## **The Nature of Labor Reallocation**

### 741 Macroeconomics Topic 5

1

Masao Fukui

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# **Firm Employment is Log-Linear in TFP**

### ■ In Hopenhayn-Rogerson, firm-level employment is given by

*n* = ( $\alpha$ 

 $\Leftrightarrow$   $\log n =$ 

### ■ Two implications:

- 1. The elasticity of firm employment w.r.t. (firm-level) TFP shock is above 1
- 2. The elasticity is symmetric to positive & negative shocks
- Is this true in the data?



$$
\equiv Z
$$

$$
z^{1-\alpha} \alpha/w)^{\frac{1}{1-\alpha}}
$$

$$
\frac{1}{1-\alpha}\log Z + const
$$



# **Ilut, Kehrig & Schneider (2018)**

- Focus on US manufacturing establishments (Census data)
- Construct firm-level TFP using Solow residual:

$$
\log sr_{it} = \log y_{it} - (\beta_n)
$$

- Construct firm-level TFP shocks, Z<sub>it</sub>, assuming  $\log sr_{it} = g \times t + \alpha^{l} + \log Z_{it}$
- Q: How does firm-level employment respond to TFP shocks?  $\Delta \log n_{it} = h(\Delta \log Z_{it}) + \gamma' X_{it} + \epsilon_{it}$
- 
- $\log n_{it} + \beta_k \log k_{it} + \beta_m \log m_{it}$



4





### In the data,

- 1. The elasticity of firm employment to TFP shock is far below 1
- 2. Elasticity is two times larger for negative shocks than positive shocks



## Hopenhayn-Rogerson with Labor Adjustment Costs







### ■ The simplest explanation:

## **Slow to Hire, Quick to Fire**

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- it is costly to hire workers
- less so to fire workers



# **Labor Adjustment Cost**

### ■ Suppose that firms face

- a flow adjustment cost in hiring  $h \geq 0$  of the form  $\Phi(h,n)$  with
- no cost from firing workers:  $\Phi(h, n) = 0$  for  $h \leq 0$

⇒ convex cost in hiring & free firing

- The firm employment evolves  $dn_t = hdt$
- Firms never want to jump *n* upward
	-
- But firms may jump *n* downward

# $h \geq 0$  of the form  $\Phi(h, n)$  with  $\partial_h \Phi > 0$ , $\partial^2_{hh} \Phi > 0$

• Why? – The cost of doing so is 
$$
\lim_{dt\to 0} \Phi(h, n)dt = \infty
$$
 with  $h = \frac{n'-n}{dt}$  and  $n' > n$ 































- The rest of the model remains the same as before
- The production function is

## **Rest of the Model**

 $f(z, n) = z^{1-\alpha}n^{\alpha}$ 

 $dz = \mu(z)dt + \sigma(z)dW$ 



- and firms incur a fixed operating cost *cf*
- Firm's productivity evolves according to a diffusion process



## **Start from Discrete Time**

- **■** Start from a discrete-time setup with time interval *dt*
- The firm's value function is

 $\nu(n,z) = \max \{v^*$ 

• the value of hiring is  $v^*(n, z) = \max_{h \ge 0} (f(n, z) - wn - c_f)$ 

10

*π*(*n*,*z*)

• the value of firing is

• the value of exit is  $\nu$ , as before

$$
-c_f - \Phi(h, n) dt + \left( e^{-rd} \mathbf{F} \left[ v(n', z') \right] \right)
$$

 $s.t.$   $n' = n + hdt$ 

$$
*(n, z), \max\{y, v^f(n, z)\}
$$

$$
v^f(n,z) = \max_{n^f \le n} v(n^f, z)
$$



## **Continuous Time Limit**

■ Add and subtract  $(1 - rdt)v(n, z)$  and defining  $dv(n, z) \equiv v(n', z') - v(n, z)$ , we have  $\nu^*(n, z) = \max_{h \ge 0} (\pi(n, z) - \Phi(h, n)) dt + (1 - rdt) \mathbb{E}[dv(n, z)] + (1 - rdt)v(n, z)$  $s.t.$   $n' = n + hdt$ 

**■** Apply Ito's lemma to  $dv(n, z)$ : ■ Substitute (5) back into (4) and dropping  $dt^2$  term  $dv(n, z) = v_n(n, z) \underline{dn}$  $\overline{\phantom{a}}$ *hdt*  $\nu^*(n, z) = \max_{h \geq 0} \left( \pi(n, z) - \Phi(h, n) + \nu_n(n, z)h + \nu_z(n, z)\mu(z) + \nu_z(n, z)h \right)$  $+v(n,z) - r dt v(n,z)$ 

2



$$
+v_z(n,z)\big(\mu(z)dt+\sigma(z)dZ\big)+\frac{1}{2}\sigma(z)^2v_{zz}(n,z)dt
$$





1 2 *σ*(*z*) 2 *vzz*(*n*,*z*) ) *dt*

## **Bellman Equation in Continuous Time**

### ■ Therefore, we have

where:



$$
v(n,z) = \max\left\{v^*(n,z), v^f(n,z)\right\}
$$

$$
v^*(n, z) = \max_{h \ge 0} \left( \pi(n, z) - \Phi(h, n) + v_n(n, z)h + v_z(n, z)\mu(z) + \frac{1}{2}\sigma(z)^2 v_{zz}(n, z) \right) dt
$$
  
+ 
$$
v(n, z) - r dt v(n, z)
$$
  

$$
\underline{v}^f(n, z) = \max \left\{ \max_{n' \le n} v(n', z), \underline{v} \right\}
$$

### ■ Three cases

- 1. Firms do not fire or exit:  $v(n, z) > v^f(n, z)$  and  $v(n, z) = v^*(n, z)$
- 2. Firms fire workers:  $v(n, z) = v^f(n, z) > v$ , and  $v(n, z) > v^*(n, z)$
- 3. Firms exit:  $v(n, z) = v > v^f(n, z)$ , and  $v(n, z) > v^*(n, z)$

$$
f(n, z) \text{ and } v(n, z) = v^*(n, z)
$$
  
> v, and 
$$
v(n, z) > v^*(n, z)
$$
  
dv(n, z) > 
$$
v^*(n, z)
$$

### **HJB-QVI**

■ Distinct from HJB-VI because now the stopping value  $\nu^{f}(n, z)$  is endogenous to  $\nu(n, z)$ 



■ **Complexly, we can write**  
\n
$$
\min \left\{ rv(n, z) - \max_{h \ge 0} \left( \pi(n, z) - \Phi(h, n) + v_n(n, z)h + v_z(n, z)\mu(z) + \frac{1}{2}\sigma(z)^2 v_{zz}(n, z) \right), \atop v(n, z) - \underline{v}^f(n, z)
$$

- This is called HJB Quasi-Variational Inequality (HJB-QVI)
- 



## **Policy Functions of HJB-QVI**





$$
dn(n, z) = \begin{cases} h(n, z)dt & \text{if } n \le n^f(n, z) \\ n^f(n, z) - n & \text{if } n > n^f(n, z) \end{cases}
$$

■ Let  $χ(n, z)$  denote an indicator function of exiting decision

lies 
$$
\partial_h \Phi(h, n) = v_n(n, z)
$$

■ When firms fire  $(h < 0)$ , firms cut down employment to  $n^f(n, z) = \arg \max_{n^f \le n} v(n^f - z)$ 







$$
\min \left\{ \begin{array}{l} rv(n,z) - \max_{h \ge 0} \left( \pi(n,z) - \Phi(h,n) + v_n(n,z)h + v_z(n,z)\mu(z) + \frac{1}{2}\sigma(z)^2 v_{zz}(n,z) \right), \\ v(n,z) - v^f(n,z) \end{array} \right\} = 0
$$

**■** When firms hire ( $h \ge 0$ ), the FOC implies  $\partial_h \Phi(h, n) = v_n(n, z)$ 

- Let  $h(n, z)$  denote the policy function
- 
- The employment evolution is



## ■ When firms enter, they draw  $(n, z)$  from cdf  $\Psi(n, z)$ ■ We assume (potentially) inelastic entry:

# **Entry**

 $\frac{1}{c^e}$  J  $v(n,z)d\Psi(n,z)$  $\int$ *ν*





$$
m_t = M \times \left(\frac{1}{\bar{c}^{\epsilon}}\right)
$$

(6)

# **Stationary Distribution**

**■** Define  $\mathcal{A}_{KFE}$  as the infinitesimal generator defined for a function  $f(n, z)$ :  $KFFf(n, z) = \mu(z)f_z(n, z) +$ 1 2 *σ*(*z*) 2 *f zz*(*n*,*z*) + (*h*(*n*,*z*) − *sn*)*f*  $+\Lambda^f(n,z)$ [ $f(n^f)$ 

where



$$
-\sigma(z)^2 f_{zz}(n, z) + (h(n, z) - sn) f_n(n, z)
$$
  

$$
u^f(n, z), z) - f(n, z) - \Lambda^e(n, z) f(n, z)
$$

 $g(n,z) + m\psi(n,z)$ 

$$
\Lambda^{f}(n,z) = \begin{cases}\n\infty & \text{if } n \ge n^{f}(n,z) \\
0 & \text{if } n < n^{f}(n,z)\n\end{cases}, \quad \Lambda^{e}(n,z) = \begin{cases}\n\infty & \text{if } \mathbb{I}^{e}(n,z) = 1 \\
0 & \text{if } \mathbb{I}^{e}(n,z) = 0\n\end{cases}
$$

**■** Let  $\mathscr{A}_{KFE}^{\dagger}$  be adjoint operator of  $\mathscr{A}_{KFE}$ . The steady-state distribution  $g(n,z)$  satisfies  $\int_{KFE}^T$  be adjoint operator of  $\mathscr{A}_{KFE}$ . The steady-state distribution  $g(n,z)$ 

$$
0 = \mathscr{A}_{KFE}^{\dagger} \xi
$$

# **Equilibrium Definition**

Equilibrium consists of  $\{v(n, z), h(n, z), n^f(n, z), \chi(n, z), g(n, z), w, m\}$  such that

- 1. Value and policy functions  $\{v(n, z), h(n, z), n^f(n, z), \chi(n, z)\}$  solve HJB-QVI
- 2. Stationary distribution  $g(n, z)$  solve KFE
- 3. Entry *m* is given by (6)
- 4. Labor market clears: ∫∫ *ng*(*n*,*z*)*dndz* = *L*





## Numerically Solving HJB-QVI — Nested Howard Algorithm







## **How to Solve HJB-QVI?**

- 1. optimization w.r.t. *h*
- 2. optimization w.r.t.  $n^f$  inside  $\nu^f(n,z)$
- Discrezie the state space  $n_1, ..., n_I$  and  $z_1, ..., z_J$
- Use short-hand notation of, e.g.,  $v_{i,j} \equiv v(n_i, z_j)$
- 

$$
|\,z_1,\ldots,z_J\,
$$

■ We will use nested Howard's algorithm (Azimzadeh, Bayraktar, Labahn, 2018)



$$
\min \left\{ rv(n, z) - \max_{h \ge 0} \left( \pi(n, z) - \Phi(h, n) + v_n(n, z)h + v_z(n, z)\mu(z) + \frac{1}{2}\sigma(z)^2 v_{zz}(n, z) \right), v(n, z) - \underline{v}^f(n, z)
$$

■ Relative to the case without adj. costs, there are two additional complications:





## **No Firing or Exit**

■ Start from the case where firms do not fire or exit,  $v^f_i$ 

*i*,*j*  $= - \infty$ 



$$
rv(n, z) - \max_{h \ge 0} \left( \pi(n, z) - \Phi(h, n) + v_n(n, z)h + v_z(n, z)\mu(z) + \frac{1}{2}\sigma(z)^2 v_{zz}(n, z) \right) = 0
$$

■ We can solve the above problem using Howard's algorithm:

- 1. Guess  $v^k(n_i, z_i)$  for each  $(n_i, z_j)$  for each  $(i, j)$
- 2. Comp  $h_i^k$
- 3. Solve

But the optimal hiring using the FOC:

\n
$$
h_{i,j}^{k} = \max\{h^*, 0\} \quad \text{where } \partial_h \Phi(h^*, n_i) = \partial_n v_{i,j}^{k}
$$
\nthe linear system to obtain

\n
$$
v_{i,j}^{k+1}
$$
\n
$$
r v_{i,j}^{k+1} - \left(\pi_{i,j} - \Phi(h_{i,j}^k, n_i) + \partial_n v_{i,j}^{k+1} h_{i,j} + \mu_j \partial_z v_{i,j}^{k+1} + \frac{1}{2} \sigma_j^2 \partial_{zz}^2 v_{i,j}^{k+1}\right) = 0
$$

4. Update  $v_{i,j}^k := v_{i,j}^{k+1}$  and repeat until convergence

## **Linear System**



 $\Delta n$ <sub>21</sub>

■ In a matrix form,  
\n
$$
(r\mathbb{I} - \mathbf{A}_{HJB}^k) v^{k+1} = \pi - \Phi^k \iff v^{k+1} = (\mathbf{B}^k)^{-1} [\pi - \Phi^k]
$$
\n•  $\mathbf{v} \equiv [v_{1,1}, ..., v_{I,1}, v_{1,2}, ..., v_{I,2}, ..., v_{I,2}, ..., v_{I,j}]'$  is a  $(I \times J)$  vector  
\n•  $\mathbf{A}_{HJB}$  is  $(I \times J) \times (I \times J)$  matrix, whose elements are  
\n
$$
\begin{cases}\nh_{i,j} \frac{1}{\Delta n} & \text{for } k = i + 1, l = j \\
-h_{i,j} \frac{1}{\Delta n} & \text{for } k = i, l = j \\
\frac{1}{2} \sigma_{j}^2 \frac{1}{(\Delta z)^2} & \text{for } k = i, l = j + 1 \\
\mu_{j} \frac{1}{\Delta z} - \sigma_{j}^2 \frac{1}{(\Delta z)^2} & \text{for } k = i, l = j - 1 \\
-\mu_{j} \frac{1}{\Delta z} + \frac{1}{2} \sigma_{j}^2 \frac{1}{(\Delta z)^2} & \text{for } k = i, l = j - 1\n\end{cases}
$$

 $s$  ince  $dn = h \geq 0$ , we always use forward approximation for  $\partial_n v: \partial_n v_{i,j} \approx 0$ 

3. Set

**However, we can interpret the result.**  
\nFor a fixed value of 
$$
y^f \equiv [y^f_{i,j}]_{i,j}
$$
, we can incorporate exit & firing a  
\n1. Guess  $v^0$   
\n2. For  $k \ge 0$ , given  $v^k$ , construct  $B^k$  as described earlier, and set  
\n
$$
d_{i,j} = \begin{cases} 0 & [B^k v^k - \pi - \Phi]_i \le v^k_{i,j} - v^f_{i,j} \\ 1 & [B^k v^k - \pi - \Phi]_i > v^k_{i,j} - v^f_{i,j} \end{cases}
$$



$$
\left[\tilde{\mathbf{B}}^k\right]_{ij,lm} = \begin{cases} \left[\mathbf{B}^k\right]_{ij,lm} & \text{if } d_{i,j} = 0\\ \left[\mathbf{I}\right]_{ij,lm} & \text{if } d_{i,j} = 1 \end{cases} \qquad \left[\mathbf{q}^k\right]_{i,j} = \begin{cases} \left[\pi - \Phi\right]_{i,j} & \text{if } d_{i,j} = 0\\ \nu_{i,j}^f & \text{if } d_{i,j} = 1 \end{cases}
$$

4. Update  $v^{k+1}$  solving

*B*

# **Firing**

as follows

$$
\tilde{B}^k v^{k+1} = q^k \quad \Leftrightarrow \quad v^{k+1} = [\tilde{B}^k]^{-1} q^k
$$

# **Nested Howard's Algorithm**

The outer loop keeps updates  $\mathbf{v}^f$  starting from  $\mathbf{v}^f = -\infty$ 

- 1. Set the value of firing to  $v_i^{f,0} = -\infty$  for all *i*,*j*  $=$   $\infty$  for all  $i,j$
- 2. For each  $k = 0, 1, \ldots$ 
	- i. Given  $\{v^{f,k}_{i,j}\}_{i,j}$ , set ii. Given  $\mathbf{v}^f \equiv [\mathbf{v}^f_{i,j}]_{i,j}$  solve HJB-VI (*not* QVI) using Howard's algorithm  $\{f, k\}_{i,j}$ , set  $\{\nu^f_{i,j} \equiv \max\left\{\nu, \nu^{f,k}_{i,j}\right\}$
	- $\left[\begin{matrix}I\,,\end{matrix}\right]$  $i,j$
	- iii. Compute the new value of firing as  $v_i^{\{f,k+1\}}$ *i*,*j*
		- If  $v_{i,j}^{f,k+1}$  is close enough to  $v_{i,j}^{f,k}$ , we are done.
		- Otherwise, set  $v_{i,j}^{f,k} := v_{i,j}^{f,k+1}$  and go back to 2.i.

- 
- $=$  max *i*′≤*i v i*,*j*



- Some use algorithms that simultaneously update  $\nu^f$  in inner loop
- Never do this. I wasted my entire summer because of it.
- At the same time, the nested Howard algorithm is inefficient
	- Need many outer loop iterations to converge
- Alternative algorithms that improve speeds have been proposed:
	- The most successful one seems to be penalized Howard algorithm (Azimzadeh and Forsyth, 2016; Azimzadeh, Bayraktar, and Labahn, 2018)
	- I tried to implement it but failed
- If you implement Penalized Howard's algorithm, I will count it as a final project

## **Can We Do Better?**



## Numerically Computing Steady State Equilibrium







# **Discretized Kolmogorov Forward Equation**

tate variables





**Discretized Kolmogorov Forward Eq**\n
$$
\begin{aligned}\n\left[D + (A_{HJB}M)' \right]g + m\psi &= 0 \\
\left[M\right]_{ij,kl} &= \begin{cases}\n1 & \text{for } n^f(n_i, z_j) = n_k, \chi(n_i, z_j) = 0, l = j \\
0 & \text{otherwise}\n\end{cases} \\
\left[D\right]_{ij,kl} &= \begin{cases}\n1 & \text{for } i = k, j = l, n^f(n_i, z_j) < n_i, \chi(n_i, z_j) = 1 \\
0 & \text{otherwise}\n\end{cases}\n\end{aligned}
$$
\nThe matrix *M* takes care of transitions associated with jump in the st

 $\blacksquare$  The matrix  $\boldsymbol{D}$  ensures that  $\left[\boldsymbol{g}\right]_{ij} = 0$  for states  $(n_i, z_j)$  that are never reached

## Macroeconomic Implications of Slow to Hire, Quick to Fire



### **Parameterization**





 $\Phi(h,n)$ 

### **■** I set  $\phi = 10$  and contrast with  $\phi = 0$

■ All the other parameters are unchanged from the lecture note 2



$$
= \frac{\phi}{2} \left(\frac{h}{n}\right)^2 n
$$





## **SS Distribution of Employment Growth**





## **SS Distribution of Employment Growth**





## **SS Distribution of Employment Growth**













### **Aggregate Positive Productivity Shocks** 0.00  $-0.02$  $\mathbf \Pi$ A log  $-0.04$  $-0.06$  $-0.05$ 0.00 0.05  $-0.10$ 0.10  $\Delta \log Z$





















Source: Ilut, Kehrig & Schneider (2018)



### **Countercyclical Volatility & Skewness** TABLE 8 (1980) cal volatility & Skewness

### Moments

 $Data$   $1.278$ Linear hiring Concave hiring

 $\textsf{Source:}\ \mathsf{llut}\ \textsf{K}$ ehrig & Schneider (2018) for the employment growth distribution are defined in equal to the employment of the employment growth distribution are defined in equal to the employment of the employment of

$$
\frac{\text{IQR}(n^i|u^a = -\sigma^a)}{\text{IQR}(n^i|u^a = +\sigma^a)}
$$
\n(1)

$$
\begin{array}{c} 1.278 \\ 1.220 \end{array}
$$

## Firing Cost and Misallocation — Hopenhayn & Rogerson (1993)





Data source: OECD 37

![](_page_42_Picture_2.jpeg)

![](_page_43_Picture_0.jpeg)

- What is the cost of strict firing regulations?
	- - US:  $\tau = 0$
		- Europe: high *τ*
- Firing costs take the form of taxes
- 

■ Suppose that in order to fire a worker, firms have to pay *τ* × annual wage salary

■ The collected tax revenue is rebated back to households as lump-sum transfers

![](_page_43_Picture_10.jpeg)

### **HJB-QVI**

![](_page_44_Picture_8.jpeg)

$$
\min\left\{rv(n,z) - \max_{h\geq 0} \left(\pi(n,z) - \Phi(h,n) + v_n(n,z)h + v_z(n,z)\mu(z) + \frac{1}{2}\sigma(z)^2v_{zz}(n,z)\right), v(n,z) - \underline{v}^f(n,z)\right\}
$$

![](_page_44_Picture_7.jpeg)

$$
\underline{v}^f(n,z) = \max\left\{\max_{n^f \le n} v(n^f,z) - \tau w(n - n^f), \underline{v}\right\}
$$

- This is the only modification
- No firing tax when exiting (maybe I should have assumed otherwise)

## **HJB-QVI**

![](_page_45_Picture_8.jpeg)

$$
\min \left\{ rv(n, z) - \max_{h \ge 0} \left( \pi(n, z) - \Phi(h, n) + v_n(n, z)h + v_z(n, z)\mu(z) + \frac{1}{2}\sigma(z)^2 v_{zz}(n, z) \right), v(n, z) - \underline{v}^f(n, z) \right\}
$$

![](_page_45_Picture_7.jpeg)

$$
v^{f}(n, z) = \max \left\{ \max_{n^{f} \leq n} v(n^{f}, z) + \tau w(n - n^{f}) \right\}
$$
   
 
$$
\left\{ \text{ring tax} \right\}
$$

- This is the only modification
- No firing tax when exiting (maybe I should have assumed otherwise)

# **Misallocation Cost of Firing Regulations**

Wage

![](_page_46_Figure_2.jpeg)

■ Firing costs lead to the misallocation of workers because

1. Unproductive firms cannot downsize

2. Productive firms become hesitant to expand

### Labor Productivity

![](_page_46_Figure_7.jpeg)

![](_page_46_Picture_9.jpeg)

![](_page_46_Picture_10.jpeg)

# **Idiosyncratic Distortion**

- Firing tax is a distortion at the aggregate level
- There is no shortage of reasons to expect firms to face idiosyncratic distortions
	- Corruption, firm-level taxes/subsidies, financial frictions, incomplete contracts
- Restuccia & Rogerson (2008) consider *wedges* in the form of

 $(1 +$ 

where 
$$
\tau_i = \begin{cases} \tau & \text{with prob } 1/2 \\ -\tau & \text{with prob } 1/2 \end{cases}
$$

■ *τ<sub><i>i*</sub> is assigned when firm *i* is born and fixed over time

![](_page_47_Picture_10.jpeg)

$$
-\tau_i) z^{1-\alpha} n^{\alpha}
$$

## **Misallocation from Idiosyncratic Distorton**

![](_page_48_Picture_7.jpeg)

### **Table 3**

Effects of idiosyncratic distortions—uncorrelated case

![](_page_48_Picture_79.jpeg)

Relative E 1*.*00 1*.*00 1*.*00 1*.*00 *Ys/Y* 0*.*72 0*.*85 0*.*93 0*.*97 Source: Restuccia & Rogerson (2008)

![](_page_48_Picture_6.jpeg)

## Non-Parametric Identification of Misallocation

— Carrillo, Donaldson, Pomeranz, & Singhal (2023)

![](_page_49_Picture_3.jpeg)

## **What is the Cost of Misallocation?**

- How large is the cost of misallocation in the data?
- Let us step back and consider a static model with a fixed mass of firms
- Each firm *i* produces using
- The efficient allocation solves
- *Y*\* ≡ max
	-
- **■** The solution features equalization of MPL:
	- $f_i^\prime$  $C_i^n(n_i) = w$  for all *i*
- $y_i = f_i(n_i)$ 
	- ${n_i}$ ∫ *f i* (*ni* )*di*
- s.t.  $\int n_i di = L$

![](_page_50_Picture_13.jpeg)

![](_page_50_Picture_12.jpeg)

## **Variance of MPL**

- Take arbitrary allocation  $\{n_i\}$ . Up to a second order around the efficient allocation *Y* − *Y*\* *Y*\*  $\approx -\frac{1}{2}$ 
	- $\mathsf{where} \; MPL_i = f'_i(n_i)$ ,  $\lambda_i = w_i n_i / Y^*$  and  $\epsilon_i \equiv -\; \frac{d \log MPL_i}{d \log n_i}$
- (Weighted) variance of MPL is the key moment for the cost of misallocation
- Testing the presence of misallocation
- How do we get the distribution of MPL?
	- 1. Assume  $f_i(n_i) = Z_i n_i^{\alpha}$ , and then  $MPL_i = \alpha \frac{N}{n_i}$  (Hsieh & Klenow, 2009)  $\hat{I}_i(n_i) = Z_i$ *nα*  $\frac{\alpha}{i}$ , and then  $MPL_i = \alpha$
	- 2. Nonparametrically identify the distribution of MPL (Carrillo et al. 2023)

$$
\varepsilon_i \equiv -\frac{d \log MPL_i}{d \log n_i}
$$

$$
\Leftrightarrow \text{testing Var}(MPL_i) = 0
$$

*yi ni*

![](_page_51_Picture_15.jpeg)

$$
\frac{1}{2}\int \lambda_i \epsilon_i \log(MPL_i/w)^2 di
$$

# **Nonparametric Identification**

- Taking the first-order approximation of equation (7),  $\Delta y_i = \beta_i \Delta n_i + \epsilon_i$ 
	- $\epsilon_i$ : technology shocks (i.e., changes in  $f_i(\cdot)$ )  $(\ \cdot\ )$
	- $\beta_i = f'_i(n_i) = MPL_i$ : treatment effect of exogenously increasing  $n_i$  on  $y_i$
- **■** With suitable instruments  $Z_i$  that exogenously shift  $n_i$ ,  $E[\beta_i^k]$  ( $k = 1, 2, ...$ ) are identified (Masten & Torgovitsky, 2016)

![](_page_52_Picture_5.jpeg)

![](_page_52_Picture_6.jpeg)

# **Empirical Implementation**

- Construction sector in Ecuador, 2009-2014
- Public construction projects were allocated through a randomized lottery
- Lottery serves as an ideal instrument
	- exogeneity: orthogonal to technology shocks  $\epsilon_i$  or  $MPL_i$
	- relevance: winning a lottery does shift *ni*

![](_page_53_Picture_9.jpeg)

![](_page_53_Picture_26.jpeg)

![](_page_54_Picture_5.jpeg)

![](_page_54_Figure_2.jpeg)

 $N$ notes: This figure 4 Panel (b), estimating the analysis of  $P$  and  $\alpha$  elements of analysis of analysis of an additional (b), estimating the monthly e $P$ 

### **Sales Contract Contra**

![](_page_54_Picture_4.jpeg)

## Heterogenous Treatment Effects by Firm Size?

sales. Dashed lines in panel (a) indicate 95% confidence intervals that allow for clustering at the firm level.

### **Sales Sales Sales**

notes: This figure extends the analysis of Figure 2, estimating the monthly e⊿ects of analysis of analysis of a<br>This figure 2, estimating the monthly e⊿ects of an additional \$1,000, estimate and \$1,000, estimate and \$1,00

### **Small Cost of Misallocation** Uniform Small Cost of Misallocation

Table 4: Estimated Cost of Misallocation 10010 T. Doulliand Cost of Minagement

![](_page_55_Picture_9.jpeg)

# Panel (a): IVCRC estimates

Panel (b): Alternative procedure assuming common scale elasticities Constant returns-to-scale  $(\gamma = 1)$ .

**■** Assume  $\epsilon_i = 3$  for all *i*  $\mathsf{sum} \, \epsilon \cdot \mathsf{S} = 3$  for all i.

■ The welfare cost of misallocation is 1.6%

■ Hsieh-Klenow type calculation implies 48% of welfare loss in the same dataset United VI american control de la computación de la control de la computación de la computación de la computació <u>23 TUNU OI WUHUIU 1033 III LIIU SUITIU UU</u>  $\overline{1.407}$   $\overline{1.407}$   $\overline{1.457}$   $\overline{1.457}$ 

![](_page_55_Picture_217.jpeg)

in the state of the<br>The state of the st

 $1.6\%$ 

*Notes:* Columns (1) and (2) report estimates of the 2008 sales-weighted expectation and variance

![](_page_56_Picture_0.jpeg)

- [Laissez-faire](https://www.google.com/search?sca_esv=2e08a6d3b8f5b541&q=Laissez-faire&spell=1&sa=X&ved=2ahUKEwjBp97zheqJAxWUlIkEHVY-A-YQkeECKAB6BAgPEAE) of Hopenhayn-Rogersion with labor adjustment costs is efficient
- But, MPL is not equalized in a static sense
- Firms hire workers until (present discounted value of hiring a worker) = (hiring cost today)
- Hiring a worker is an investment
- 
- How do we incorporate dynamics without imposing strong assumptions? ■ How do we incorporate entry & exit dynamics?

## **Questions**

![](_page_56_Picture_8.jpeg)