Consumption, Wealth, and Income Inequality

Gaillard, Hellwig, Wangner, and Werquin (2024)

704 Macroeconomics II
Topic 8

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Inequality: Model vs. Data

- We have covered two classes of incomplete market models:
  - Idiosyncratic shock to return of savings (Moll, 2014, with many predecessors)
  - Idiosyncratic shock to labor income (Belwley-Hugget-Aiyagari)
- These models naturally generate inequality in income, wealth, and consumption
- Are they consistent with the data?
- We focus on **top** inequality ($\approx$ top 10%) because
  (i) This is where the theory has strong predictions
  (ii) This might be on its own interest as they drive the aggregate

Top 10% Wealth Share

Consumption, Wealth, and Income Inequality in the Data

Data

- Panel Study of Income Dynamics (PSID) 2004-2021
- Wealth refers to net worth = assets - liabilities
- Labor income: gross of taxes, benefits & employee payroll deduction
- Capital income: dividends, interests, business income, rents, capital gains, etc
- Consumption: total expenditure including various categories

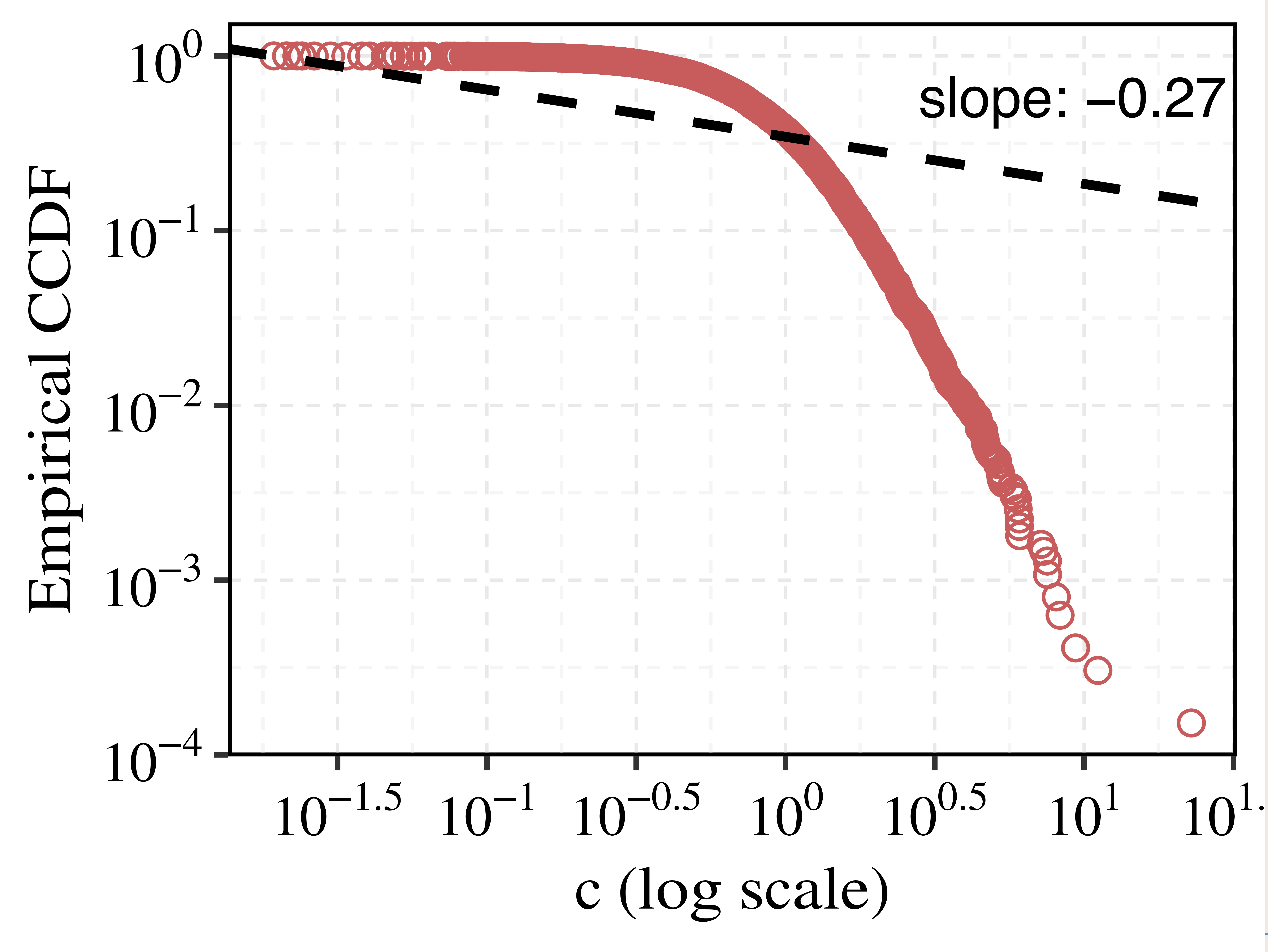
Complmentary CDF

- We are interested in the relationship between
  1. Level of x (consumption, wealth, and income)
  2. Ranking of x in the distribution:

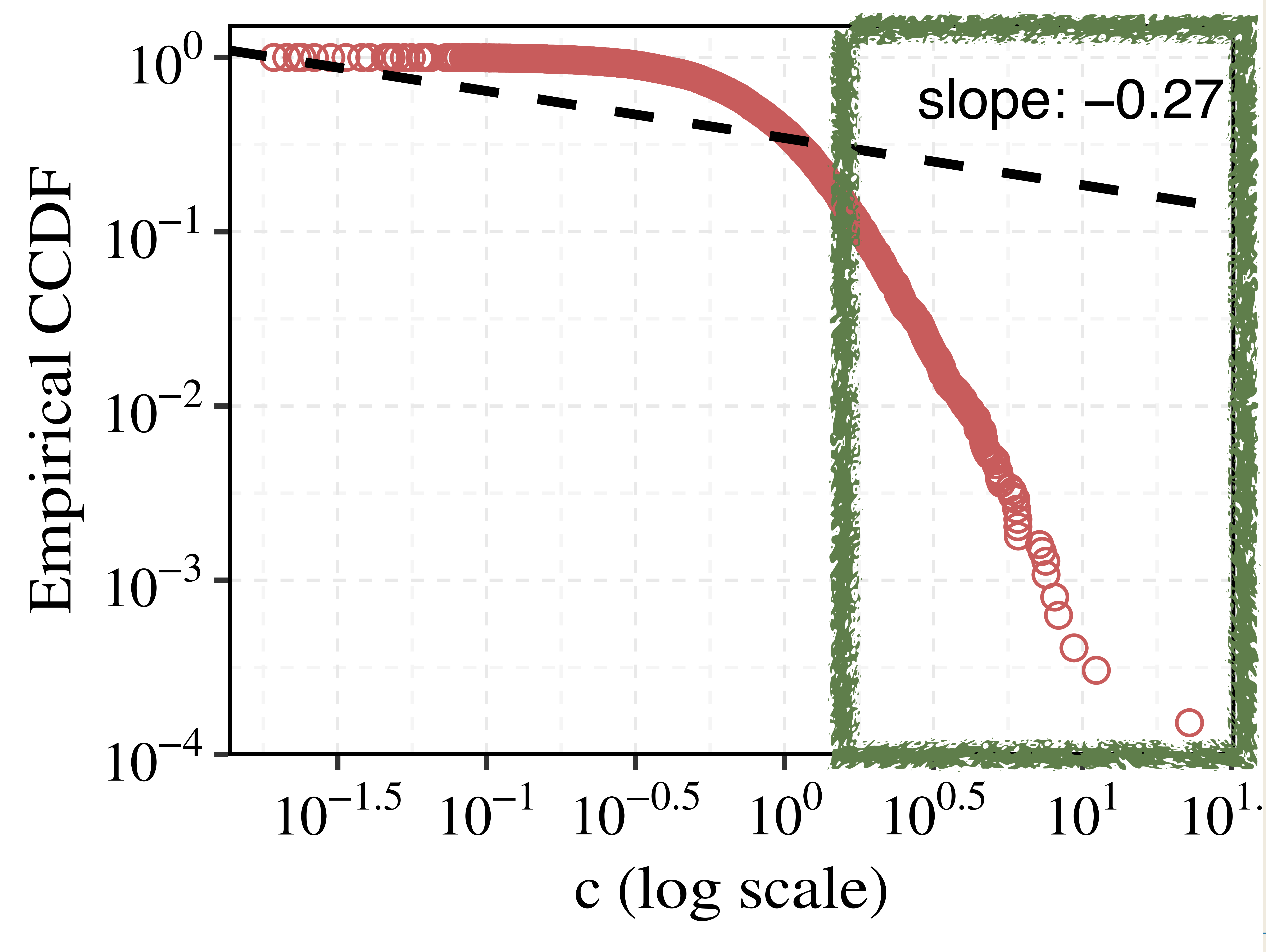
$$\text{Prob}(\tilde{x} > x) = \underbrace{1 - F(x)}_{\text{Complementary CDF}}$$

- Moreover, we look at log-log relationships
- This answers the following question:
  "If x increases by 1%, how much does the ranking increase in percentage terms?"

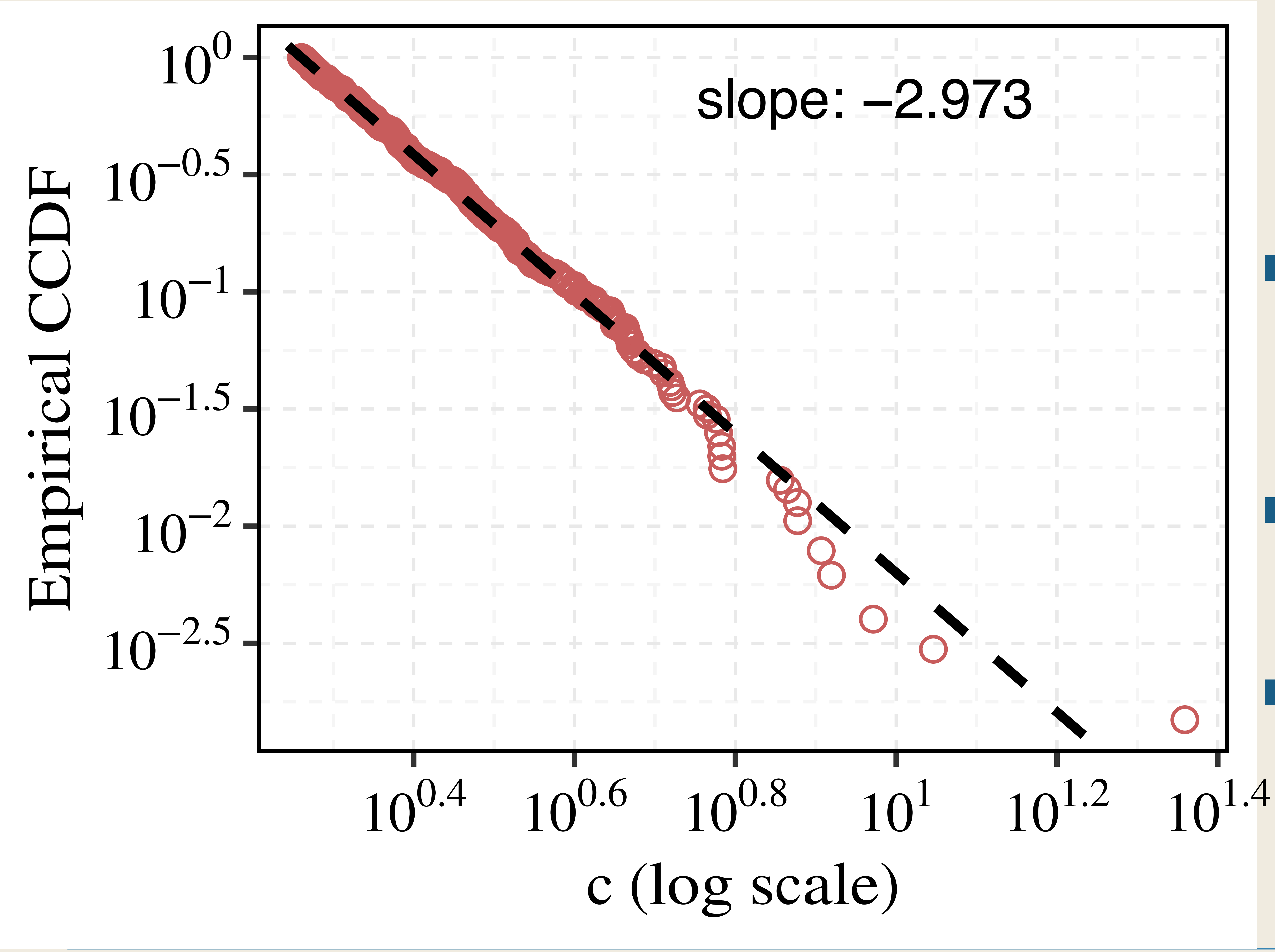
Consumption Distribution in 2004



Consumption Distribution in 2004



Top 10% Consumption Distribution in 2004



- log-rank approximately log-linear:

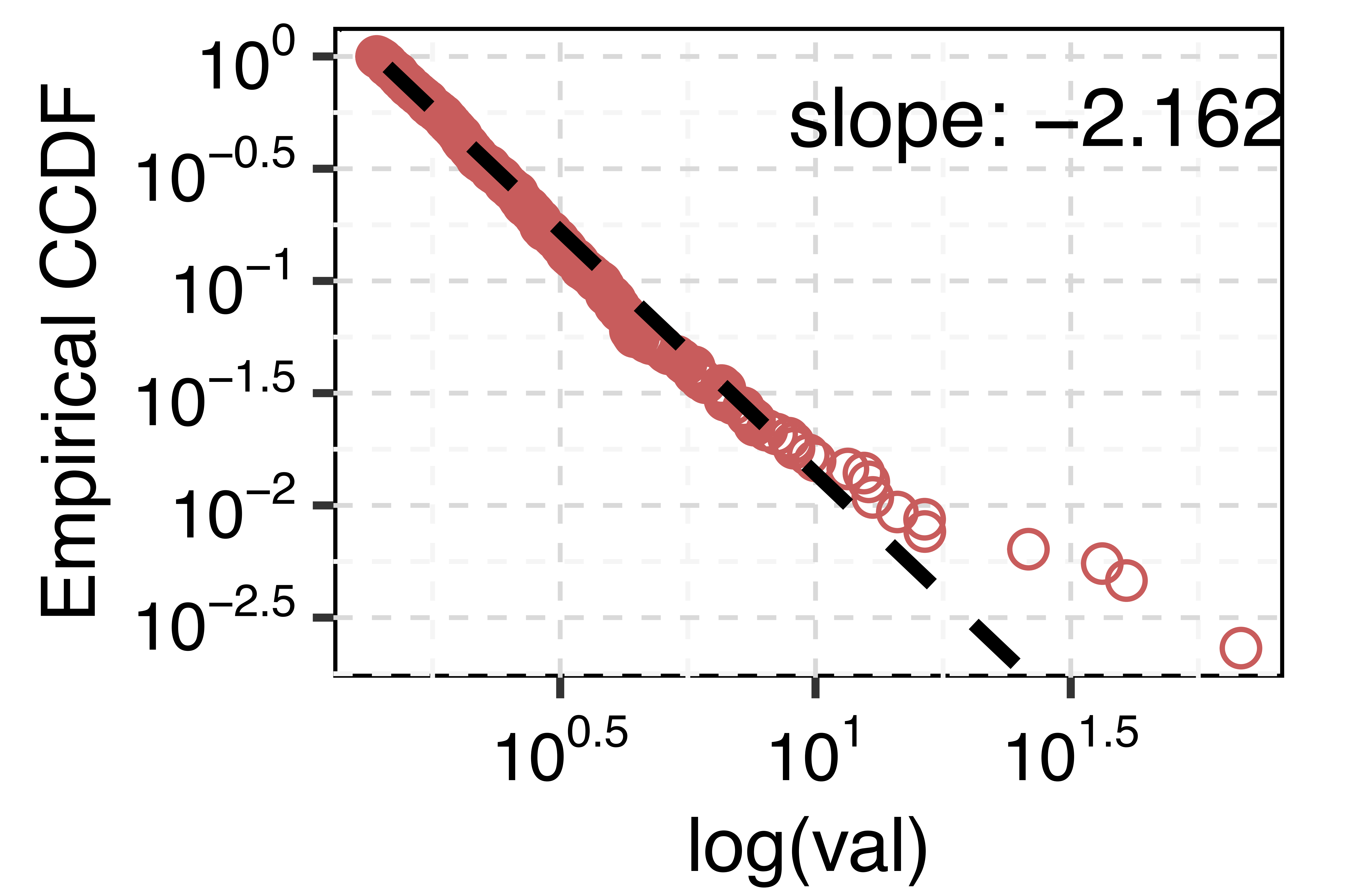
  $\log \Pr(\tilde{x} > x) \approx -\zeta \log x + const$

- What is this distribution?

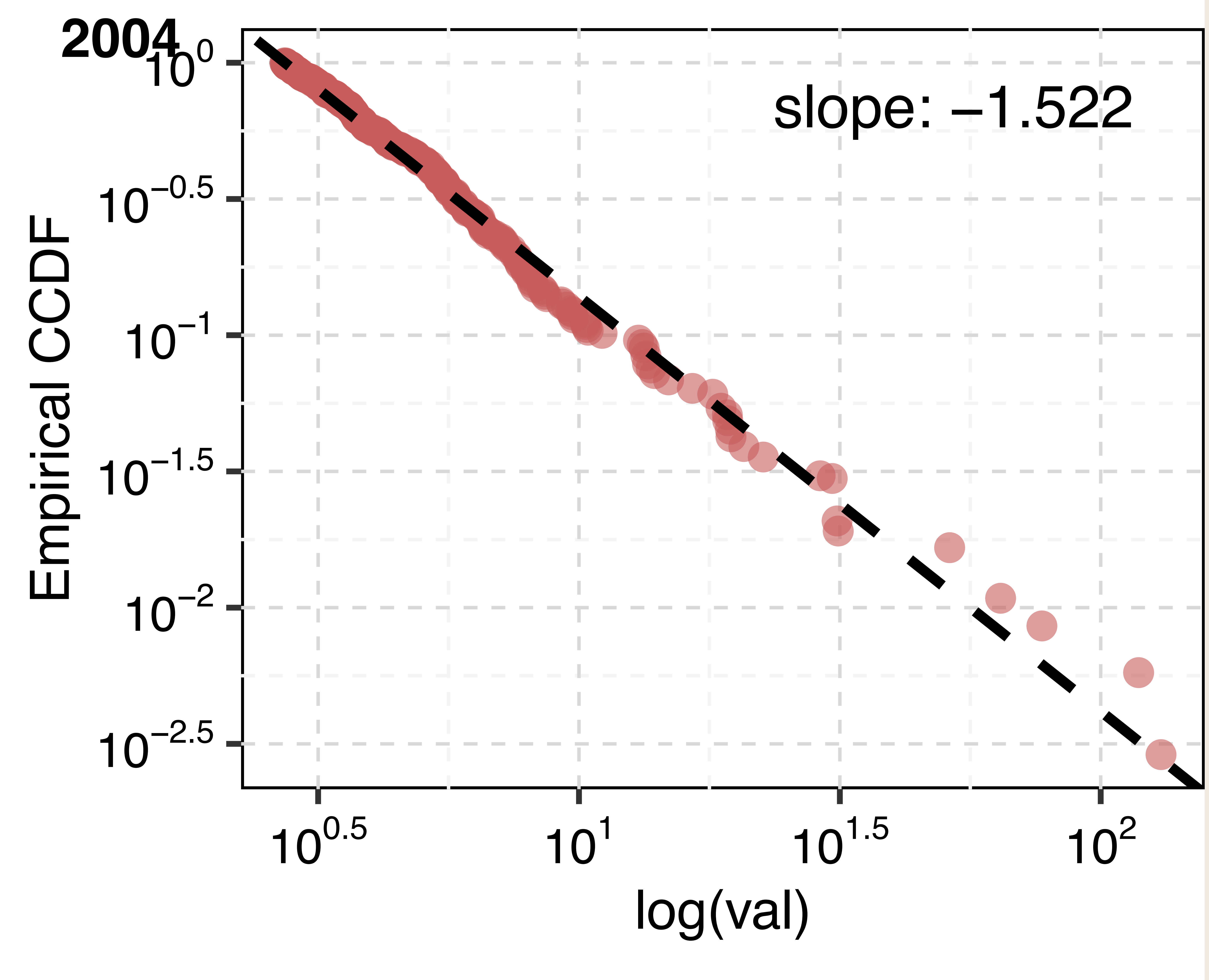
  – Pareto: $\Pr(\tilde{x} > x) = (x/\underline{x})^{-\zeta}$

- This is called "power law"

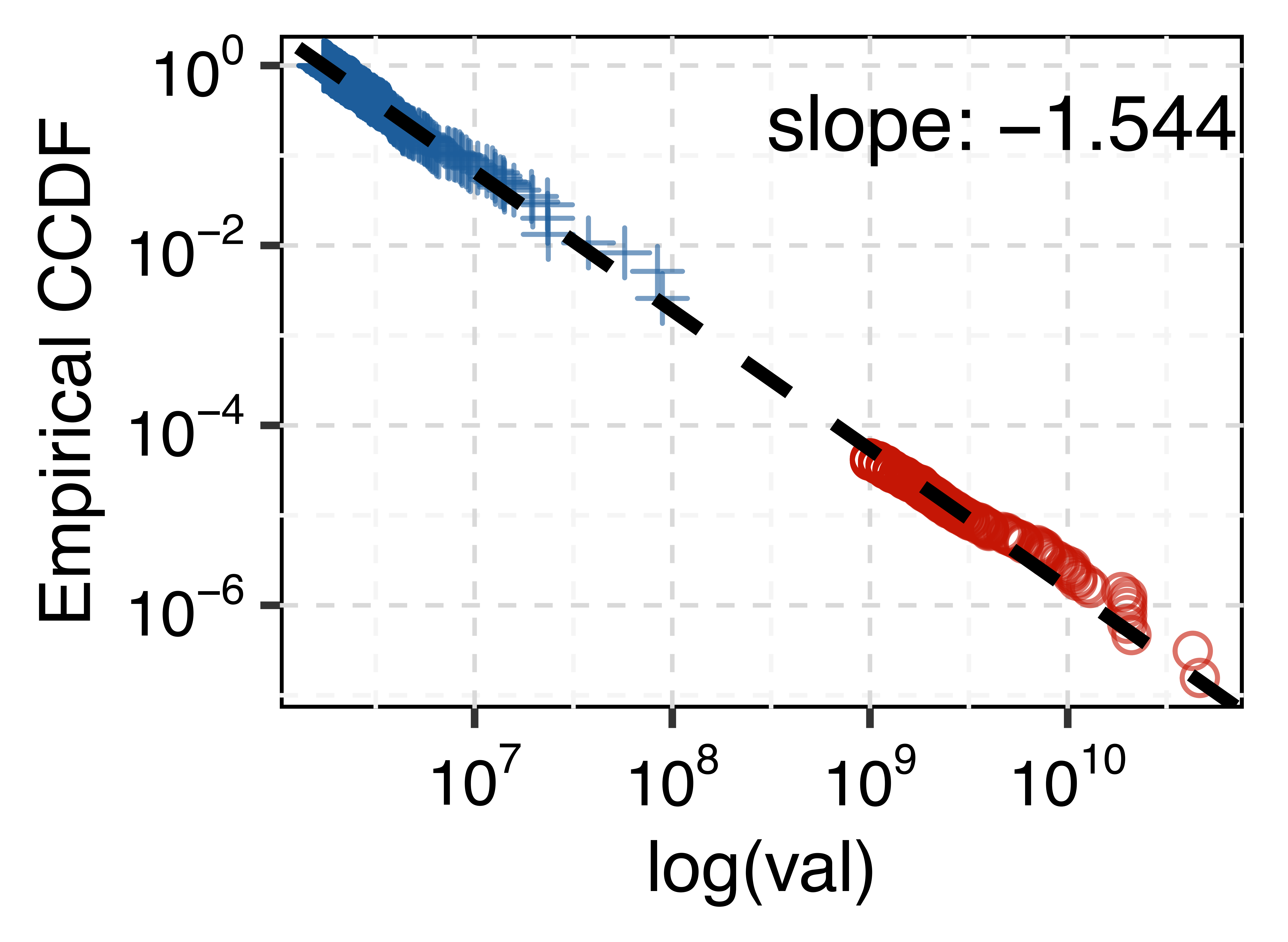
Top 10% Labor Income Distribution in 2004



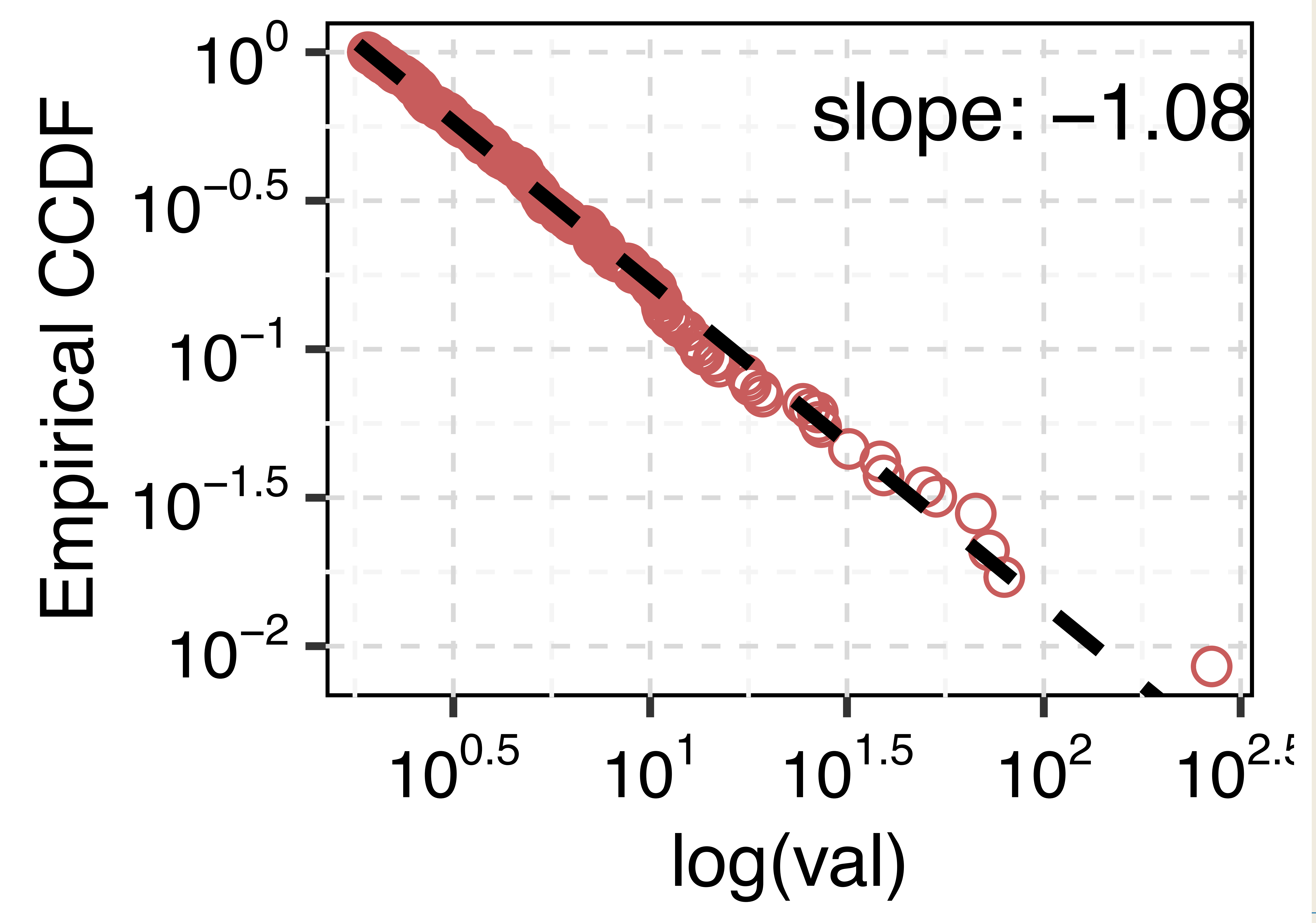
Top 10% Wealth Distribution in 2004



Forbes 500 Rich List



Top 10% Capital Income Distribution in 2004



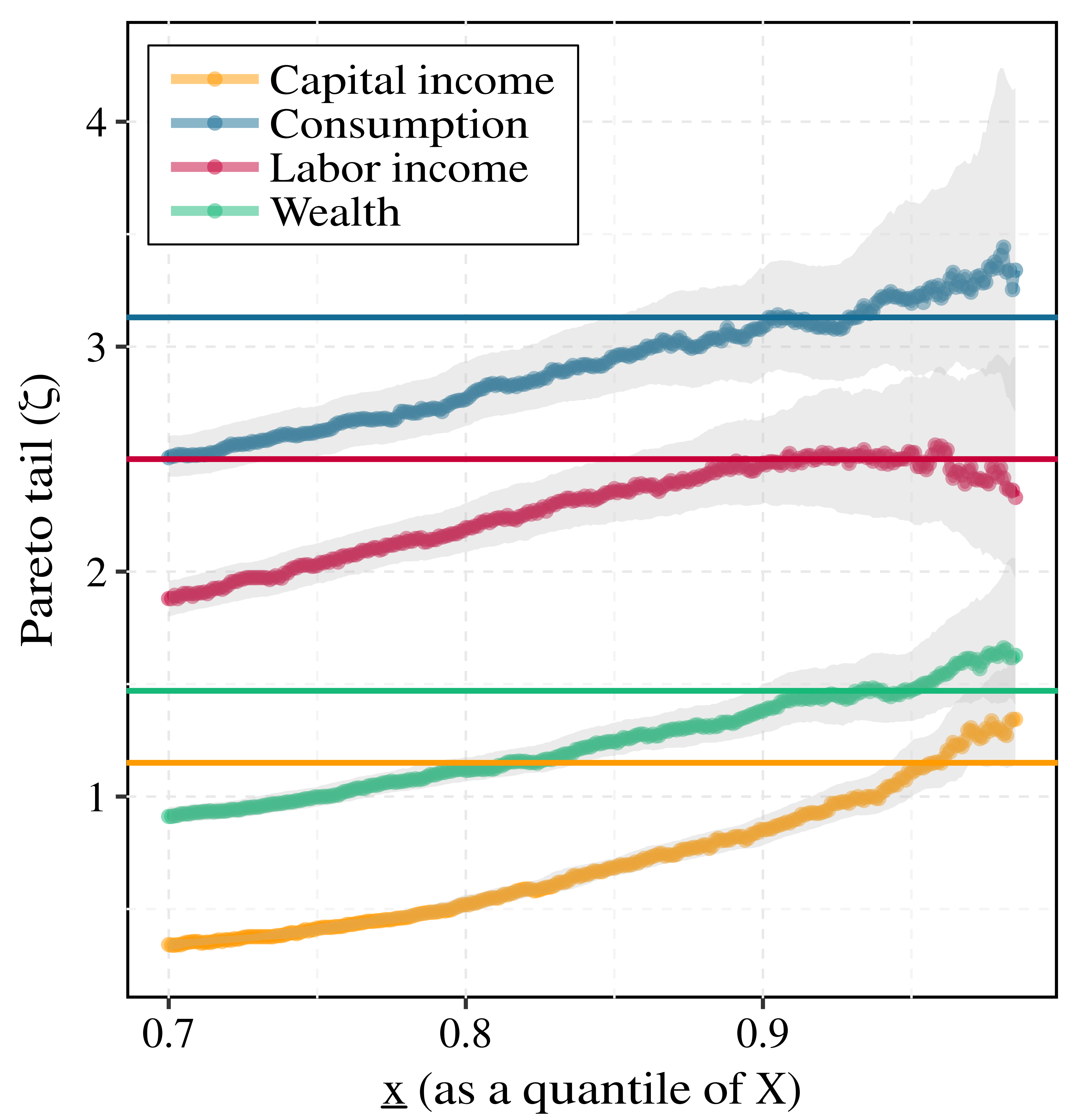
Power Laws in Economics

"Paul Samuelson (1969) was once asked by a physicist for a law in economics that was both nontrivial and true… Samuelson answered, 'the law of comparative advantage.'

A modern answer to the question posed to Samuelson would be that a series of power laws count as actually nontrivial and true laws in economics."

— Gabaix (2016)

Ranking of Pareto Tail



- Ranking of Pareto tails:

  1. Consumption
  2. Labor income
  3. Wealth
  4. Capital income

  from less to more unequal

Consumption, Wealth, and Income Inequality in the Model

Bewley-Hugget-Aiyagari

$$V(a, y) = \max_{c, a' \geq -\phi} \frac{c^{1-\gamma}}{1-\gamma} + \beta \mathbb{E}_{y'} V(a', y')$$

$$\text{s.t.} \quad c + a' = (1 + r)a + y$$

- We will take labor income distribution as an input to the model,

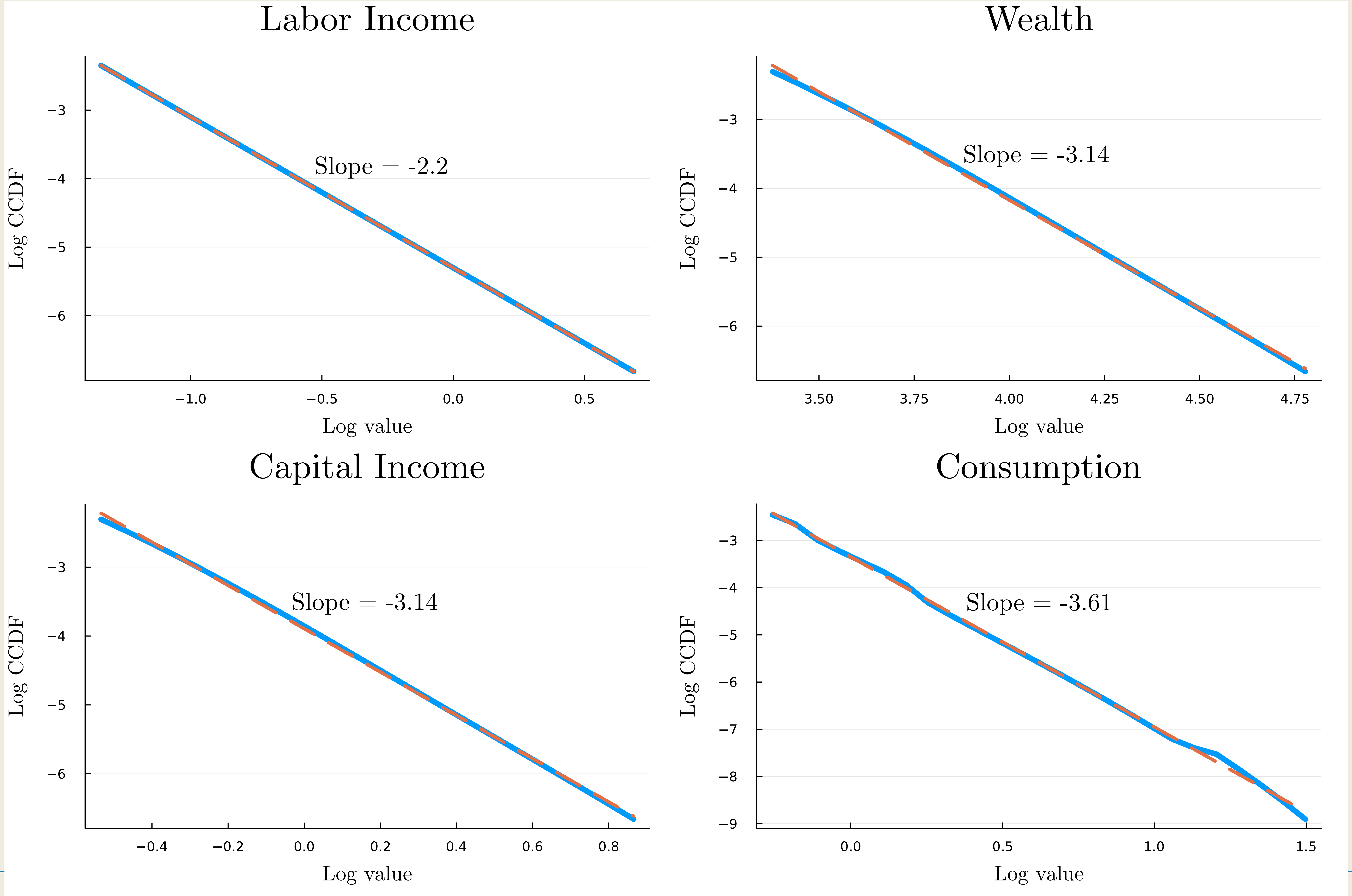
$$y' = \begin{cases} y & \text{with prob. } p \\ \tilde{y} \sim \text{Pareto}(\zeta_y, \underline{y}) & \text{with prob. } 1 - p \end{cases}$$

  where ζ_y is the shape parameter, and $\underline{y}$ is the scale parameter

- Can it generate cons. (c), wealth (a), and capital income (ra) inequality in the data?

- Throughout, assume $\beta(1 + r) < 1$

Power Laws in Bewley-Hugget-Aiyagari



Failure of Canonical Models

Consider the canonical incomplete market model described earlier. Suppose that the stationary distribution of y features asymptotic power law with Pareto tail ζ_y:

$$\lim_{y\to\infty} \frac{\log \Pr(\tilde{y} > y)}{\log y} = -\zeta_y$$

Then, the stationary distribution features

$$\underbrace{\lim_{a\to\infty} \frac{\log \Pr(\tilde{a} > a)}{\log a}}_{\equiv -\zeta_a} = \underbrace{\lim_{c\to\infty} \frac{\log \Pr(\tilde{c} > c)}{\log c}}_{\equiv -\zeta_c} = \underbrace{\lim_{ra\to\infty} \frac{\log \Pr(\widetilde{ra} > ra)}{\log (ra)}}_{\equiv -\zeta_{ra}} = -\zeta_y \, .$$

- See Stachursk and Toda (2019) and Gaillard et al. (2024) for proofs
- Tail behavior of a, c, ra inherits the tail behavior of y in BHA

Intuition

Why $\zeta_a = \zeta_y$?

- Loosely speaking, this is because

  1. $a \propto$ sum of y (in BHA, the richest households = high labor income for a while)
  2. a sum of Pareto asymptotically follows Pareto with the same tail

Why $\zeta_a = \zeta_c$?

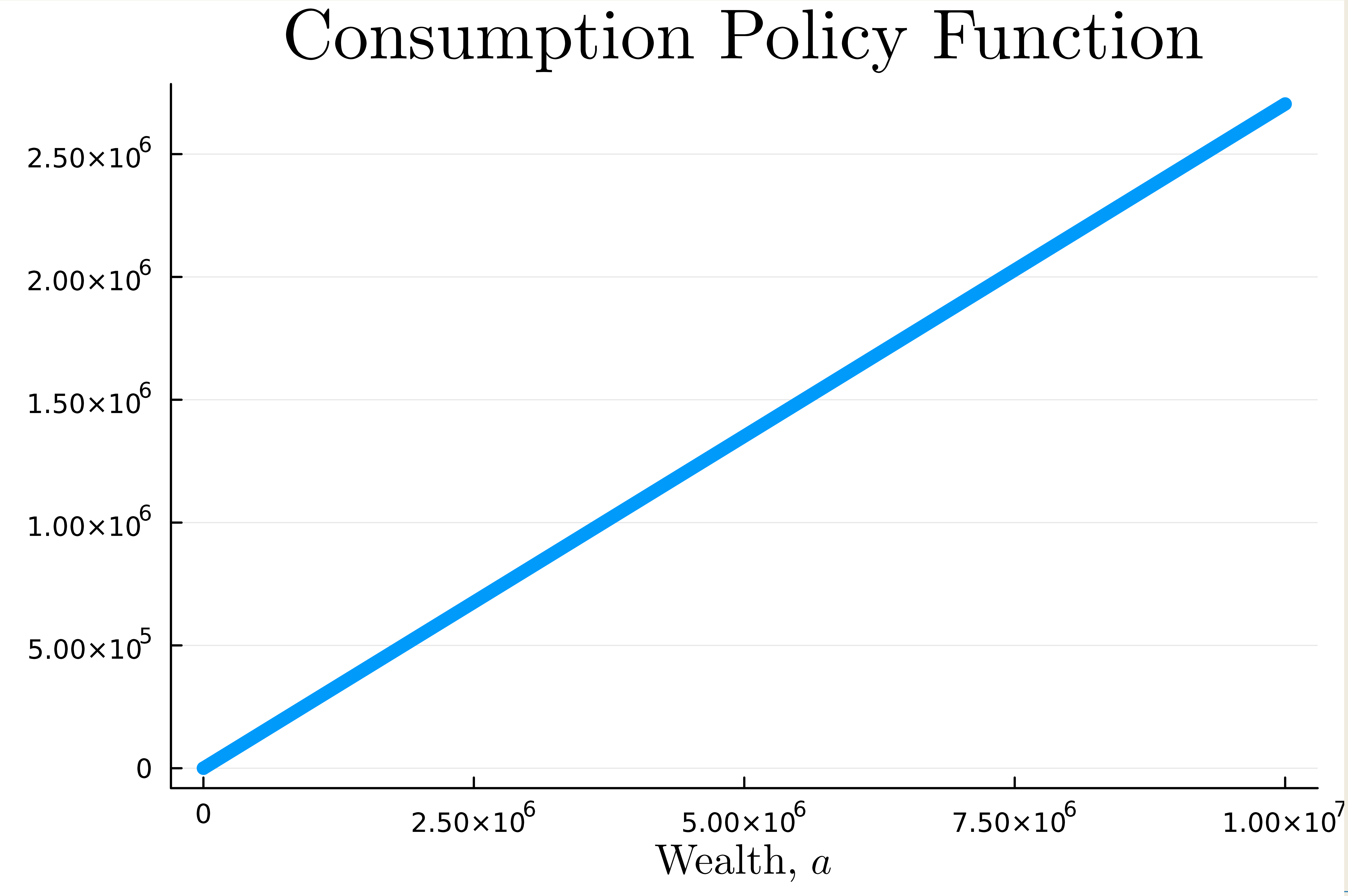
- This is because $c \propto a$ as $a \to \infty$

- As $a \to \infty$, precautionary saving motive disappears and acts on permanent income

Why $\zeta_a = \zeta_{ra}$?

- This quite mechanically follows since r is a constant

Consumption Policy Linear in Wealth



Return Heterogeneity

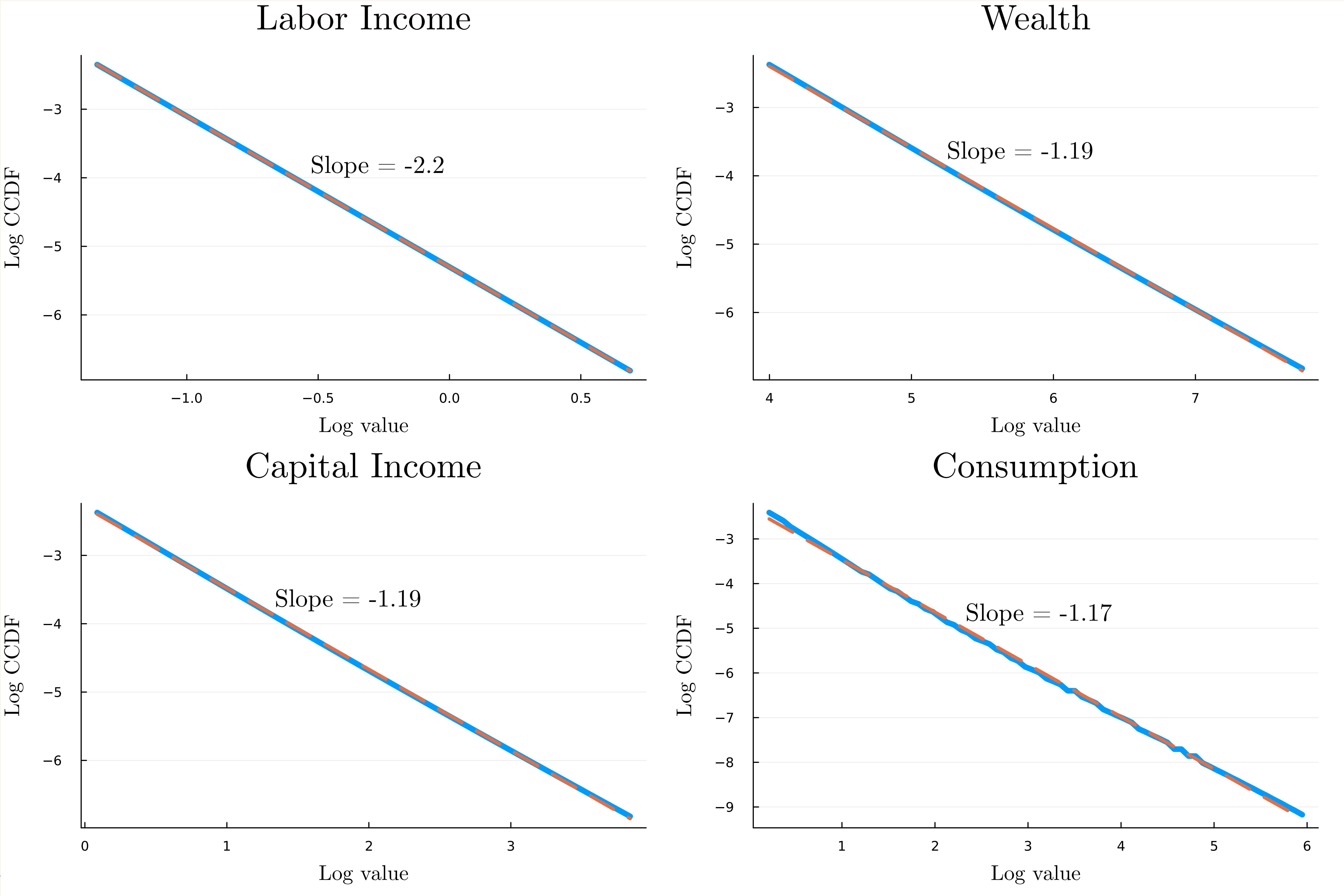
- In the data, heterogeneity in return plays a much more important role
  - See Hubmer et al. (2024) for the most recent evidence from Norway
- Let us add idiosyncratic shocks to return like we did in Moll (2014):

$$V(a, y, z) = \max_{c, a' \geq -\phi} \frac{c^{1-\gamma}}{1-\gamma} + \beta \mathbb{E}_{z', y'} V(a', y', z')$$

$$\text{s.t.} \quad c + a' = (1 + r z) a + y$$

where $z \in \{z_1, \ldots, z_K\}$ is drawn independently over time and across households

With Return Heterogeneity



Return Heterogeneity Leads to Concentrated Wealth

Consider the model with return heterogeneity described earlier.

The stationary distribution, if it exists, features

1. For sufficiently high $\mathbb{E}[z]$ or $\mathrm{Var}(z)$, $\zeta_a < \zeta_y$
2. $\zeta_c = \zeta_{ra} = \zeta_a$

- See Beare and Toda (2022) and Gaillard et al. (2024) for proofs
- Return heterogeneity provides a powerful force for wealth inequality
  - Unlike labor income, return keeps multiplying wealth

Intuition

- Even with return heterogeneity, $c \propto a$ as $a \to \infty$
  - Consumption is an asymptotically linear function of wealth
  - Consequently, the tail of c inherits the tail behavior of a
- Capital income is rza
  - A product of random variables follows Pareto with the thicker tail of the two
  - Here, z is bounded, hence rza asymptotically follows Pareto with tail ζ_a

Solving the Puzzle

- Two extensions:
  1. nonhomothetic wealth in the utility
  2. scale-dependent return
- Bellman equation:

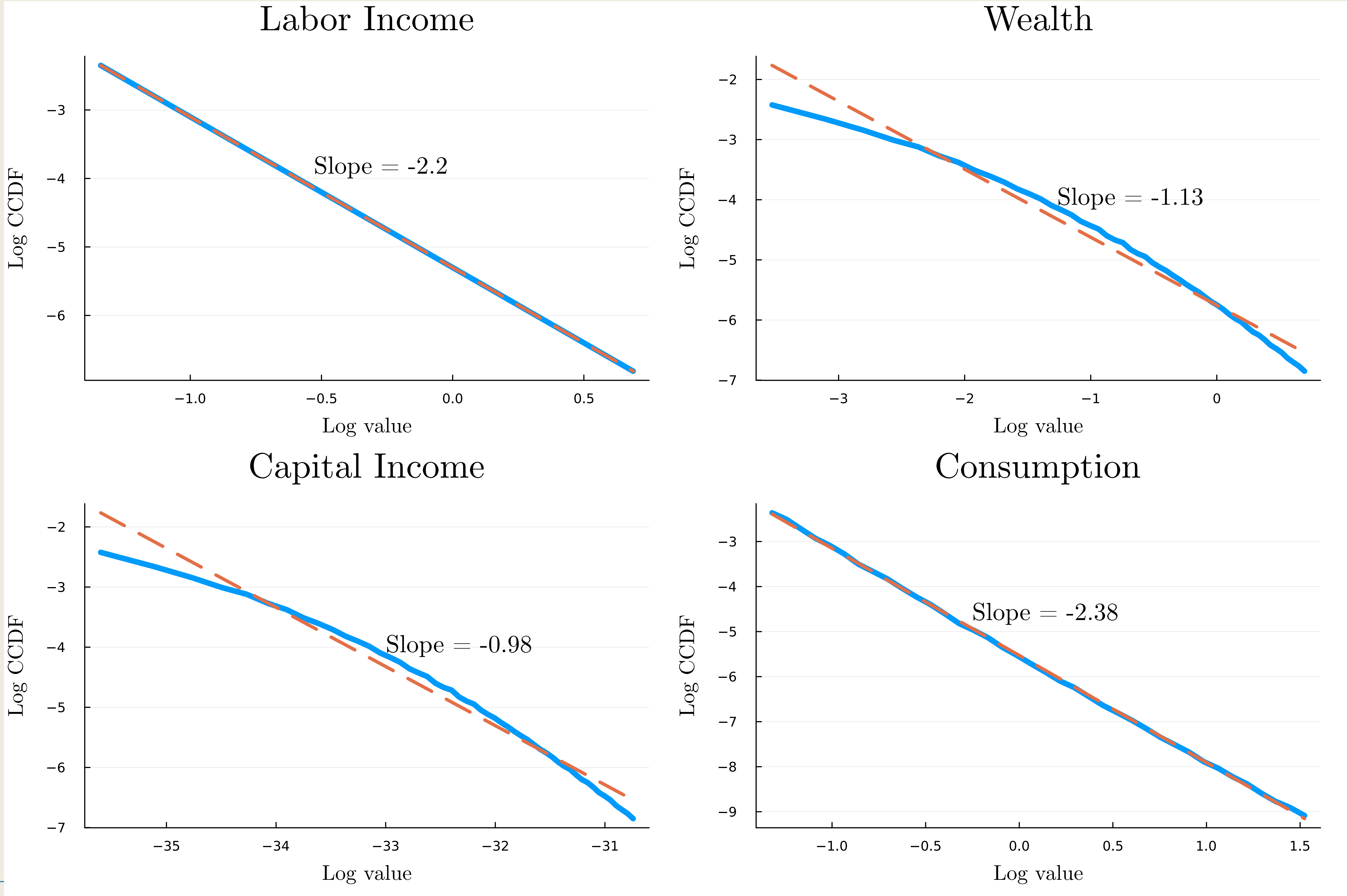
$$V(a, y, z) = \max_{c, a' \geq -\phi} \frac{c^{1-\gamma}}{1-\gamma} + \kappa \frac{(a')^{1-\nu}}{1-\nu} + \beta \mathbb{E}_{z', y'} V(a', y', z')$$

$$\text{s.t.} \quad c + a' = (1 + \hat{r}(a)\, z) a + y$$

$$\hat{r}(a) \equiv \bar{r} a^{\eta}$$

- Assume $\gamma > 1, \nu \leq \gamma$, and $\nu \notin [1-\eta, 1]$ for technical reasons

With Wealth-in-Utility & Scale Dependent Return



$\zeta_c \neq \zeta_a \neq \zeta_{ra}$

Consider the model with wealth-in-utility and scale dependent return described earlier.

The stationary distribution, if it exists, features

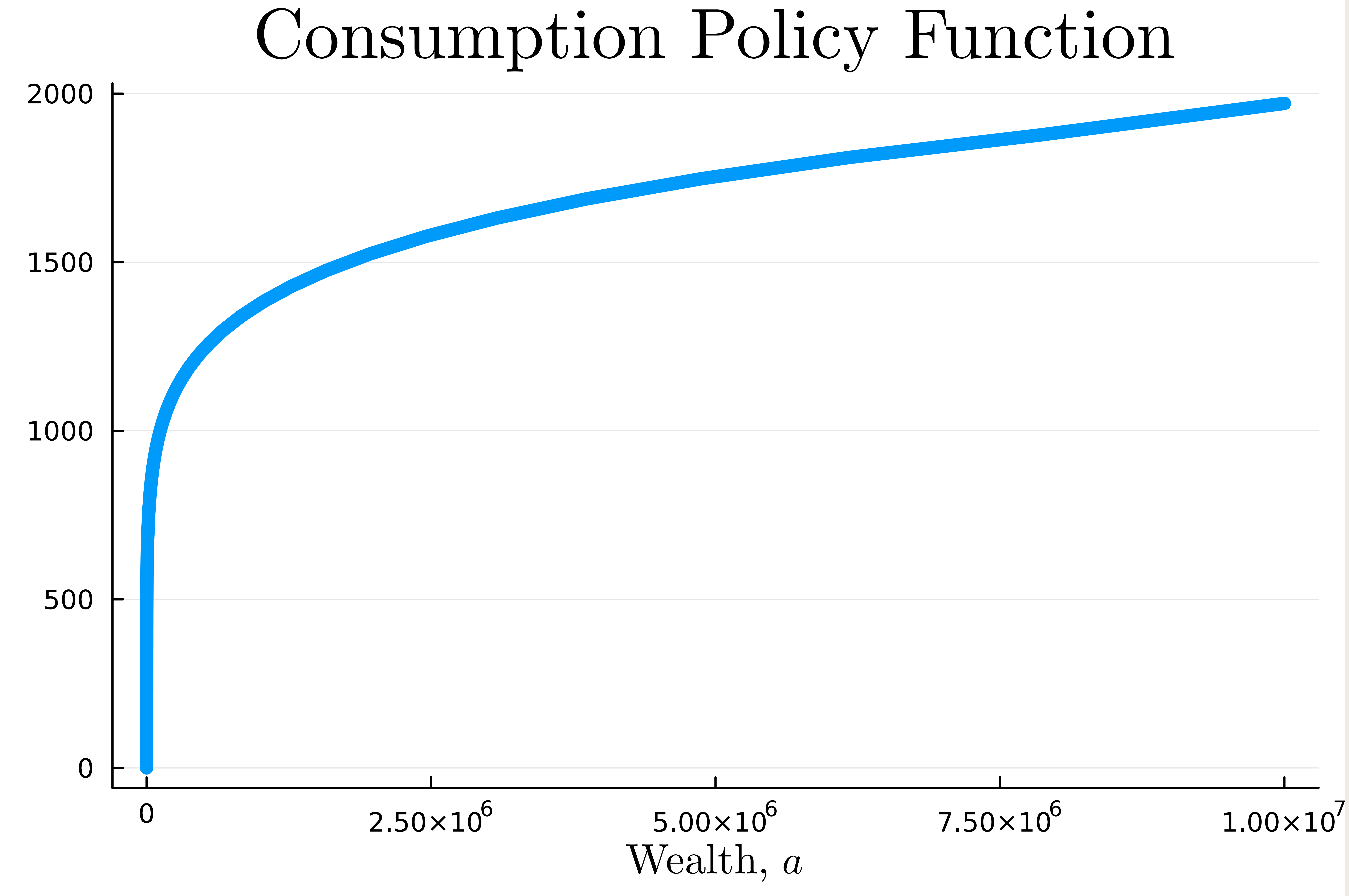
1. $\zeta_c = \frac{\gamma}{\nu + \eta} \zeta_a$
2. $\zeta_{ra} = \frac{1}{1 + \eta} \zeta_a$

- $\nu < \gamma - \eta \Rightarrow \zeta_c > \zeta_a$
- $\eta > 0 \Rightarrow \zeta_{ra} < \zeta_a$

Why $\zeta_c > \zeta_a$?

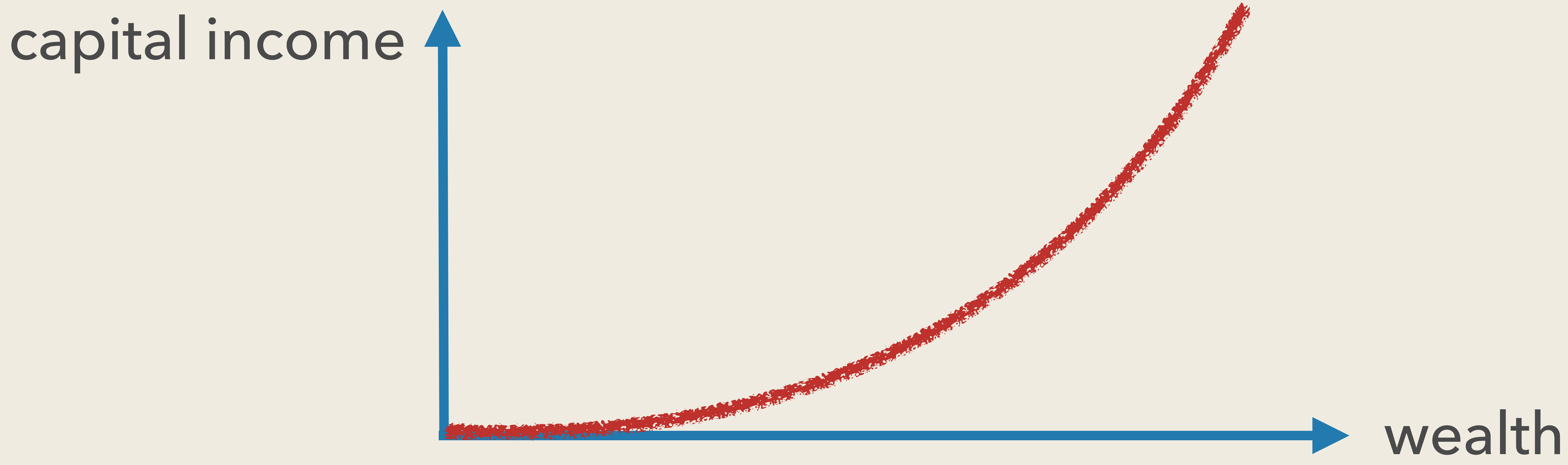
- When $\nu \ll \gamma$, consumption has a thinner tail than wealth
- MU of consumpion, $c^{-\gamma}$, diminishes faster than MU of wealth, $a^{-\nu}$
- Consumption is non-homothetic and *concave* in wealth
    - Doubling the wealth less than doubles the consumption
- Consumption distribution is more equal than wealth distribution

Consumption Policy Concave in Wealth



Why $\zeta_{ra} < \zeta_a$?

- When $\eta > 0$, capital income has a fatter tail than wealth
- Capital income is *convex* in wealth
  - Doubling the wealth more than doubles capital income
- Capital income distributed even more unequally than wealth



Conclusion

- Bewley-Hugget-Aiyagari is a workhorse model in macroeconomics
- However, the model faces a challenge in jointly matching the four tails of inequality
- The data strongly favors
  1. non-homothetic wealth-in-utility:
     rich households save more because they are rich
  2. scale-dependent return:
     rich households earn higher return from their wealth because they are rich