Firm Wage

741 Macroeconomics
Topic 6

Masao Fukui

2024 Spring

Is the Labor Market Competitive?

- In Hopenhayn-Rogerson, all firms pay the same wage to all workers
- This is a natural consequence of the competitive labor market
- Of course, in the data, average wages differ greatly across firms
- Is this a rejection of the competitive labor market?
  – not necessarily because firms employ different workers

AKM Model

- Consider the following statistical model by Abowd, Kramarz, and Margolis (1999):

$$\ln w_{it} = \alpha_i + \psi_{j(i,t)} + \epsilon_{it}$$

  - w_{it}: wage of worker i at time t
  - $j(i, t)$: firm employing worker i at time t
  - ψ_j: wage premium of firm j

- Assume $\mathbb{E}[\epsilon_{it} \,|\, j(i, s)] = 0$ for all i, t. This embeds:

  1. Worker's mobility decisions are not driven by time-varying wage fluctuations
  2. log wages are additively separable between worker- and firm-components

- Then, worker's movements across firms identify ψ_j (up to a constant):

$$\mathbb{E}[\ln w_{it'} - \ln w_{it} \,|\, j(i, t') = j, j(i, t) = k] = \psi_j - \psi_k$$

Firm Wage and Wage Inequality (US)

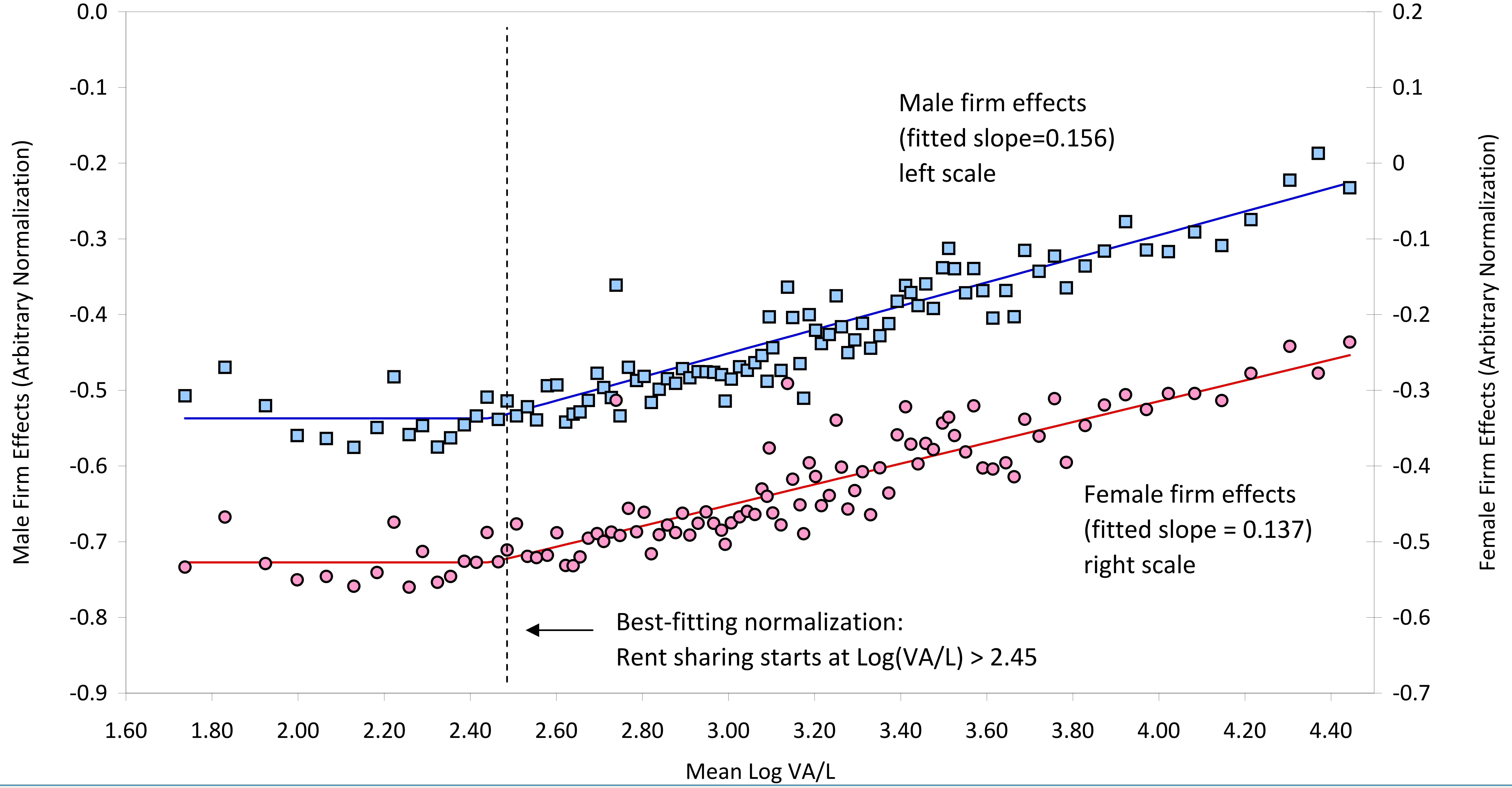
		Interval 1 (1980–1986)		Interval 2 (1987–1993)		Interval 3 (1994–2000)		Interval 4 (2001–2007)		Interval 5 (2007–2013)		Change from 1 to 5	
		Comp. (1)	Share (2)	Comp. (3)	Share (4)	Comp. (5)	Share (6)	Comp. (7)	Share (8)	Comp. (9)	Share (10)	Comp. (11)	Share (12)
Total variance	Var(y)	0.708	—	0.776	—	0.828	—	0.884	—	0.924	—	0.216	—
Components of variance	Var(WFE)	0.330	46.6	0.375	48.3	0.422	51.0	0.452	51.2	0.476	51.5	0.146	67.6
	Var(FFE)	0.084	11.9	0.075	9.7	0.067	8.1	0.075	8.5	0.081	8.7	−0.003	−1.6
	Var(Xb)	0.055	7.8	0.065	8.4	0.079	9.5	0.061	6.9	0.059	6.4	0.004	1.8
	Var(ϵ)	0.154	21.7	0.148	19.1	0.146	17.6	0.149	16.8	0.136	14.7	−0.018	−8.2
	2*Cov(WFE, FFE)	0.033	4.7	0.057	7.3	0.076	9.2	0.094	10.6	0.108	11.7	0.075	34.8

Source: Song, Price, Guvenen, Bloom, and Wachter (2019)

- Variance in Firm FE accounts for 8-12% of wage inequality
- Cov(Worker FE, Firm FE) > 0, more so in the recent periods
  - Often interpreted as “high-wage workers work for high-wage firms”

Higher Value Added, Higher Firm Wage (Portugal)

Figure IV: Firm Fixed Effects vs. Log Value Added/Worker



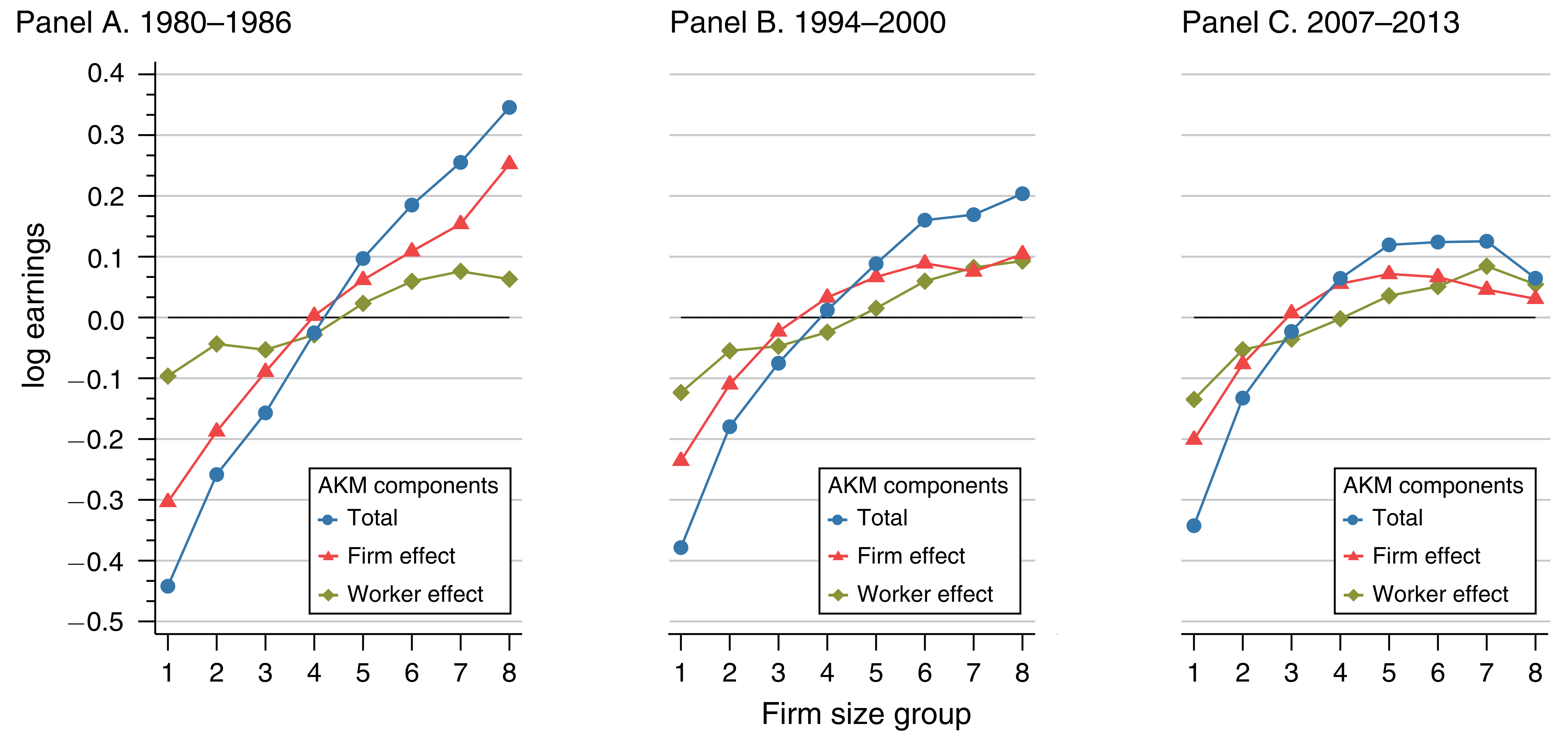
Male firm effects
(fitted slope=0.156)
left scale

Female firm effects
(fitted slope = 0.137)
right scale

Best-fitting normalization:
Rent sharing starts at Log(VA/L) > 2.45

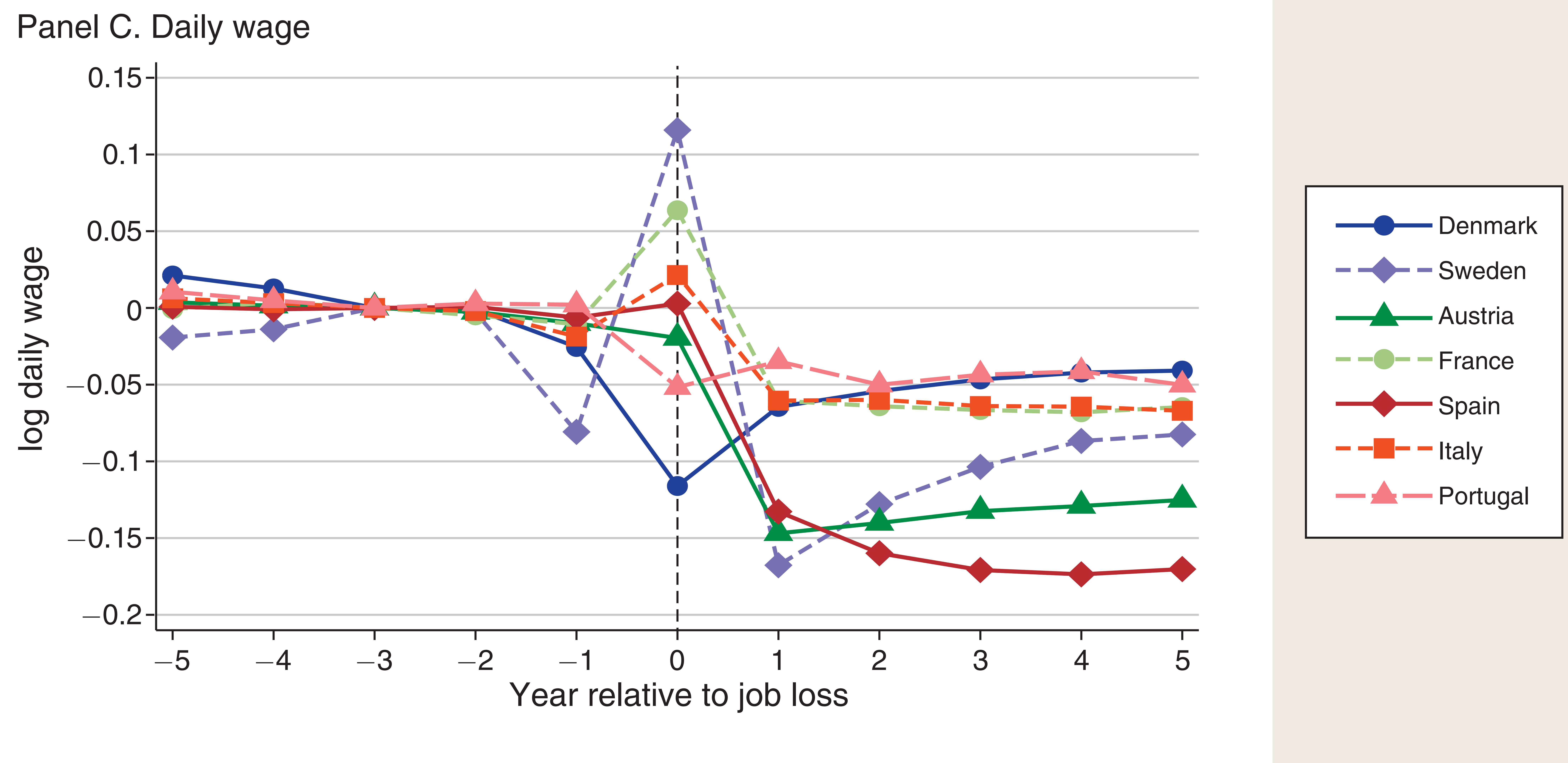
Source: Card, Cardoso, & Kline (2016).

Larger Firm, Higher Firm Wage? (US)



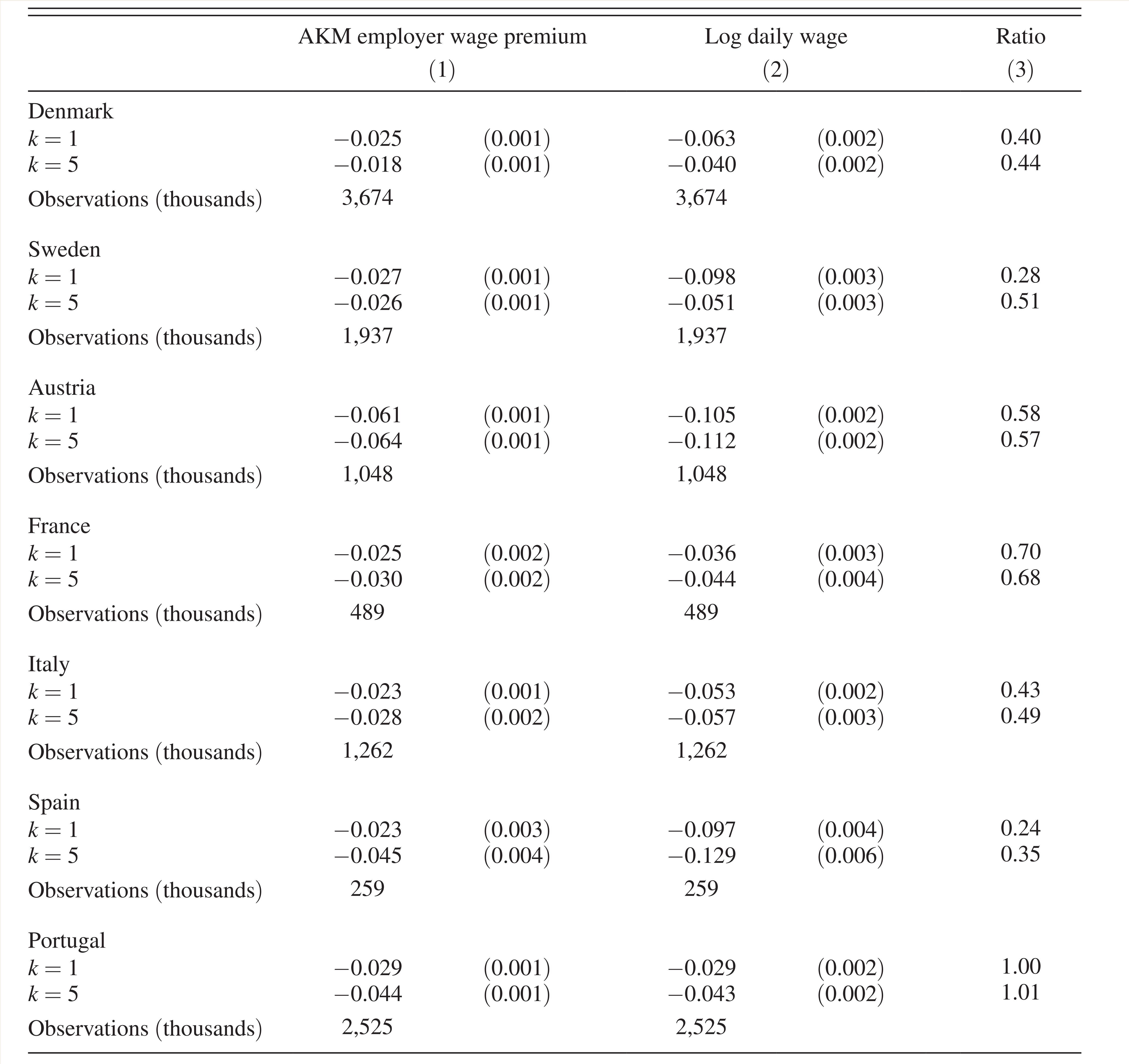
Source: Card, Cardoso, & Kline (2016).

Dispaced Workers Suffer Wage Losses…



Source: Bertheau, Accabi, Barcelo, Gulyas, Lambardi & Saggio (2023).

… Because Workers Move to Firms with Lower Firm FE



	AKM employer wage premium (1)		Log daily wage (2)		Ratio (3)
Denmark					
$k = 1$	−0.025	(0.001)	−0.063	(0.002)	0.40
$k = 5$	−0.018	(0.001)	−0.040	(0.002)	0.44
Observations (thousands)	3,674		3,674		
Sweden					
$k = 1$	−0.027	(0.001)	−0.098	(0.003)	0.28
$k = 5$	−0.026	(0.001)	−0.051	(0.003)	0.51
Observations (thousands)	1,937		1,937		
Austria					
$k = 1$	−0.061	(0.001)	−0.105	(0.002)	0.58
$k = 5$	−0.064	(0.001)	−0.112	(0.002)	0.57
Observations (thousands)	1,048		1,048		
France					
$k = 1$	−0.025	(0.002)	−0.036	(0.003)	0.70
$k = 5$	−0.030	(0.002)	−0.044	(0.004)	0.68
Observations (thousands)	489		489		
Italy					
$k = 1$	−0.023	(0.001)	−0.053	(0.002)	0.43
$k = 5$	−0.028	(0.002)	−0.057	(0.003)	0.49
Observations (thousands)	1,262		1,262		
Spain					
$k = 1$	−0.023	(0.003)	−0.097	(0.004)	0.24
$k = 5$	−0.045	(0.004)	−0.129	(0.006)	0.35
Observations (thousands)	259		259		
Portugal					
$k = 1$	−0.029	(0.001)	−0.029	(0.002)	1.00
$k = 5$	−0.044	(0.001)	−0.043	(0.002)	1.01
Observations (thousands)	2,525		2,525		

Source: Bertheau, Accabi, Barcelo, Gulyas, Lambardi & Saggio (2023).

Discussions

1. Even if one believes in AKM model, there are lots of econometrics issues
    - Take labor sequence, or see Kline (2024) for an excellent survey
    - Frontier: clustering approach by Bonhomme, Lamadon & Manresa (2019)
2. Do we believe in AKM model?
    - Easy to write down a model that leads to AKM equation
    - But, if all workers equally benefit from high-wage firms, why do high-wage workers work for high-wage firms?
    - See Borovicková & Shimer (2024) for a beautiful criticism of AKM model
3. Did we reject the competitive labor market in the end?
    - I am not sure…, but let's pretend we did and move on

Hopenhayn-Rogerson with Search Friction

– Based on McCrary (2022)

Environment

- Firms
    - hire workers by posting a vacancy v
- Workers
    - search for a job while unemployed
    - No on-the-job search for simplicity
- Random matching market with CRS matching function $M(u, v)$
- Wages are determined by Nash bargaining ⇒ "firm wage"

Technology

- Firms hire workers by posting vacancies v
  - Each vacancy meets with a worker at rate $q(\theta) = M(1/\theta,1)$ where $\theta \equiv v/u$
  - The vacancy cost is $\Phi(v, n)$
- Worker separations occur either (i) at exogenous rate s or (ii) firing
- The firm size evolves according to

$$dn_t = (q(\theta)v - sn)dt - \text{firing}$$

- Firms' technology is $y(z, n) = z^{1-\alpha}n^{\alpha}$, where z follows a diffusion process
- Firms can exit to obtain $\underline{J} \equiv 0$
- Firms pay wages w, which are determined through bargaining

Firm Value and Policy Functions

- Firm's policy functions:
  - wage: $w(n, z)$
  - vacancy: $v(n, z)$
  - size of retained workers post-firing: $n^f(n, z)$
  - exit: $\chi(n, z)$

- When firms do not fire/exit, the HJB equation of a firm for a given wage $w(n, z)$ is

$$rJ(n, z) = y(n, z) - c_f - w(n, z)n - \Phi(v(n, z), n) + (q(\theta)v(n, z) - sn)J_n(n, z)$$

$$+\mu(z)J_z(n, z) + \frac{1}{2}\sigma(z)^2 J_{zz}(n, z)$$

- When firms fire: $J(n, z) = J(n^f(n, z), z)$

- When firms exit: $J(n, z) = 0$

Worker's HJB

- Unemployed workers receive UI benefits of b, and find jobs at rate $\lambda(\theta)$
- Let U denote the unemployment value
- When firms do not fire or exit in state (n, z): employed worker's HJB solves

$$rW(n,z) = w(n,z) + s(U - W(n,z)) + (q(\theta)v - s)W_n(n,z) + \mu(z)W_z(n,z) + \frac{1}{2}\sigma(z)^2 W_{zz}(n,z)$$

- When firms fire: $W(n,z) = \frac{n^f(n,z)}{n} W(n^f(n,z), z) + \left(1 - \frac{n^f(n,z)}{n}\right) U$
- When firms exit: $W(n,z) = U$

Wage Barganing

- In each period, a coalition of workers and a firm bargain to determine w, v, n^f, χ
- We assume Nash bargaining with worker bargaining power γ
- The Nash bargaining problem in state (n, z) is

$$\max_{w,v,\chi,n^f \leq n} \left(W(n^f, z)n^f - Un^f\right)^\gamma J(n^f, z)^{1-\gamma} \tag{1}$$

- Noting $\frac{\partial W(n^f, z)}{\partial w} = -\frac{\partial J(n^f, z)}{\partial w}$, FOC w.r.t. w is

$$(1-\gamma)\left(W(n^f, z)n^f - Un^f\right) = \gamma J(n^f, z)$$

- Defining joint match surplus $S(n, z) \equiv J(n, z) + (W(n, z) - U)n$,

$$\left(W(n^f, z)n^f - Un^f\right) = \gamma S(n^f, z), \quad J(n^f, z) = (1-\gamma)S(n^f, z) \tag{2}$$

Bilateral Efficiency

- Substituting (2) back into (1), we have

$$\max_{v,\chi,n^f \leq n} \gamma^\gamma (1-\gamma)^{1-\gamma} S(n^f, z)$$

- **Result**: vacancy, firing, and exit policies maximize joint match surplus

Joint Match Surplus

- Recall when there is no firing or exit

$$rJ(n,z) = y(n,z) - c_f - w(n,z)n - \Phi(v,n) + (q(\theta)v - sn)J_n(n,z)$$

$$+\mu(z)J_z(n,z) + \frac{1}{2}\sigma(z)^2 J_{zz}(n,z)$$

$$rW(n,z)n - rUn = w(n,z)n - rUn + s(Un - W(n,z)n) + (q(\theta)v - sn)W_n(n,z)n$$

$$+\mu(z)W_z(n,z)n + \frac{1}{2}\sigma(z)^2 W_{zz}(n,z)n$$

Joint Match Surplus

- Recall when there is no firing or exit

$$rJ(n,z) = y(n,z) - c_f - w(n,z)n - \Phi(v,n) + (q(\theta)v - sn)J_n(n,z)$$

$$+\mu(z)J_z(n,z) + \frac{1}{2}\sigma(z)^2 J_{zz}(n,z)$$

$$rW(n,z)n - rUn = w(n,z)n - rUn + s(Un - W(n,z)n) + (q(\theta)v - sn)W_n(n,z)n$$

$$+\mu(z)W_z(n,z)n + \frac{1}{2}\sigma(z)^2 W_{zz}(n,z)n$$

- Adding up the above two, and noting $S_n(n,z) = J_n(n,z) + W_n(n,z)n + (W(n,z) - U)$

$$rS(n,z) = y(n,z) - c_f - \Phi(v,n) - rnU - s(W(n,z) - U)n$$

$$+(q(\theta)v - sn)\big(S_n(n,z) - (W(n,z) - U)\big) + \mu(z)S_z(n,z) + \frac{\sigma(z)^2}{2}S_{zz}(n,z)$$

Joint Match Surplus

- Recall when there is no firing or exit

$$rJ(n,z) = y(n,z) - c_f - w(n,z)n - \Phi(v,n) + (q(\theta)v - sn)J_n(n,z)$$

$$+\mu(z)J_z(n,z) + \frac{1}{2}\sigma(z)^2 J_{zz}(n,z)$$

$$rW(n,z)n - rUn = w(n,z)n - rUn + s(Un - W(n,z)n) + (q(\theta)v - sn)W_n(n,z)n$$

$$+\mu(z)W_z(n,z)n + \frac{1}{2}\sigma(z)^2 W_{zz}(n,z)n$$

- Adding up the above two, and noting $S_n(n,z) = J_n(n,z) + W_n(n,z)n + (W(n,z) - U)$

$$rS(n,z) = y(n,z) - c_f - \Phi(v,n) - rnU - s\underbrace{(W(n,z) - U)n}_{\gamma S(n,z)}$$

$$+(q(\theta)v - sn)\big(S_n(n,z) - \underbrace{(W(n,z) - U)}_{\gamma S(n,z)/n}\big) + \mu(z)S_z(n,z) + \frac{\sigma(z)^2}{2}S_{zz}(n,z)$$

HJB-QVI

- Since v, n^f, χ maximize the joint match surplus, $S(n, z)$ solve the following HJB-QVI:

$$\min \left\{ rS - \max_{v} \left[y(n, z) - c_f - \Phi(v, n) - rnU + (q(\theta)v - sn)S_n - q(\theta)v\frac{1}{n}\gamma S + \mu(z)S_z + \frac{\sigma(z)^2}{2}S_{zz} \right], S - \underline{S}^f \right\} = 0$$

where dependence on (n, z) is omitted for brevity, and

$$\underline{S}^f(n, z) = \max \left\{ \max_{n^f \leq n} S(n^f, z), 0 \right\}$$

- Entrants draw (n, z) from cdf $\Psi(n, z)$. The entry is given by

$$m_t = M \times \left(\frac{1}{\bar{c}^e} \int \underbrace{(1 - \gamma)S(n, z)}_{J(n,z)} d\Psi(n, z) \right)^{\nu}$$

Wage Formula

- **Result**: The wage function $w(n, z)$ is given by

$$w(n,z) = \gamma \frac{1}{n}\left(y(n,z) - c_f - \Phi(v,n)\right) + (1-\gamma)\left(rU + q(\theta)\frac{v}{n}\gamma S(n,z)\frac{1}{n}\right)$$

  - **Proof**: Since $(W(n,z) - U)n = \gamma S(n,z)$, worker's HJB can be written as

$$\begin{aligned} r\gamma S(n,z)n = {} & w(n,z)n - rUn - s\gamma S(n,z) + (q(\theta)v - sn)\left(\gamma S_n(n,z) - \gamma S(n,z)/n\right) \\ & + \mu(z)\gamma S_z(n,z)n + \frac{\sigma(z)^2}{2}\gamma S_{zz}(n,z)n \end{aligned} \tag{3}$$

  - The surplus solves

$$\begin{aligned} rS(n,z)n = {} & y(n,z) - c_f - \Phi(v,n) - rUn + (q(\theta)v - sn)S_n - q(\theta)v\gamma S(n,z)/n \\ & + \mu(z)S_z(n,z) + \frac{\sigma(z)^2}{2}S_{zz}(n,z) \end{aligned} \tag{4}$$

  - Multiply (4) by γ and subtract from (3) gives the formula

Stationary Distribution

- Define $\mathscr{A}_{KFE}$ as the infinitesimal generator defined for a function $f(n,z)$:

$$\mathscr{A}_{KFE}f(n,z) = \mu(z)f_z(n,z) + \frac{1}{2}\sigma(z)^2 f_{zz}(n,z) + dn(n,z)f_n(n,z)$$
$$+\Lambda^{fire}(n,z)\left[f(n^f(n,z),z) - f(n,z)\right] - \Lambda^{exit}(n,z)f(n,z)$$

where $dn(n,z) \equiv q(\theta)v(n,z) - s$,

$$\Lambda^{fire}(n,z) = \begin{cases} \infty & \text{if } n > n^f(n,z) \\ 0 & \text{if } n \leq n^f(n,z) \end{cases}, \quad \Lambda^{exit}(n,z) = \begin{cases} \infty & \text{if } \chi(n,z) = 1 \\ 0 & \text{if } \chi(n,z) = 0 \end{cases}$$

- Let $\mathscr{A}^{\dagger}_{KFE}$ be adjoint operator of $\mathscr{A}_{KFE}$. The steady-state distribution $g(n,z)$ satisfies

$$0 = \mathscr{A}^{\dagger}_{KFE}g(n,z) + m\psi(n,z)$$

Rest of the Model

- Aggregate employment and unemployment in this economy is

$$N = \int\int ng(n,z)dndz$$

$$u = 1 - N$$

- Aggregate vacancy and market tightness are

$$V = \int v(n,z)g(n,z)dndz$$

$$\theta = \frac{V}{u}$$

- The value of unemployment can be written as

$$rU = b + \lambda(\theta)\gamma\int S(n,z)\frac{1}{n}dg(n,z) + \frac{m\int nd\Psi(n,z)}{u}\int\gamma\frac{n}{\int nd\Psi(n,z)}S(n,z)d\Psi(n,z)$$

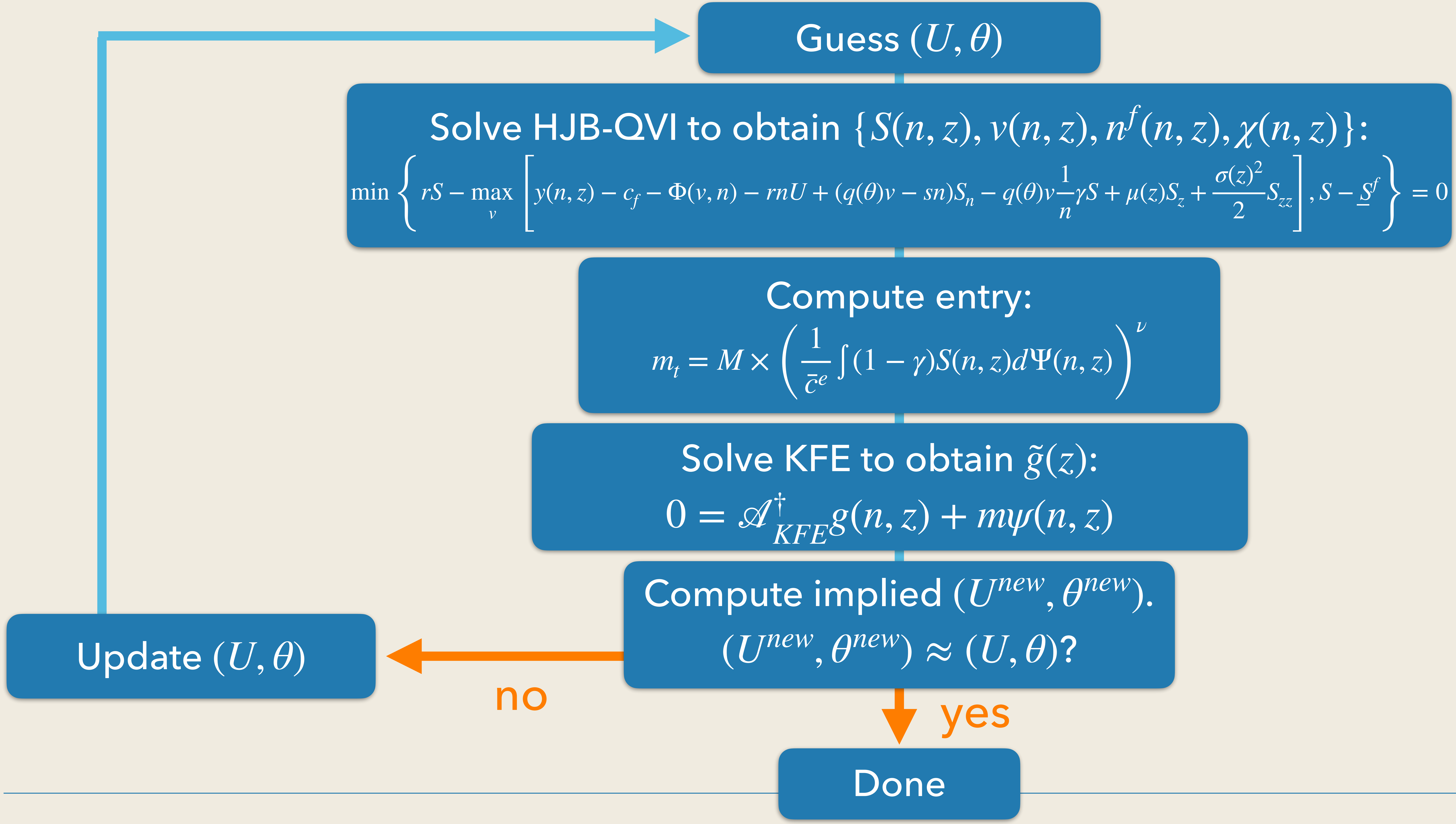
Numerical Illustration

Fixed Point Problem

$$\min \left\{ rS - \max_{v} \left[y(n,z) - c_f - \Phi(v,n) - rnU + (q(\theta)v - sn)S_n - q(\theta)v\frac{1}{n}\gamma S + \mu(z)S_z + \frac{\sigma(z)^2}{2}S_{zz} \right], S - \underline{S}^f \right\} = 0$$

- Firm's problem depends on the aggregate through two endogenous variables:
  1. Market tightness, θ
  2. Unemployment value, U
- These two have to be in turn consistent with equilibrium:
  1. $\theta = V/u$
  2. $rU = b + \lambda(\theta)\gamma \int S(n,z)\frac{1}{n}dg(n,z) + \frac{m \int n d\Psi(n,z)}{u} \int \gamma \frac{n}{\int n d\Psi(n,z)} S(n,z) d\Psi(n,z)$
- Two-dimensional fixed point problem

Steady State Algorithm



Guess (U, θ)

Solve HJB-QVI to obtain $\{S(n,z), v(n,z), n^f(n,z), \chi(n,z)\}$:

$$\min\left\{ rS - \max_v \left[y(n,z) - c_f - \Phi(v,n) - rnU + (q(\theta)v - sn)S_n - q(\theta)v\frac{1}{n}\gamma S + \mu(z)S_z + \frac{\sigma(z)^2}{2}S_{zz} \right], S - \underline{S}^f \right\} = 0$$

Compute entry:

$$m_t = M \times \left(\frac{1}{\bar{c}^e} \int (1-\gamma)S(n,z)d\Psi(n,z) \right)^{\nu}$$

Solve KFE to obtain $\tilde{g}(z)$:

$$0 = \mathscr{A}^{\dagger}_{KFE}g(n,z) + m\psi(n,z)$$

Compute implied (U^{new}, θ^{new}).

$(U^{new}, \theta^{new}) \approx (U, \theta)$?

no

Update (U, θ)

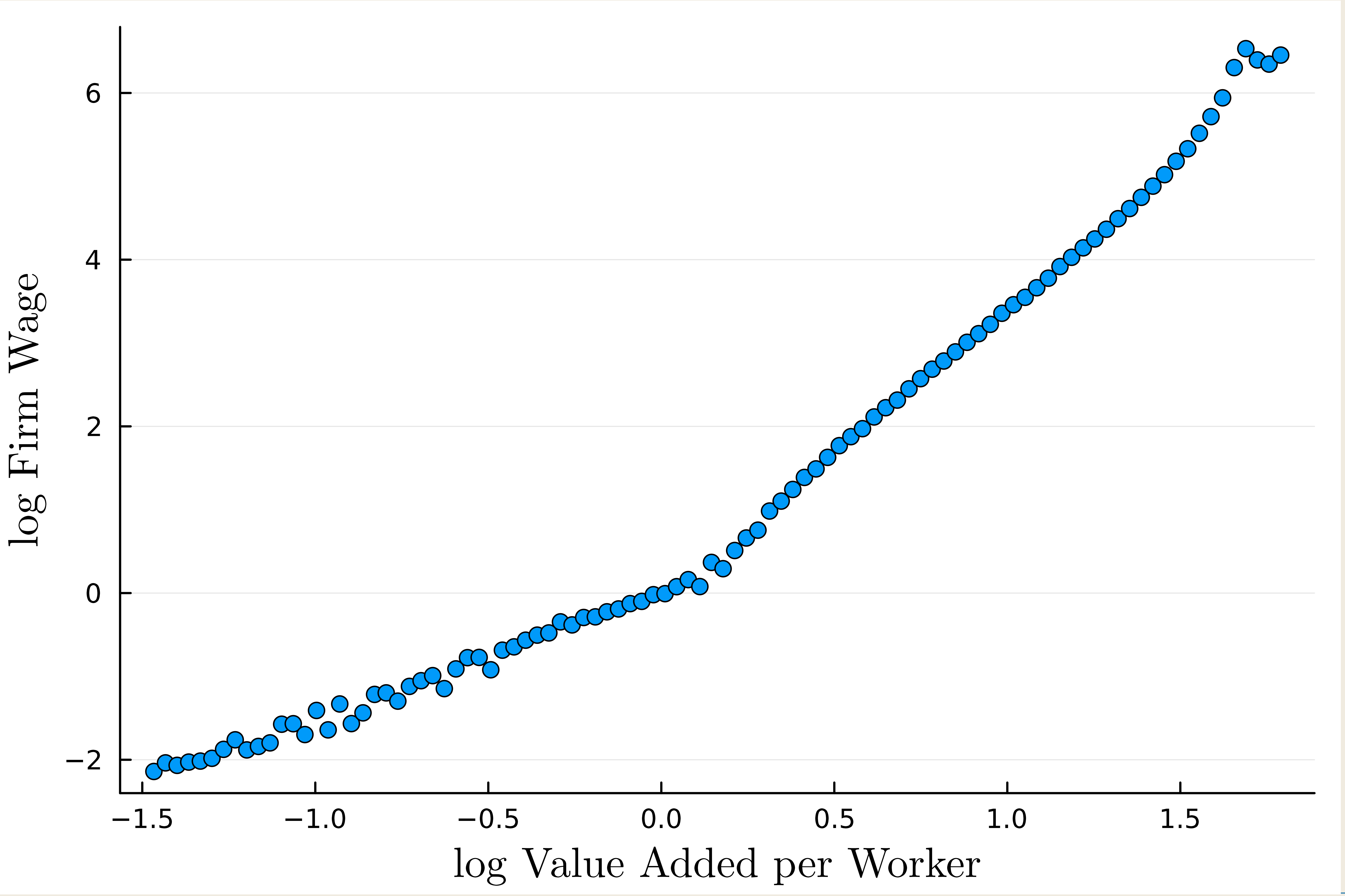
yes

Done

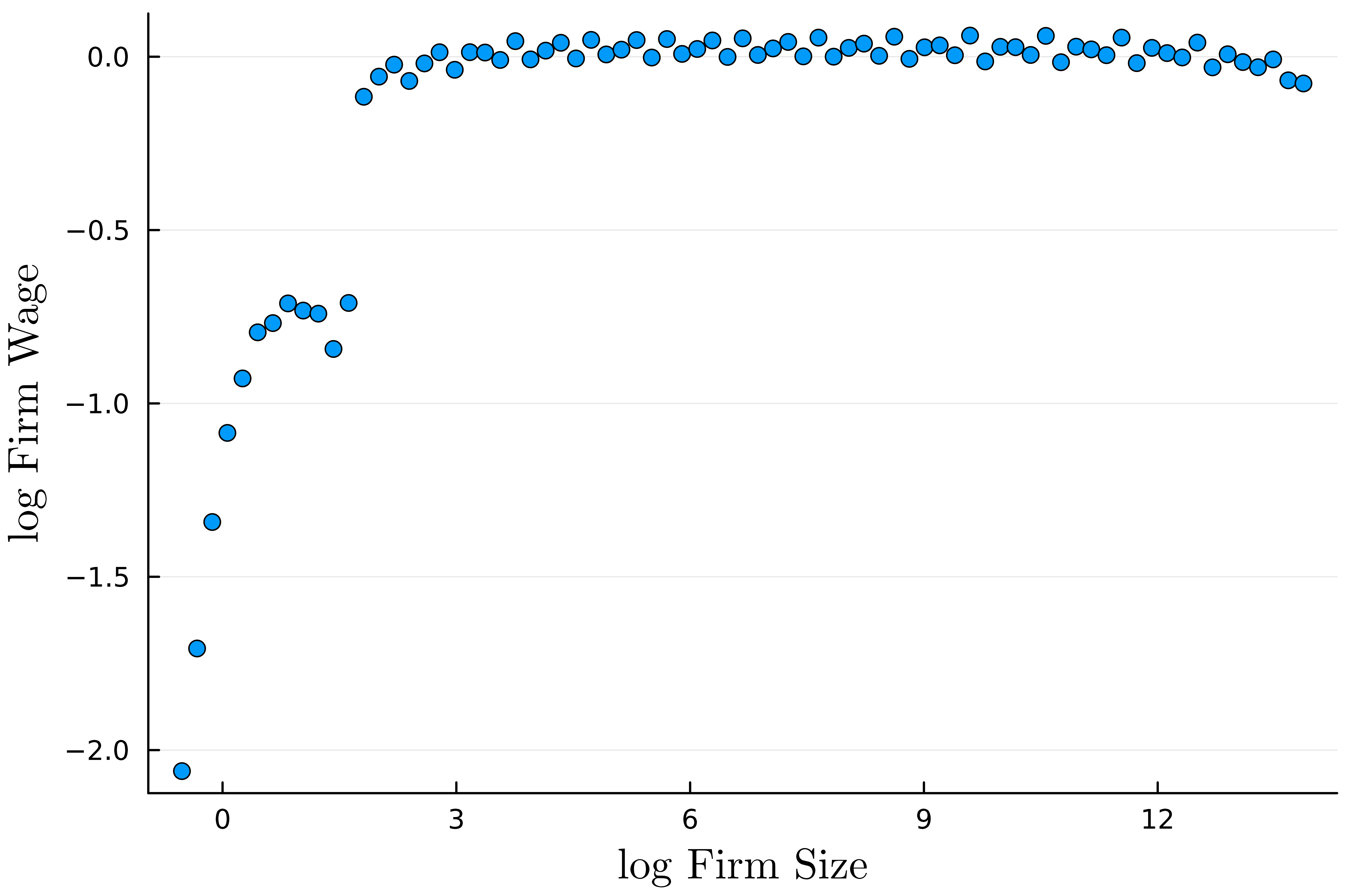
Parameterization

- Assume $\Phi(v,n) = \frac{\phi}{\kappa}(v/n)^{\kappa}n$, and set $\phi = 0.1, \kappa = 2$
- Assume $M(u,v) = u^{\eta}v^{1-\eta}$, and set $\eta = 0.5$
- Set $\gamma = 0.5$
- Set c_e so that $\theta = 1$, and set b so that $U = 5$
- The rest of the parameters are the same as in the lecture note 2

Higher Value Added, Higher Firm Wage



Larger Firm, Higher Firm Wage?



Labor Share


0.9
0.8
0.7
0.6
Labor Share
0
3
6
9
12
log Firm Size
1.0
0.9
0.8
0.7
0.6
0.5
Labor Share
0
3
6
9
12
log Firm Value Added