Declining Business Dynamism

(based on Karahan, Pugsley, and Şahin, 2024)

741 Macroeconomics

Topic 3

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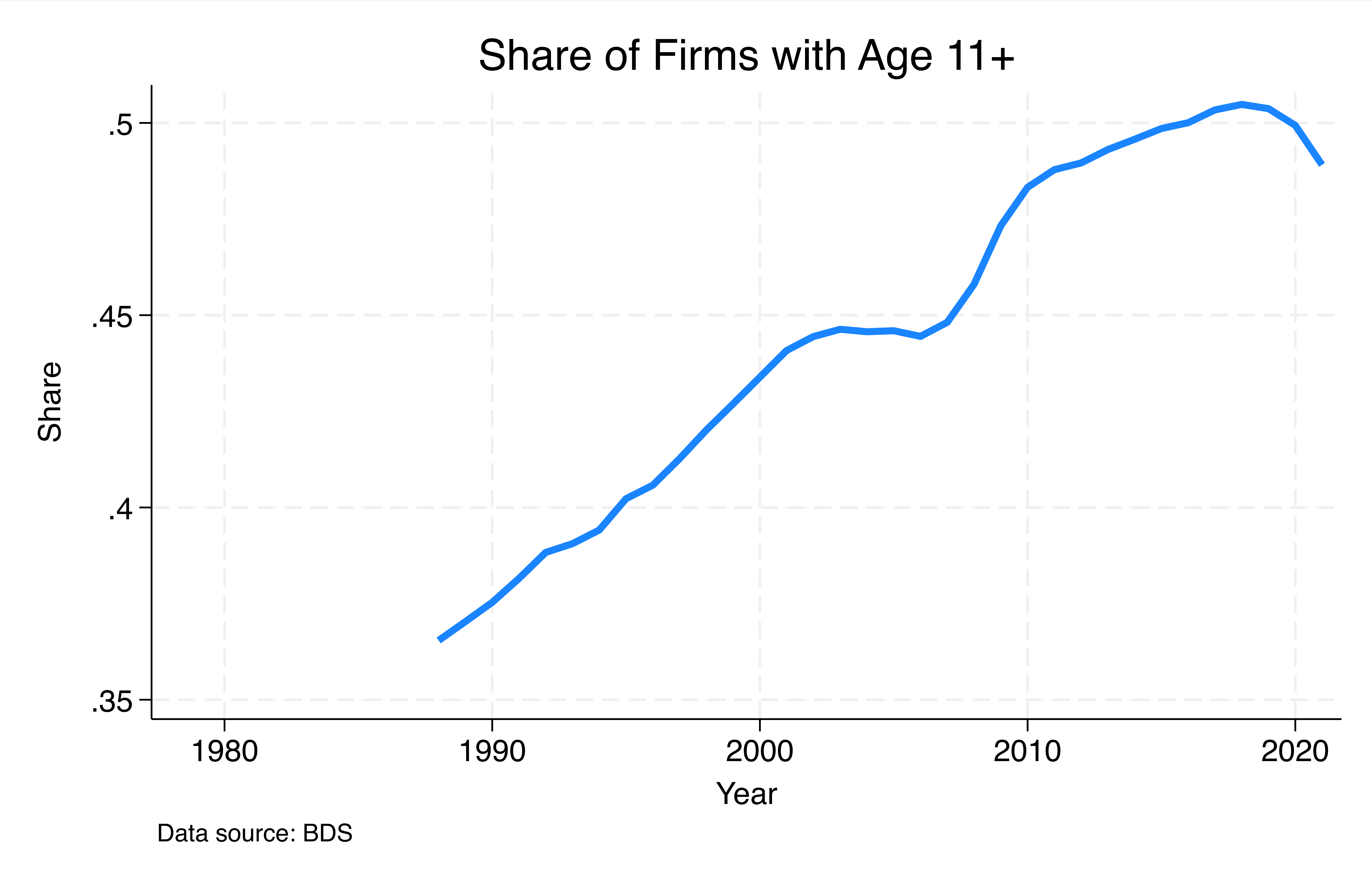
Fall 2024

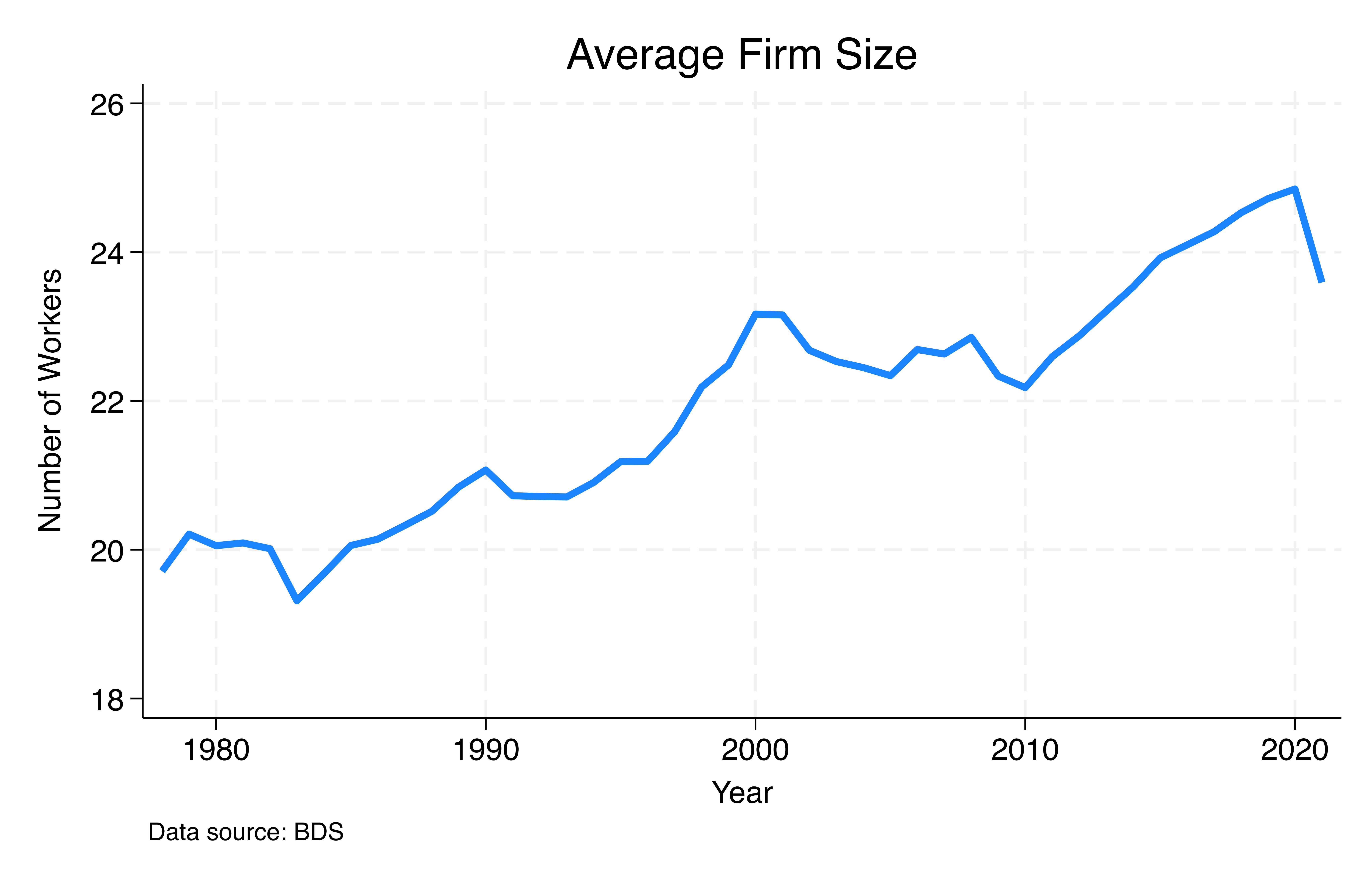
Declining Entry and Exit Rates

Data source: BDS

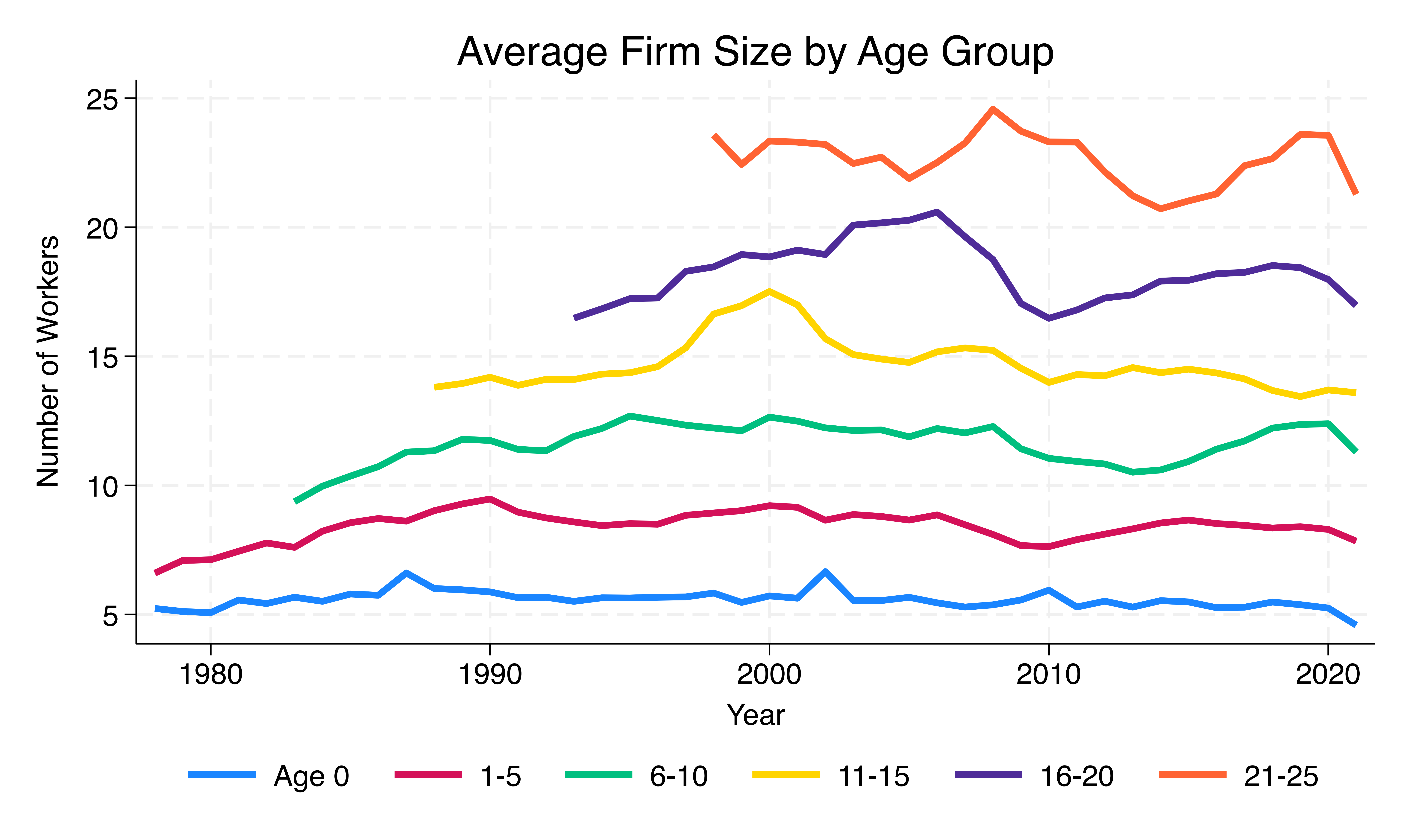
Firms are Getting Older



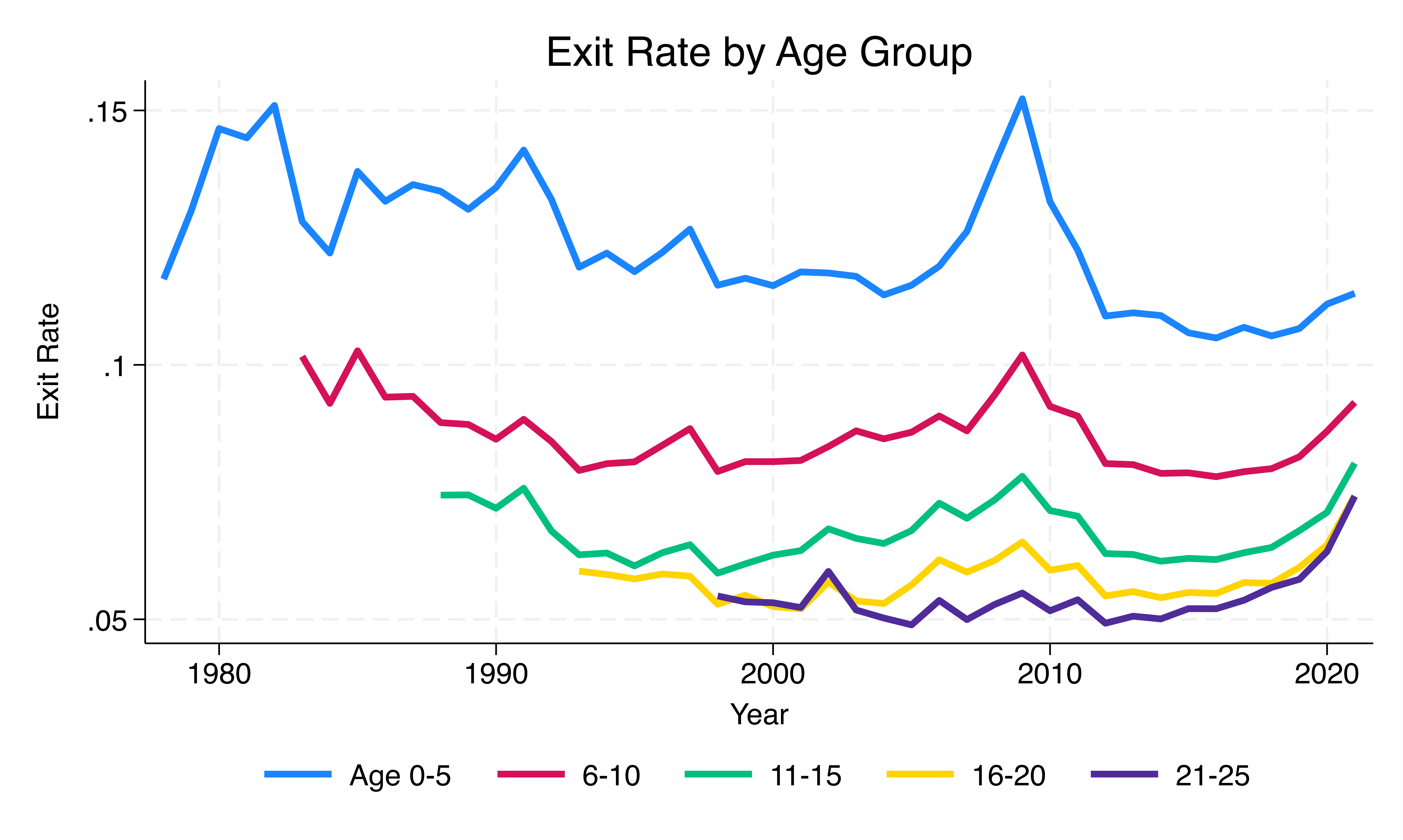
Firms are Getting Larger



Conditional on Age, Firm Size Remains Stable



Conditional on Age, Exit Rates Remain Stable

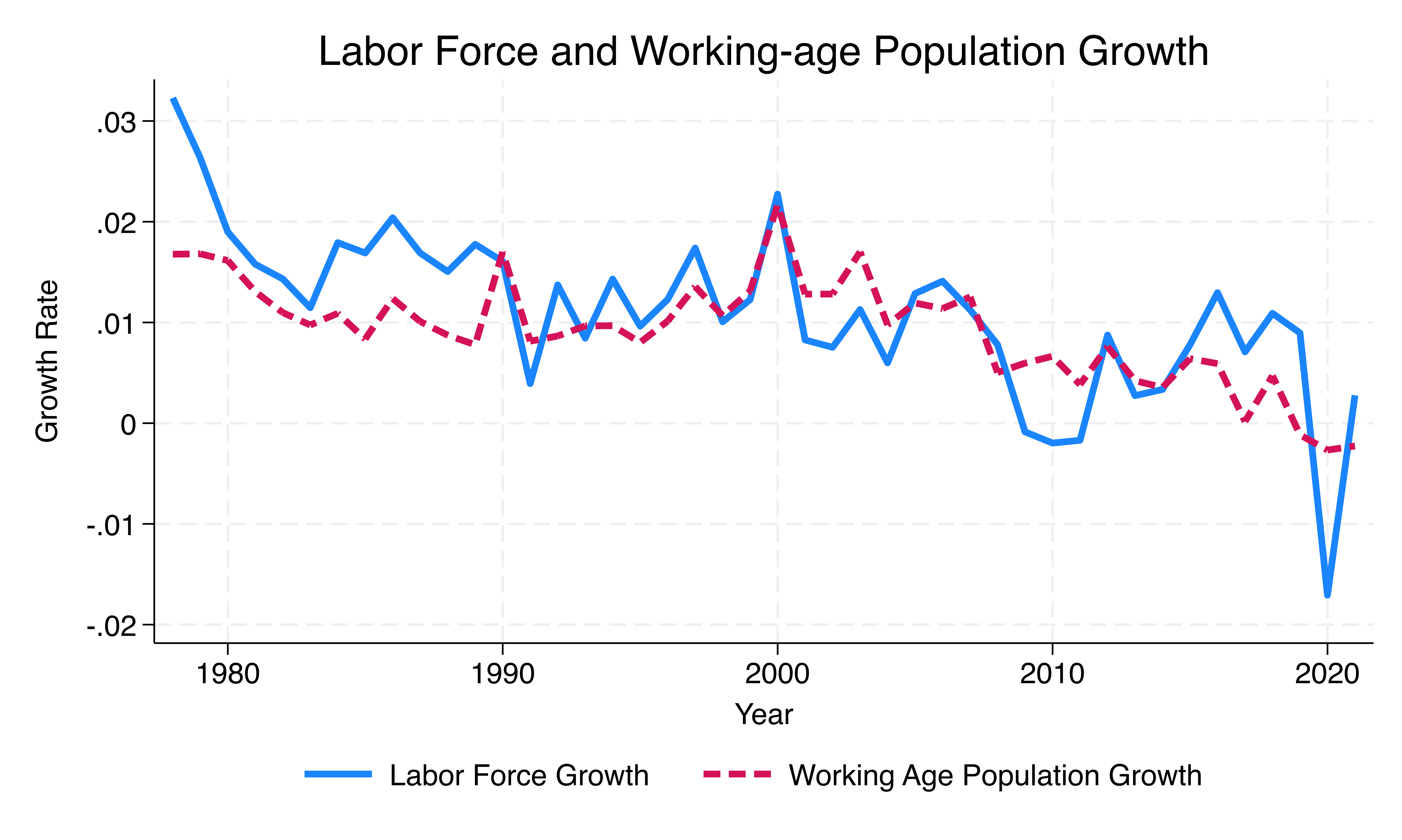


Empirical Facts

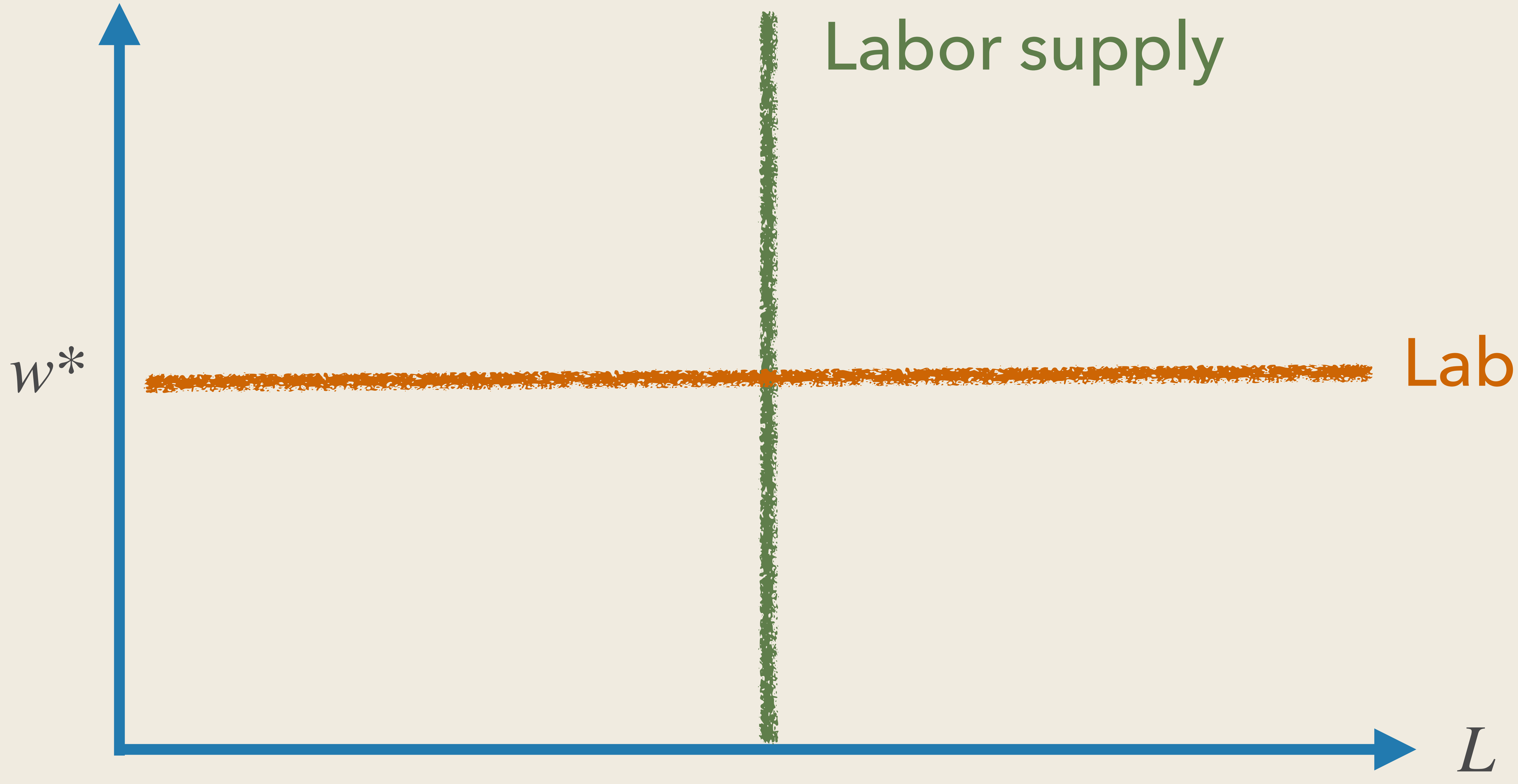
1. Entry rates have been declining, and consequently, firms are getting older
2. The firm's life-cycle dynamics (conditional on age) have little changed

Why?

Falling Labor Supply Growth

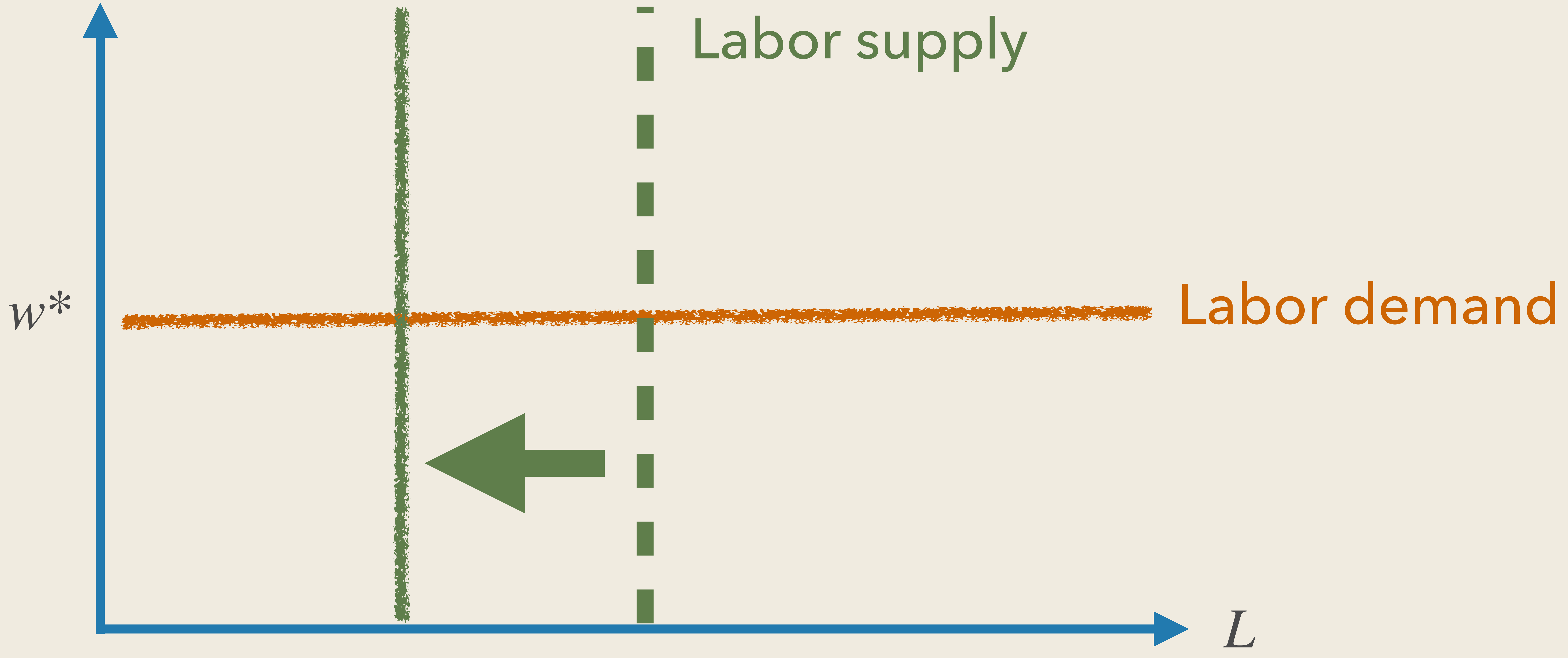


Fall in Labor Supply ⇒ Decline in Entry



- If labor supply falls, labor demand needs to fall in equilibrium
  - In Hopenhayn-Rogerson, wages do not rise
  - Then what adjusts? – Entry

Fall in Labor Supply ⇒ Decline in Entry



- If labor supply falls, labor demand needs to fall in equilibrium
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Hopenhayn-Rogerson with Labor Supply Growth

Households

- Households solve

$$\max_{\{C_t\}} \int_0^\infty e^{-rt} C_t L_t dt$$

$$\text{s.t.} \quad C_t L_t = w_t L_t + \int \pi(z; w) g(z) dz - m_t c_e$$

$$\partial_t L_t = \eta_t L_t$$

$\eta_t \geq 0$: population growth rate

- This is the only modification to the previous model

HJB Block

- HJB block remains completely unchanged

$$\min \left\{ rv(z) - \pi(z; w) - \mu(z)v'(z) - \frac{1}{2}\sigma(z)^2 v''(z), v(z) - \underline{v} \right\} = 0$$

$$v(\underline{z}) = \underline{v}$$

$$\int v(z)\psi(z)dz = c_e$$

- Consequently, real wage is a constant despite labor supply growth

KFE Block

- The distribution satisfies

$$\partial_t g_t(z) = -\partial_z[\mu(z)g_t(z)] + \frac{1}{2}\partial^2_{zz}\left[\sigma(z)^2 g_t(z)\right] + m_t\psi(z) \quad \text{for } z > \underline{z}$$

$$\int n(z;w)g_t(z)dz = L_t$$

- Conjecture $\tilde{g}(z) = g_t(z)/L_t$ and $\tilde{m} = m_t/L_t$ are stationary so that

$$\frac{1}{L_t}\partial_t g_t(z) = -\partial_z[\mu(z)\tilde{g}_t(z)] + \frac{1}{2}\partial^2_{zz}\left[\sigma(z)^2\tilde{g}_t(z)\right] + \tilde{m}\psi(z) \quad \text{for } z > \underline{z}$$

$$\underbrace{\frac{1}{L_t}\partial_t g_t(z) - \frac{g_t(z)}{L_t}\frac{\partial_t L_t}{L_t}}_{=\,\partial_t\tilde{g}_t(z)} + \frac{g_t(z)}{L_t}\frac{\partial_t L_t}{L_t} = \partial_t\tilde{g}_t(z) + \tilde{g}_t(z)\eta_t$$

Normalized Distribution

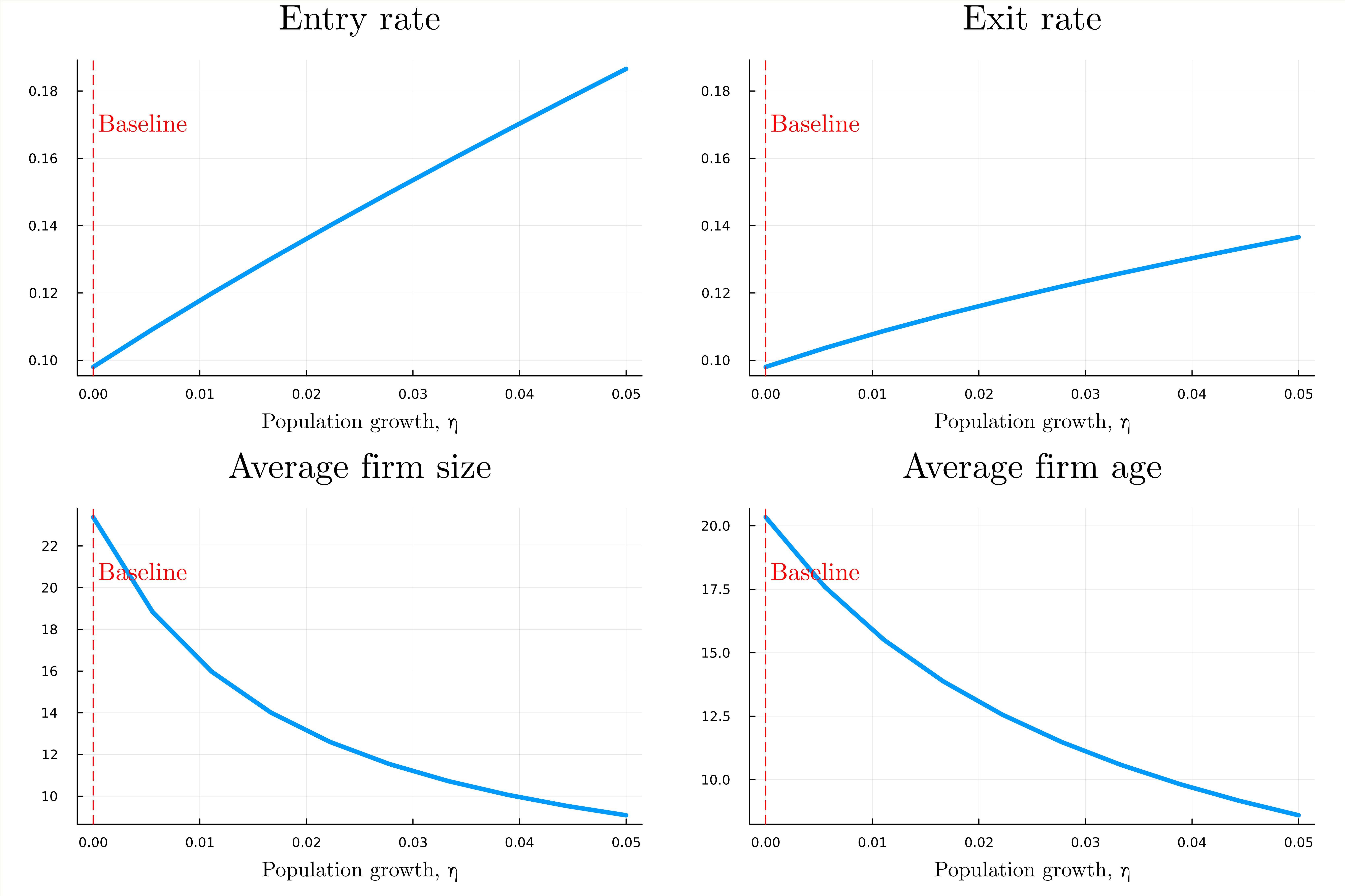
$$\partial_t \tilde{g}_t(z) = -\eta_t \tilde{g}_t(z) - \partial_z[\mu(z)\tilde{g}_t(z)] + \frac{1}{2}\partial_{zz}^2\left[\sigma(z)^2\tilde{g}_t(z)\right] + \tilde{m}_t\psi(z) \quad \text{for } z > \underline{z}$$

$$\int n(z;w)\tilde{g}_t(z)dz = 1$$

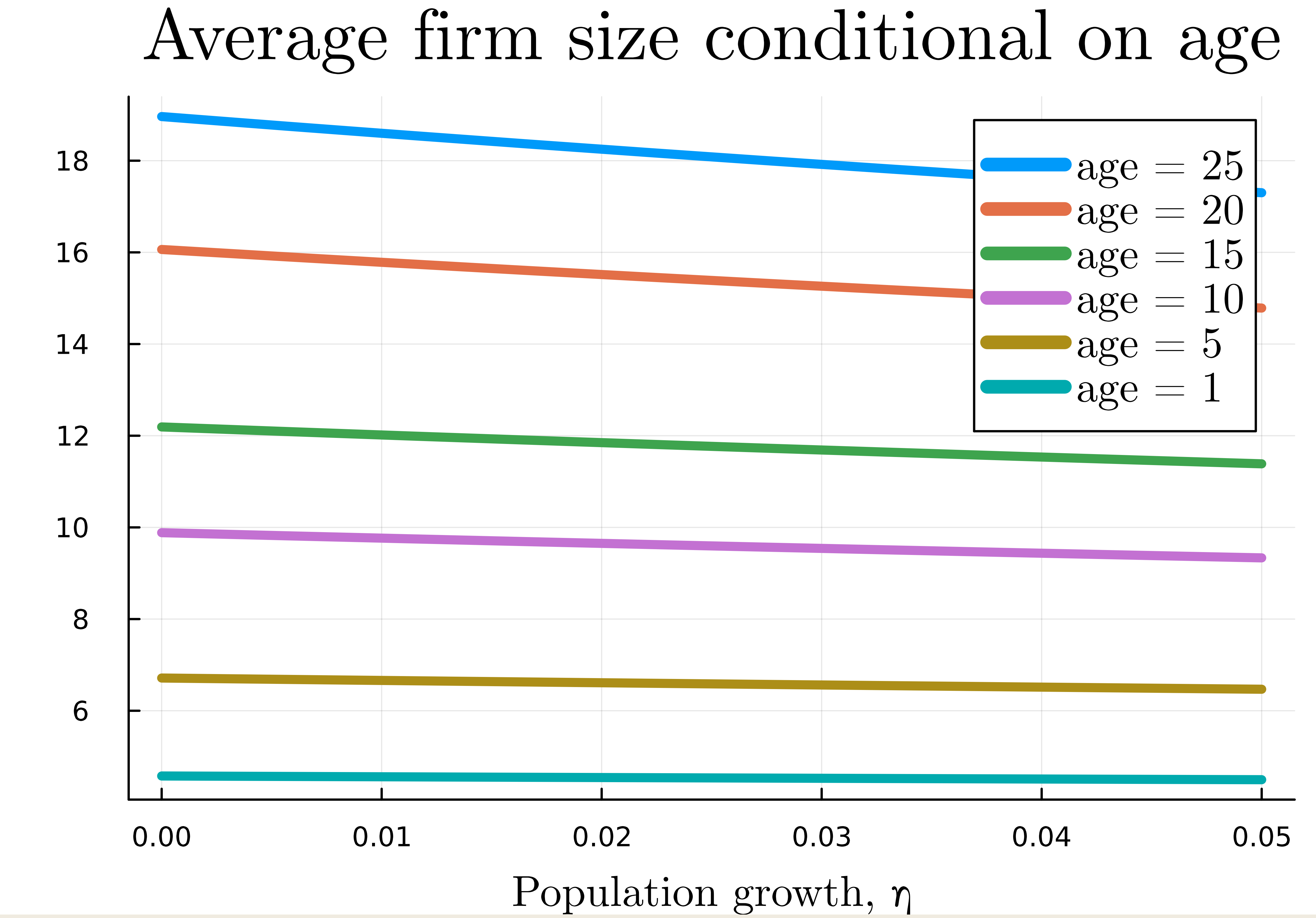
- This does not involve non-stationary variable,
  $\Rightarrow$ confirming our conjecture that $\tilde{g}(z)$ and $\tilde{m}$ are stationary

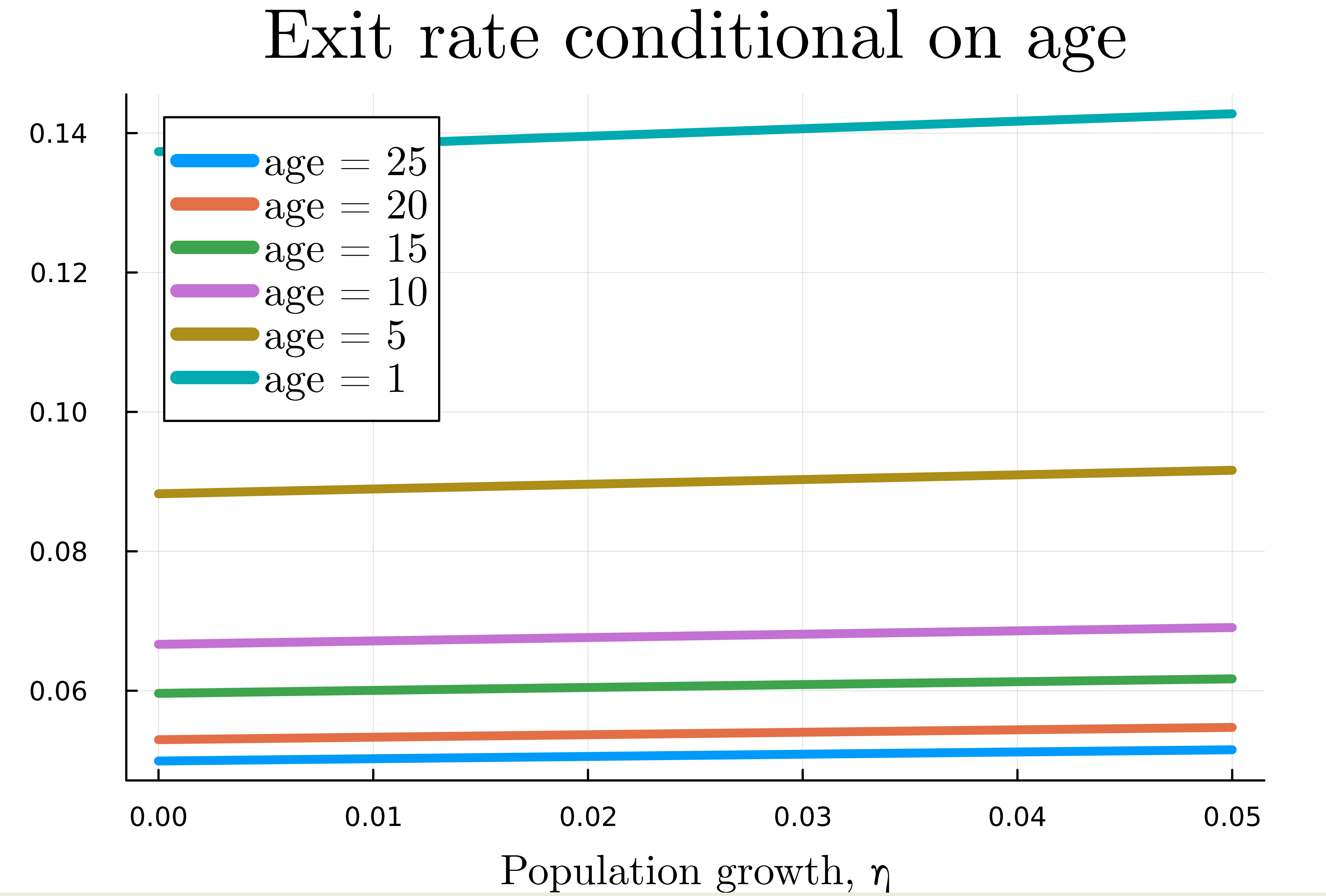
Comparative Statics Across Steady State

Demographic Origins of Startup Deficits

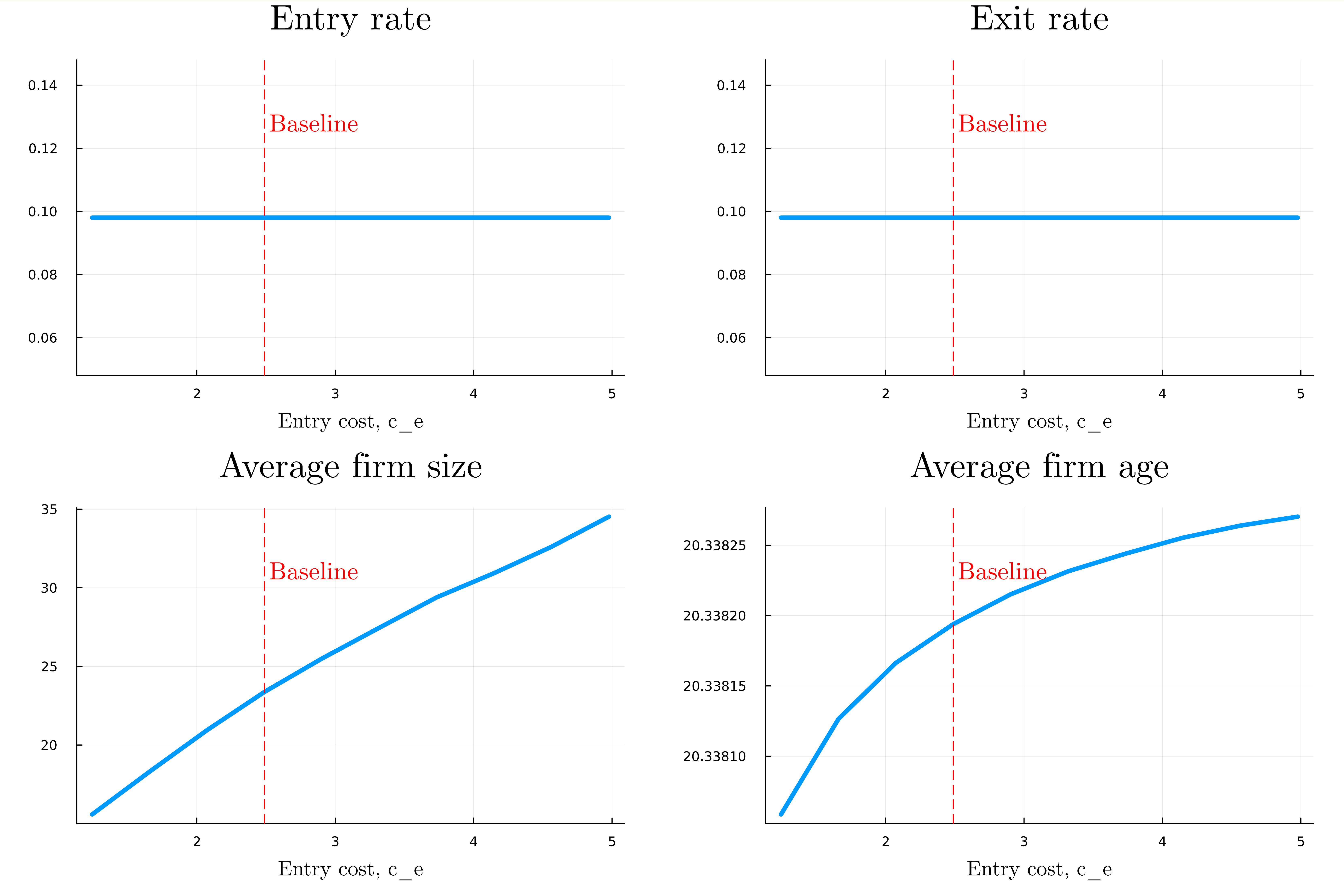


No Changes in Firm Life Cycle

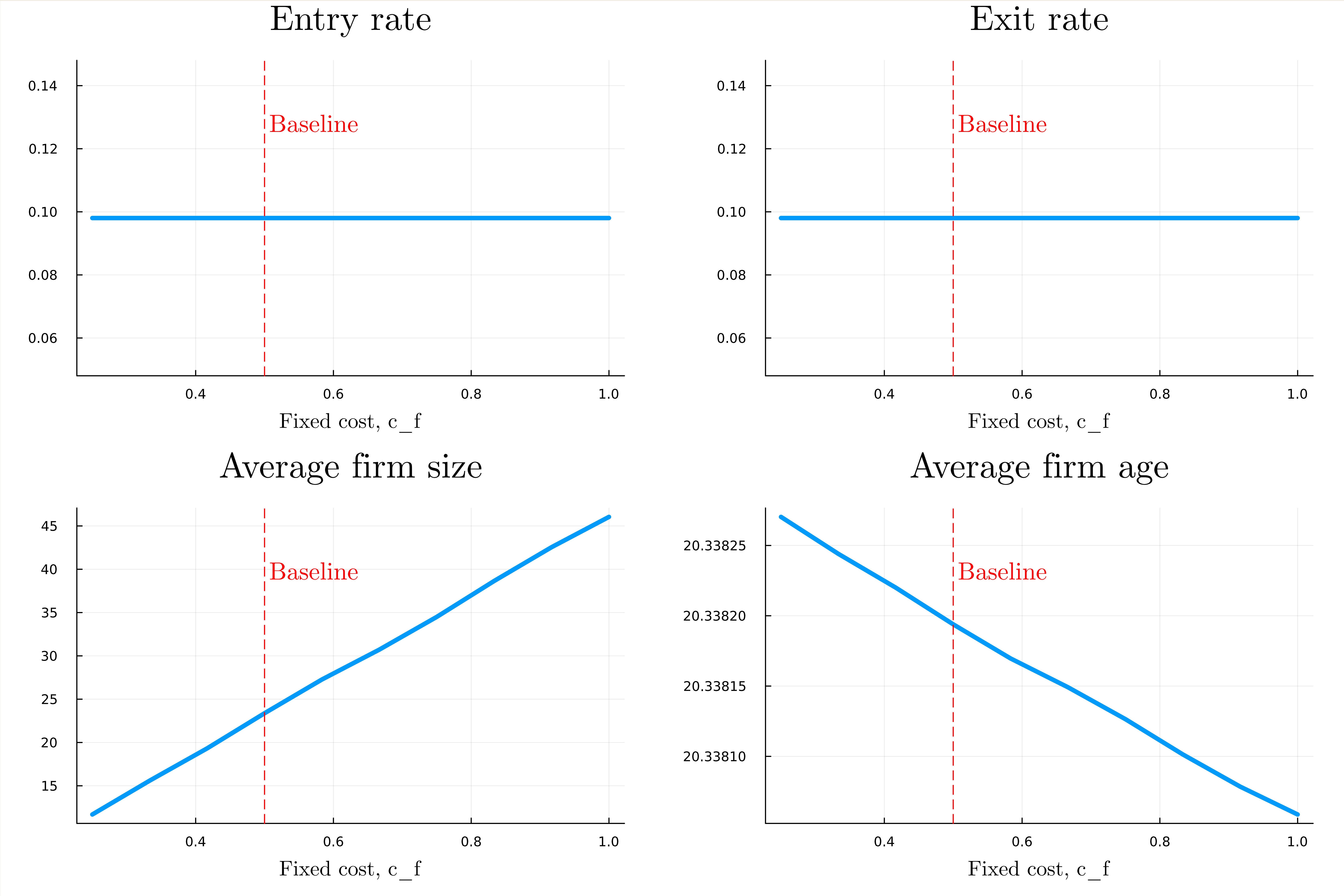




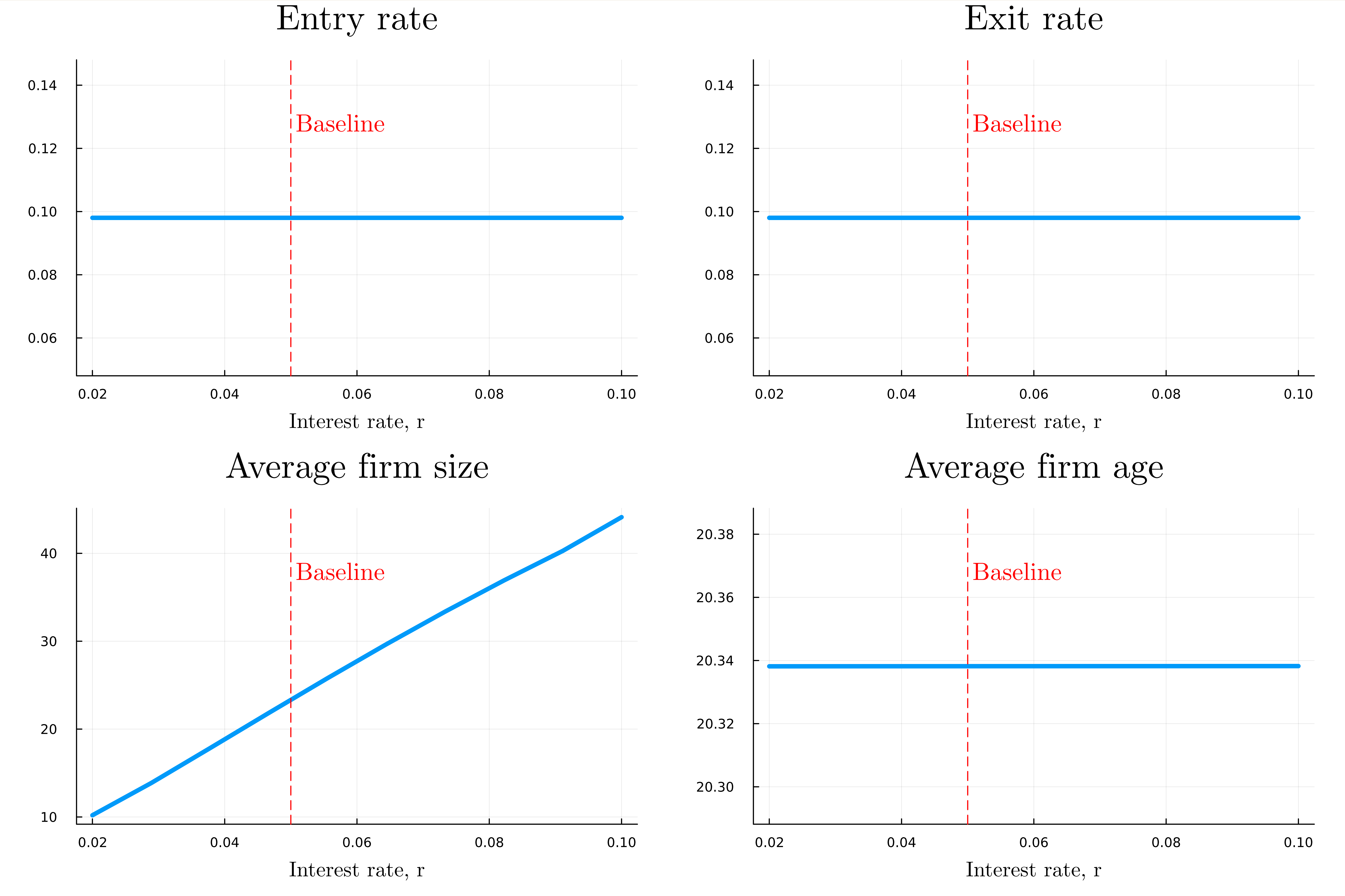
Alternative Explanation: Changes in Entry Cost, c_e



Alternative Explanation: Changes in Fixed Cost, c_f



Alternative Explanation: Changes in Real Rate, r



Original Results

	Actual change	Potential channels: Entry cost, c_e	Operating cost, c_f	Labor supply growth, η
Panel A. Explaining the long-run decline in the start-up rate				
Required parameter change	—	122.6%	–55.7%	–2.1(pp)
Panel B. Implied change in each margin				
Start-up rate (pp)	–2.9	–2.9	–2.9	–2.9
Economy-wide exit rate (pp)	–0.8	–3.0	–3.0	–1.0
Average firm size (emp)	2.0	6.4	–10.7	3.7
Start-up size (emp)	0.1	3.3	–3.4	0.0
Young small exit rate (pp)	–0.1	–3.2	–3.6	0.0
Young small growth rate (pp)	0.5	–0.4	–0.8	0.0

Source: Karahan-Pugsley-Sahin (2024)

Conjecture

Conjecture

Suppose the following two holds.

(i) The productivity distribution of entrants follows Pareto

(ii) The productivity process is given by geometric Brownian motion

Then, entry and exit rates in the steady state are invariant to changes in any parameter that only enters into HJB-VI

- If you prove the above conjecture, I will count it as a final project
- Another thought: Is there a restriction that $c_e, c_f \uparrow$ leads to entry & exit rates $\uparrow$?

Transition Dynamics

How Fast is the Transition?

- Comparing across steady states is potentially misleading
- What if it takes a thousand years to reach from one steady state to another?
- To address this issue, we would like to simulate the transition dynamics
- How do we do that?

Block Recursive Property, Again

- In Hopenhayn-Rogerson, this is extremely easy
- Recall that the HJB block is independent of the KFE block:

$$\min\left\{rv(z) - \pi(z; w) - \mu(z)v'(z) - \frac{1}{2}\sigma(z)^2 v''(z), v(z) - \underline{v}\right\} = 0$$

$$v(\underline{z}) = \underline{v}$$

$$\int v(z)\psi(z)dz = c_e$$

  - η_t absent $\Rightarrow$ there is no need to solve the HJB block along the transition
  - More generally, the HJB block can be solved without solving for the KFE block
  - Again, this is “block recursive property” (see Kaas (2023) for a general treatment)

Solving the Transition of KFE Block

- The only part we need to simulate is the KFE block:

$$\partial_t \tilde{g}_t(z) = -\eta_t \tilde{g}_t(z) - \partial_z[\mu(z)\tilde{g}_t(z)] + \frac{1}{2}\partial_{zz}^2\left[\sigma(z)^2\tilde{g}_t(z)\right] + \tilde{m}_t\psi(z) \quad \text{for } z > \underline{z}$$

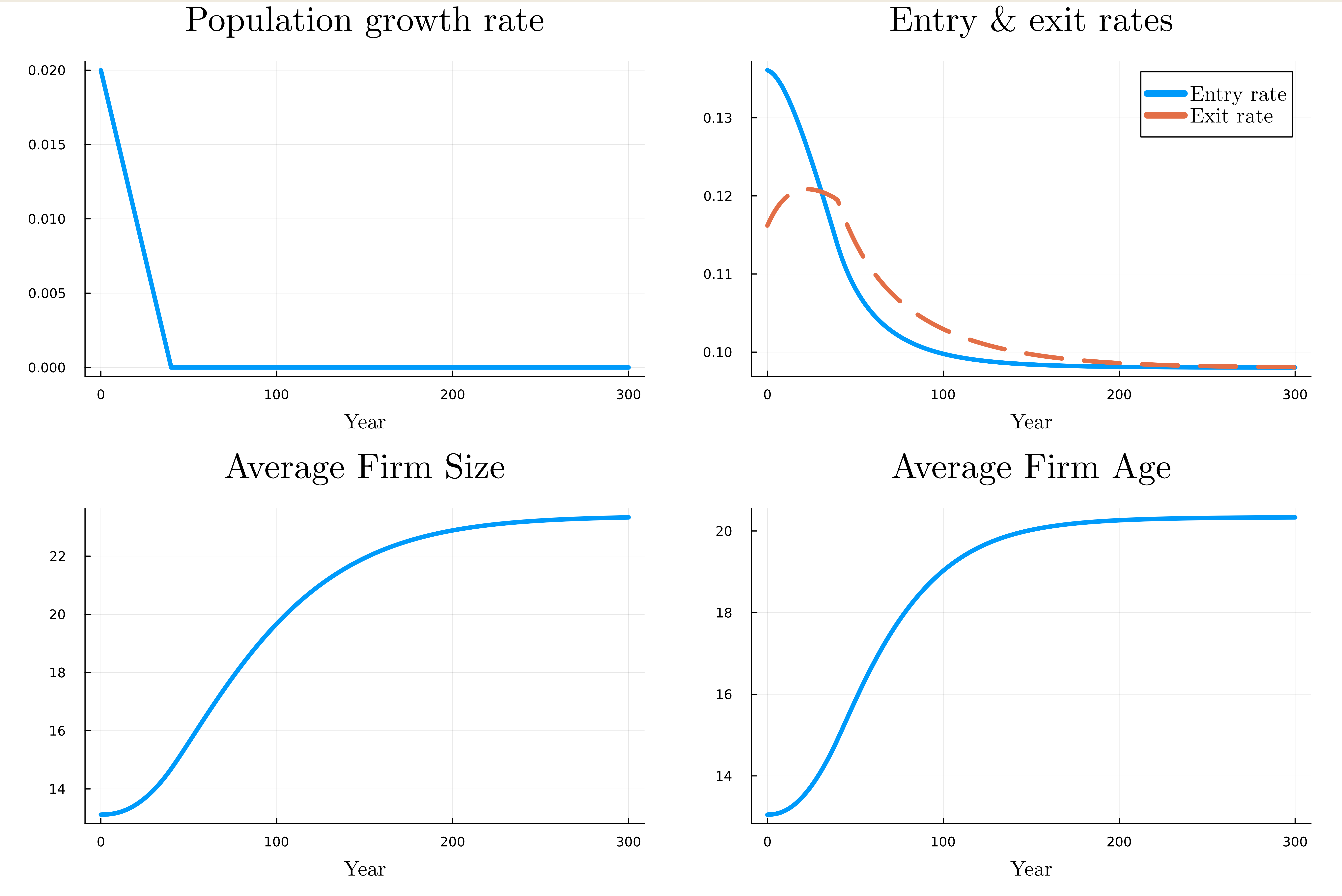
$$\int n(z; w)\tilde{g}_t(z)dz = 1$$

- Starting from $\{\tilde{g}_0(z)\}$, we can simulate any $\{\tilde{g}_t(z)\}_{t\geq 0}$
- We already know how to do this!

$$\frac{\tilde{\boldsymbol{g}}_t - \tilde{\boldsymbol{g}}_{t-\Delta t}}{\Delta t} = \tilde{\boldsymbol{A}}_t^T \tilde{\boldsymbol{g}}_t + \tilde{\boldsymbol{\psi}}$$

$$\Leftrightarrow \quad \tilde{\boldsymbol{g}}_t = \left[\boldsymbol{I} - \Delta t \times \tilde{\boldsymbol{A}}_t^T\right]^{-1}\left[\tilde{\boldsymbol{g}}_{t-\Delta t} + \Delta t \times \tilde{\boldsymbol{\psi}}\right]$$

Transition Dynamics



Empirical Support for “Demographic Origin of Startup Deficits” in the Cross-Section

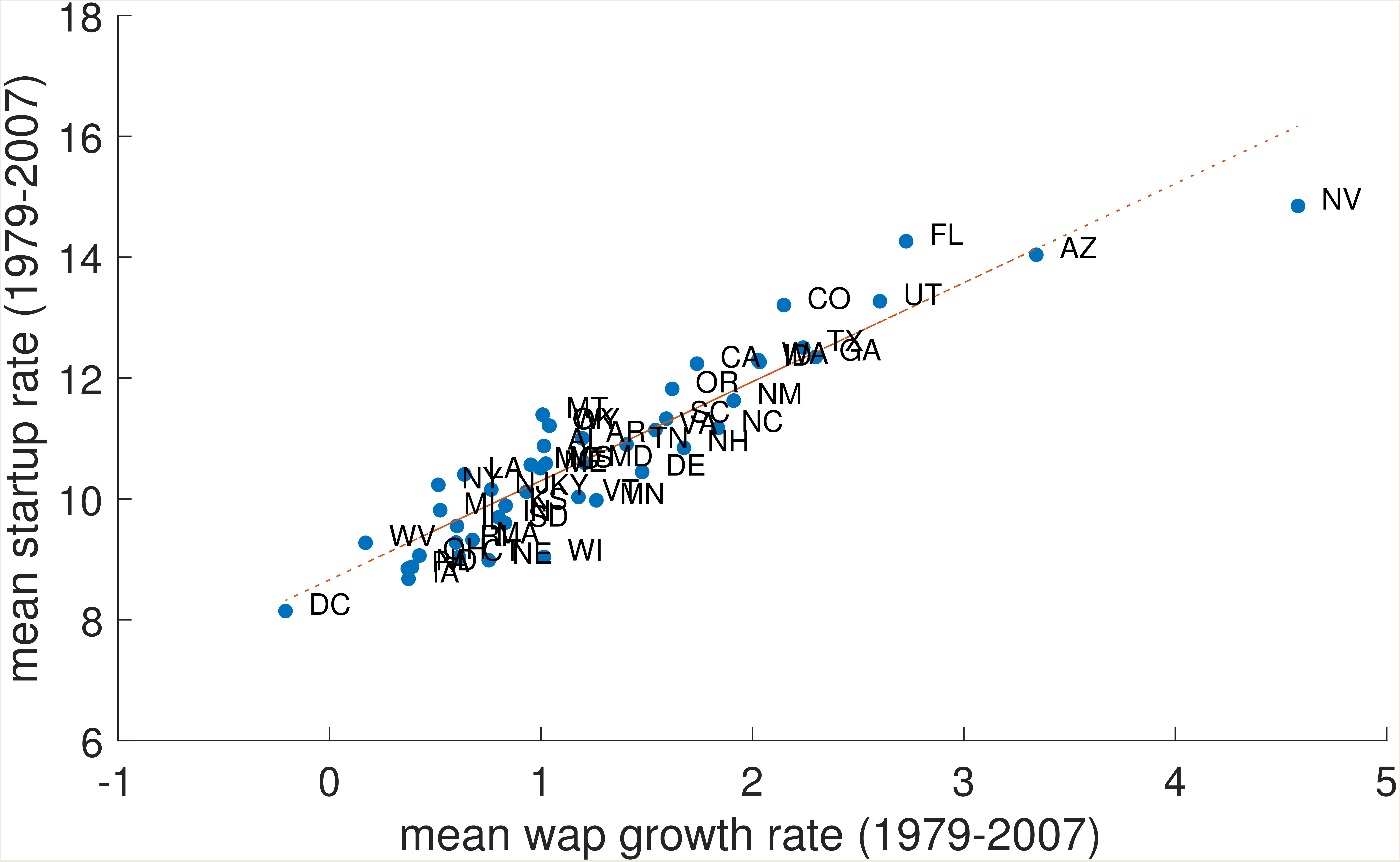
From Time-Series to Cross-Section

- Demographic origins of startup deficit:

$$\text{Labor supply growth} \downarrow \; \Rightarrow \text{entry rates} \downarrow$$

- Evaluate the mechanism in the cross-section across U.S. states

Startup Rates & Labor Supply Growth in the Cross-Section



Empirical Speficification

$$SR_{st} = \beta g_{st} + \alpha_s + \gamma_t + \boldsymbol{\delta}' \boldsymbol{X}_{st} + \epsilon_{st}$$

- SR_{st}: startup rate in state s in year t
- g_{st}: labor supply growth rate
- α_s, γ_t: state- and year- fixed effects
- OLS estimates of β can be biased:
  - A positive TFP shock can bring both new firms and new workers $\Rightarrow \mathbb{E}[g_{st}\epsilon_{st}] \neq 0$
- Need an IV that is (i) correlated with g_{st}; (ii) uncorrelated with ϵ_{st}

Two Instruments

1. Lagged fertility instrument:

$$IV_{1,st} = \text{Fertility}_{s,t-20}$$

- Fertility rate 20 years ago is a strong predictor of labor supply growth
- Exclusion restriction:
  Higher fertility 20 years ago affects firm creation only through labor supply

2. Migration instrument:

$$IV_{2,st} = \sum_{k \neq s} \omega^k_{s,t-10} \times g_{kt}$$

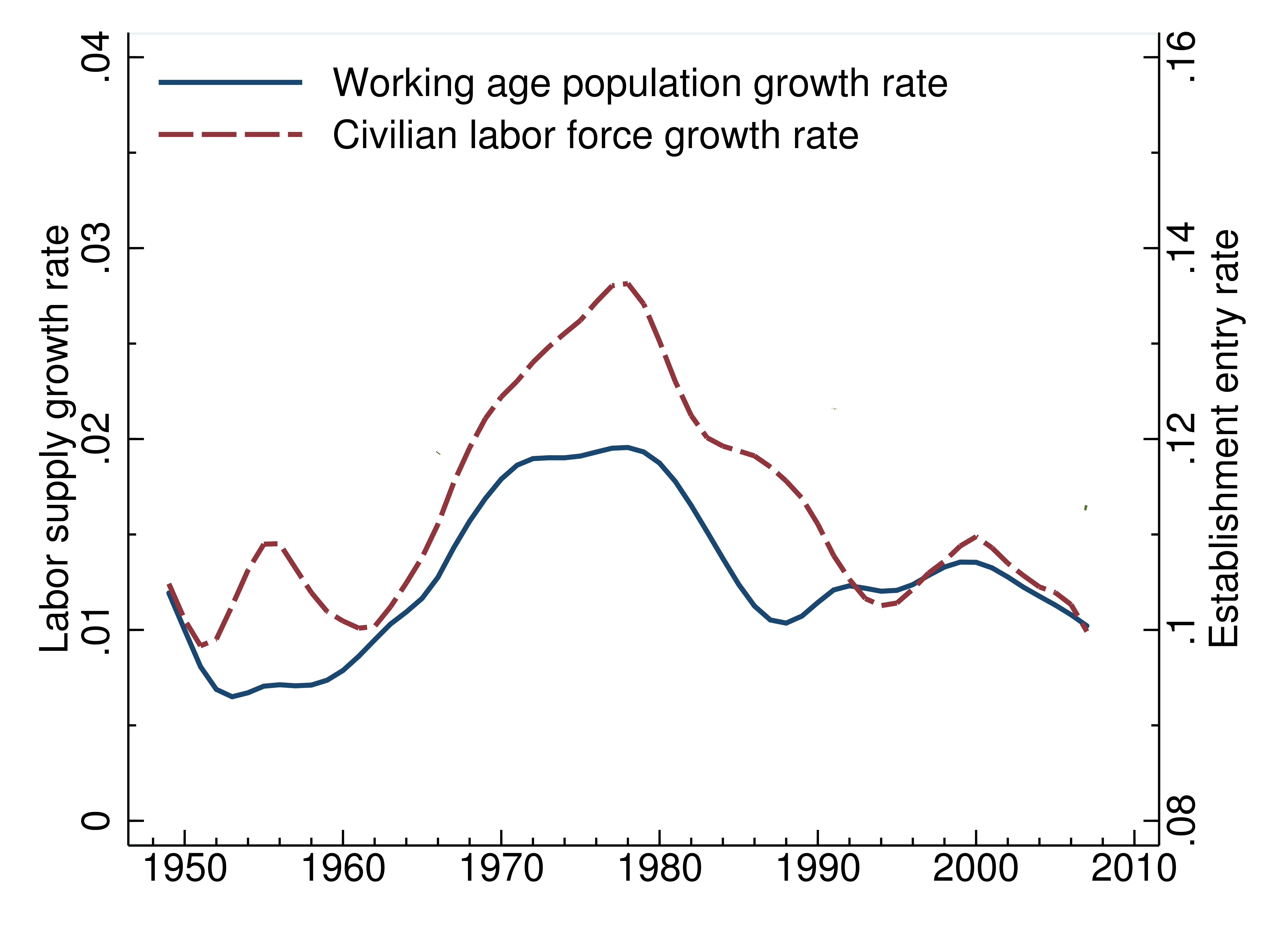
- $\omega^k_{s,t-10}$: share of residents in state s born in state k measured 10 years ago
- Labor supply growth predicted by "push" factors and historical migration patterns

Startup Rate and Labor Supply Growth

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	First Stage			OLS	IV_1	IV_2	IV_1&IV_2	Model
WAP Growth (%)				0.61	1.09	1.27	1.19	1.10
				(0.05)	(0.32)	(0.22)	(0.22)	(0.00)
Birthrate IV	1.36		1.11					
	(0.24)		(0.24)					
Migration IV		1.04	0.87					
		(0.30)	(0.28)					
N	1,421	1,421	1,421	1,421	1,421	1,421	1,421	1,421
R^2	0.64	0.64	0.65	0.90	0.87	0.85	0.86	0.90
F-test	32.63	11.82	17.46					
p-value of J test							0.55	

Empirical Support for “Demographic Origin of Startup Deficits” in the Time-Series

Labor Supply Growth Prior to 1980



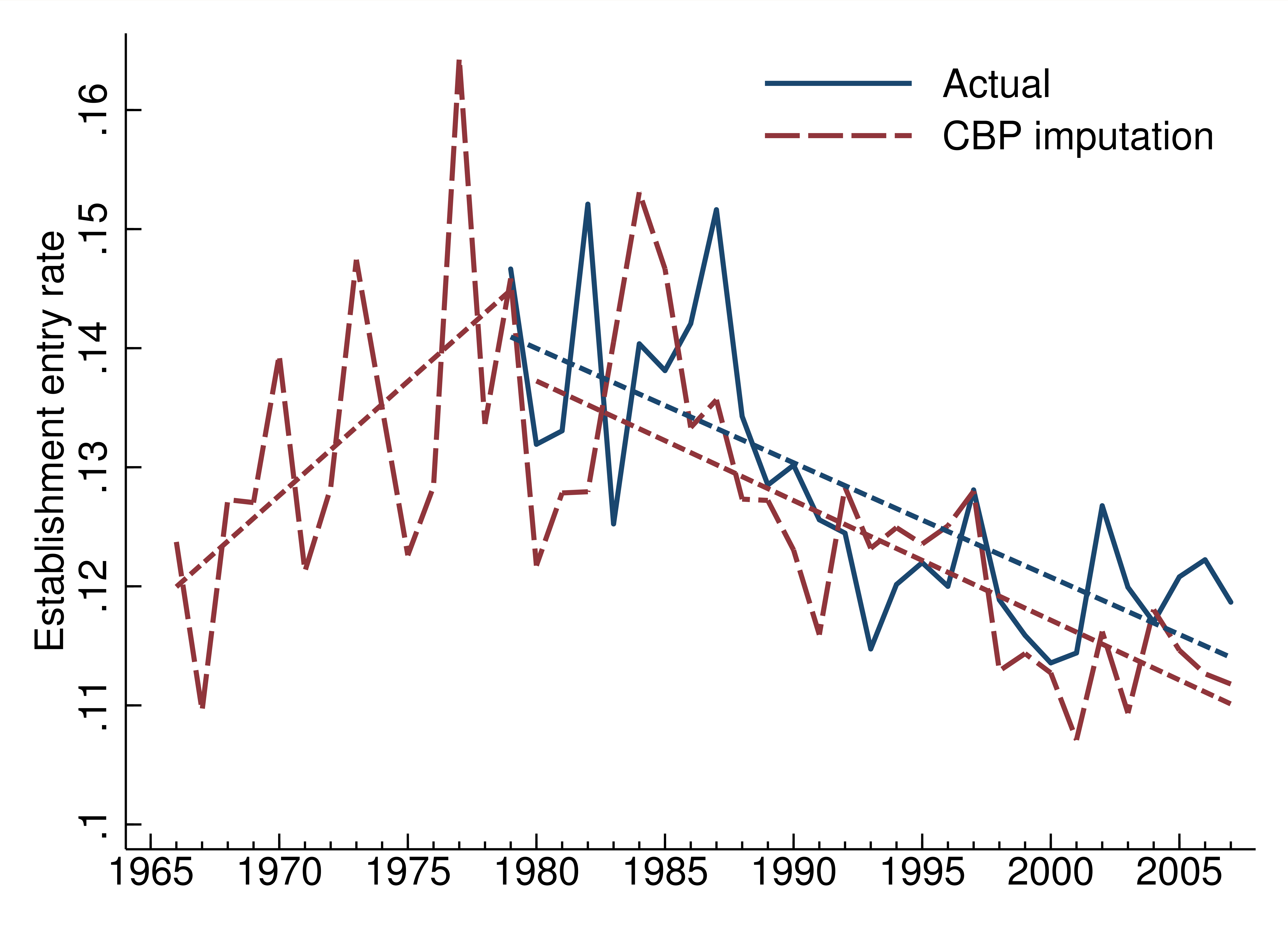
Imputing Historical Entry Rates

- How do the entry rates look like before 1980?
- There is no direct measure
- However, County Business Patterns record the number of establishments since 1965
- The flow-stock equation is (in discrete time)

$$e_t = (1 - x_t)e_{t-1} + s_t$$

  - e_t: # of establishments
  - s_t: establishment entry rates (unobserved)
  - x_t: establishment exit rates $\Rightarrow$ predict using 1980-2007 data

Imputed Entry Rates



Taking Stock

- Over the past 40 years,
  1. Entry rates have been declining
  2. Firm life-cycle dynamics have little changed

- Evaluate the demographic origins of startup deficit through
  1. structural model of firm dynamics
  2. cross-section
  3. time-series

Appendix A: Transition Dynamics from Interest Rate Shock

Changes in Interest Rate

- Changes in population growth $\{\eta_t\}$ only shows up in KFE block
- This is why we didn't need to resolve HJB block
- What if we consider a shock that enters into HJB block such as $\{r_t\}$?

HJB Block with Time-Varying Interest Rates

$$\min\left\{r_t v(z) - \pi(z; w_t) - \mu(z)v_t'(z) - \frac{1}{2}\sigma(z)^2 v_t''(z) - \partial_t v_t(z), v_t(z) - \underline{v}\right\} = 0$$

$$v_t(\underline{z}_t) = \underline{v}$$

$$\int v_t(z)\psi(z)dz = c_e$$

- Again, HJB block alone pins down the path of equilibrium wages $\{w_t\}$
- How do we solve time-dependent HJB-VI?

Moving HJB-VI Backward in Time

- We first assume that, at $t = T$, the economy is in the steady state, $v_T = v(z)$
- We use forward approximation to approximate the time derivative:

$$\partial_t v_t(z) \approx \frac{v_{t+\Delta t}(z) - v_t(z)}{\Delta t}$$

  - Can use backward approximation but requires small Δt

- In a matrix form,

$$\min\left\{[r_t\mathbf{I} - \boldsymbol{A}]\boldsymbol{v}_t - \boldsymbol{\pi}_t(w_t) - \frac{\boldsymbol{v}_{t+\Delta} - \boldsymbol{v}_t}{\Delta t}, \boldsymbol{v}_t - \underline{v}\mathbf{1}\right\} = 0$$

Computational Algorithm

$$\min\left\{[r_t\mathbf{I} - \boldsymbol{A}]\boldsymbol{v}_t - \boldsymbol{\pi}_t(w_t) - \frac{\boldsymbol{v}_{t+\Delta} - \boldsymbol{v}_t}{\Delta t}, \boldsymbol{v}_t - \underline{v}\mathbf{1}\right\} = 0 \qquad \text{(HJB-VI)}$$

$$(\boldsymbol{v}_t \cdot \boldsymbol{\psi}) \times \Delta z = c_e \qquad \text{(Free-entry)}$$

- For $t = T - \Delta t, T - 2\Delta t, \ldots, 0$
  - Given $\boldsymbol{v}_{t+\Delta t}$, guess w_t
    - Solve (HJB-VI) to obtain $\boldsymbol{v}_t$ using Howard's algorithm
    - Check (Free-entry)
      - If $|(\boldsymbol{v}_t \cdot \boldsymbol{\psi}) \times \Delta z - c_e| < tol$, break
      - If $(\boldsymbol{v}_t \cdot \boldsymbol{\psi}) \times \Delta z - c_e > 0$, raise w_t
      - If $(\boldsymbol{v}_t \cdot \boldsymbol{\psi}) \times \Delta z - c_e < 0$, lower w_t

Appendix B: Joint Distribution of Productivity and Age

Two-Dimensional KFE

- Let $g_t(z, a)$ be the density of the joint distribution of (z, a)
- The KFE is given by

$$\partial_t g_t(z, a) = -\partial_z[\mu(z) g_t(z, a)] - \partial_a[g_t(z, a)] + \frac{1}{2}\partial_{zz}^2 \left[\sigma(z)^2 g_t(z)\right] + m_t \psi(z, 0) \quad \text{for } z > \underline{z}$$

which follows from the fact that

$$da = dt$$

(firms age by dt within a time interval dt)

Exit Rate by Age

- The exit rate by age is given by

$$
\begin{aligned}
\frac{1}{g_t(a)}\frac{g_{t-dt}(a-dt)-g_t(a)}{dt} &= \frac{1}{L_t\tilde{g}(a)}\frac{L_{t-dt}\tilde{g}(a-dt)-L_t\tilde{g}(a)}{dt} \\
&= \frac{1}{L_t\tilde{g}(a)}\frac{L_{t-dt}\tilde{g}(a-dt)-L_{t-dt}\tilde{g}(a)+L_{t-dt}\tilde{g}(a)-L_t\tilde{g}(a)}{dt} \\
&= \frac{1}{L_t\tilde{g}(a)}\left(-L_t\partial_a\tilde{g}(a)-\eta L_t\tilde{g}(a)\right) \\
&= -\eta-\frac{\partial_a\tilde{g}(a)}{\tilde{g}(a)}
\end{aligned}
$$