Monoposony Models of Frictional Labor Market

704 Macroeconomics II
Topic 5

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Firms as Wage-Setters

- In DMP, workers and firms bargain over wages
- Are wages really bargained in the data?

Hall and Krueger (2012)

- Survey 1,300 workers
- **Q:** When you were offered your job, did your employer make a “take-it-or-leave-it” offer or was there some bargaining that took place over the pay?

  A: 33% bargained

  - 25% for women. 85% for professional degree. 6% for blue-color workers.
- **Q:** At the time that you were first interviewed for your job, did you already know exactly how much it would pay?

  A: 23% yes

  - 23% for women. 14% for professional degree. 57% for blue-color workers.

Wage Posting

- In the data, the majority of workers receive “take-it-or-leave-it” offers
- Now let us replace wage bargaining with wage posting in DMP

	Wage Bargaining	**Wage Posting**
Random Search	DMP	**Today**
Directed Search		Competitive Search (Moen, 1997)

Diamond (1971) Paradox

DMP with Wage Posting

- Consider the DMP model in continuous time with discount rate $r > 0$
- To focus on the wage settings, let us assume q and f are both exogenous
- The unemployed workers value function:

$$rU = b + f \max\{E(w) - U, 0\}$$

- The employed workers:

$$rE(w) = w + s(U - E(w))$$

- Workers accept the job offer if $w \geq w^R$, where $E(w^R) = U$

Extreme Monopsony

- Firms decide what wages to offer to workers:

$$rV = -c + \max_w q\mathbb{1}(w \geq w^R)J(w)$$

$$rJ(w) = z - w + s(V - J(w))$$

- What is the firm's optimal wage setting? Clearly,

$$w = w^R$$

since there is no reason to offer $w > w^R$

- Solving for w^R, the unique equilibrium features all firms offering

$$w = w^R = b$$

- Firms set wages so that workers are exactly indifferent to unemployment – an extreme form of "monopsony"

Heterogenous Firms

- The result extends even when firms have differing productivity $z_i \in \{z_1, z_2, \dots, z_J\}$

- Workers problem unchanged

- Firms with productivity z_i solves

$$rV_i = -c + \max_{w_i} q\mathbb{1}(w_i \geq w^R)J_i(w_i)$$
$$rJ_i(w_i) = z_i - w_i + s(V_i - J_i(w_i))$$

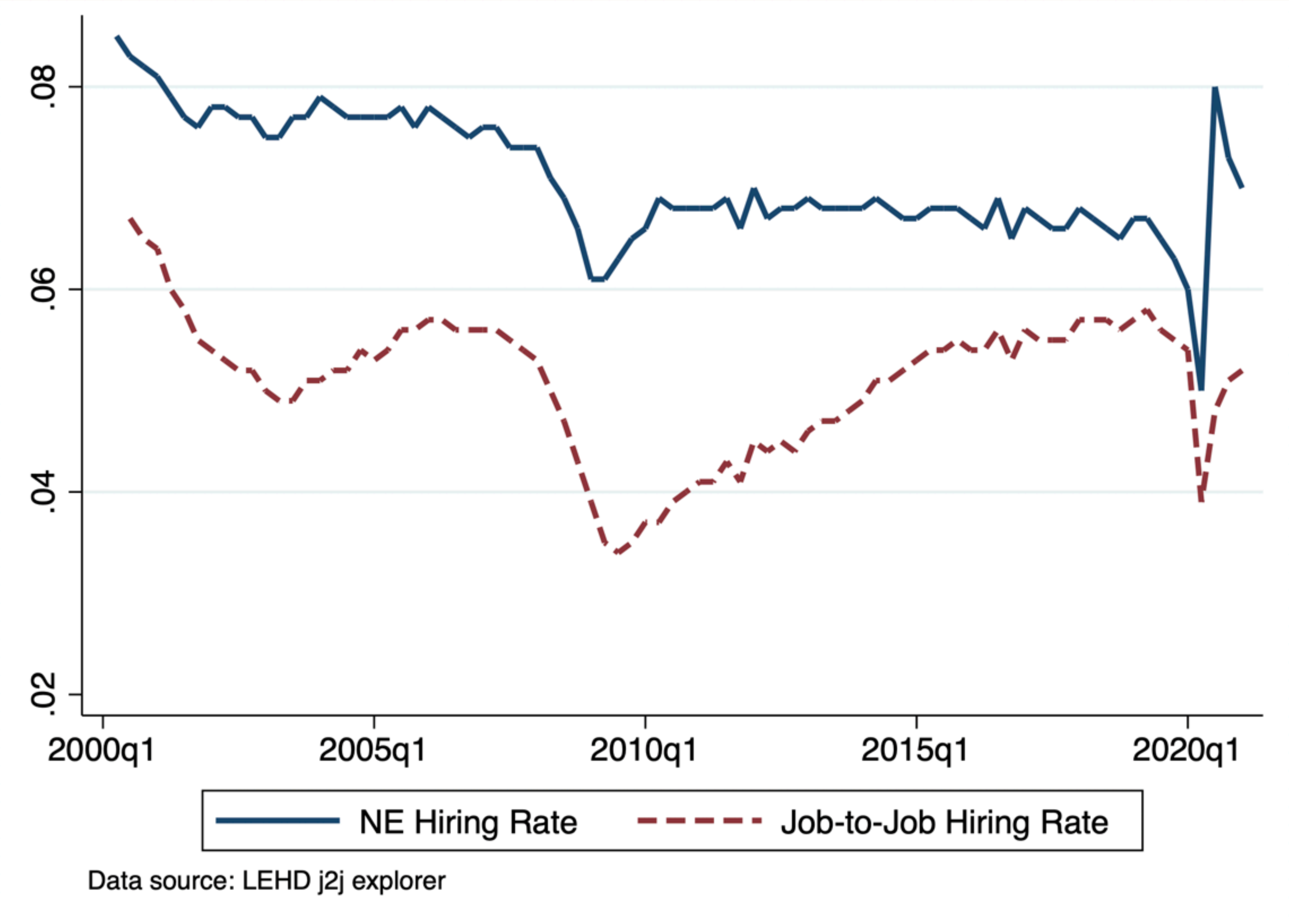
- Despite firm heterogeneity,

$$w_i = w^R = b \quad \text{for all } i$$

Diamond Paradox

- This is called Diamond (1971) Paradox
- Why is this a paradox? Why is this surprising?
- A tiny deviation from perfect competition results in an extreme form of monopsony!
- Firms capture all the rents even when $f, q \to \infty$
- This spurred subsequent research. Solutions to the paradox:
  1. Heterogenous workers (Albrecht and Axell, 1984)
  2. Multiple job applications at a time (Burdett and Judd, 1983)
  3. On-the-job search (Burdett and Mortensen, 1998)
- We focus on Burdett and Mortensen (1998)

On-the-Job Search in the Data



Burdett and Mortensen (1998) Model

Environment

- Let us introduce on-the-job to the previous model
- Unemployed workers receive job-offer at the arrival rate f^U
- Employed workers receive at a rate $f^E \equiv \zeta f^U$
- Firms with measure $m \equiv 1$ post v vacancy (exogenous) and meets worker at rate q
- Firms post wage w that applies to all employees ("firm-wage")
- Start from homogenous firm case with common productivity z
- In the background, think of a matching function that determines (f^U, f^E, q):

$$f^U = \frac{M(u + \zeta(1-u), v)}{u + \zeta(1-u)}, \quad f^E = \zeta \frac{M(u + \zeta(1-u), v)}{u + \zeta(1-u)}, \quad q = \frac{M(u + \zeta(1-u), v)}{v}$$

Worker's Problem

- Unemployed workers value function:

$$rU = b + f^U \int \left[\max\{E(w) - U, 0\}\right] dG(w) \tag{1}$$

- Employed workers with wage w:

$$rE(w) = w + f^E \int \max\{E(w') - E(w), 0\} dG(w') - s(E(w) - U) \tag{2}$$

- Worker's policy:
  - Unemployed: accept job offer iff $w \geq w^R$ where $E(w^R) = U$
  - Employed: accept job offer iff $w' \geq w$

Reservation Wage $\underline{w}$

- Combining $E(w^R) = U$, (1), and (2),

$$
\begin{aligned}
w^R - b &= (f^U - f^E) \int_{w^R} (E(w) - U) dG(w) \\
&= (f^U - f^E) \int_{w^R} E'(w)(1 - G(w)) dw \\
&= (f^U - f^E) \int_{w^R} \frac{1 - G(w)}{r + s + f^E(1 - G(w))} dw
\end{aligned}
$$

  where the second line uses integration by parts

- When $f^U = f^E$, $w^R = b$
- When $f^U > f^E$, $w^R > b$ because accepting a job offer lowers future job opportunity

Worker Flow

- The unemployment flow equation is

$$\partial_t u = -f^U u + s(1 - u_t)$$

- In the steady state,

$$u = \frac{s}{s + f^U}$$

- Let $\tilde{H}(w)$ be the **mass** of employed workers with wages below w, which follows

$$\partial_t \tilde{H}(w) = f^U G(w) u - [s + f^E(1 - G(w))]\tilde{H}(w)$$

- In the steady state,

$$G(w) = \frac{(s + f^E)\tilde{H}(w)}{f^U u + f^E \tilde{H}(w)}$$

$$= \frac{(s + f^E)H(w)}{s + f^E H(w)}$$

where $H(w) = \frac{\tilde{H}(w)}{1 - u}$ is the **fraction** of employed workers with wages below w

(■)

Labor Supply Function

- Employment at a firm offering wage $w \geq w^R$ evolves

$$\partial_t l(w) = qv(\chi + (1-\chi)H(w)) - sl(w) - f^E l(w)(1 - G(w))$$

where $\chi \equiv u/(u + \zeta(1-u)) = s/(s + f^E)$ is prob. of meeting u conditional on meeting

- In the steady state

$$l(w) = \frac{qv(\chi + (1-\chi)H(w))}{s + f^E(1 - G(w))}$$
$$= \frac{qv(s + f^E H(w))^2}{s(s + f^E)^2}$$

- $l(w)$ is increasing in w: higher $w \Rightarrow$ poach more and poached less

Search Friction as a source of Monopsony Power

- To simplify our life, let $r \to 0$ so that the firms maximize the steady-state profit
- The firms solve

$$\max_{w} (z - w) l(w)$$

  - At this point, this is a typical "monoposny" problem:
    $\Rightarrow$ firms face an upward-sloping labor supply curve.
  - Search frictions give a full microfoundation of $l(w)$
  - Other microfoundations:
    - Job differentiation
    - Firm-specific skill

Frictional Wage Dispersion

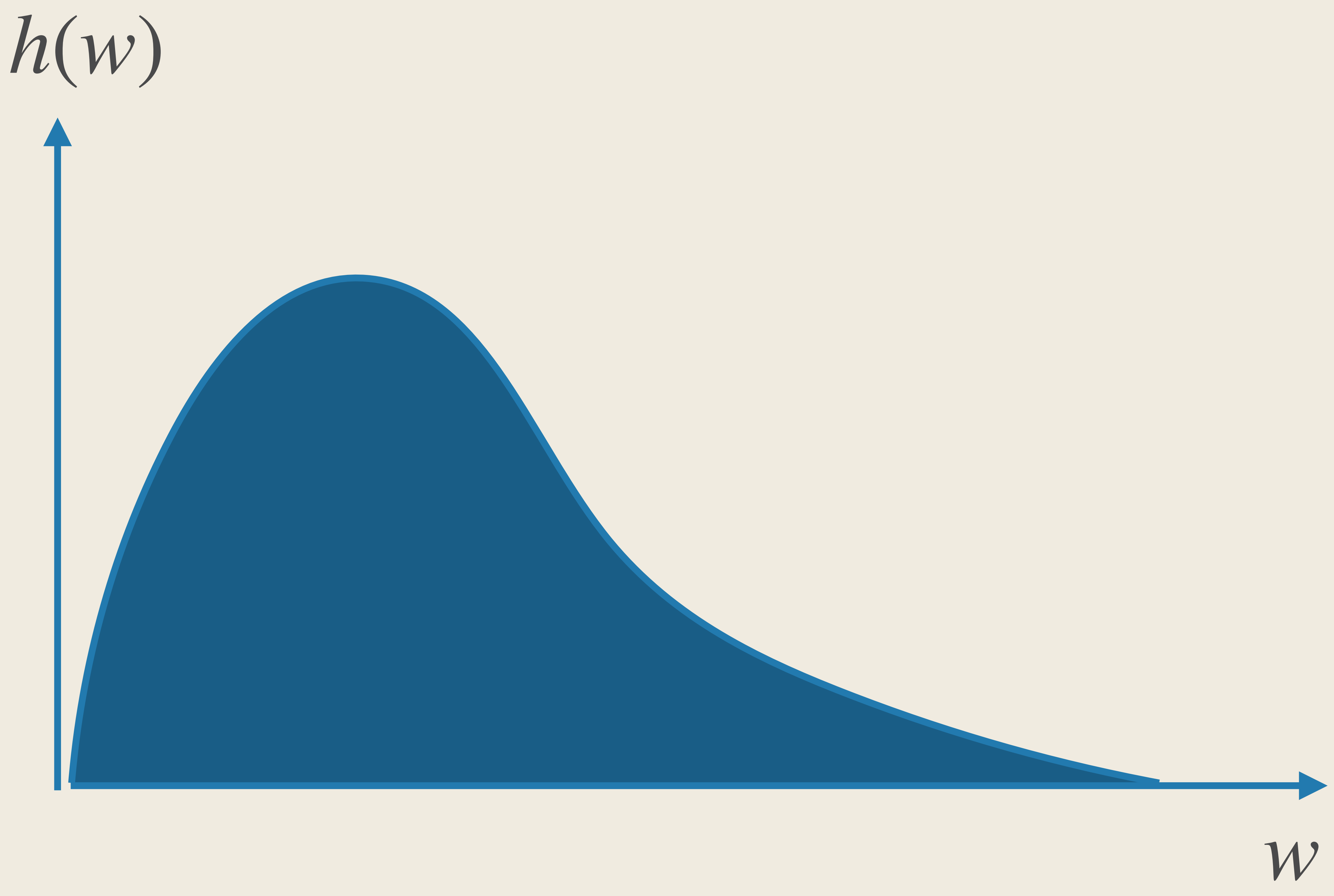
- Since all firms are homogenous, tempted to think we have a symmetric eqm
- Suppose all firms offer $w = \hat{w} \in [w^R, z)$
- Then, a firm can profitably deviate by offering $\hat{w} + \epsilon$
  - The cost of doing so is continuous in ϵ
  - But it attracts a discontinuously larger amount of workers
    - The firm can poach all workers
    - No other firms can poach workers from the firm
- All firms offering $\hat{w} \geq z$ cannot be an eqm because $w = z - \epsilon$ gives higher profits
- Therefore, equilibrium has to be a mixed-strategy equilibrium
  $\Rightarrow$ "Frictional wage dispersion"

Smooth Wage Distribution

More generally,

1. There cannot be a mass point

2. There cannot be a gap



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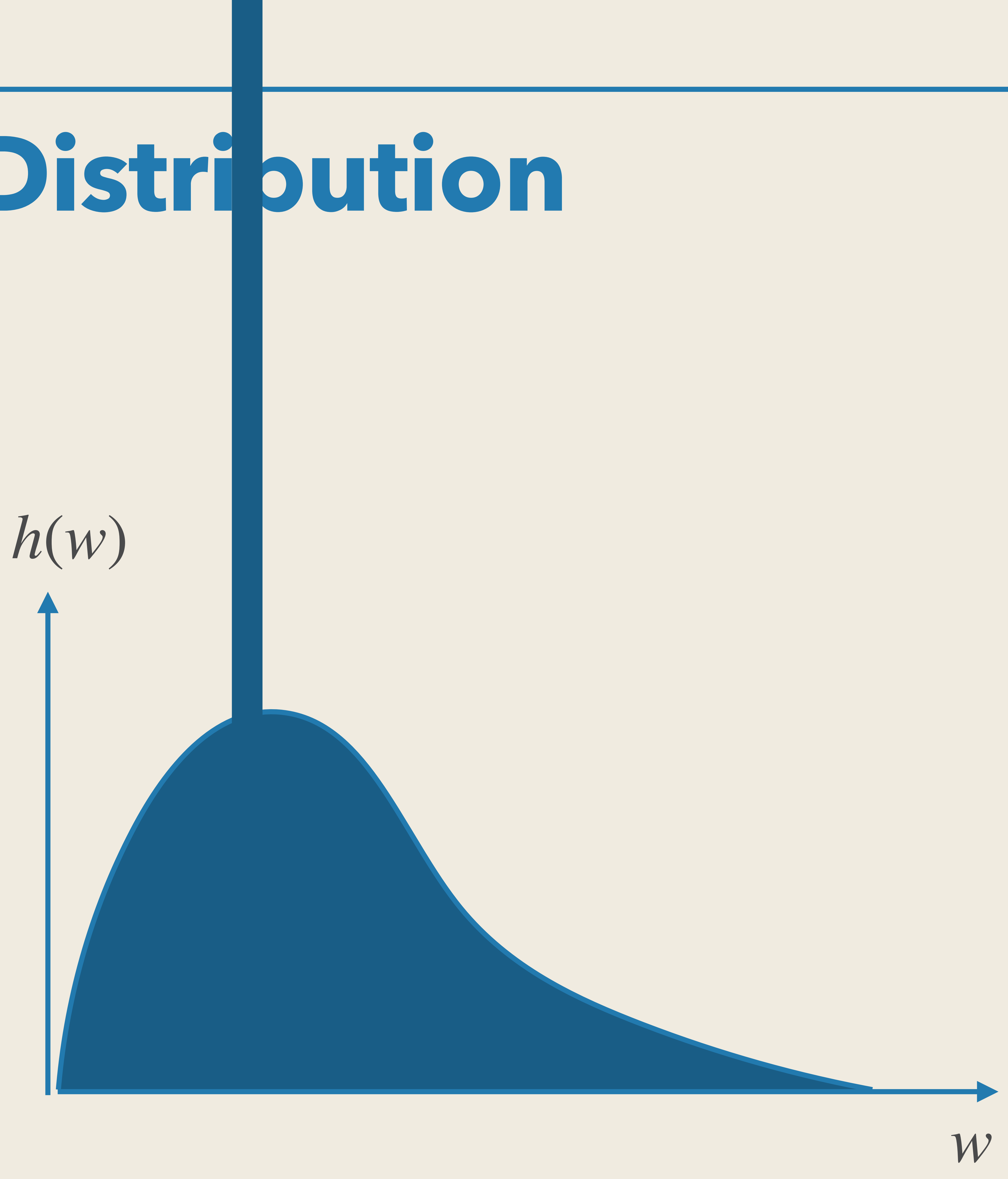

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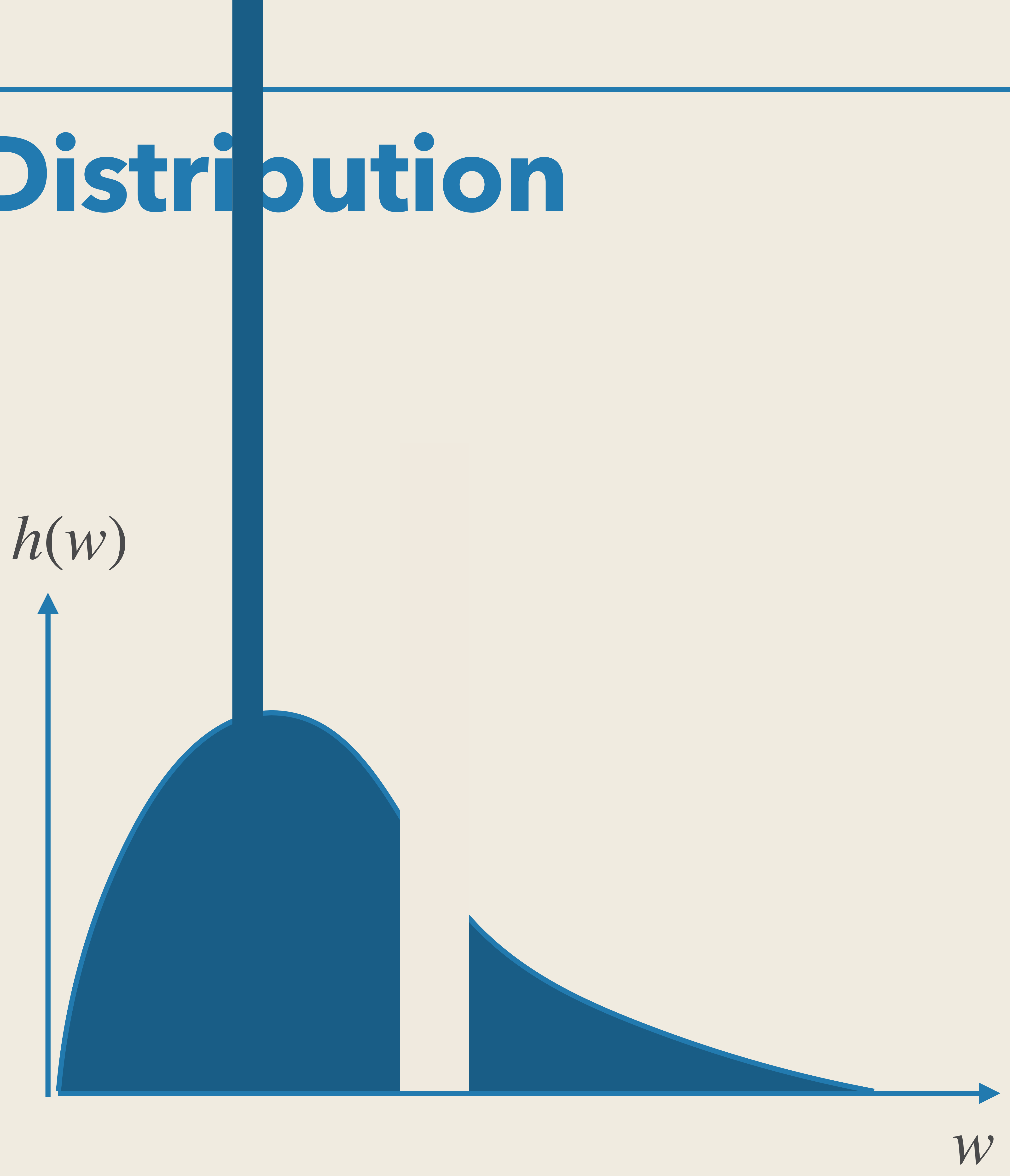


Wage Offer Distribution

- Firms with the lowest wage offer must find it optimal to offer w^R
- All the other firms must be indifferent to offering w^R, implying

$$(z - w)l(w) = (z - w^R)l(w^R) \quad \text{for all } w \text{ in the support of } G$$

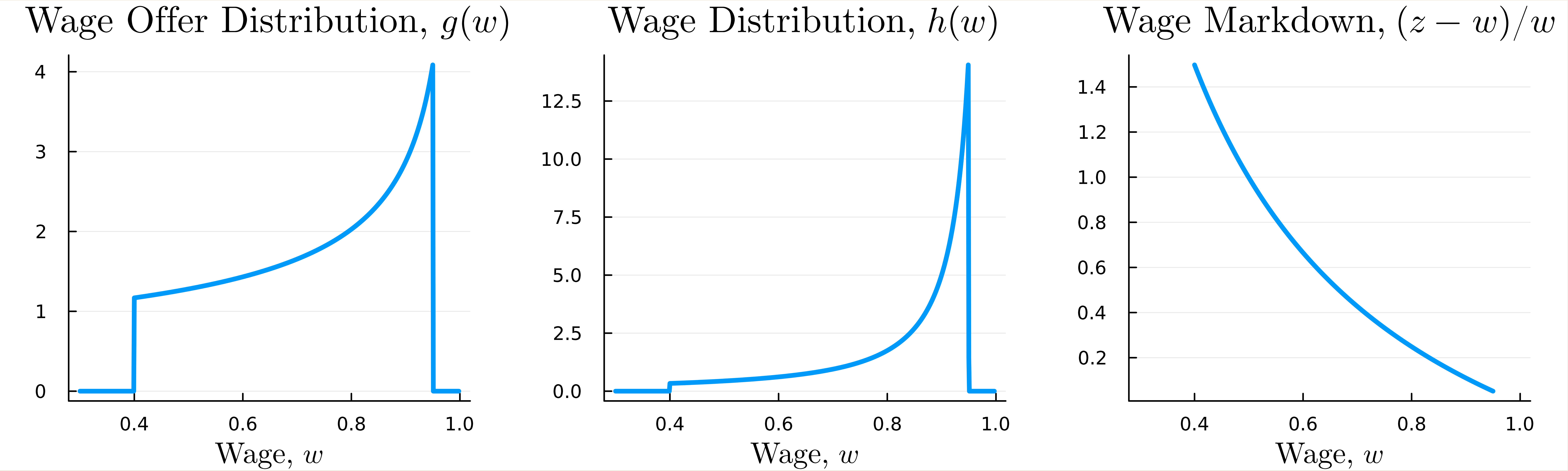
- Since $l(w^R) = qvs/(f^E + s)^2$, the wage distribution $H(w)$ must satisfy

$$H(w) = \frac{s}{f^E}\left(\sqrt{\frac{(z - w^R)}{(z - w)}} - 1\right) \quad \text{for } w \in [w^R, \bar{w}], \quad \text{where} \quad G(\bar{w}) = 1$$

- Plug back into (■) to recover the expression for offer distribution $G(w)$:

$$G(w) = (1 + s/f^E)\left(1 - \sqrt{\frac{(z - w)}{(z - w^R)}}\right)$$

Numerical Example



Numerical Example ($f^E \to \infty$)

Numerical Example ($f^E \to 0$)

Heterogenous Firms

Heterogenous Firm Setup

- Now the firm's productivity distribution is given by $J_0(z)$ with full support $z \in [0,\infty)$
- Firms with $z < w^R$ cannot make profits so inactive. Let $J(z) \equiv \dfrac{J_0(z) - J_0(w^R)}{1 - J_0(w^R)}$
- Firms with $z \geq w^R$ solve

$$\max_w (z - w) l(w) \qquad (\star)$$

- The first-order condition is ($\epsilon_w \equiv l'(w)w/l(w)$)

$$\underbrace{\frac{z-w}{w}}_{\text{wage markdown}} = \underbrace{\frac{1}{\epsilon_w}}_{\text{inv. LS elasticitiy}} \Rightarrow \begin{aligned}(z-w)2(s+f^E H(w))f^E H'(w) \\ = (s+f^E H(w))^2\end{aligned}$$

- At this point, what do we know about $H(w)$? – Nothing… big fixed point problem!

Rank-Preserving Property

- Realize that ($\star$) is strictly supermodular in (z, w)

  $\Rightarrow$ $w(z)$ is strictly increasing in z

- Then we know a lot about $H(w)$

$$H(w(z)) = \hat{H}(z) \qquad \Rightarrow \qquad H'(w(z))w'(z) = \hat{H}'(z)$$

- $\hat{H}(z)$ is the fraction of employed workers at firms productivity below z:

$$\hat{H}(z) = \frac{sJ(z)}{s + f^E(1 - J(z))}$$

  where we used (■) and $G(w(z)) = J(z)$

- It is this **rank-preserving property** that makes all the job-ladder models tractable
  - More productive firms poach from less productive firms

Solving for Wage Function

- Combine with FOCs to obtain an ODE that characterizes the equilibrium wage

$$w'(z) = (z - w(z))\frac{2f^E\hat{H}'(z)}{(s + f^E\hat{H}(z))}$$

- Solving the ODE with boundary condition $w(w^R) = w^R$

$$w(z) = \frac{s^2 w^R + \int_{w^R}^{z} 2\tilde{z}\left(s + f^E\hat{H}(\tilde{z})\right)f^E\hat{H}'(u)\, d\tilde{z}}{\left(s + f^E\hat{H}(z)\right)^2}$$

- One can check the second-order condition is also satisfied

Understanding Wage Function

- One can rewrite the wage function as

$$w(z) = \frac{\hat{F}^2(w^R)w^R + \int_{w^R}^{z} \tilde{z} d\hat{F}^2(\tilde{z})}{\hat{F}^2(z)}$$

$$\equiv \mathbb{E}_{\hat{F}^2}[\tilde{z} \,|\, \tilde{z} \leq z]$$

where $\hat{F}^2(z)$ is the cdf of the quadratic of productivity distribution:

$$\hat{F}^2(z) \equiv \left(u + (1-u)\zeta \hat{H}(z)\right)^2$$

weighted by employment and search intensity

- Firms pay the average productivity among firms below them on the job ladder!
  - Why quadratic? – Retention and hiring!
- It turns out this is a very general formula behind the BM-type model

Closed Form Solution

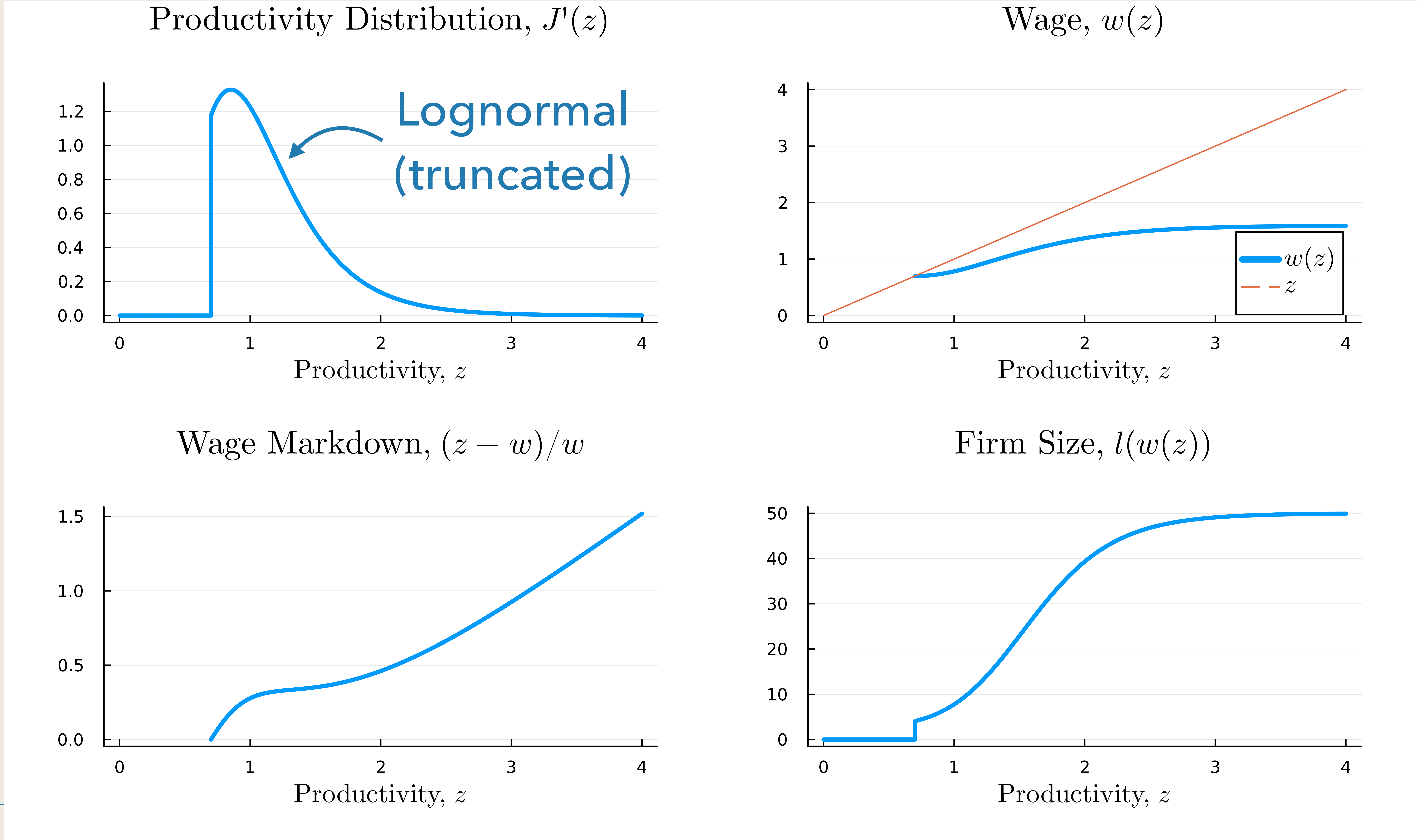
- Using $\hat{H}(z) = \frac{sJ(z)}{s + f^E(1 - J(z))}$, one can express the wage function in closed forms

$$w(z) = z - \int_{w^R}^{z} \frac{\left(s + f^E(1 - J(z))\right)^2}{\left(s + f^E(1 - J(\tilde{z}))\right)^2} d\tilde{z}$$

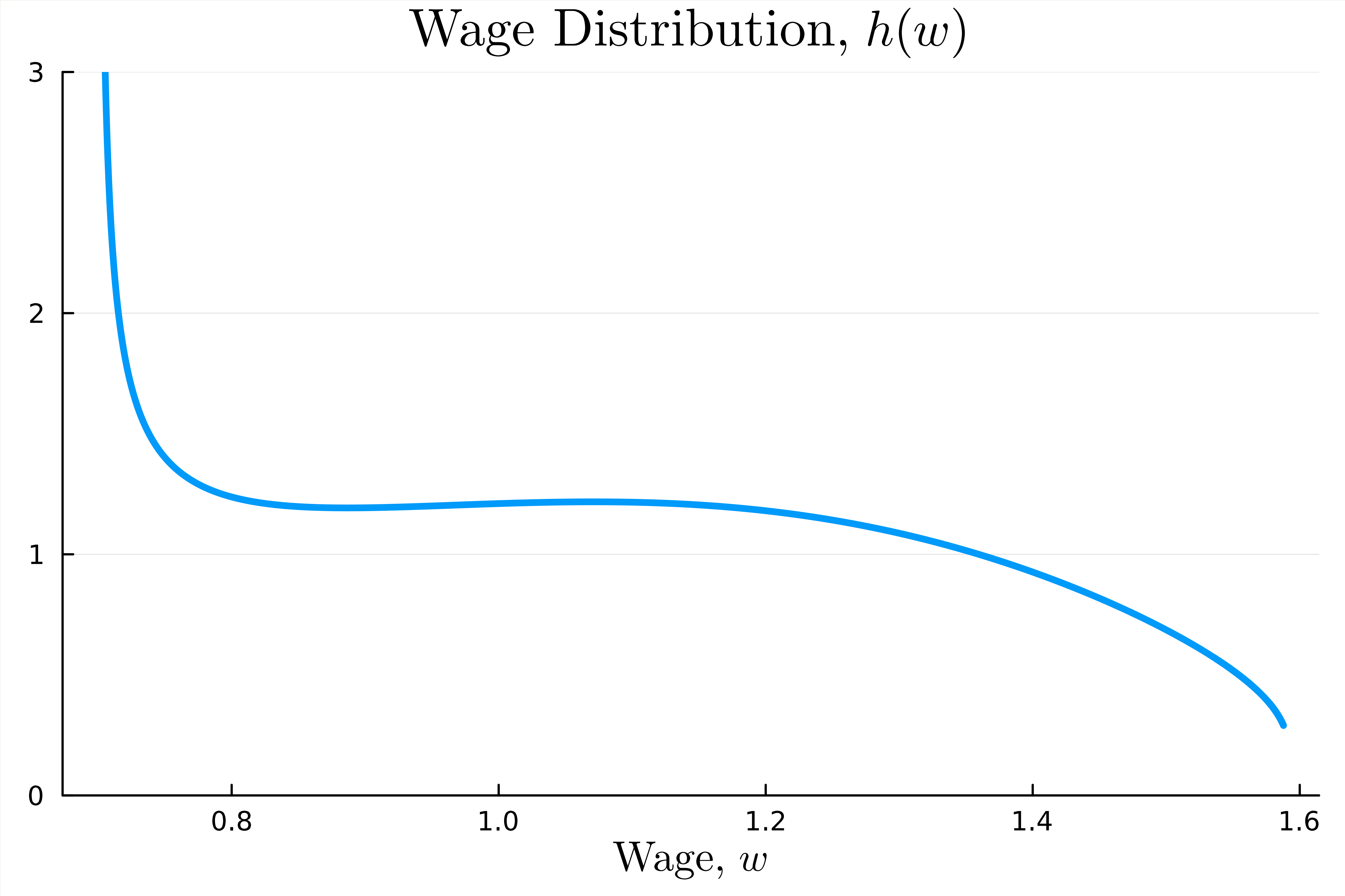
  which is the expression in the original paper

- I used to teach this way, but this is much harder to see the economics behind it…

Numerical Example



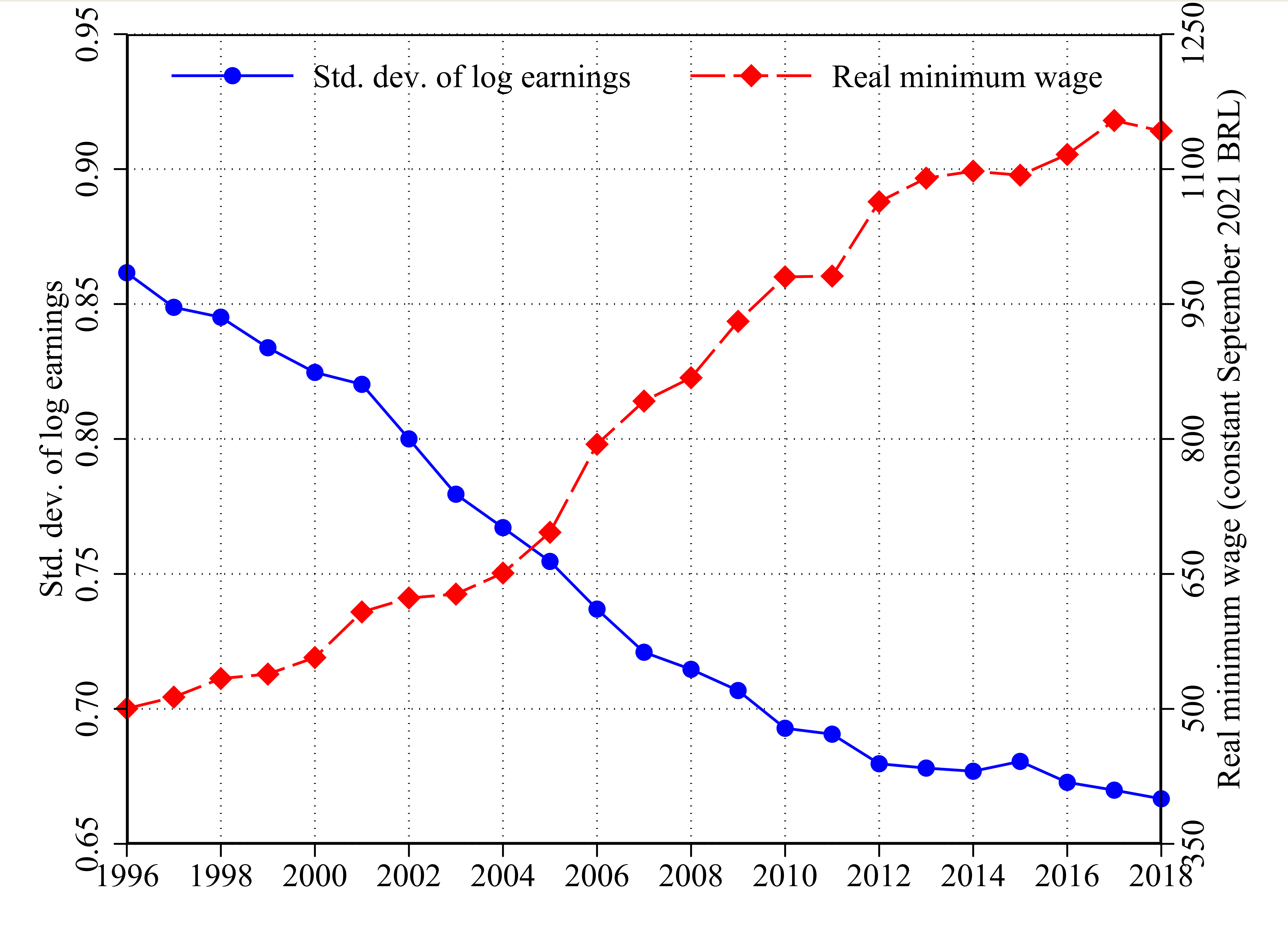
Wage Distribution



Spillover Effect of Minimum Wage

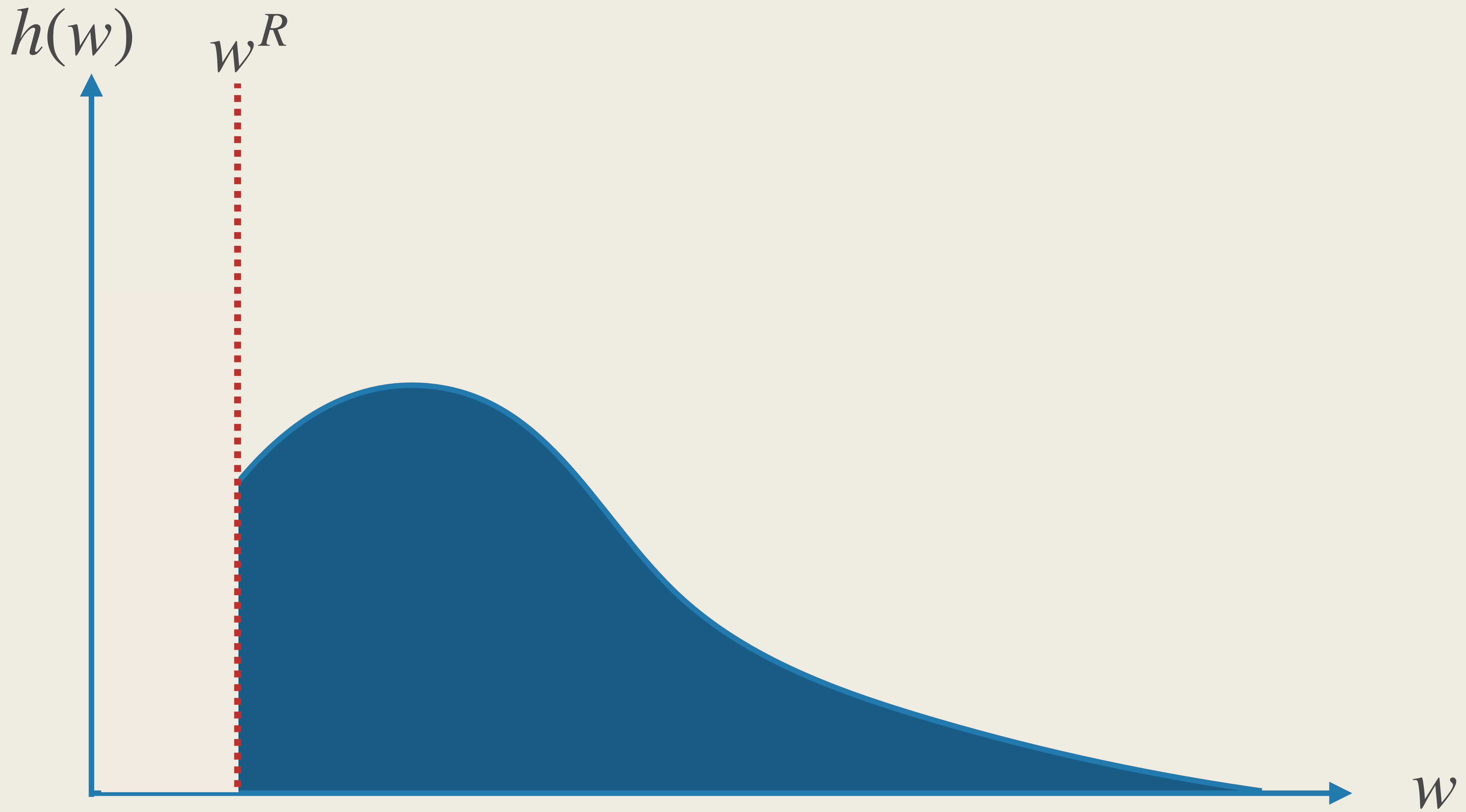
– Engbom and Moser (2021)

Earnings Inequality and Minimum Wage in Brazil



Minimum Wage Spillover

- Interpret w^R as the minimum wage
- Suppose we raise w^R. What would happen to the wage distribution?



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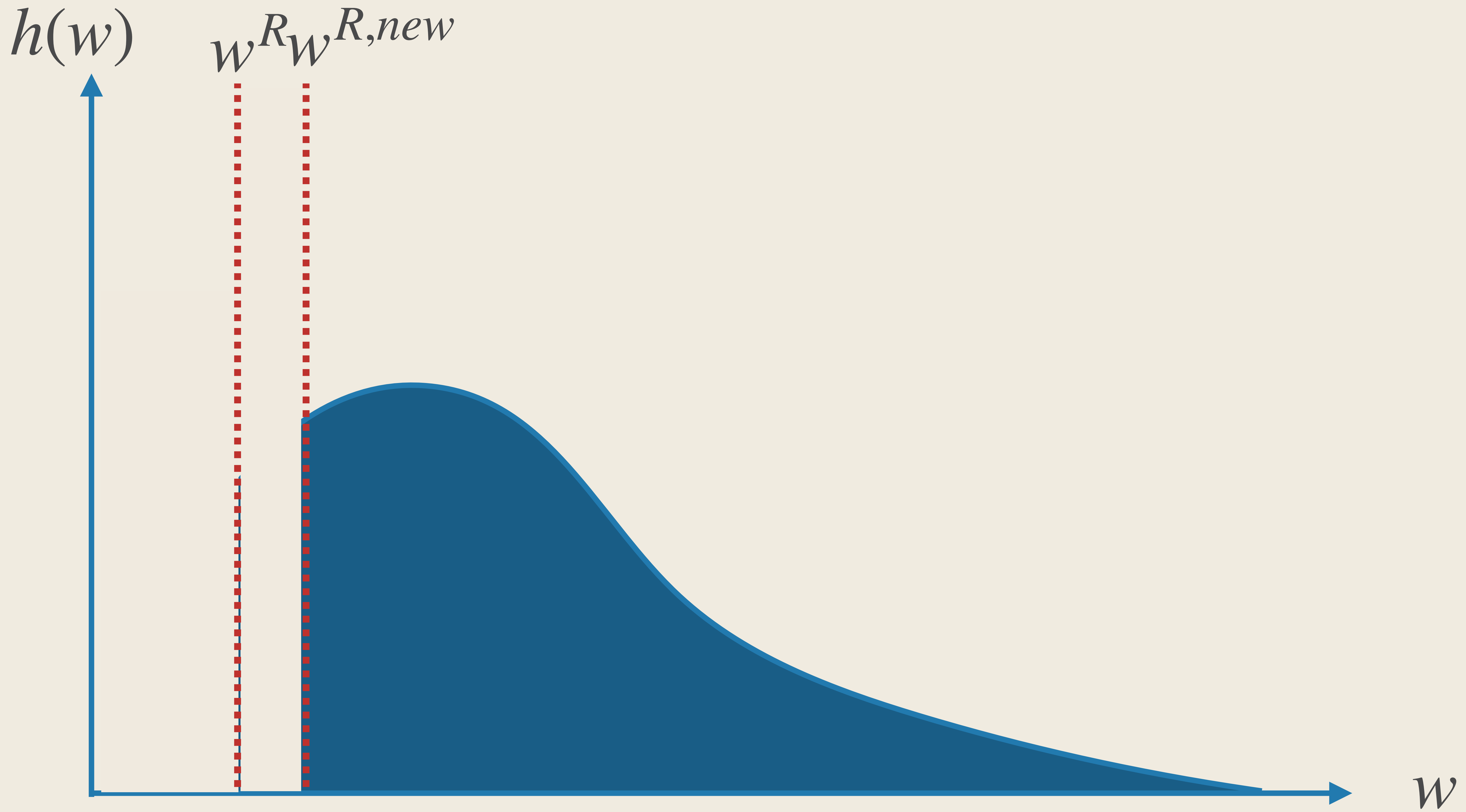

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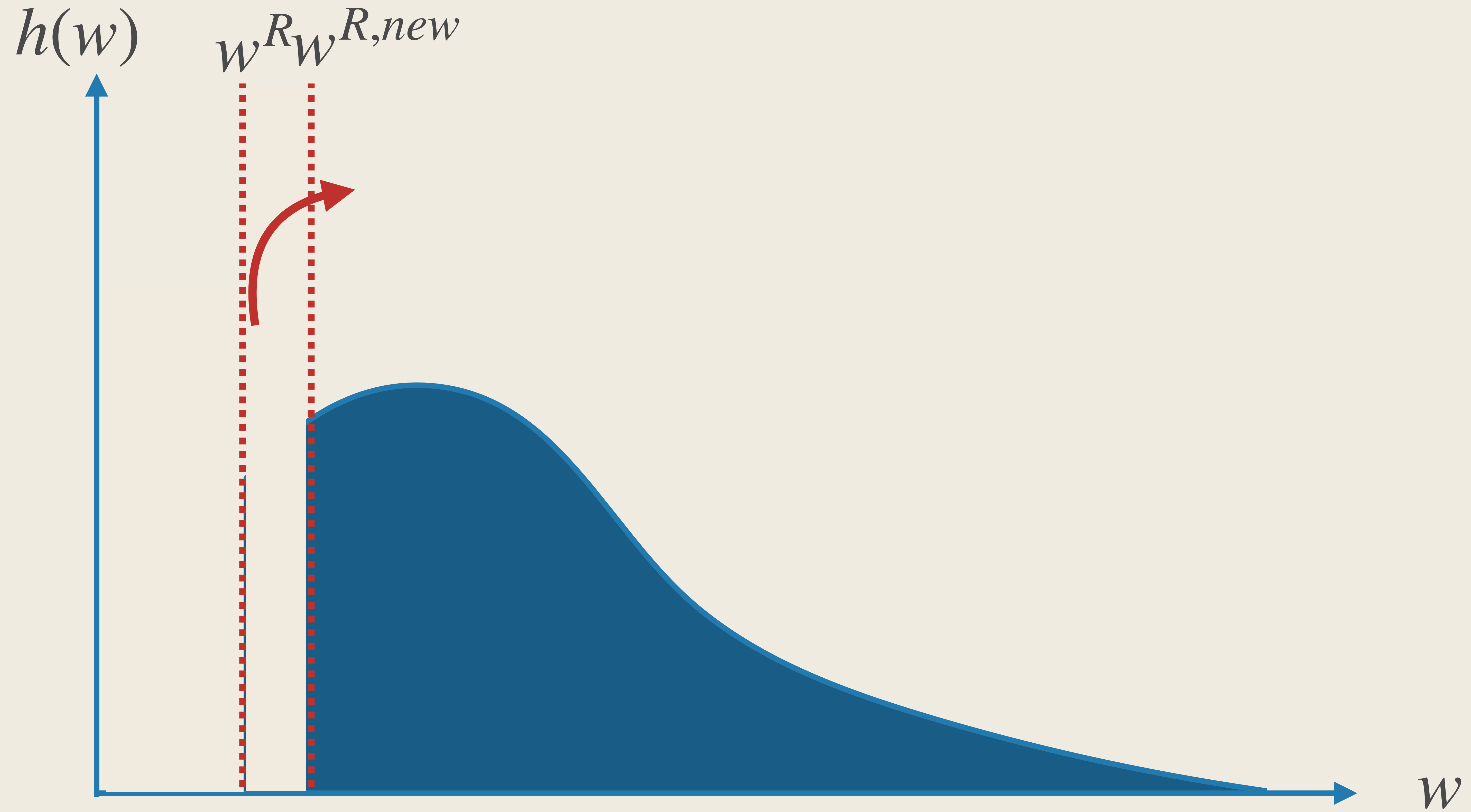

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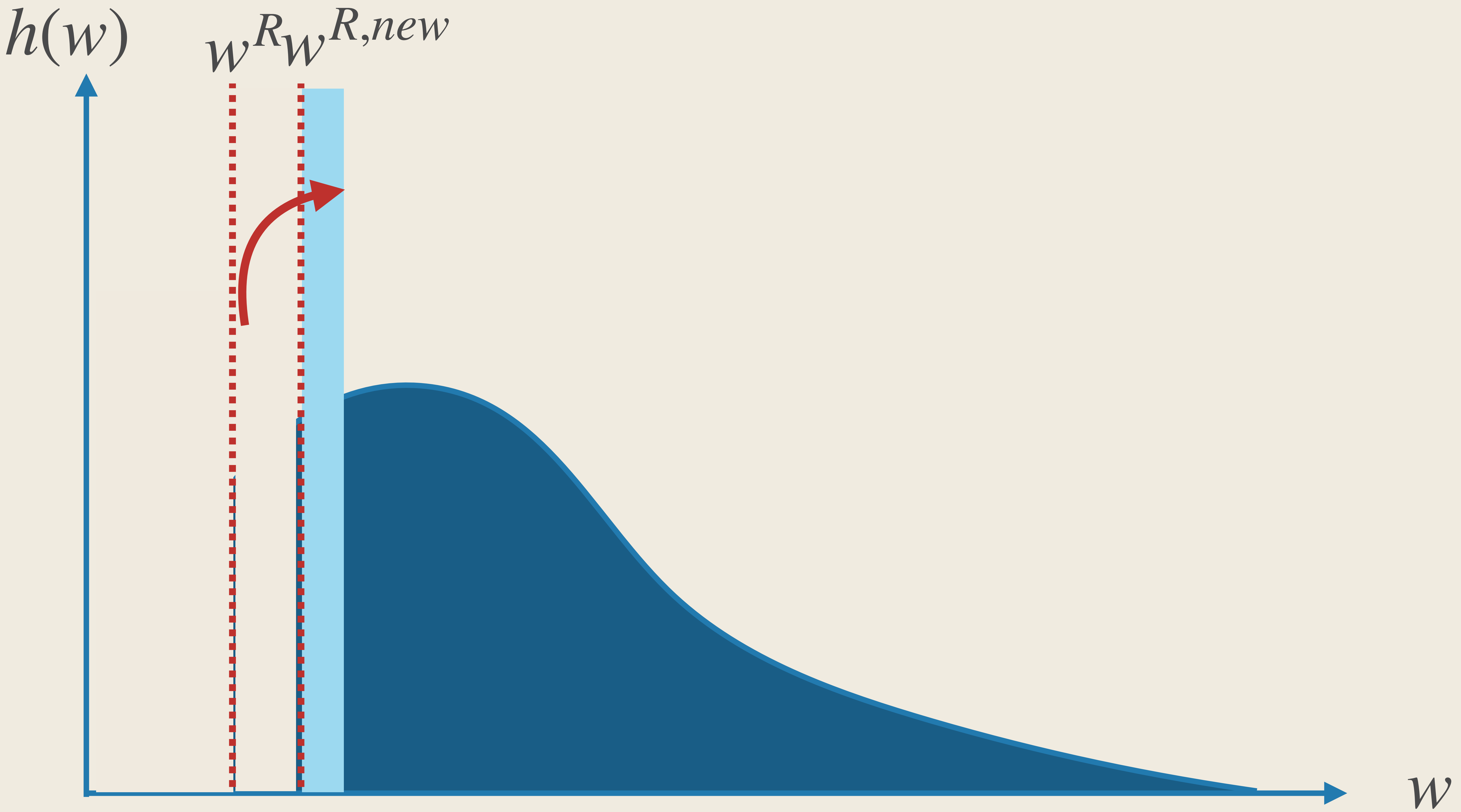

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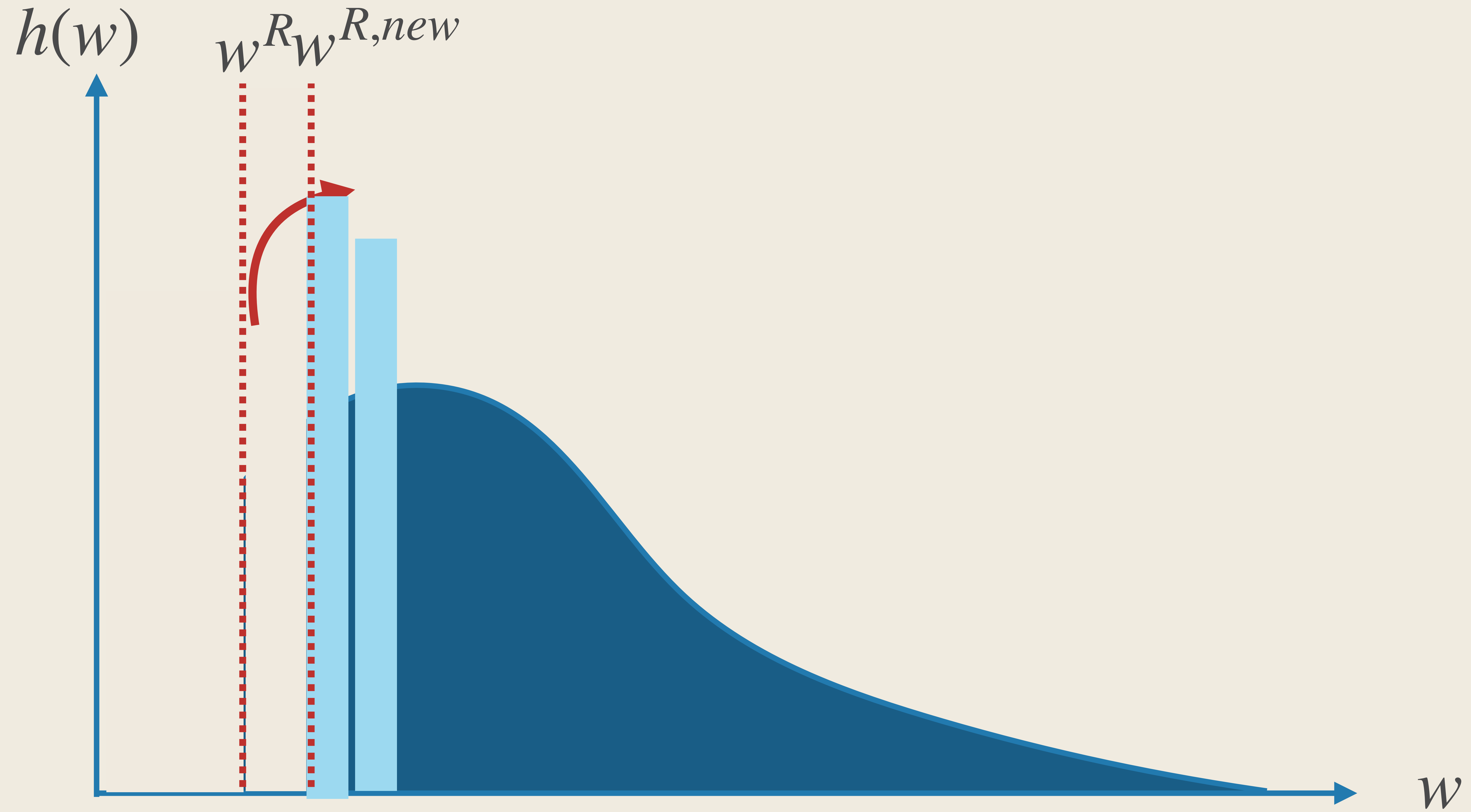

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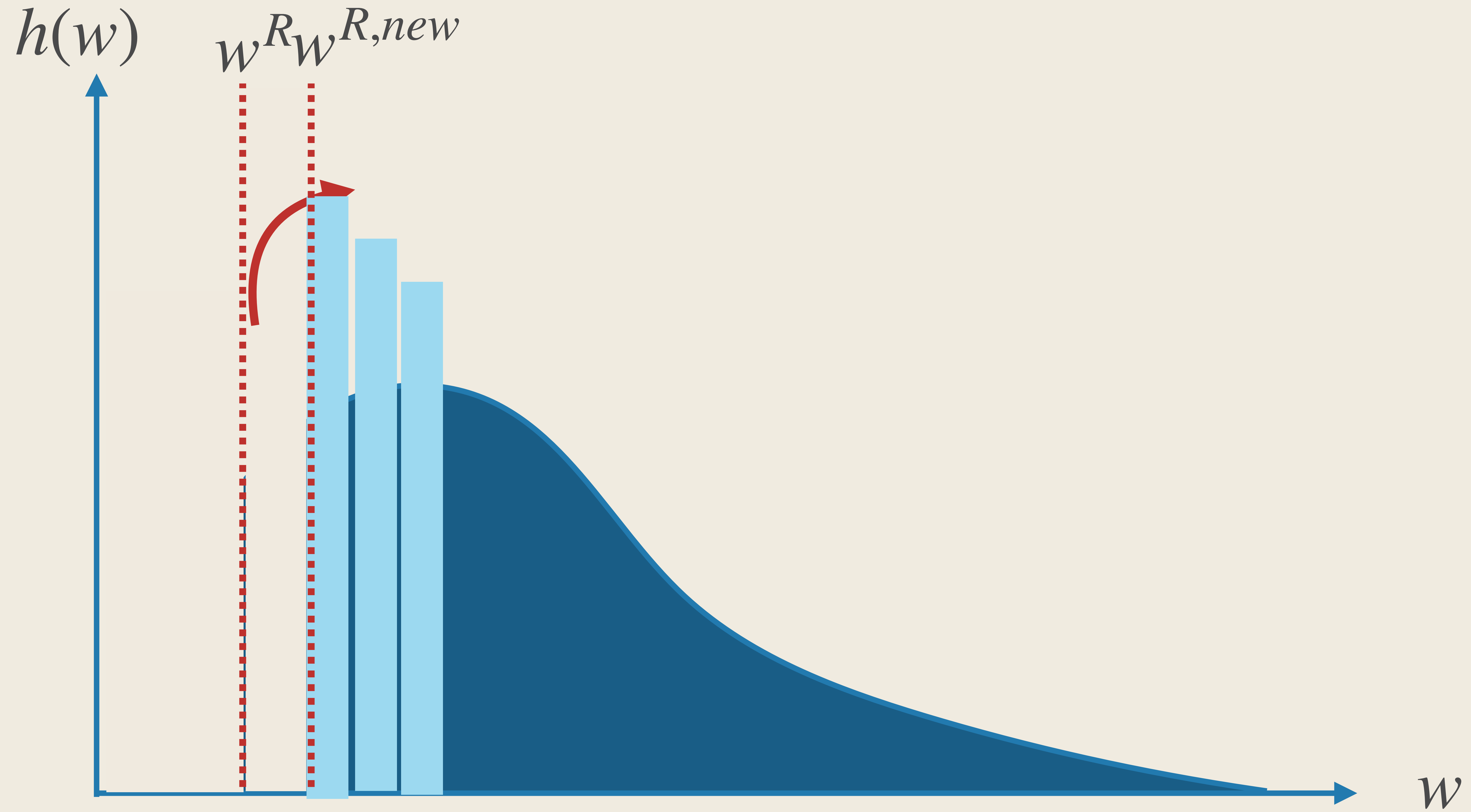

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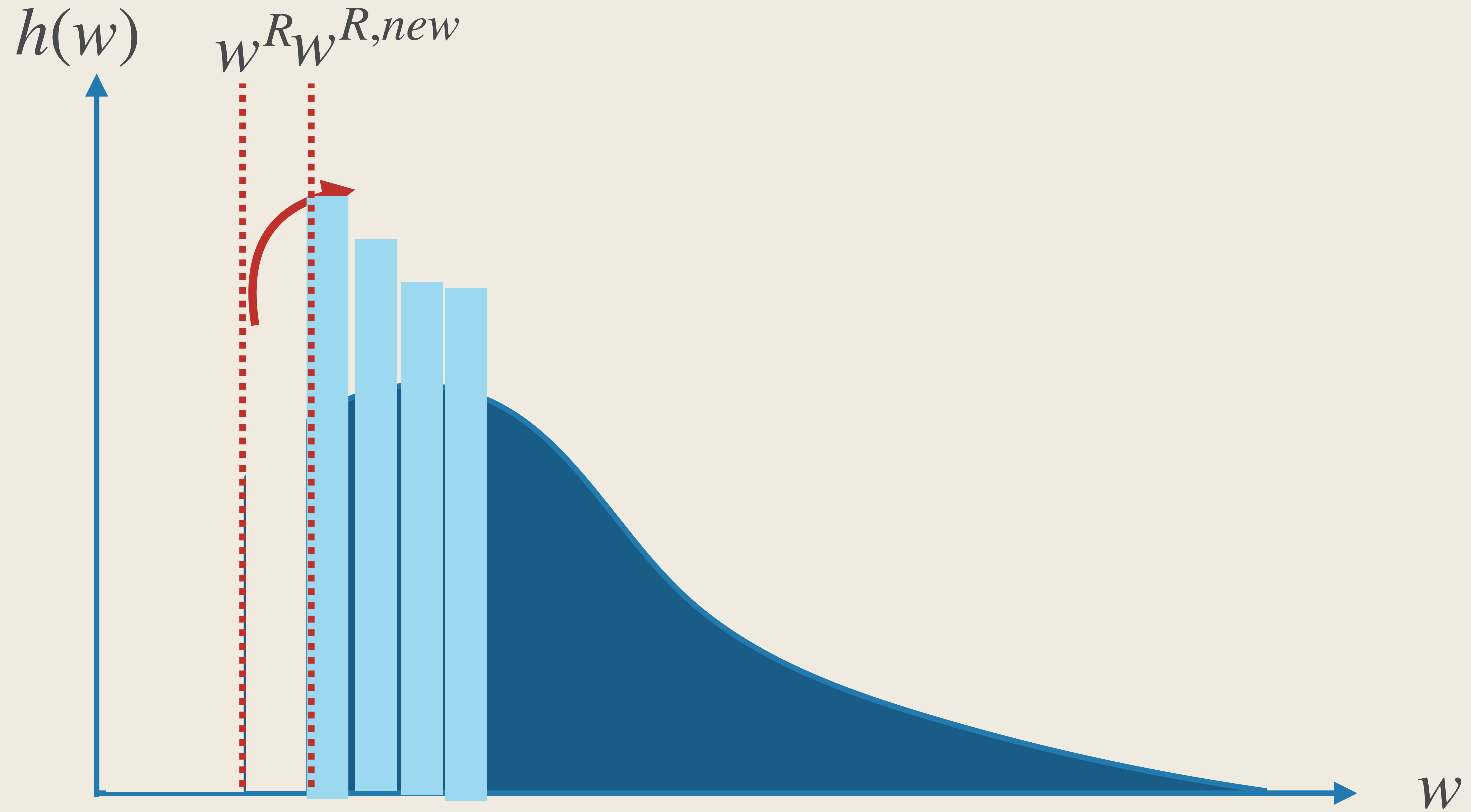

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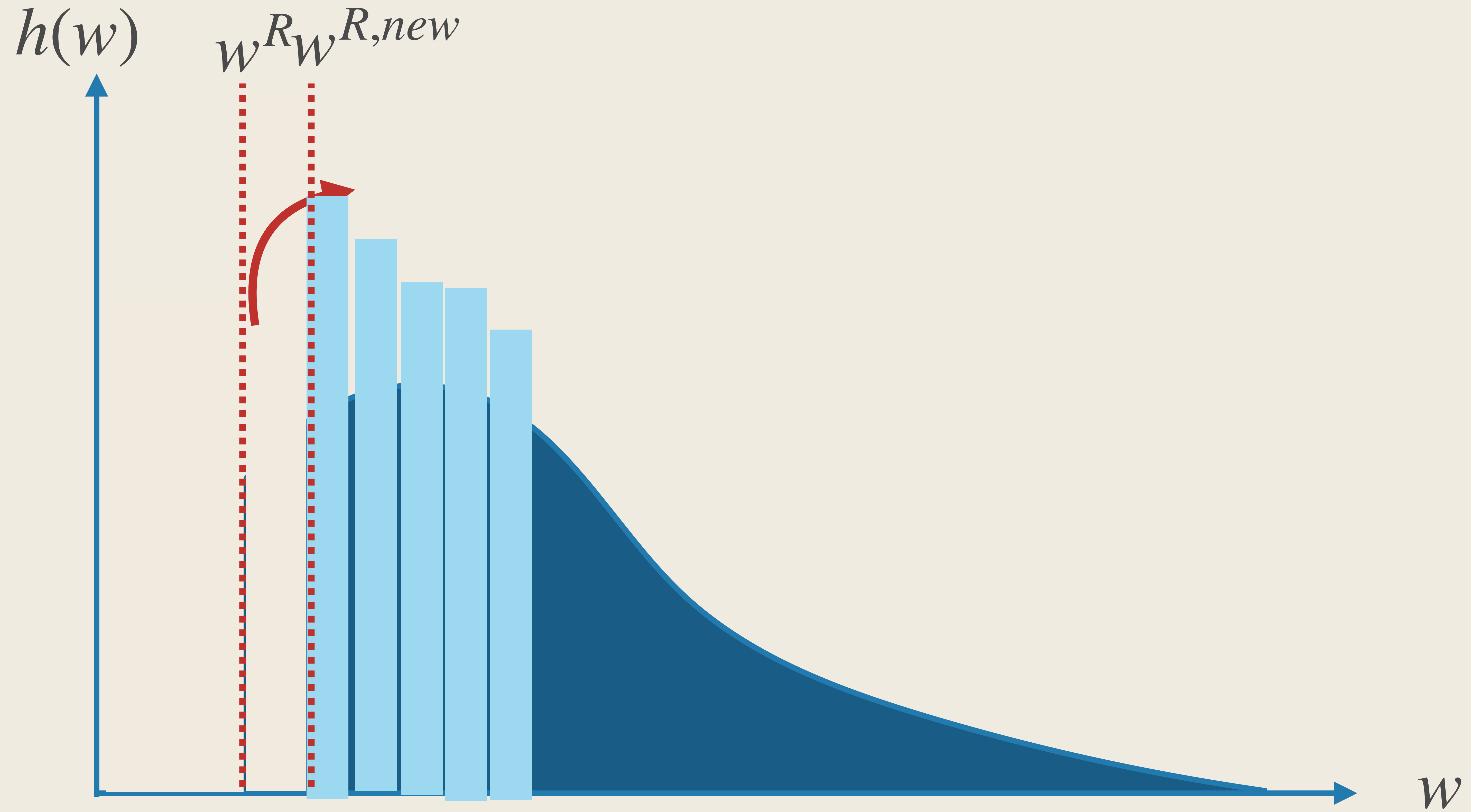

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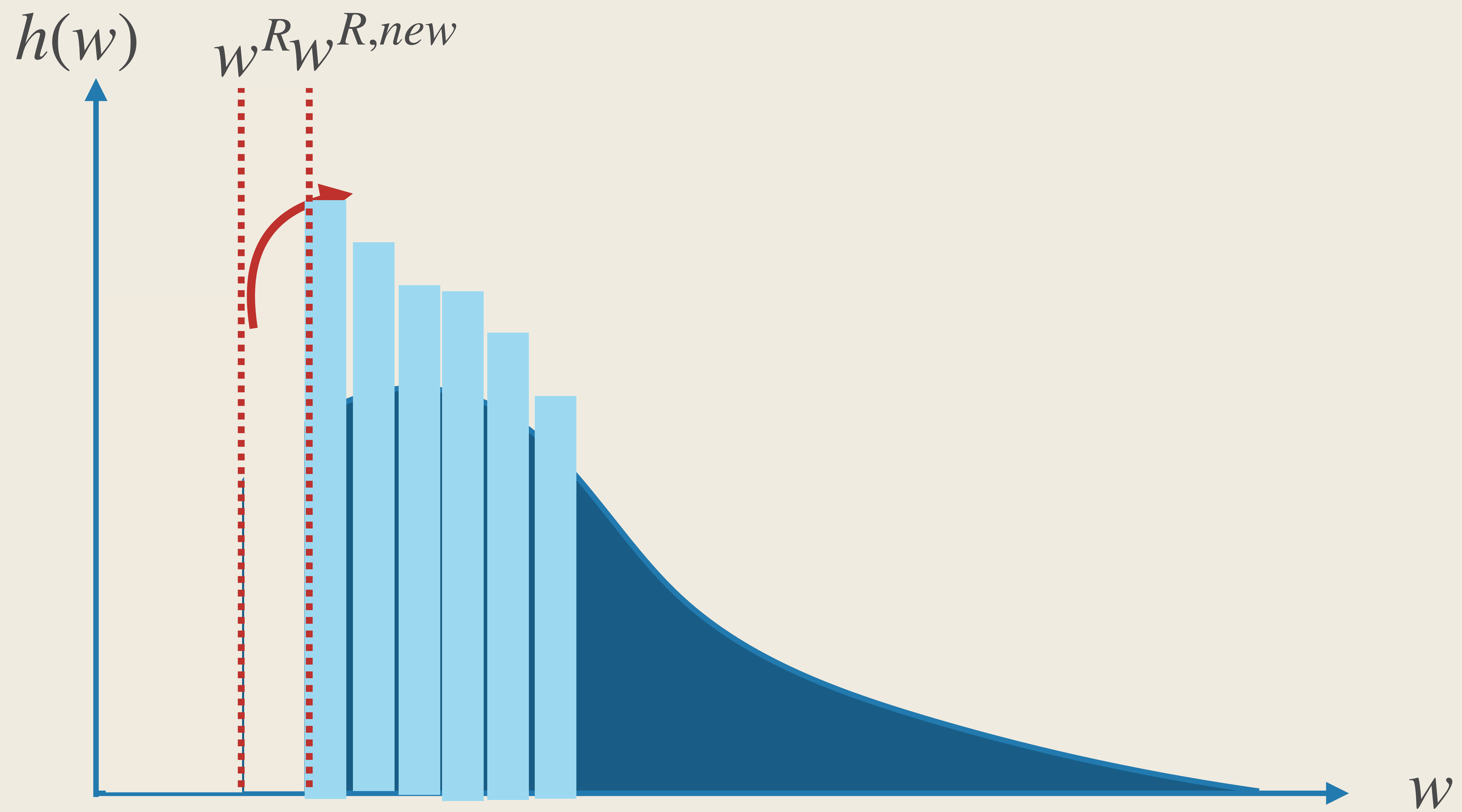

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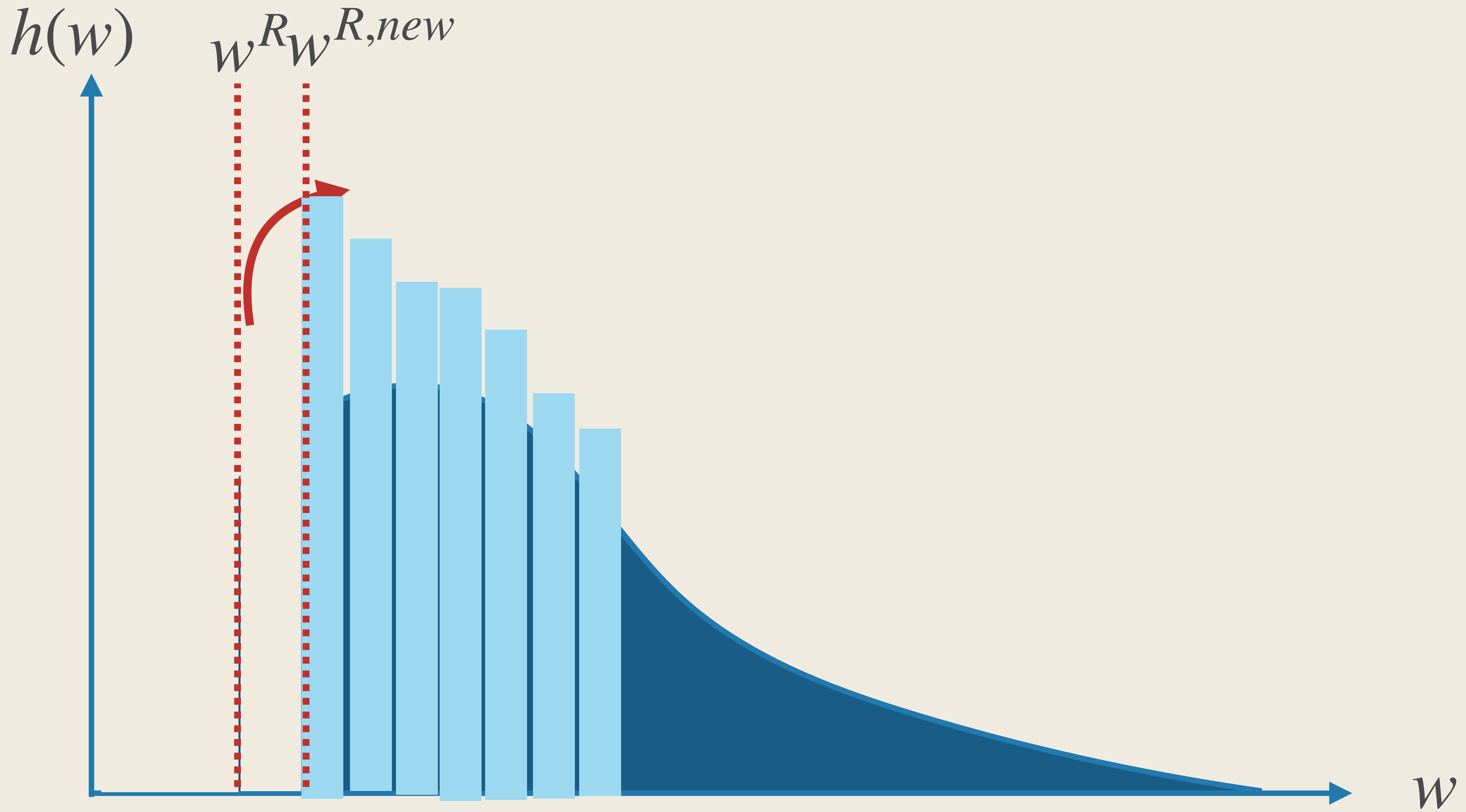
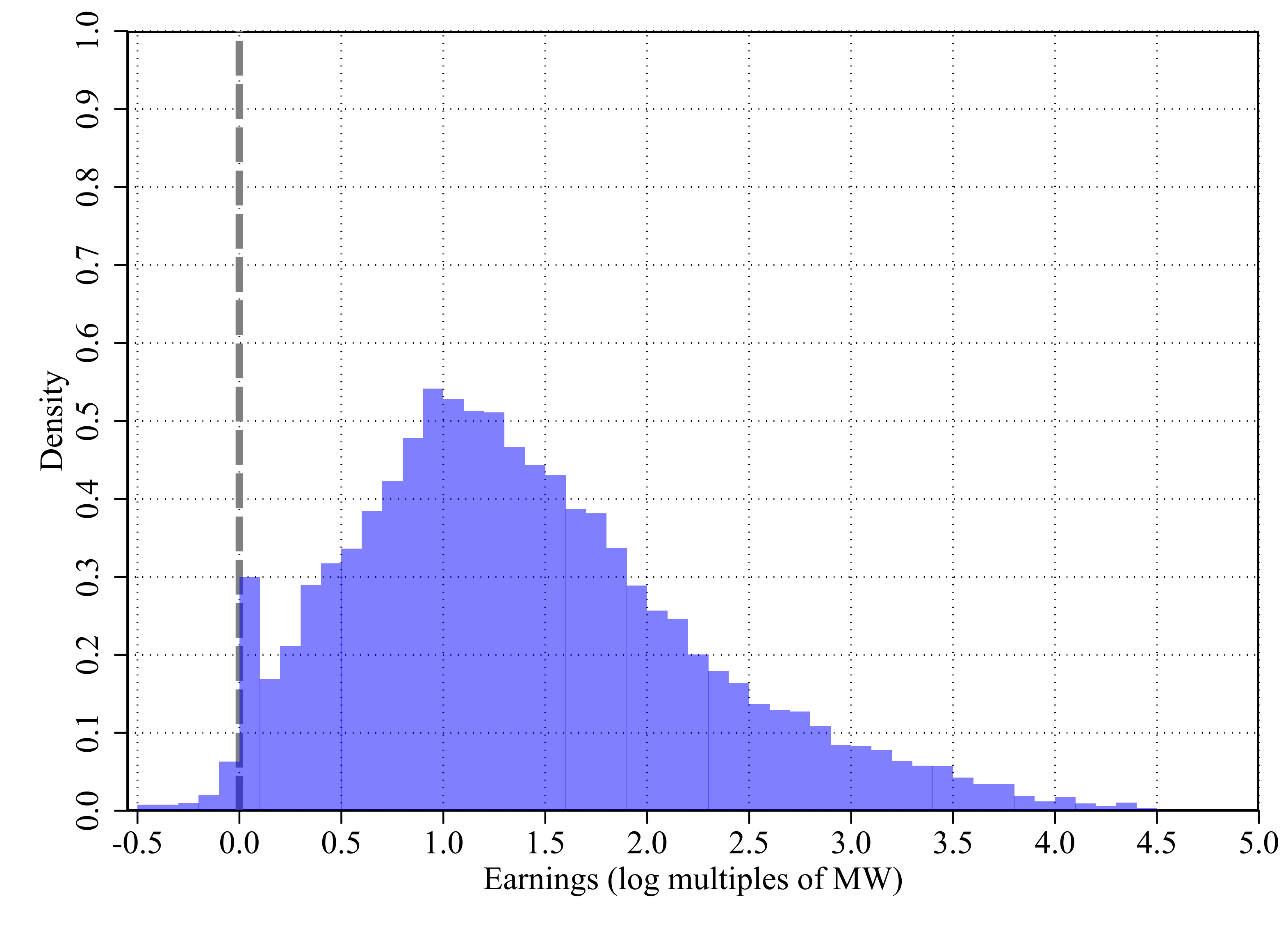
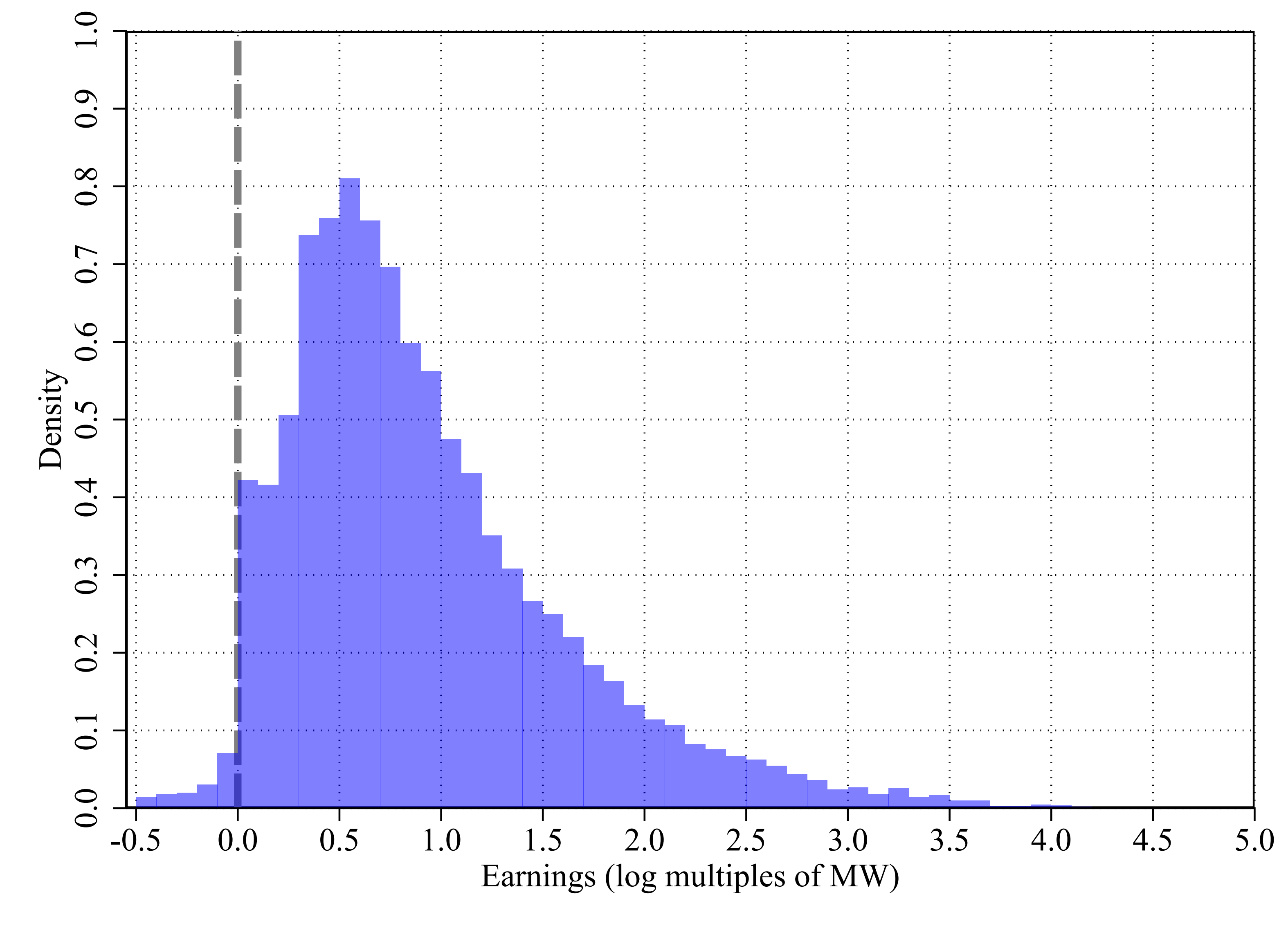


Falling Inequality at the Bottom

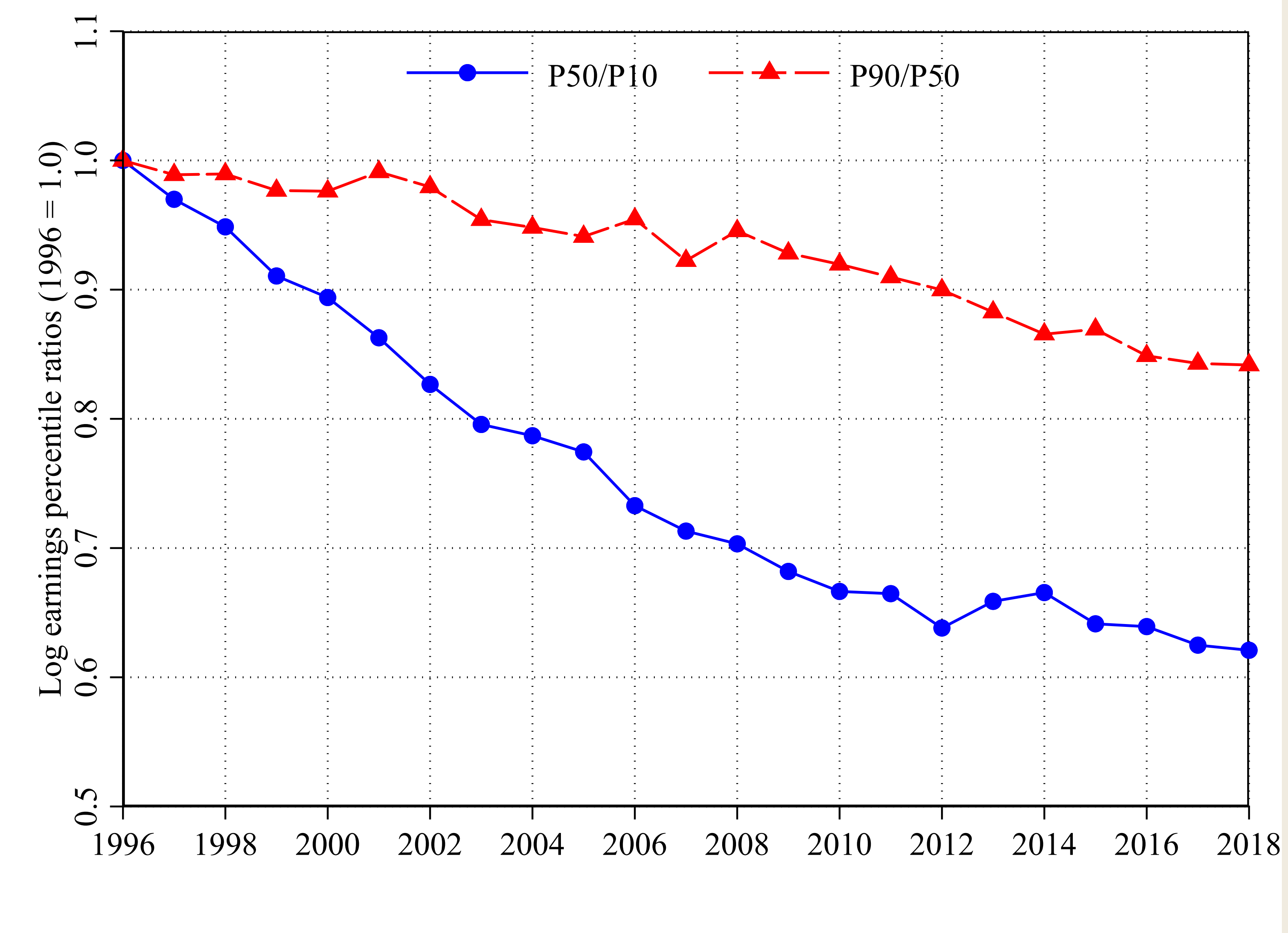
A. Histogram of log wages, 1996



B. Histogram of log wages, 2018



C. Percentile ratios, 1996–2018

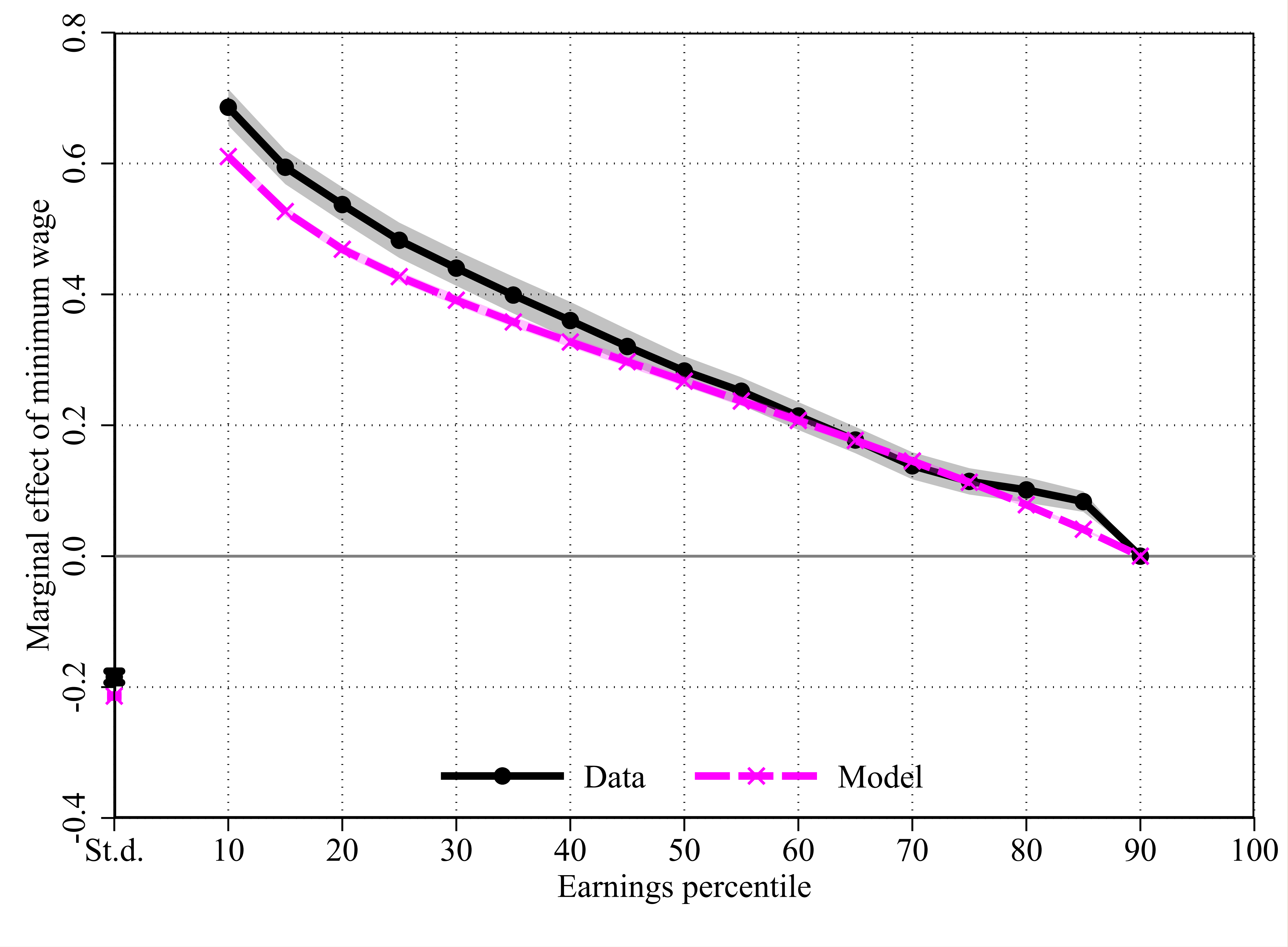


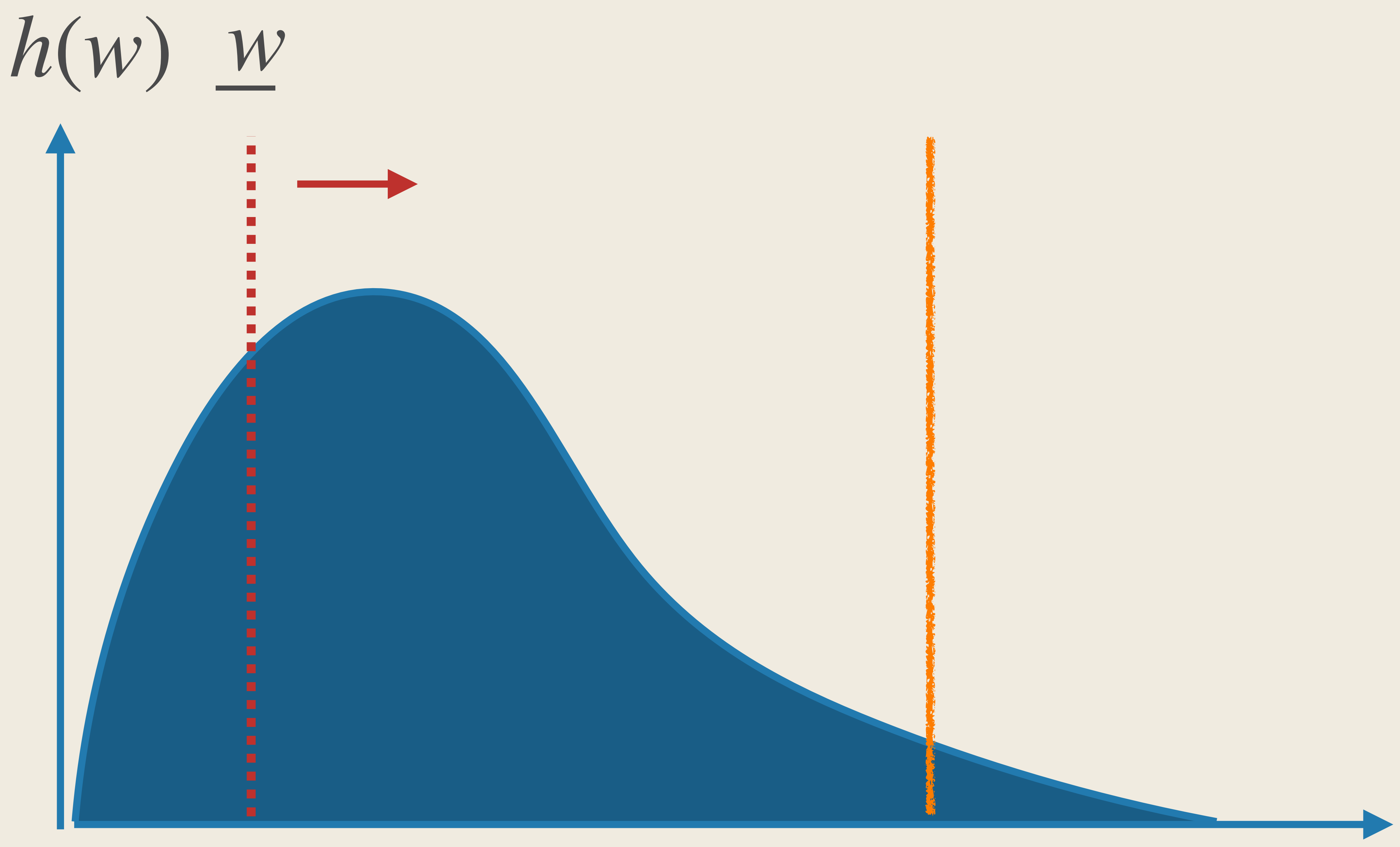
Notes: Panels A and B show histograms of log wages in multiples of the current minimum wage based on 60 equispaced bins for population of male workers aged 18–54 for 1996 and 2018, respectively. Panel C plots lower- and upper-tail wage inequality, as measured by the P50/P10 and the P90/P50 log wage percentile ratios between 1996 and 2018, normalized to 1.0 in 1996. *Source:* RAIS, 1996–2018.

Data versus Model

$$\ln w^p_{st} - \ln w^{90}_{st} = \beta^p[\ln w^{\min}_{st} - \ln w^{90}_{st}] + \gamma^p_s + \delta^p_s \times t + \epsilon^p_{st}$$

B. Relative to P90





90th percentile (control group)

Impact of Minimum Wage on Inequality

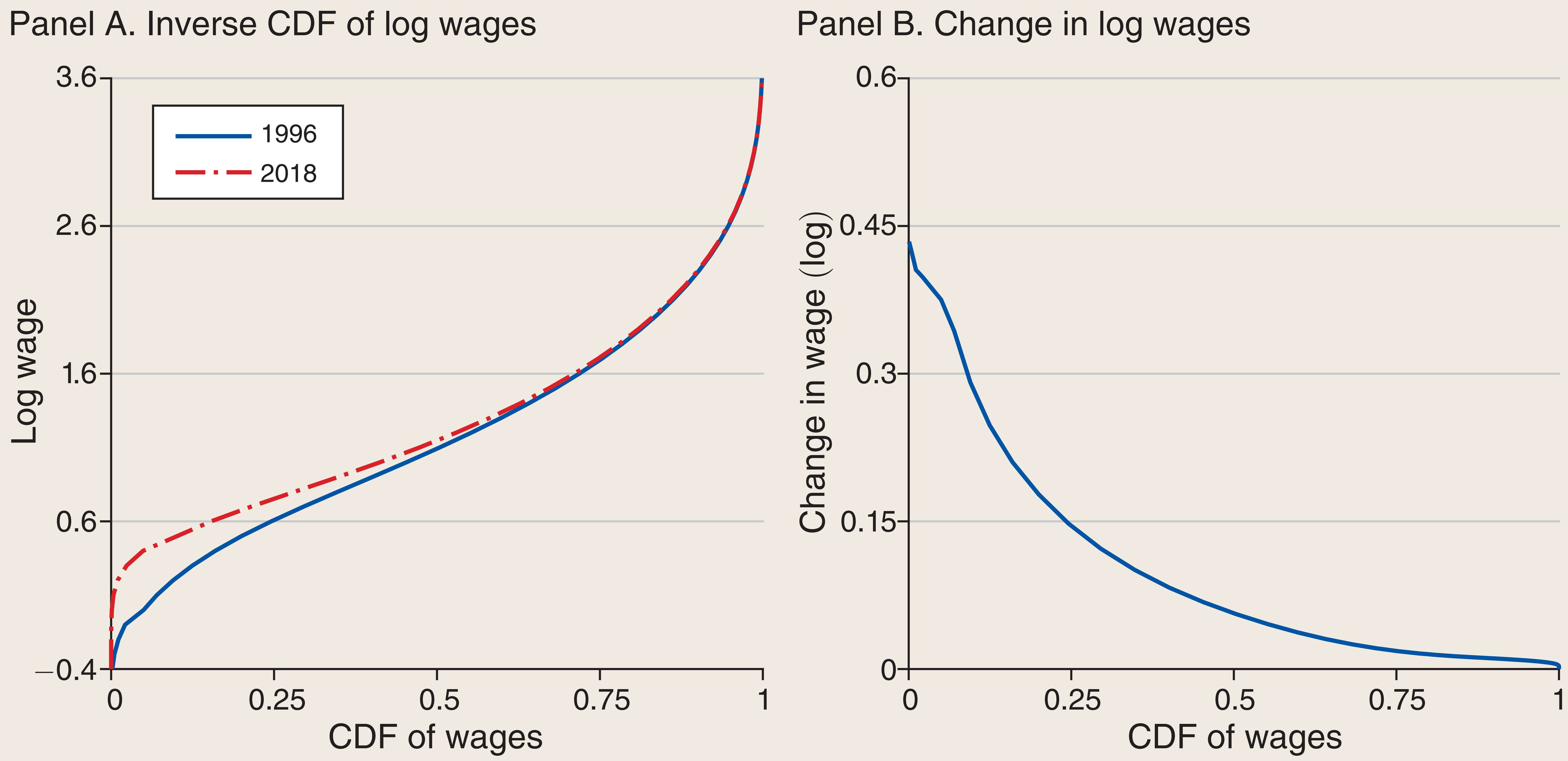


FIGURE 9. IMPACT OF THE MINIMUM WAGE THROUGHOUT THE WAGE DISTRIBUTION IN THE MODEL

- Increases in MW account for 45% of the reduction in Var($\ln w$) over 1996-2018

Taking Stock

- Burdett-Mortensen model with wage-posting instead of bargaining
  - Tractable framework with many empirical predictions
- However, we have restricted the contract space significantly
  - firms offer a single wage to all workers
  - why not wage-tenure contracts?
  - why not counteroffer?
- Active research going on how firms set wages