Offer-matching and Counter-offers: the Role of Origin Firms?

741 Macroeconomics
Topic 3

Masao Fukui

2025 fall

Counter-Offers?

- In wage-posting models, firms are passive to outside offers
- Firms let workers leave even when counter-offers are profitable
- Is this true in the data?

NY Fed Job Search Suvery (2013-2021)

Q [already accepted an offer]. Did your previous employer match the wage that was offered on your new job, or do you think they would have if you asked?

A. 10% matched, 10% would have matched if asked, 80% no

Q [received an offer but not accepted]. Do you think your current employer would match the wage that was offered (on your best job offer)?

A. 63% yes, 37% no

Q [hypothetical]. Suppose you are offered the same line of job with a 10% higher salary. Would you ask your current employer for a counteroffer? If yes, what is the chance they will match?

A. 46% yes, 54% no; 37% chance on average conditional on asking

Perceived Counter-Offer Probability

	(1)	(2)	(3)	(4)	(5)	(6)
Female	-0.035					-0.028
	(0.009)					(0.010)
Non-white		-0.030				-0.032
		(0.011)				(0.011)
Log wage			0.024			0.022
			(0.004)			(0.004)
College and above				0.021		0.000
				(0.008)		(0.009)
Blue-collar occupation					-0.031	-0.038
					(0.015)	(0.015)
Observations	6146	6138	6114	6147	5794	5757

Notes: Dependent variable is the perceived probability that the current employer matches an outside offer. Robust standard errors in parentheses.

A Model of Counter-Offers (a.k.a "Sequential Auction")

based on

Postel-Vinay & Robin (2002)
Cahuc, Postel-Vinay & Robin (2006)
Fukui & Mukoyama (2025)

Preferences and Technology

- Continuous time & focus on the stationary environment
- Workers $i \in [0,1]$ with heterogeneous productivity z with measure μ_z
- Firms with heterogenous productivity p
  - Let $F(p)$ denote vacancy-weighted CDF (exogenous)
  - Support $[\underline{p}, \infty)$, where $\underline{p}$ is the productivity below which firms exit
- Technology:
  - The match (z, p) produces $z \times p$ units of output
  - Unemployed produces $z \times b$
- Both workers and firms are risk-neutral with discount rate ρ

Search Friction

- Search is random:
  - All unemployed workers receive a job offer at an exogenous rate λ^U
  - All employed workers receive a job offer at an exogenous rate λ^E
- All jobs exogenously separate at rate δ
- Value functions:
  - U_{it}: unemployment value of worker i at time t
  - $W_{it}(p)$: employment value of worker i at firm p at time t
  - $J_{it}(p)$: value of a filled job at firm p employing worker i at time t
  - $V(p)$: value of vacancy after meeting, which we assume is zero, $V(p) = 0$
    - Assumption in the background is that a vacancy is not durable

Bargaining

- Match surplus:

$$S_{it}(p) \equiv W_{it}(p) + J_{it}(p) - U_{it} - V(p)$$

- We assume the split of the pie is determined by a version of Nash bargaining:

$$\max_{w} \Big(W_{it}(p) - U_{it} - O_{it}(p) \Big)^{\gamma} \Big(J_{it}(p) - V(p) \Big)^{1-\gamma}$$

  - γ: bargaining power of workers, w: wage
  - $O_{it}(p)$: outside option of worker in addition to U_{it} (to be determined)
  - Standard Nash: $O_{it}(p) = 0$

Sequential Auction

- When an unemployed worker meets a firm, $O_{it}(p) = 0$
- When an employed worker at firm p meets firm p':
  - Two firms compete for a worker
  - If $S_{it}(p) > S_{it}(p')$, incumbent firm wins & the worker stays
    - Worker can use the outside option $O_{it}(p) = S_{it}(p')$
    - Worker can move to the poaching firm and extract full surplus there
    - if the worker already has higher outside option, nothing happens
  - If $S_{it}(p) < S_{it}(p')$, poaching firm wins & the worker moves
    - Worker bargains with the poacher with the outside option $O_{it}(p) = S_{it}(p)$
    - Worker can go back to the previous firm and extract full surplus there

Guess and Verify

- We will guess and verify that the following property holds
  1. $S_{it}(p) = S_z(p)$
     - Suprlus is only a function of the productivity of the current match
     - Sequential auction only matters in how to split the pie
  2. $W_{it}(p) = W_z(p, O)$ and $J_{it}(p) = J_z(p, O)$
     - The current outside option O summarizes the history
     - $O =$ the second-best offer the worker has received besides the current firm
- We write $w_z(p, O)$ as the wage of worker z at firm p with outside option O

Bellman Equations

$$\rho U_z = bz + \lambda^U \int_{\underline{p}}^{\infty} \max\{W_z(p,0) - U_z, 0\} dF(p)$$

$$\rho W_z(p, O) = w_z(p, O) + \lambda^E \int_{p}^{\infty} \left[W_z\left(\tilde{p}, S_z(p)\right) - W_z(p, O) \right] dF(\tilde{p})$$

$$+ \lambda^E \int_{\underline{p}}^{p} \left[W_z\left(p, \max\{S_z(\tilde{p}), O\}\right) - W_z(p, O) \right] dF(\tilde{p}) + \delta(U_z - W_z(p, O))$$

$$\rho J_z(p, O) = pz - w_z(p, O) - \lambda^E \int_{p}^{\infty} dF(\tilde{p}) J_z(p, O)$$

$$+ \lambda^E \int_{\underline{p}}^{p} \left[J_z\left(p, \max\{S_z(\tilde{p}), O\}\right) - J_z(p, O) \right] dF(\tilde{p}) - \delta J_z(p, O)$$

Match Surplus

- Nash bargaining implies:

$$W_z(p,O) = U_z + O + \gamma[S_z(p) - O]$$
$$J_z(p,O) = (1-\gamma)[S_z(p) - O]$$

- Imposing these conditions, the match surplus $S_z(p)$ solves

$$(\rho+\delta)S_z(p) = pz - bz + \lambda^E\gamma\int_p^\infty \left[S_z(\tilde{p}) - S_z(p)\right] dF(\tilde{p}) - \lambda^U\gamma\int_0^\infty S_z(\tilde{p})dF(\tilde{p})$$

  - This confirms that $S_z(p)$ is only a function of (p,z) and not a function of O

- Boundary condition: $S_z(\underline{p}) = 0$

Match Surplus Solution

Match surplus is given by

$$S_z(p) = z \int_{\underline{p}}^{p} \frac{1}{\rho + \delta - \lambda^E \gamma (1 - F(\tilde{p}))} d\tilde{p}$$

where $\underline{p}$ solves

$$0 = \underline{p} - b - (\lambda^U - \lambda^E)\gamma \int_{\underline{p}}^{\infty} \int_{\underline{p}}^{\tilde{p}} \frac{1}{\rho + \delta - \lambda^E \gamma (1 - F(p'))} dp' dF(\tilde{p})$$

Proof

1. We can guess and verify that $S_z(p) = z\,S(p)$, where $S(p)$ solves

$$(\rho+\delta)S(p) = p - b + \lambda^E\gamma\int_p^\infty \left[S(\tilde{p}) - S(p)\right] dF(\tilde{p}) - \lambda^U\gamma\int_0^\infty S(\tilde{p})dF(\tilde{p}) \qquad \text{(S-1)}$$

2. Differentiate w.r.t. p to obtain

$$(\rho+\delta)S'(p) = 1 + \lambda^E\gamma(1 - F(p))S'(p)$$

3. With the boundary condition $S(\underline{p}) = 0$, we solve the above ODE to obtain

$$S(p) = \int_{\underline{p}}^{p} \frac{1}{(\rho+\delta-\lambda^E\gamma(1-F(\tilde{p}))}d\tilde{p}$$

4. Evaluate (S-1) at $p = \underline{p}$ to obtain

$$0 = \underline{p} - b - (\lambda^U - \lambda^E)\gamma\int_{\underline{p}}^\infty S(\tilde{p})dF(\tilde{p})$$

Wage Equation

The wage of worker z at firm p with previous offer from firm $q < p$ is:

$$
\begin{aligned}
w_z(p, S_z(q)) = \rho U_z + \gamma & \left[(\rho + \delta) S_z(p) - \lambda^E \int_p^\infty \partial_p S_z(\tilde{p})(1 - F(\tilde{p})) d\tilde{p} \right] \\
+ (1 - \gamma) & \left[(\rho + \delta) S_z(q) - \lambda^E \int_q^p \partial_p S_z(\tilde{p}) \big(1 - F(\tilde{p})\big) \, d\tilde{p} \right]
\end{aligned}
$$

Wage Equation

Option value from finding a better match

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$$+ (1 - \gamma) \left[(\rho + \delta) S_z(q) - \lambda^E \int_q^p \partial_p S_z(\tilde{p}) \big(1 - F(\tilde{p})\big)\, d\tilde{p} \right]$$

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$$+ (1 - \gamma) \left[(\rho + \delta) S_z(q) - \lambda^E \int_q^{p} \partial_p S_z(\tilde{p}) \big(1 - F(\tilde{p})\big)\, d\tilde{p} \right]$$

Option value from outside-offer matching

Special Cases

- Under $\gamma = 0$, the expression dramatically simplifies (Postel-Vinay & Robin, 2002)
- $\gamma = 0 \quad \Rightarrow \quad S_z(p) = z\frac{p - \underline{p}}{\rho + \delta}$, $\partial_p S_z(p) = \frac{z}{\rho + \delta}$, and $\underline{p} = b$
- Consequently,

$$w_z\big(p, S_z(q)\big) = z\left(q - \frac{\lambda^E}{\rho + \delta}\int_q^p [1 - F(\tilde{p})]d\tilde{p}\right)$$

$\Rightarrow$ wage at firm p strongly depends on q not so much on p!

- Notice that workers may accept wage cut, expecting the pay rise in the future
- When $\gamma = 1$, $w_z(p, S_z(q))$ is independent of origin firm q:

$$w_z(p, S_z(q)) = \rho U_z + \left[(\rho + \delta)S_z(p) - \lambda^E \int_p^\infty \partial_p S_z(\tilde{p})(1 - F(\tilde{p}))d\tilde{p}\right]$$

Empirical Content of Sequential Auction Protocol?

– DiAddario, Kline, Saggio & Sølvsten (2020)

Empirical Application of Sequential Auction

Postel-Vinay & Robin (2002):

- Assume $\gamma = 0$ and argue AKM model is "mis-specified"
- Part of what AKM attributes to "worker effect" is persistent outside option

Cahuc, Postel-Vinay & Robin (2006):

- Estimate γ using French employee-employer matched data
- The majority of workers have low γ but extract surplus through counter-offers

Bagger, Fontaine, Postel-Vinay & Robin (2014):

- Incorporate human capital accumulation to decompose the source of wage growth
- Counter-offers relatively unimportant compared to human capital & job-ladder

Putting Sequential Auction to the Test

- All studies in the previous slide *assume* a sequential auction wage-setting protocol
- This leaves the sequential auction untested
- In fact, the survey evidence suggests many receive no offer-matching
- Can we test sequential auction vs. no offer matching (e.g., Burdett-Mortensen)?

AKM with Orign Firm

- To a first-order approximation around $p = q = \bar{p}$ (mass point at $\bar{p}$), we have

$$\ln w_z\big(p, S_z(q)\big) \approx \ln z + \psi(p) + \lambda(q)$$

where

$$\psi(p) \equiv \ln \Gamma(\bar{p}) + S'(\bar{p})\gamma \frac{\rho + \delta + \lambda^E}{\rho U + (\rho + \delta) S(\bar{p})}(p - \bar{p})$$

$$\lambda(q) \equiv (1 - \gamma) S'(\bar{p}) \frac{\rho + \delta + \lambda^E}{\rho U + (\rho + \delta) S(\bar{p})}(q - \bar{p})$$

Empirical Implementation

$$\ln w_{im} = \alpha_i + \underbrace{\psi_{j(i,m)}}_{\text{destination effect}} + \underbrace{\lambda_{h(i,m)}}_{\text{origin effect}} + X'_{im}\gamma + \epsilon_{im}$$

- w_{im}: hiring wage of worker i in her m-th match
- $j(i,m)$: firm employing worker i in the m-th match
- $h(i,m) = \begin{cases} j(i,m-1) & \text{if employed at } m-1 \\ U & \text{if unemployed at } m-1 \\ N & \text{if no labor market experience} \end{cases}$
- Implement using Italian employer-employee matched data 2005-2015
- Call this model DWL (dual wage-ladder) model

DWL Yields <1pp Improvement in R^2

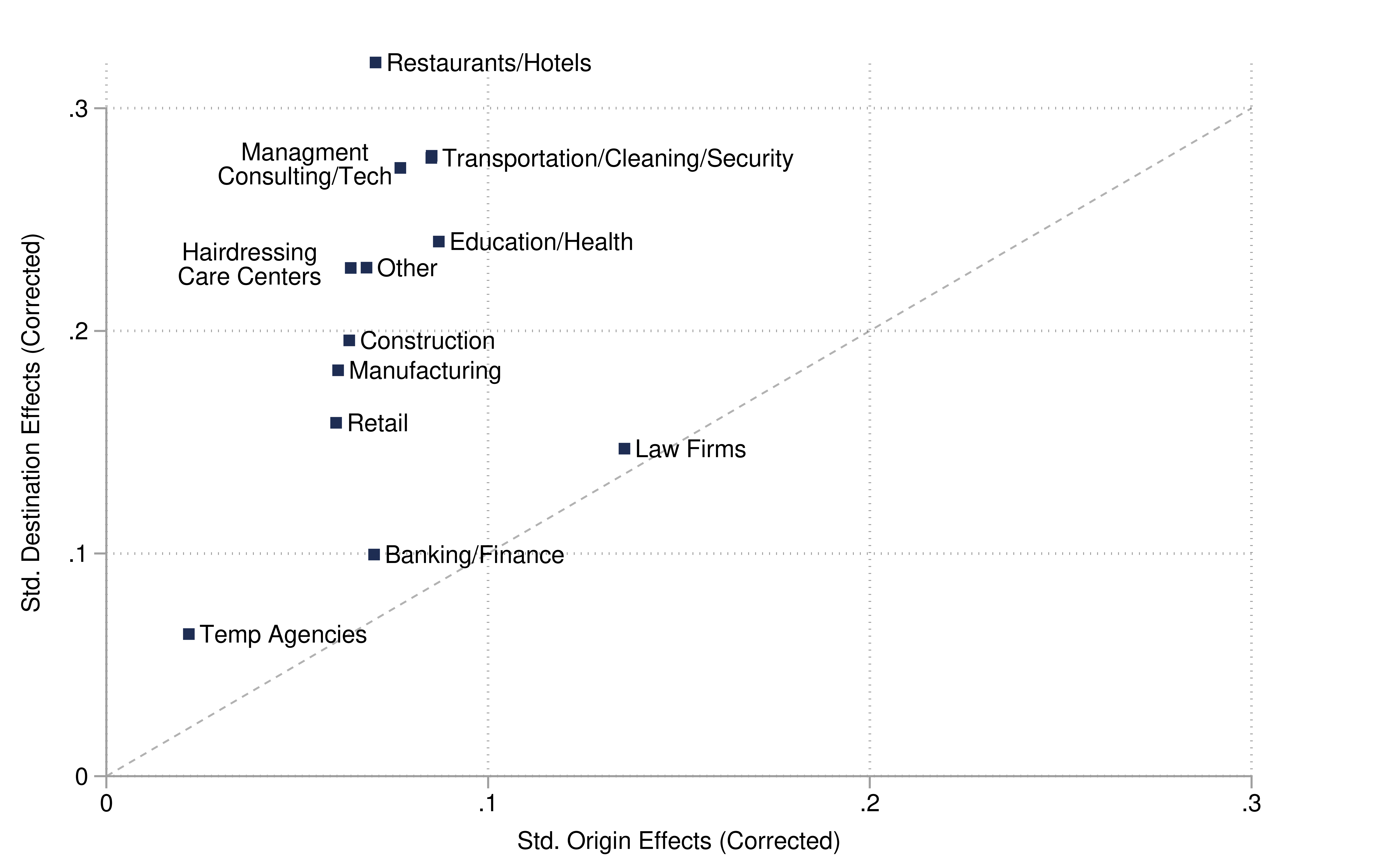
Goodness of fit, R^2

	Pooled	Men	Women
AKM	0.7199	0.7311	0.6822
AKM (Gender-interacted)	0.7349		
Origin effects	0.5809	0.5660	0.5452
Origin effects (Gender-interacted)	0.5871		
DWL	0.7245	0.7370	0.6854
DWL (Gender-interacted)	0.7427		

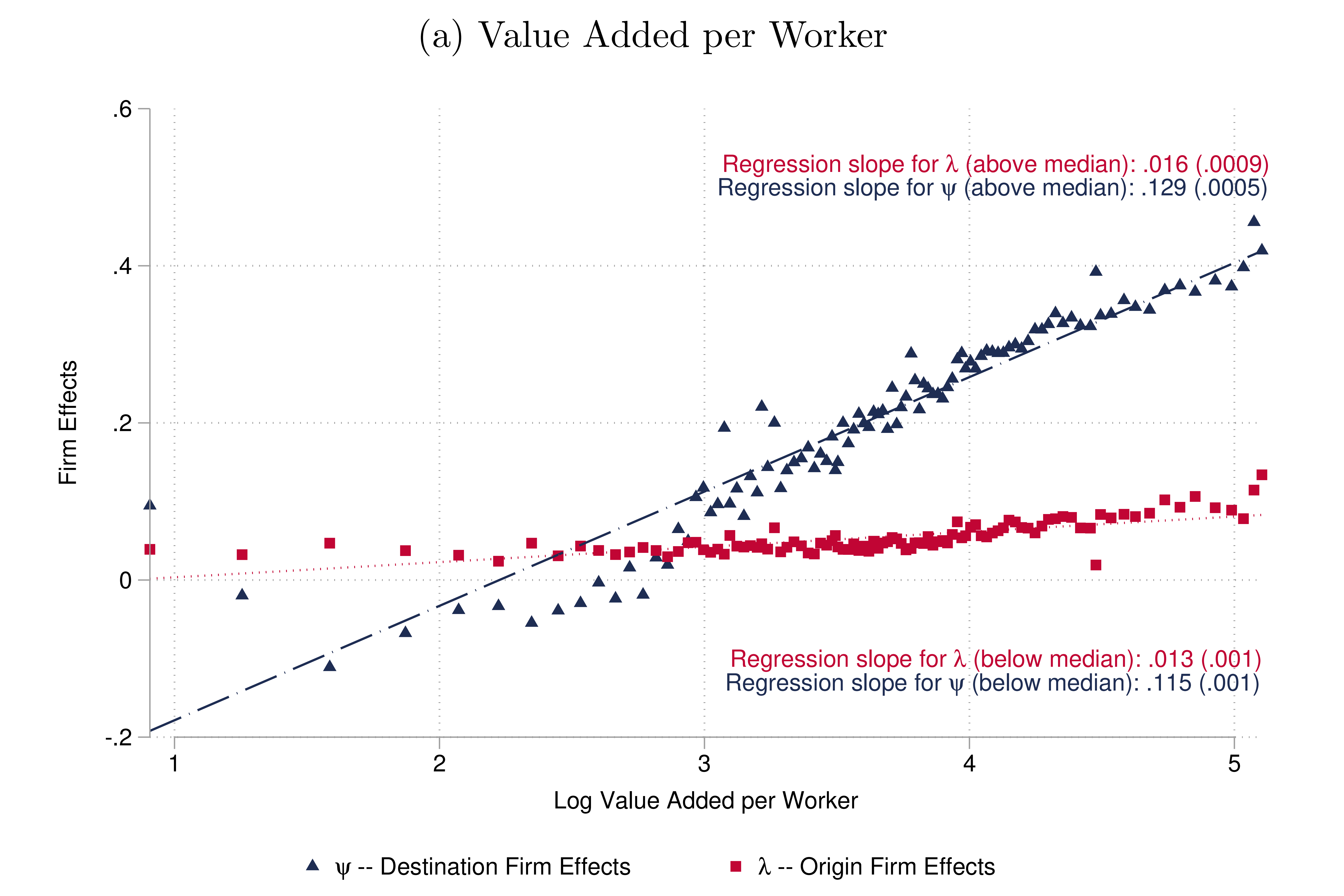
It ain't where you're from...

	Pooled	Men	Women
Std Dev of log hiring wages	0.5286	0.4706	0.5623
Mean $\lambda_{j(i,m-1)}$ among displaced workers	0.0414	0.0536	0.0687
Mean $\lambda_{j(i,m-1)}$ among poached workers	0.0508	0.0543	0.0690
Origin effect when hired from non-employment (λ_U)	0.0163	0.0136	0.0220
Percent of Total Variance Explained by			
Worker effects	28.52%	27.75%	24.77%
Destination firm effects	23.81%	26.74%	25.29%
Origin effects	0.69%	0.93%	0.59%
Covariance of worker, destination	16.46%	12.81%	17.23%
Covariance of worker, origin	1.06%	1.66%	0.58%
Covariance of destination, origin	0.26%	0.31%	0.00%
X'δ and associated covariances	1.66%	3.51%	0.09%
Residual	27.55%	26.30%	31.46%

Variance of Origin/Destination Effects by Sectors



Origin Effect Barely Varies with Value-Added



Summary

- Sequential auction wage-setting protocol has been extremely popular
- Until very recently, the assumption has never been tested
- DKSS: hiring wage is almost unrelated to the worker's origin
  $\Rightarrow$ "It ain't where you're from, it's where you're at"!
- This casts doubt on the relevance of sequential auction protocol… or not?