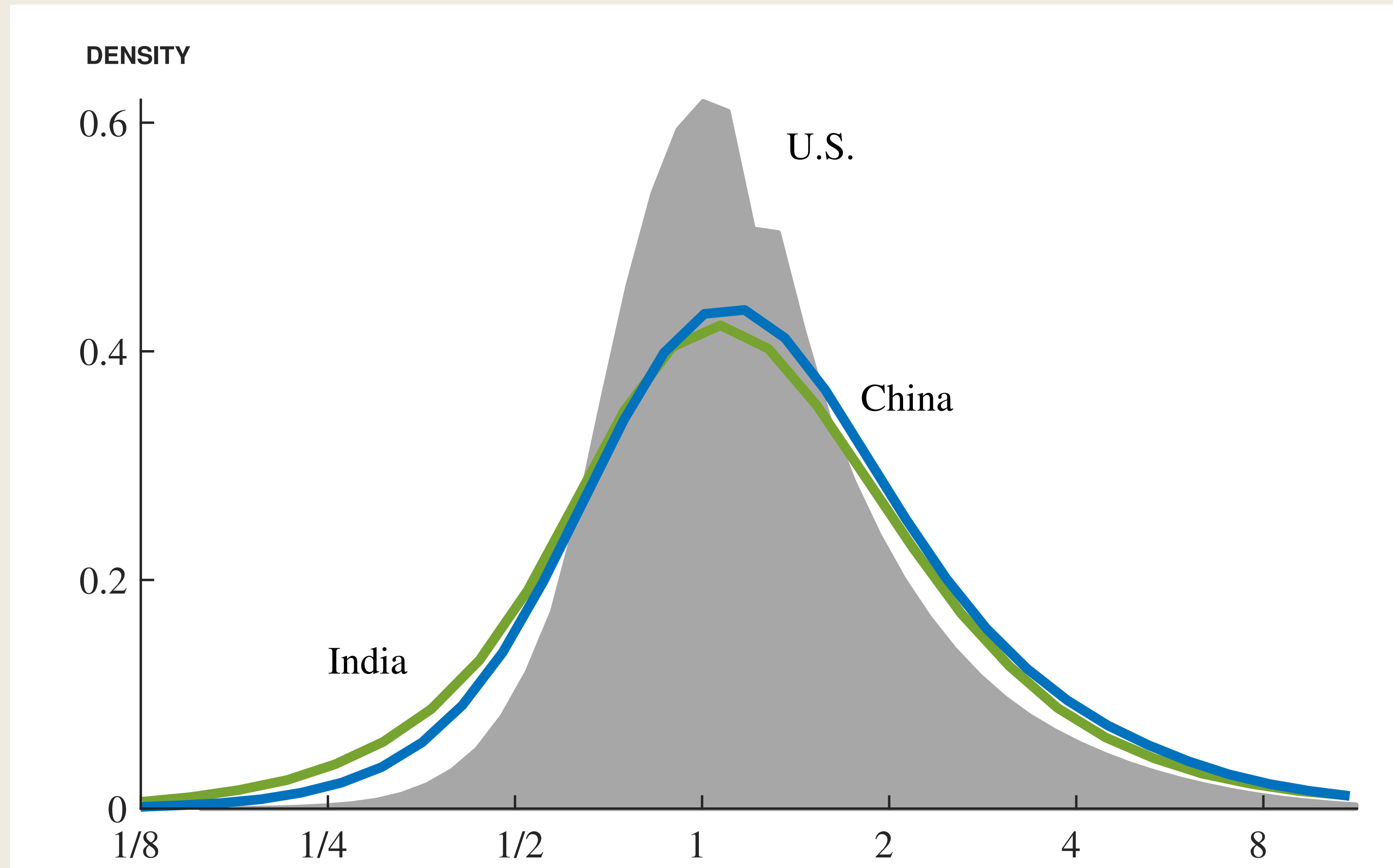


---

# Misallocation Hypothesis

- Large cross-country TFP differences. Why?
- Hsieh & Klenow (2007): misallocation
  - Measure marginal product of capital at the firm level:
$$MPK_i = f'_i(k)$$
  - Efficiency requires  $MPK_i = \overline{MPK}$  for all  $i$
  - If  $f_i(k) = A_i k^\alpha$ , then  $MPK_i = \alpha y_i / k_i \Rightarrow$  can measure  $MPK_i$  from microdata
- Implement in the context of manufacturing in the US, India, and China

# MPK Dispersion



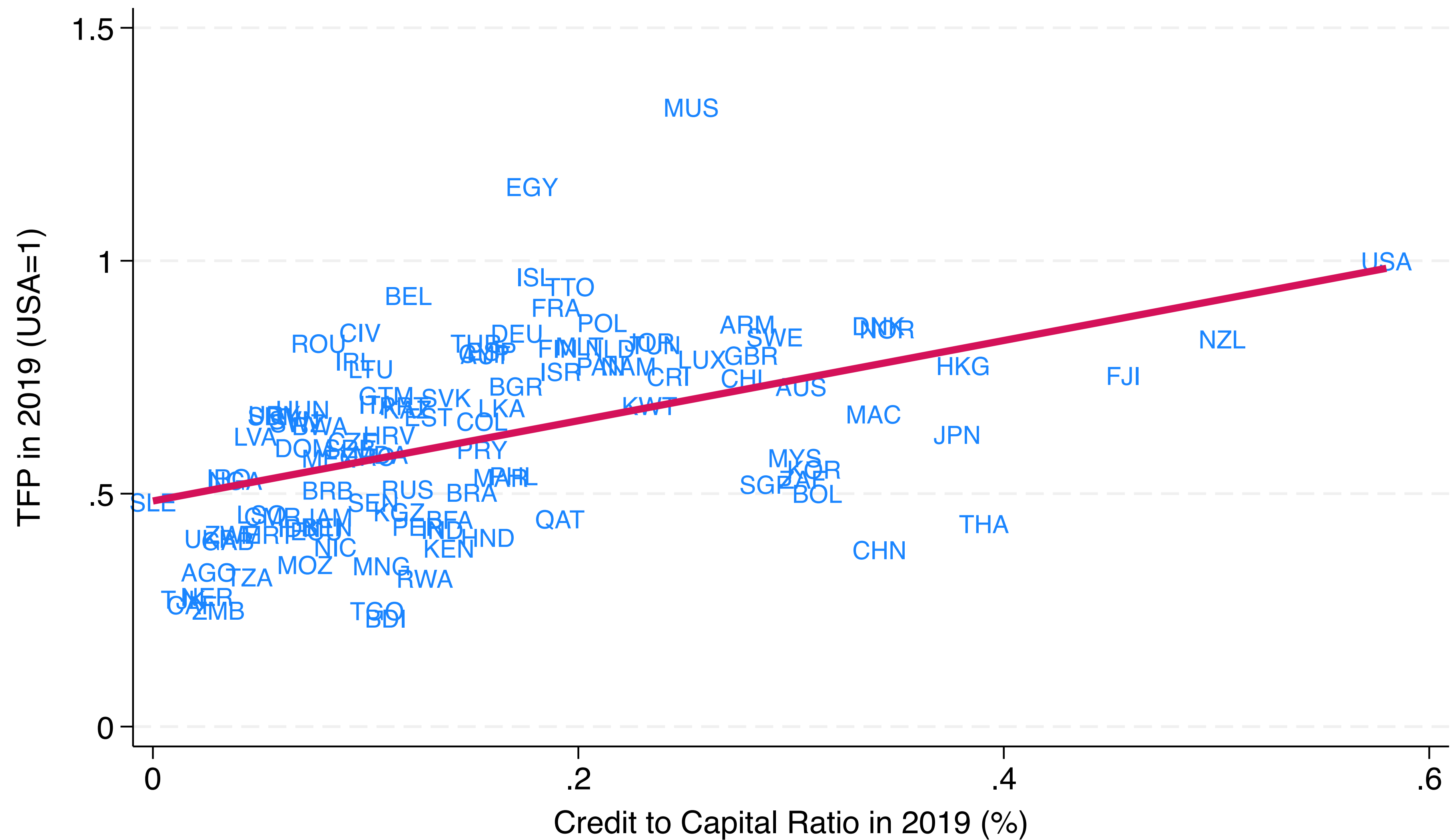


---

# Financial Friction

- Why are MPK not equalized?
- A potentially important source is financial friction
- Firms cannot borrow as much as they want
  - Financially constrained firms have higher MPK
  - Unconstrained firms have lower MPK

# TFP and Credit





---

# **Financial Frictions and Misallocation**

**—Based on Moll (2015)**

---

# Entrepreneurs

- The economy is populated by a unit mass of entrepreneurs indexed by  $i \in [0,1]$
- Preferences:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \ln c_t^i$$

- The technology of an entrepreneur with productivity  $z_t^i$  is:

$$y_t^i = z_t^i k_t^i$$

- Assume no depreciation of capital  $k_t^i$
- Productivity  $z_t$  evolves according to a Markov process
  - Let  $f(z' | z)$  denote the probability density of  $z'$  conditional on  $z$
  - Assume  $z_t \in [0, \bar{z}]$  (bounded)

# Borrowing Constraint

## ■ Budget constraint:

$$c_t^i + a_{t+1}^i = z_t^i k_t^i - r_t k_t^i + (1 + r_t) a_t^i$$

- $a_t^i$ : networth,  $k_t^i$ : capital,  $r_t$ : rental price of capital

## ■ Borrowing constraint:

$$k_t^i \leq \lambda a_t^i$$

- Can only rent capital up to  $\lambda \geq 1$  times networth

## ■ Microfoundation:

- borrowers can steal  $1/\lambda$  fraction of the rented capital  $k^i$
- if borrowers steal, lenders can seize the networth of borrowers  $a^i$
- In equilibrium, lenders are willing to lend

$$(1/\lambda)k^i \leq a^i \quad \Leftrightarrow \quad k^i \leq \lambda a^i$$



---

# Equilibrium Definition

- Given  $\{r_t\}_{t=0}^{\infty}$ , entrepreneurs choose  $\{c_t^i, a_{t+1}^i, k_t^i\}_{t=0}^{\infty}$  to maximize utility
- Markets clear

$$\int_0^1 a_t^i di = \int_0^1 k_t^i di$$

---

# No Financial Friction

# No Financial Friction

- Suppose there is no financial friction  $\lambda = \infty$
- Entrepreneur's problem in a recursive form:

$$V_t(a_t, z_t) = \max_{k_t \geq 0, c_t, a_{t+1}} \ln c_t + \beta \mathbb{E}_t V_{t+1}(a_{t+1}, z_{t+1})$$
$$\text{s.t.} \quad c_t + a_{t+1} = z_t k_t - r_t k_t + (1 + r_t) a_t$$

- Consider a sub-problem where entrepreneurs choose  $k_t$  to solve

$$\max_{k_t \geq 0} z_t k_t - r_t k_t$$

Solutions:

$$k_t = \begin{cases} \infty & \text{if } z_t > r_t \\ \tilde{k} \in [0, \infty] & \text{if } z_t = r_t \\ 0 & \text{if } z_t < r_t \end{cases}$$



# Equilibrium Interest Rate

- In the absence of borrowing constraints,

$$r_t = \bar{z}$$

- If  $r_t > \bar{z}$ , everyone will lend
- If  $r_t < \bar{z}$ , entrepreneurs with  $z \in (r_t, \bar{z}]$  will infinitely borrow

- As a result, all agents solve:

$$\begin{aligned} V(a_t) &= \max_{c_t, a_{t+1}} \ln c_t + \beta V(a_{t+1}) \\ \text{s.t.} \quad c_t + a_{t+1} &= \underbrace{\max_{k_t \geq 0} \{z_t k_t - r_t k_t\} + (1 + r_t) a_t}_{(1 + \bar{z}) a_t} \end{aligned}$$

- Guess and verify:

$$c_t(a_t) = (1 - \beta)(1 + \bar{z})a_t, \quad a_{t+1}(a_t) = \beta(1 + \bar{z})a_t$$

---

# No Financial Friction: Aggregation

- The economy follows

$$Y_t = \bar{z}K_t$$

$$K_{t+1} = \beta(1 + \bar{z})K_t$$

- Exogenous TFP. This is a standard AK economy

---

# Financial Friction



# Frictional Financial Market

- Now consider financial friction  $\lambda < \infty$

$$\max_{k_t \in [0, \lambda a_t]} z_t k_t - r_t k_t$$

Solutions:

$$k_t(a_t, z_t) = \begin{cases} \lambda a_t & \text{if } z_t > r_t \\ \tilde{k} \in [0, \lambda a_t] & \text{if } z_t = r_t \\ 0 & \text{if } z_t < r_t \end{cases}$$

- The budget constraint of entrepreneur with productivity  $z_t$  is

$$c_t + a_{t+1} = (1 + \pi_t(z_t))a_t$$

where

$$\pi(z; r_t) \equiv \begin{cases} (z - r_t)\lambda + r_t & \text{for } z \geq r_t \\ r_t & \text{for } z < r_t \end{cases}$$

- Entrepreneurs with  $z > r_t$  earn (finite) excess returns

---

# Bellman Equation

$$V_t(a_t, z_t) = \max_{c_t, a_{t+1}} \ln c_t + \beta \mathbb{E}_t V_{t+1}(a_{t+1}, z_{t+1})$$
$$\text{s.t.} \quad c_t + a_{t+1} = (1 + \pi(z_t; r_t))a_t$$

- Expectation is taken over  $z_{t+1}$
- Guess and verify:

$$c_t(a_t, z_t) = (1 - \beta)(1 + \pi(z, r_t))a_t, \quad a_{t+1}(a_t, z_t) = \beta(1 + \pi(z; r_t))a_t$$

# Aggregation

- Let  $g_t(a, z)$  denote the density of the joint distribution of  $(a, z)$

- The capital market clearing implies

$$\int_{\underline{z}}^{\bar{z}} \int_0^{\infty} a g_t(a, z) da dz = \int_{r_t}^{\bar{z}} \int_0^{\infty} \lambda a g_t(a, z) da dz = K_t \quad (1)$$

- Define wealth share held by entrepreneurs with productivity  $z$  as

$$\omega_t(z) = \frac{1}{K_t} \int_0^{\infty} a g_t(a, z) da \quad (2)$$

Note  $\int_{\underline{z}}^{\bar{z}} \omega_t(z) dz = 1$

- Using (2) to rewrite (1) as

$$\lambda \int_{r_t}^{\bar{z}} \omega_t(z) dz = 1$$

- Given  $\{\omega_t(z)\}_z$ , this pins  $r_t$ : lower  $\lambda \Rightarrow$  lower  $r_t$
- Financial friction depresses interest rate



# Aggregate Output

- The aggregate output is

$$\begin{aligned} Y_t &= \int_{r_t}^{\bar{z}} \int_0^\infty z \lambda a_t g_t(a, z) da dz \\ &= \lambda \int_{r_t}^{\bar{z}} z \omega_t(z) dz K_t \\ &= \underbrace{\frac{1}{\int_{r_t}^{\bar{z}} \omega_t(z) dz} \int_{r_t}^{\bar{z}} z \omega_t(z) dz}_{\equiv \mathbb{E}_\omega[z|z \geq r_t]} K_t \\ &\equiv Z_t K_t \end{aligned}$$

- Total factor productivity  $Z_t$  is endogenous to wealth distribution:
  - Wealth weighted average of  $z$  conditional on  $z \geq r_t$
- Depressed interest rate  $r_t \Rightarrow$  low  $z$  produce  $\Rightarrow$  misallocation

# Evolution of Capital Stock

- The evolution of capital stock is

$$\begin{aligned} K_{t+1} &= \int_{\underline{z}}^{\bar{z}} \int_0^{\infty} a_{t+1}(a, z) g_t(a, z) da dz \\ &= K_t \int_{\underline{z}}^{\bar{z}} \beta(1 + \pi(z; r_t)) \underbrace{\frac{1}{K_t} \int_0^{\infty} a g_t(a, z) da}_{= \omega_t(z)} dz \\ &= K_t \int_{\underline{z}}^{\bar{z}} \beta(1 + \pi(z; r_t)) \omega_t(z) dz \end{aligned}$$

# Evolution of Distribution

- Law of motion for  $g_t(a, z)$

$$\Pr(a_{t+1} \leq a, z_{t+1} = z) = \int_{\underline{z}}^{\bar{z}} \int_0^{\infty} g_t(\tilde{a}, \tilde{z}) \mathbb{I}[a_{t+1}(\tilde{a}, \tilde{z}) \leq a] f(z | \tilde{z}) d\tilde{a} d\tilde{z}$$

- Recalling  $a_{t+1}(\tilde{a}, \tilde{z}) = \beta(1 + \pi(\tilde{z}; r_t))\tilde{a}$ ,

$$\Pr(a_{t+1} \leq a, z_{t+1} = z) = \int_{\underline{z}}^{\bar{z}} \int_0^{\frac{a}{\beta(1 + \pi(\tilde{z}; r_t))}} g_t(\tilde{a}, \tilde{z}) f(z | \tilde{z}) d\tilde{a} d\tilde{z}$$

- Since  $g_{t+1}(a, z) \equiv \partial_a \Pr(a_{t+1} \leq a, z_{t+1} = z)$

$$g_{t+1}(a, z) = \int_{\underline{z}}^{\bar{z}} \frac{1}{\beta(1 + \pi(\tilde{z}; r_t))} g_t\left(\frac{a}{\beta(1 + \pi(\tilde{z}; r_t))}, \tilde{z}\right) f(z | \tilde{z}) d\tilde{a} d\tilde{z}$$

# Evolution of Wealth Share

- Using the previous relationship

$$\begin{aligned}
 \omega_{t+1}(z) &\equiv \frac{1}{K_{t+1}} \int_0^\infty a g_{t+1}(a, z) da \\
 &= \frac{1}{K_{t+1}} \int_0^\infty \int_{\underline{z}}^{\bar{z}} \frac{1}{\beta(1 + \pi(\tilde{z}; r_t))} a g_t \left( \frac{a}{\beta(1 + \pi(\tilde{z}))}, \tilde{z} \right) f(z | \tilde{z}) d\tilde{z} da \\
 &= \frac{K_t}{K_{t+1}} \int_{\underline{z}}^{\bar{z}} \beta(1 + \pi(\tilde{z}; r_t)) \underbrace{\frac{1}{K_t} \int_0^\infty \tilde{a} g_t(\tilde{a}), \tilde{z}) d\tilde{a}}_{\equiv \omega_t(z)} f(z | \tilde{z}) d\tilde{z} \\
 &= \frac{K_t}{K_{t+1}} \int_{\underline{z}}^{\bar{z}} \beta(1 + \pi(\tilde{z}; r_t)) \omega_t(\tilde{z}) f(z | \tilde{z}) d\tilde{z}
 \end{aligned}$$

Change of variable:

$$\tilde{a} = \frac{a}{\beta(1 + \pi(\tilde{z}))}$$



# System of Equations

- Given  $\{\omega_0(z)\}$  and  $K_0$ , equilibrium  $\{Y_t, Z_t, K_{t+1}, r_t, \omega_{t+1}(z)\}$  solve

$$Y_t = Z_t K_t$$

$$Z_t = \mathbb{E}_\omega[z \mid z \geq r_t] \equiv \frac{1}{\int_{r_t}^{\bar{z}} \omega_t(z) dz} \int_{r_t}^{\bar{z}} z \omega_t(z) dz$$

$$K_{t+1} = K_t \int_{\underline{z}}^{\bar{z}} \beta(1 + \pi(z; r_t)) \omega_t(z) dz$$

$$\lambda \int_{r_t}^{\bar{z}} \omega_t(z) dz = 1$$

$$\omega_{t+1}(z) = \frac{K_t}{K_{t+1}} \int_{\underline{z}}^{\bar{z}} \beta(1 + \pi(\tilde{z}; r_t)) \omega_t(\tilde{z}) f(z \mid \tilde{z}) d\tilde{z}$$

---

# Balanced Growth Path

- We define the balanced growth path (BGP) of this economy as the one
  - $\{Z_t, r_t, \omega_t(z)\}$  are constant over time:  $Z_t = Z, r_t = r, \omega_t(z) = \omega(z)$
  - $K_t$  and  $Y_t$  keep growing at the constant rate,  $1 + g \equiv Y_{t+1}/Y_t = K_{t+1}/K_t$

---

# Long-Run Cost of Financial Friction

---

# Calibration

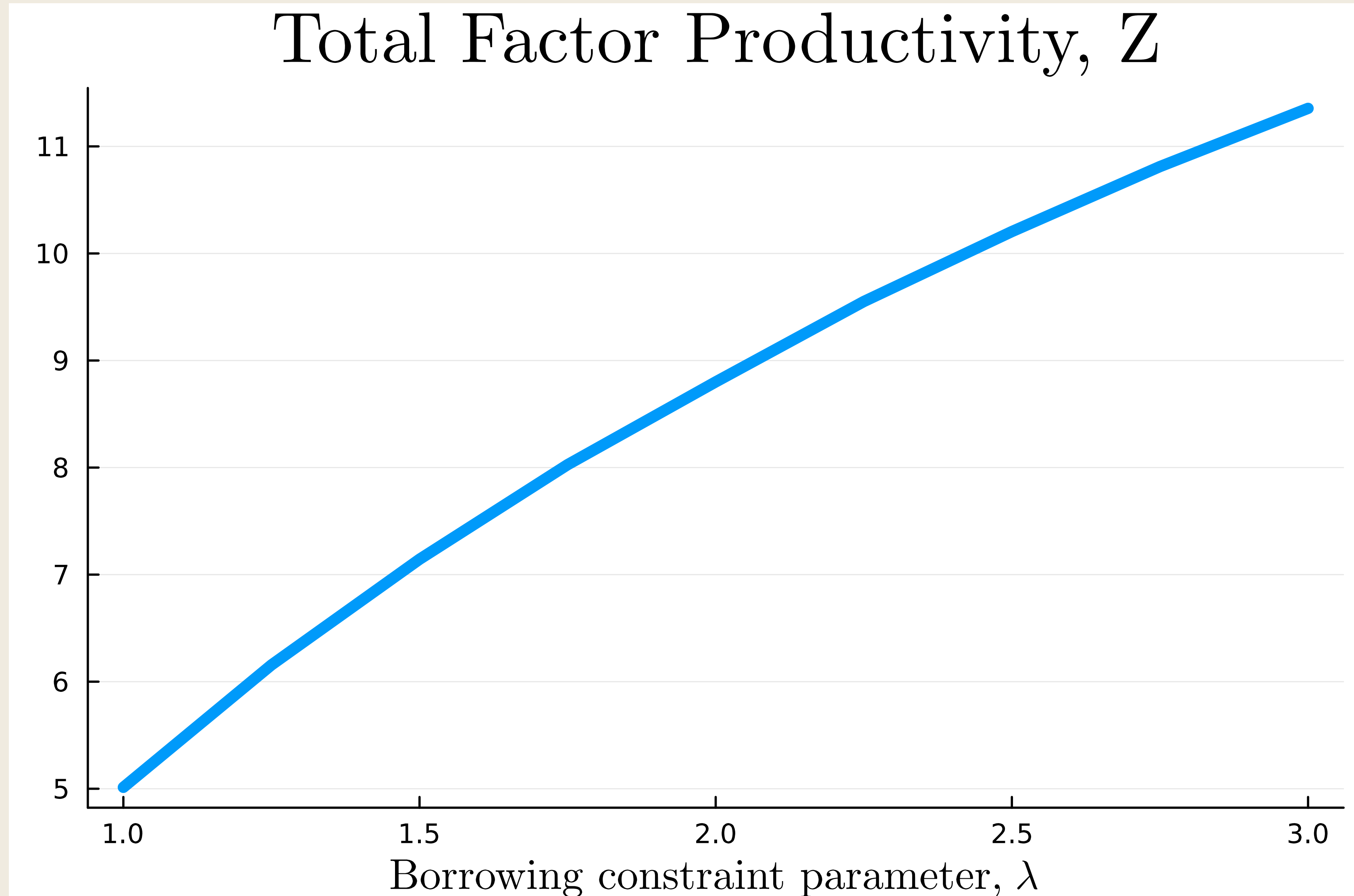
- A period is a year. Set  $\beta = 0.96$
- Parameterize the productivity process  $f(z' | z)$  as

$$\log z_{t+1} = \rho_z \log z_t + \epsilon_{t+1}, \quad \epsilon_{t+1} \sim N(0, (1 - \rho_z^2) \sigma_z^2)$$

- $\rho_z \in [0, 1)$  governs the persistence, and  $\sigma_z$  governs the variance
  - The unconditional distribution of  $\log z$  is  $\log z \sim N(0, \sigma_z^2)$
  - We truncate the distribution at  $[-6\sigma_z, 6\sigma_z]$
- Set  $\rho_z = 0.85$  and  $\sigma_z = 0.56$ 
    - The average reported in Asker, Collard-Wexler & De Loecker (2013)
  - Focus on the BGP and ask:  
How does financial friction,  $\lambda$ , affect the total factor productivity  $Z$ ?

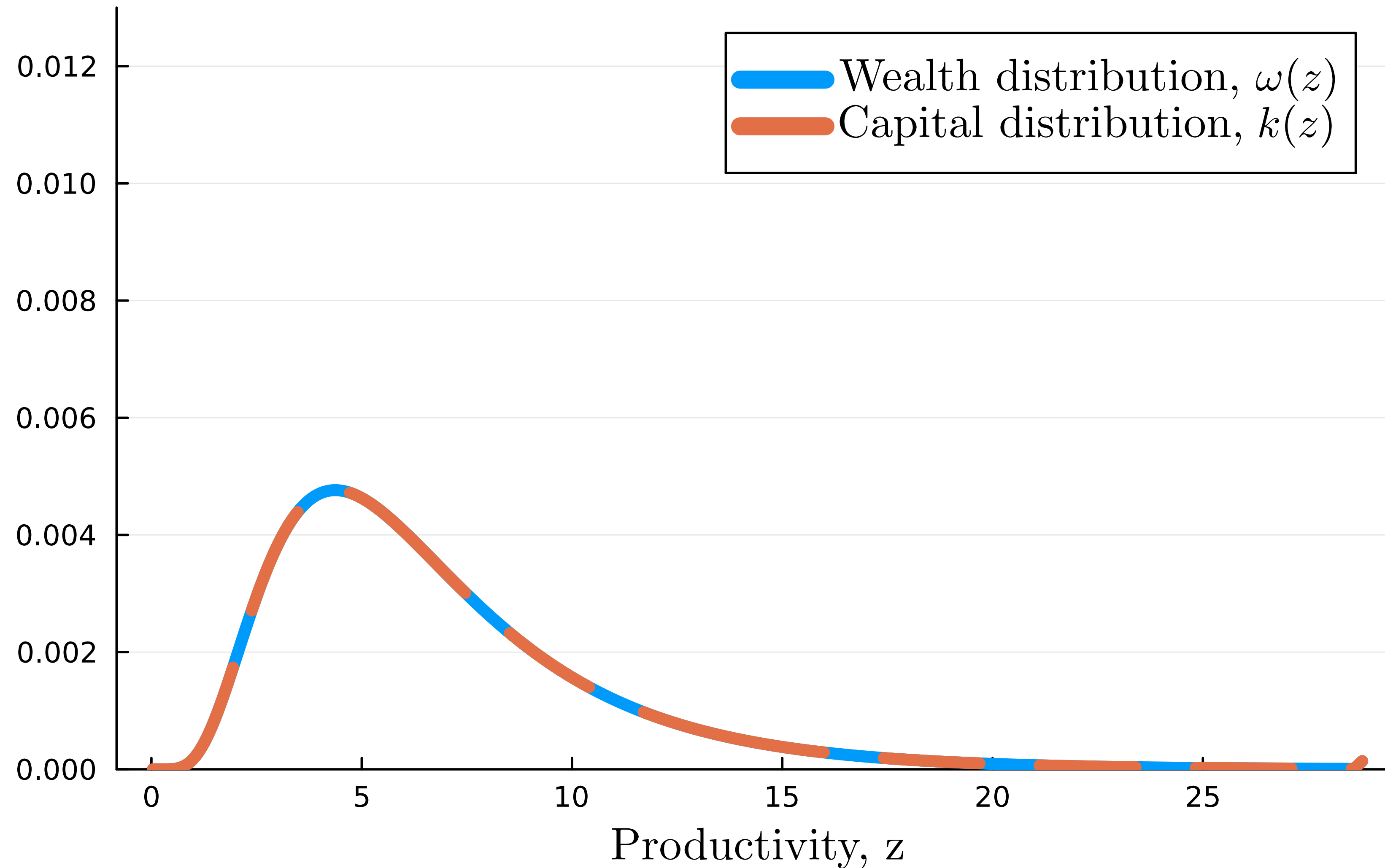


# TFP Losses from Financial Friction



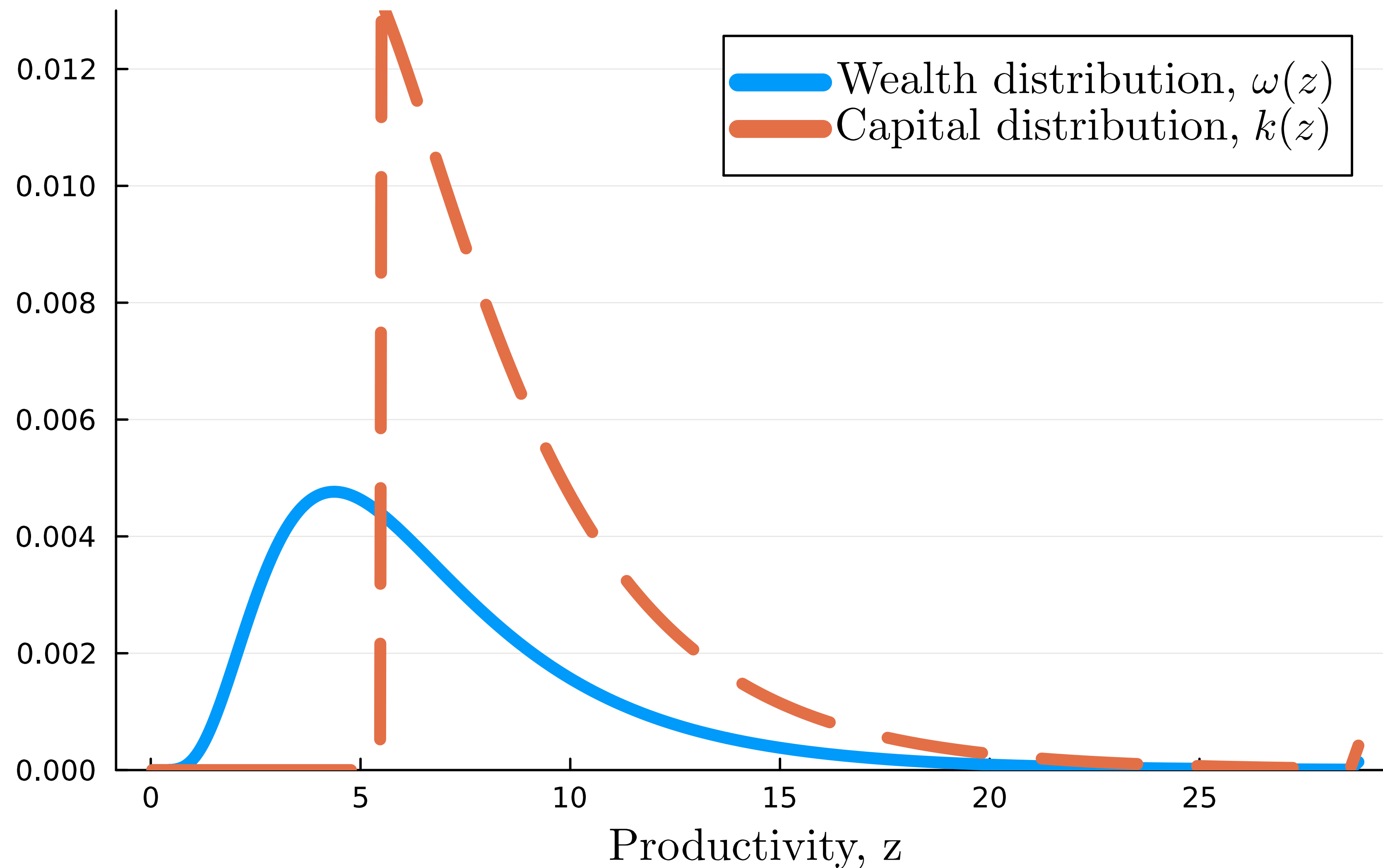
# Wealth and Capital Distribution

$$\lambda = 1.0$$



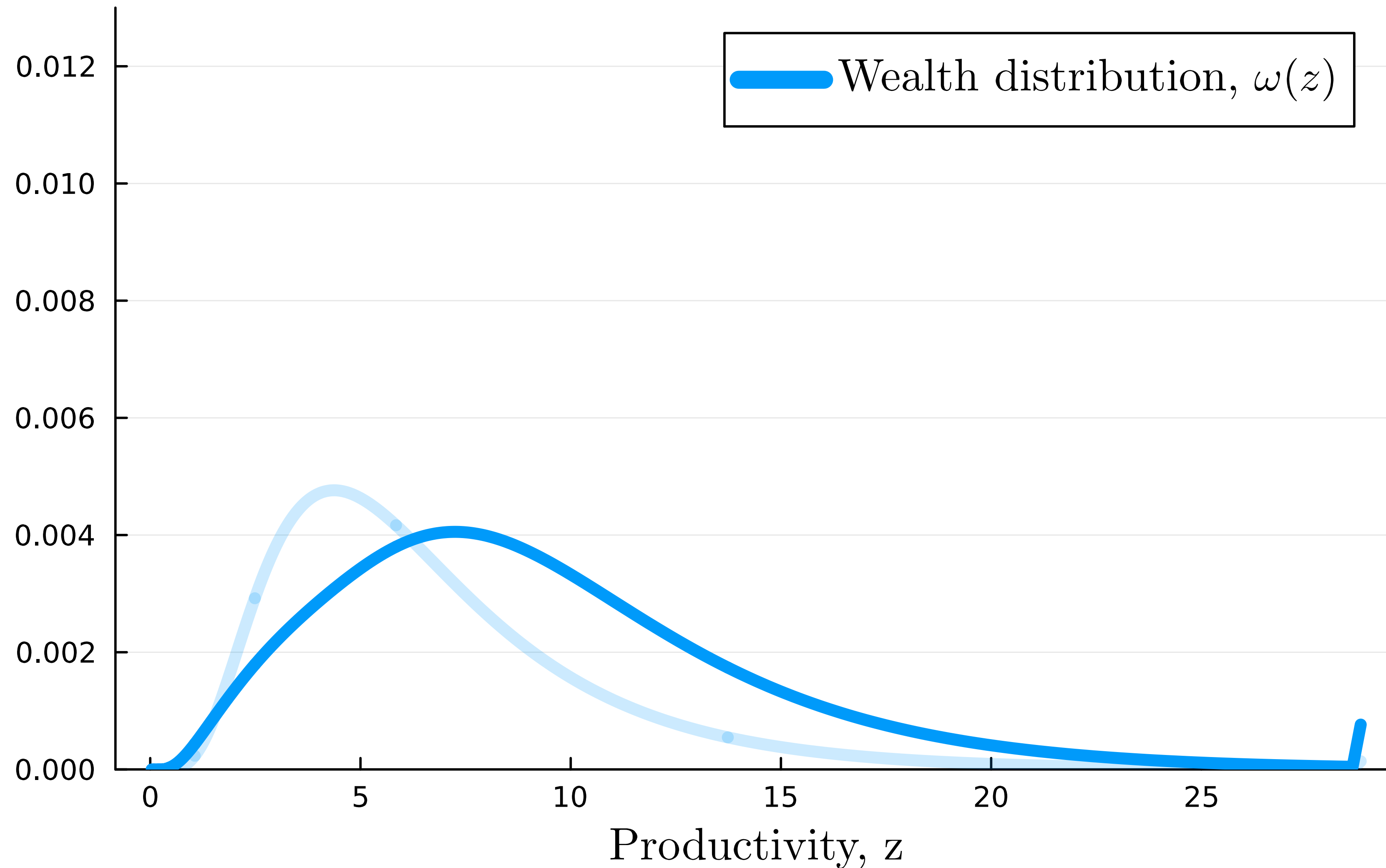
# Short-Run Effect of Higher $\lambda$

$$\lambda = 3.0$$



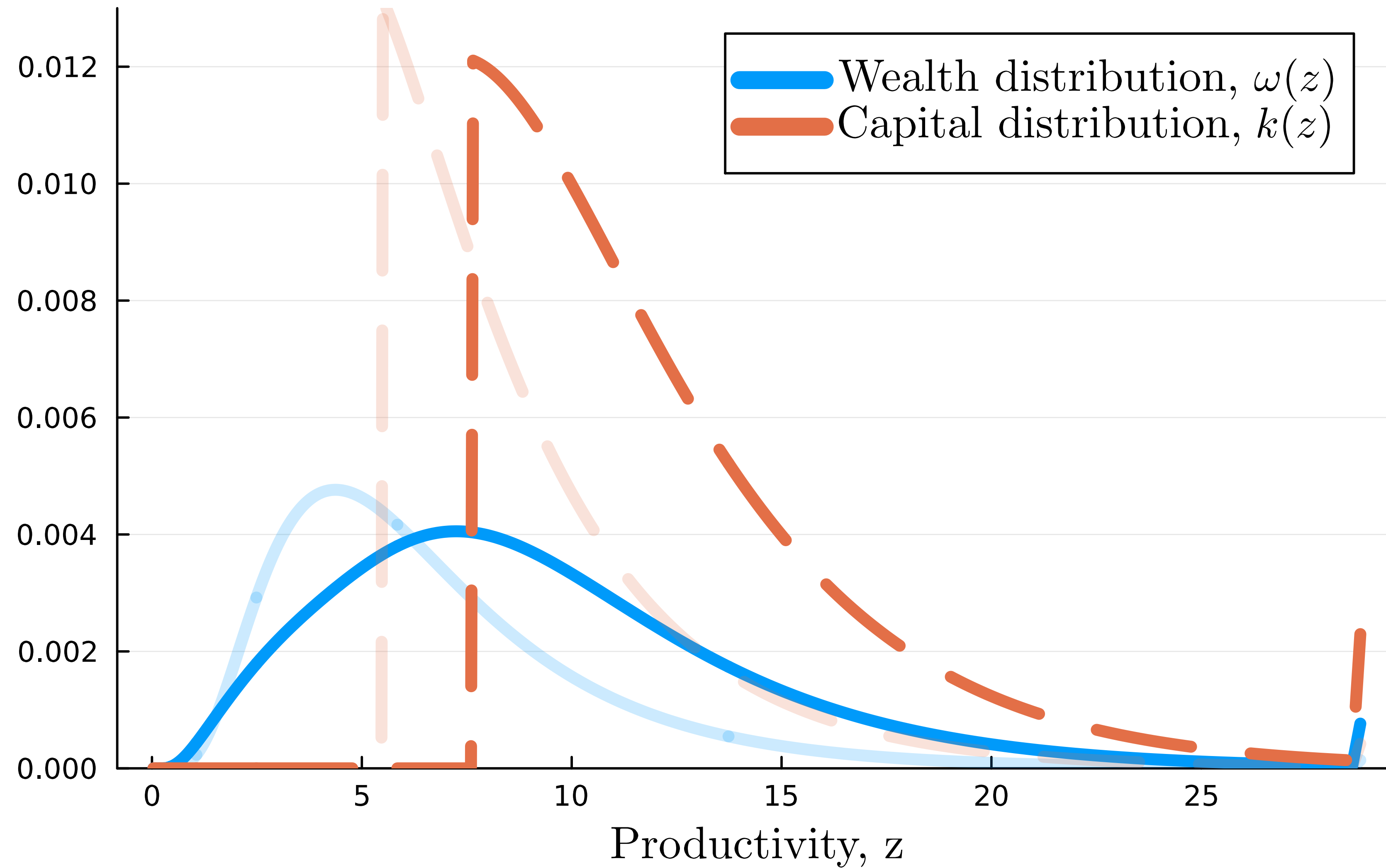
# Long-Run Effect of Higher $\lambda$

$$\lambda = 3.0$$



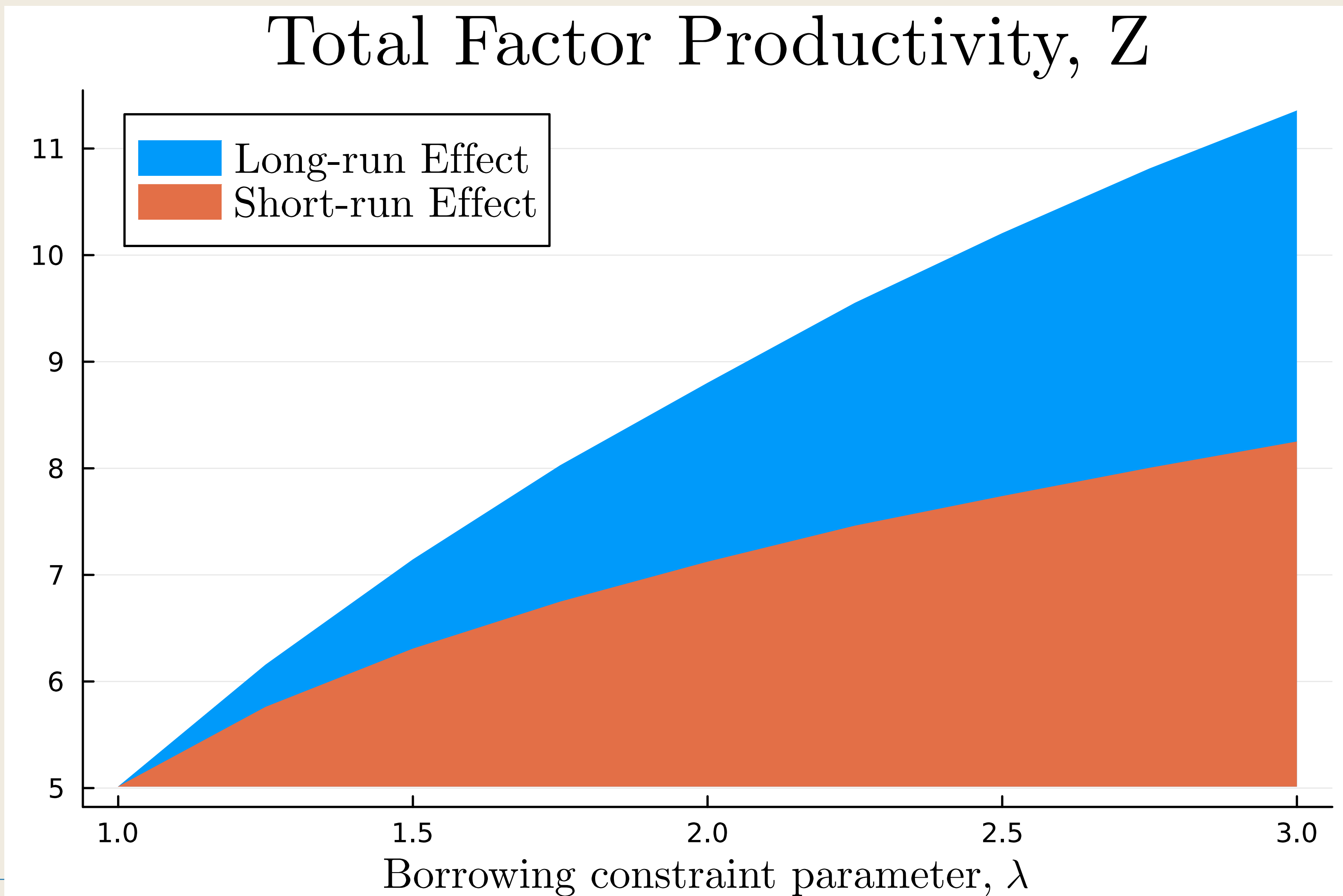
# Long-Run Effect of Higher $\lambda$

$$\lambda = 3.0$$





# Decomposition

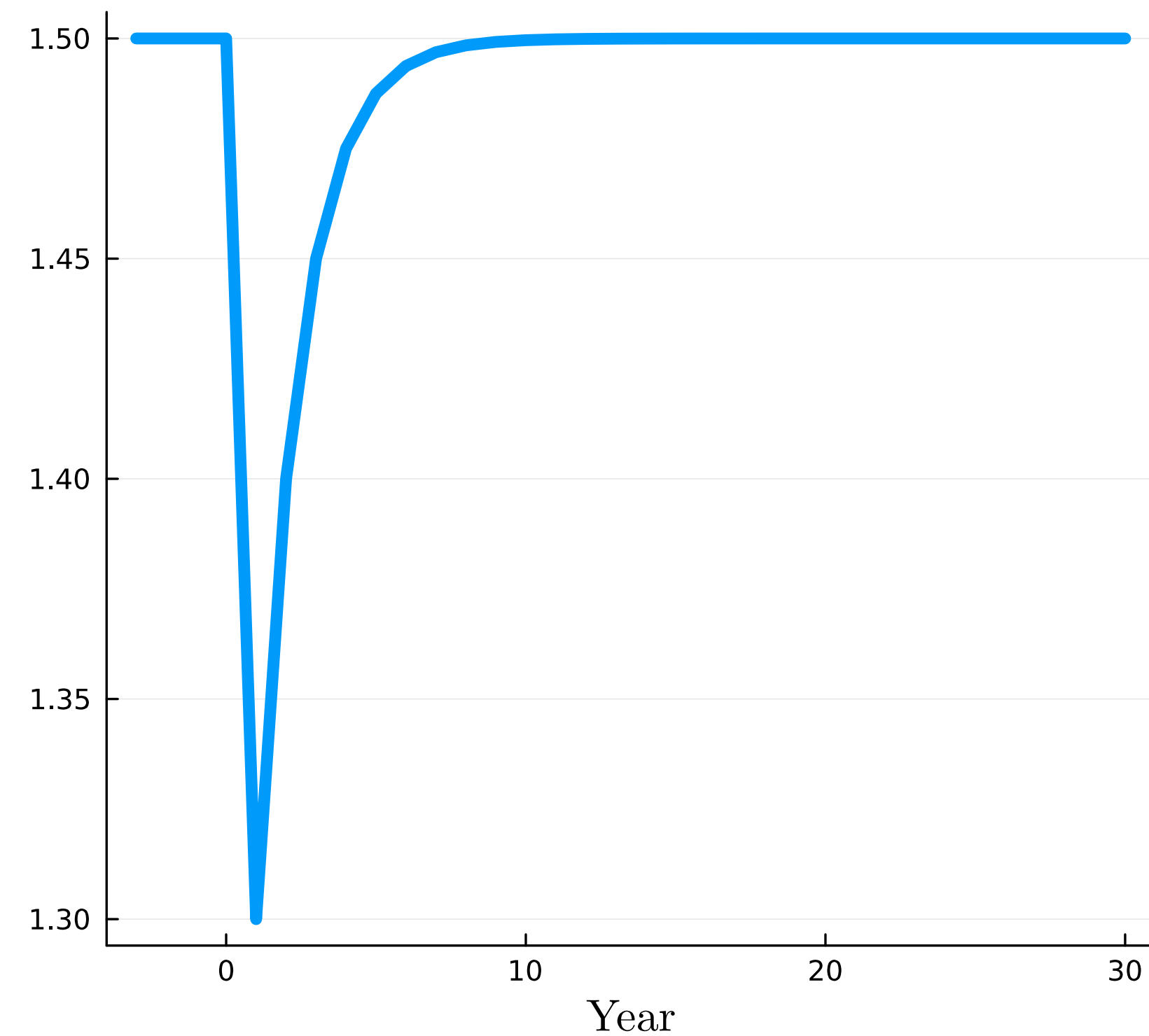


---

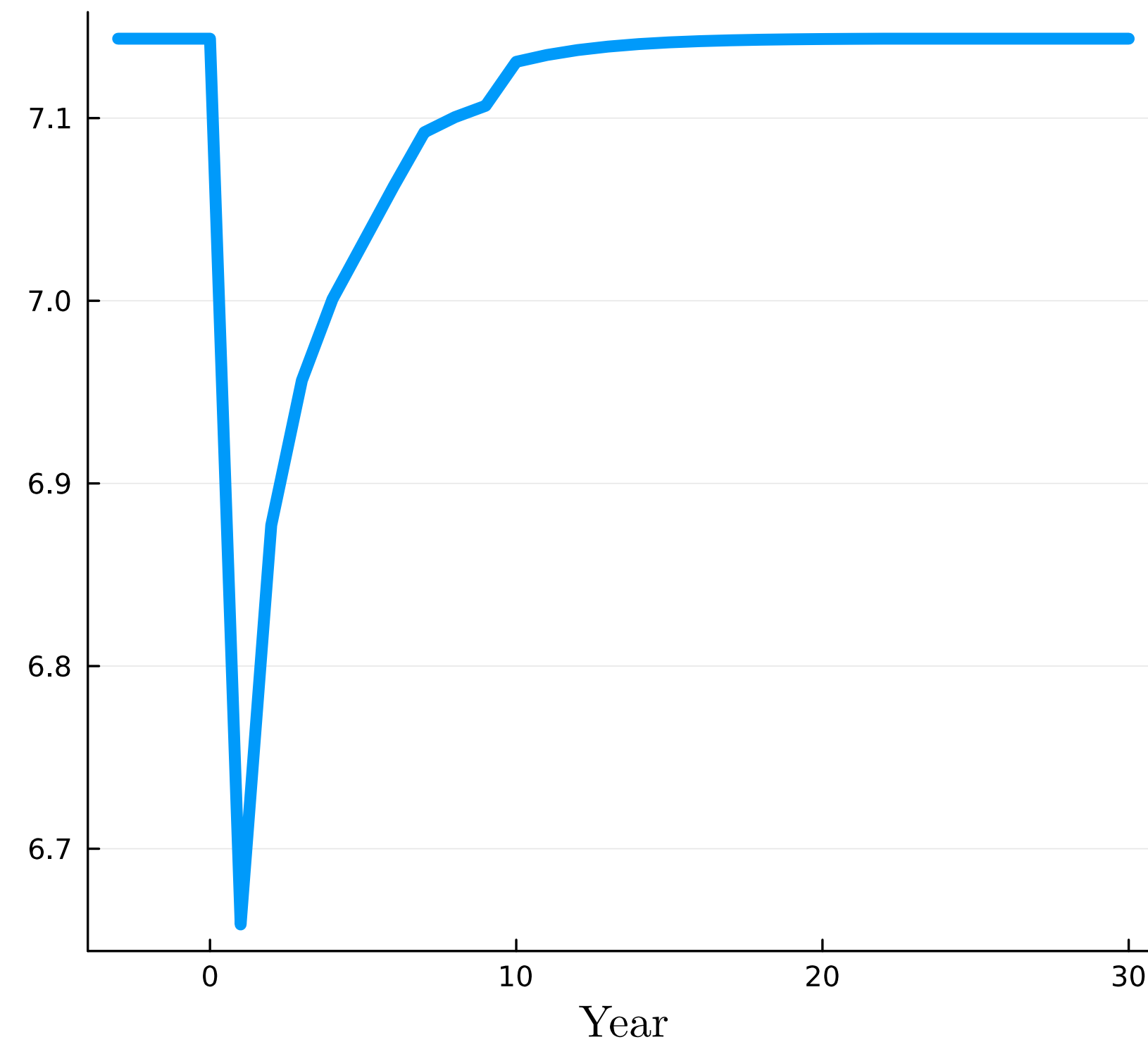
# **Short-Run Impact of Disruption in Financial Intermediation**

# Impulse Response to Credit Crunch

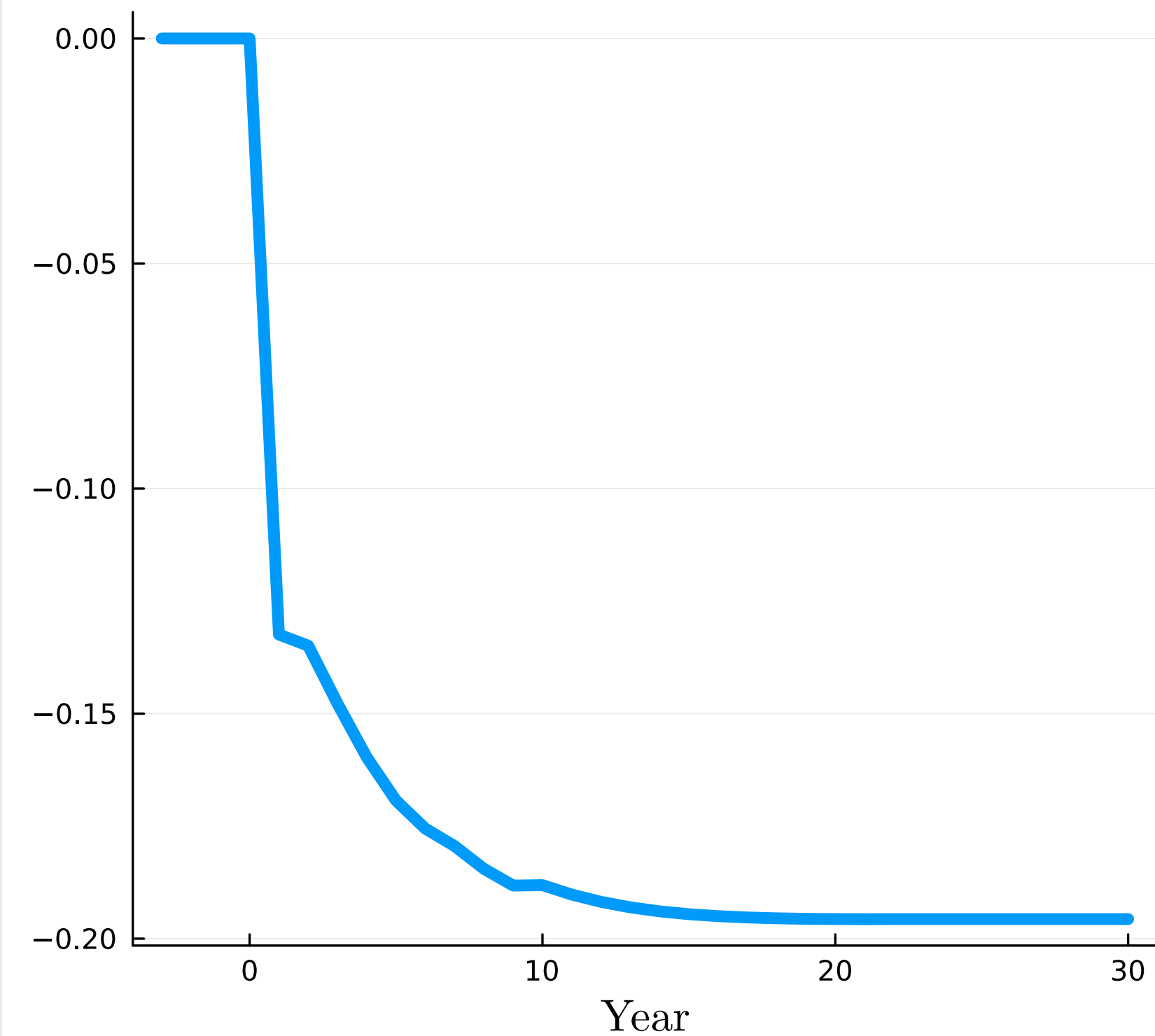
Borrowing constraint,  $\lambda$



Total Factor Productivity,  $Z$



Log Output,  $\log(Y)$



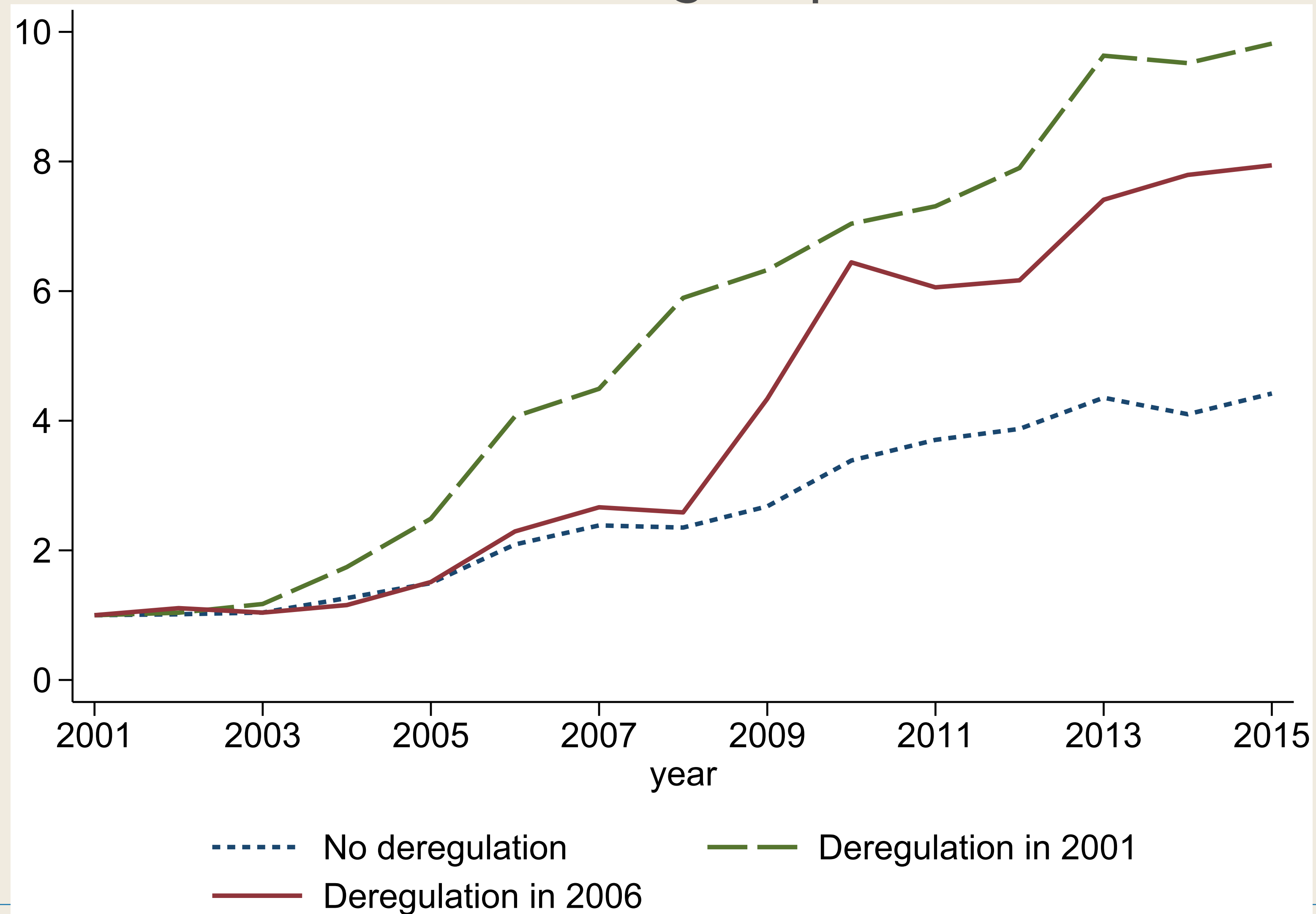
---

# Misallocation and Capital Market Integration

– Bau and Matray (2023)

# India's FDI Deregulation

## Flow of Foreign Equities





---

# Econometric Model

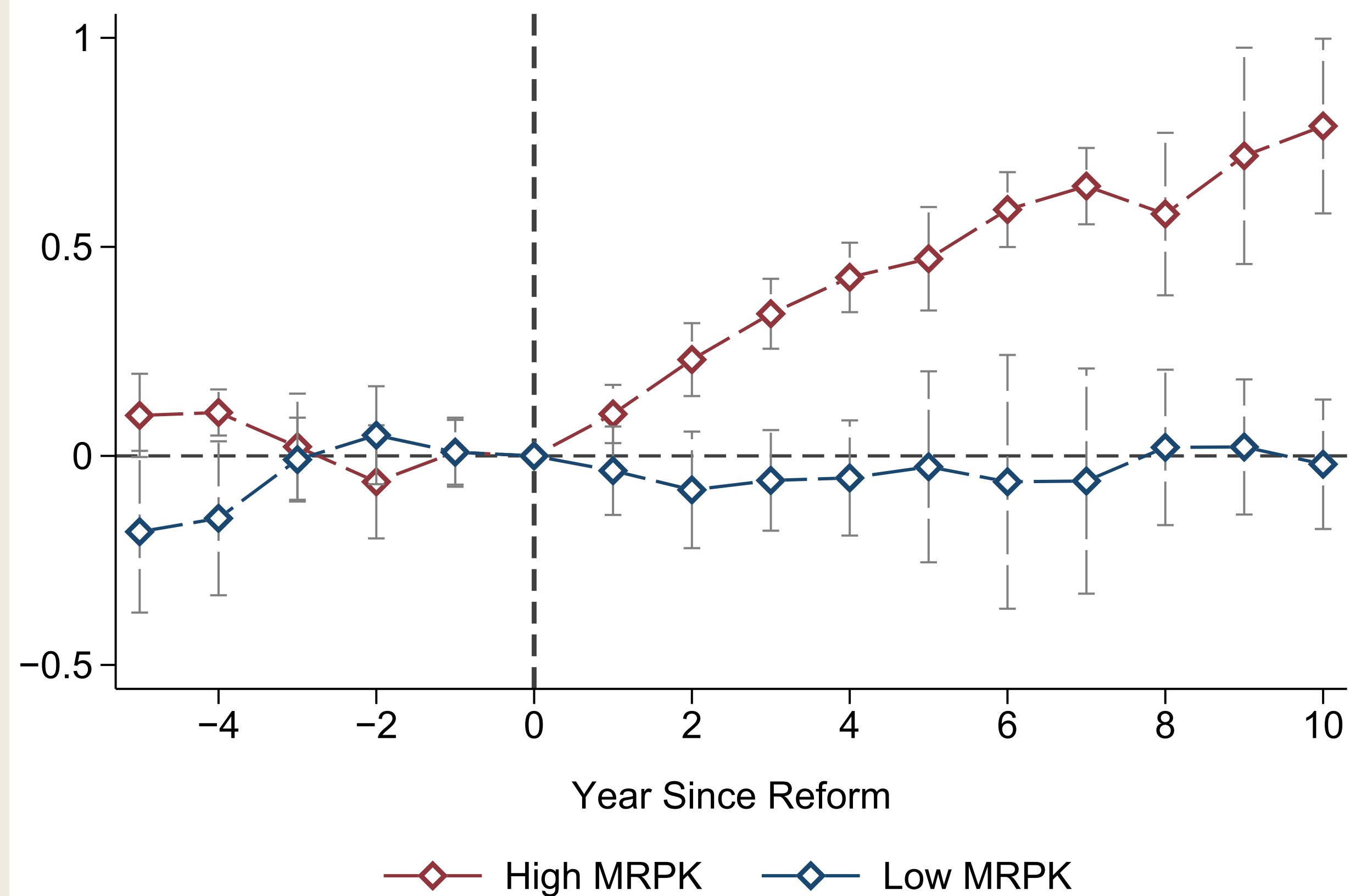
$$Outcome_{ijt} = \beta_1 Reform_{jt} + \beta_2 Reform_{jt} \times I_i^{High\ MRPK} + \mathbf{\Gamma X}_{it} + \theta_i + \delta_t + \epsilon_{ijt},$$

$i$ : firm,  $j$ : industry,  $t$ : year,  $MRPK$ : proxied by  $ARPK$  (valid under Cobb-Douglas)

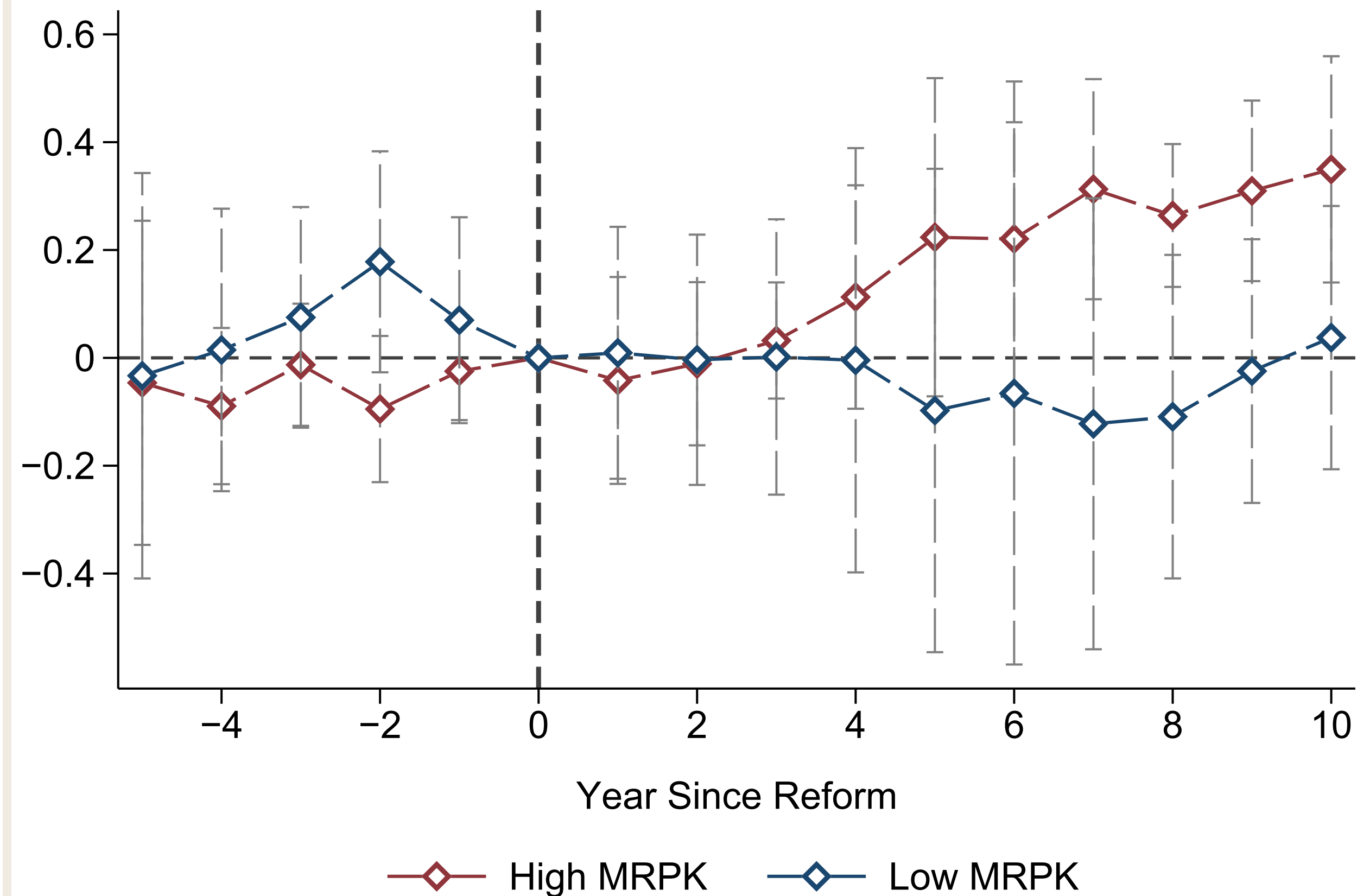
- FDI deregulation  $\approx$  relaxation of borrowing limit  $\lambda$
- Model predicts:
  - More productive (high MPRK) firms expand
  - Less productive firms should see no effect or contract

# Main Result

Physical Assets



Revenues



---

# Aggregate Impact of FDI Deregulation

- A simple aggregation:  
India's FDI deregulation in the 2000s increased TFP by 3-16%