Labor Reallocation and Misallocation

741 Macroeconomics
Topic 7

Masao Fukui

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Firm Employment is Log-Linear in TFP

- In Hopenhayn-Rogerson, firm-level employment is given by

$$n = (\underbrace{z^{1-\alpha}}_{\equiv Z}\alpha/w)^{\frac{1}{1-\alpha}}$$

$$\Leftrightarrow \quad \log n = \frac{1}{1-\alpha}\log Z + const$$

- Taking the first difference,

$$\Delta \log n = \frac{1}{1-\alpha}\Delta \log Z$$

$\Rightarrow$ Firms react symmetrically to positive and negative TFP shocks

- Is this true in the data?

Ilut, Kehrig & Schneider (2018)

- Focus on US manufacturing establishments (Census data)
- Construct firm-level TFP using Solow residual:

$$\log sr_{it} = \log y_{it} - \left(\beta_n \log n_{it} + \beta_k \log k_{it} + \beta_m \log m_{it}\right)$$

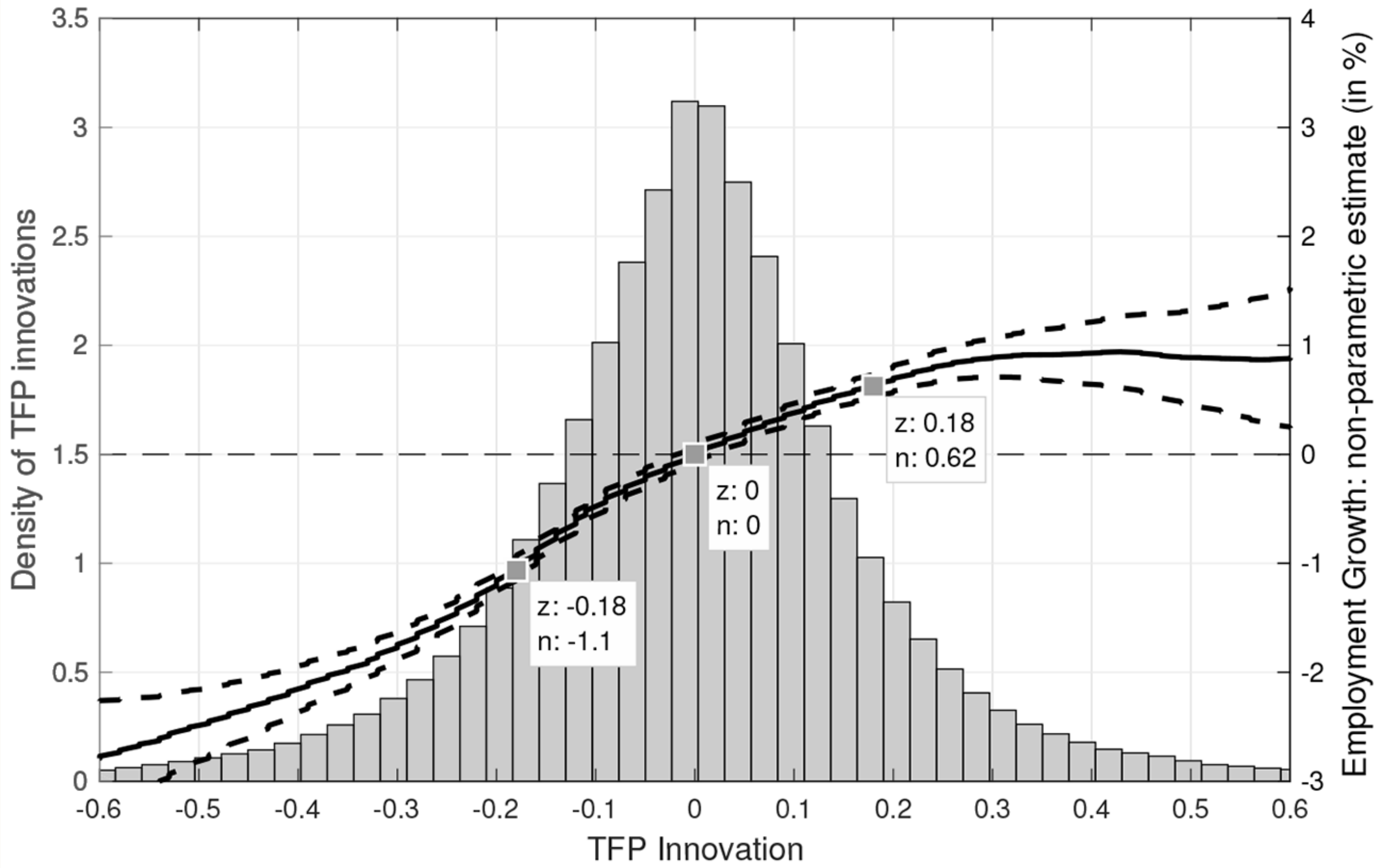
- Construct firm-level TFP shocks, Z_{it}, assuming

$$\log sr_{it} = g \times t + \alpha^i + \log Z_{it}$$

- Q: How does firm-level employment respond to TFP shocks?

$$\Delta \log n_{it} = h(\Delta \log Z_{it}) + \gamma' \boldsymbol{X}_{it} + \epsilon_{it}$$

Concave Hiring Rule



Firm Dynamics with Labor Adjustment Costs – Hopenhayn & Rogerson (1993)

Slow to Hire, Quick to Fire

- The simplest explanation:
  - it is costly to hire workers
  - less so to fire workers

- We incorporate employment adjustment costs into Hopenhayn-Rogerson

Labor Adjustment Cost

- Suppose that employment stock is costly to adjust
- Every period, $\delta \in [0,1]$ fraction of workers exogenously separate
- Firms can hire $h \times n$ workers with hiring cost $\Phi(h, n)$
    - $h < 0$ corresponds to firing
- The stock-flow equation of employment:

$$n_t = n_{t-1}(1 - \delta + h_t)$$

Bellman Equation

- Bellman equation:

$$v(n_{-1}, z) = \max \left\{ v^*(n_{-1}, z), \; -\Phi(-(1-\delta), n_{-1}) \right\}$$

where v^* is the continuation value

$$v^*(n_{-1}, z) = \max_{h, n} \; f(n, z) - wn - c_f - \Phi(h, n_{-1}) + \beta \mathbb{E} v(n, z')$$

$$\text{s.t.} \quad n = n_{-1}(1 - \delta + h)$$

- Policy functions:
  - $\chi(n_{-1}, z) \in \{0,1\}$: whether to exit or not
  - $h(n_{-1}, z)$: hiring rate
  - $n(n_{-1}, z)$: employment

Rest of the Equilibrium Conditions

- Assume that the initial firm size is given by n_0

- The free-entry condition is

$$\int v(n_0, z)\psi_0(z)dz = c_e$$

- Let $g(n_{-1}, z)$ denote the steady-state distribution, which satisfies

$$g(n, z') = \int\int \Pi(z'|z)(1 - \chi(n_{-1}, z))\mathbb{I}[n(n_{-1}, z) = n]g(n_{-1}, z)dzdn_{-1} + m\psi_0(z)\mathbb{I}[n = n_0]$$

- The labor market clearing condition is

$$\int\int n(n_{-1}, z)g(n_{-1}, z)dn_{-1}dz = L$$

Equilibrium Definition

- Recursive equilibrium: $\{v(n,z), \chi(n,z), n'(n,z), w\}$ and $\{g(n_{-1},z), m\}$ such that:
  1. Given w, $\{v(n,z), \chi(n,z), n'(n,z)\}$ solve the Bellman equation
  2. Free entry holds, $\int v(n_0,z)\psi_0(z)dz = c_e$
  3. $\{g(n_{-1},z), m\}$ satisfies the steady state law of motion
  4. Labor market clears

- The equilibrium retains the same structure as before:
  1. Block recursive property: value and policy functions independent of distribution
  2. $g(n_{-1},z)$ homogenous in m:
     - can solve for $\hat{g} \equiv g/m$ first $\Rightarrow$ solve for m using labor market clearing

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1 & 2 alone

$\Rightarrow \{v(n,z), \chi(n,z), n'(n,z), w\}$

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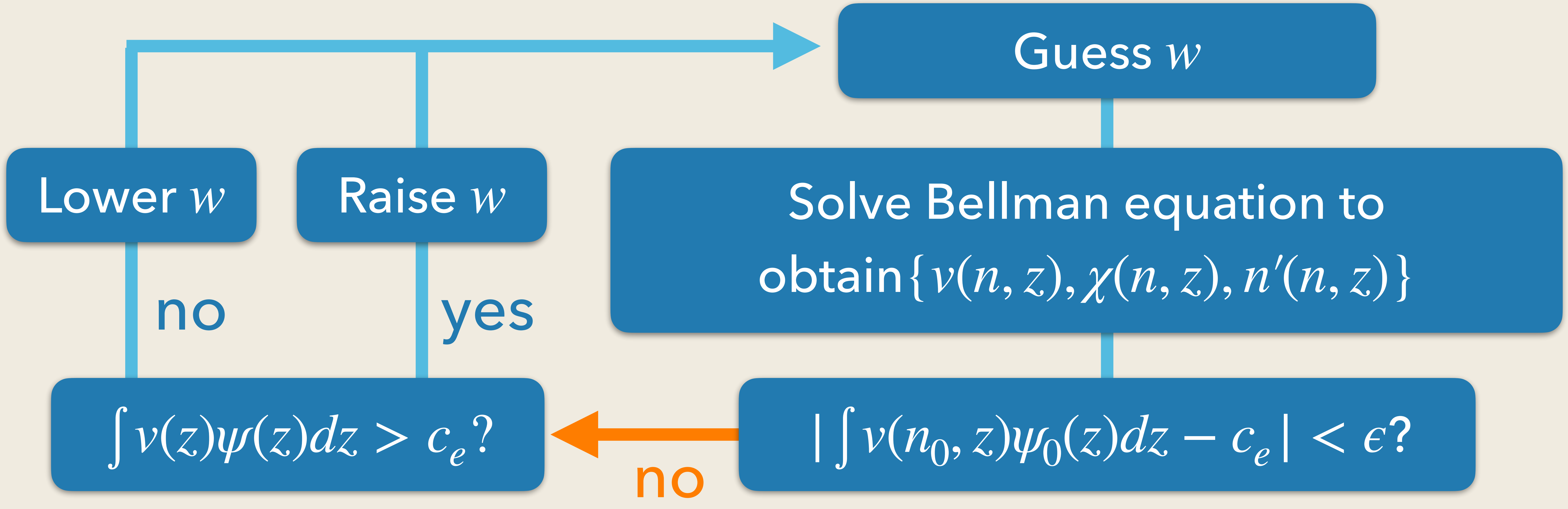
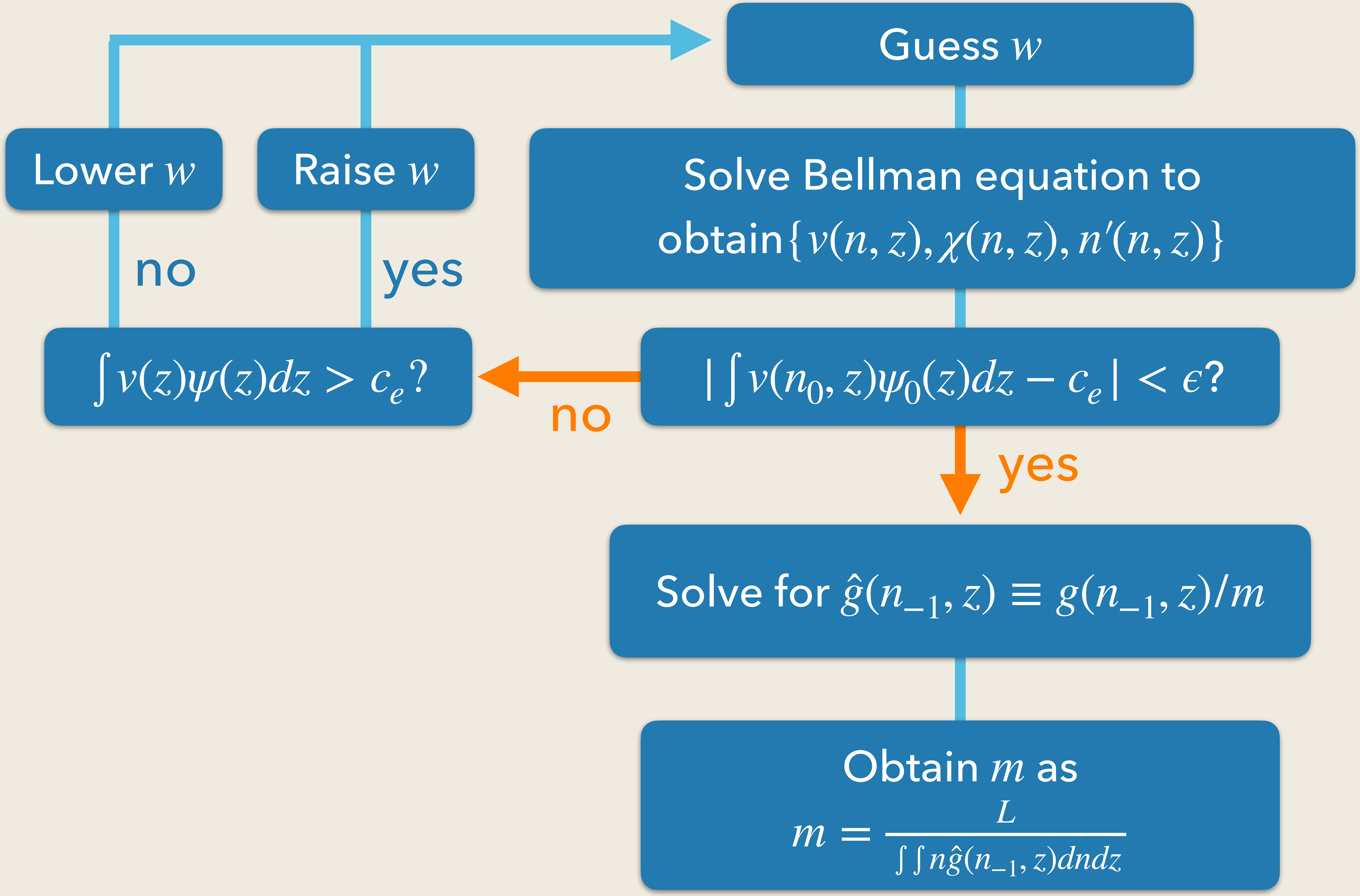
Computational Algorithm



Computational Algorithm



Calibration

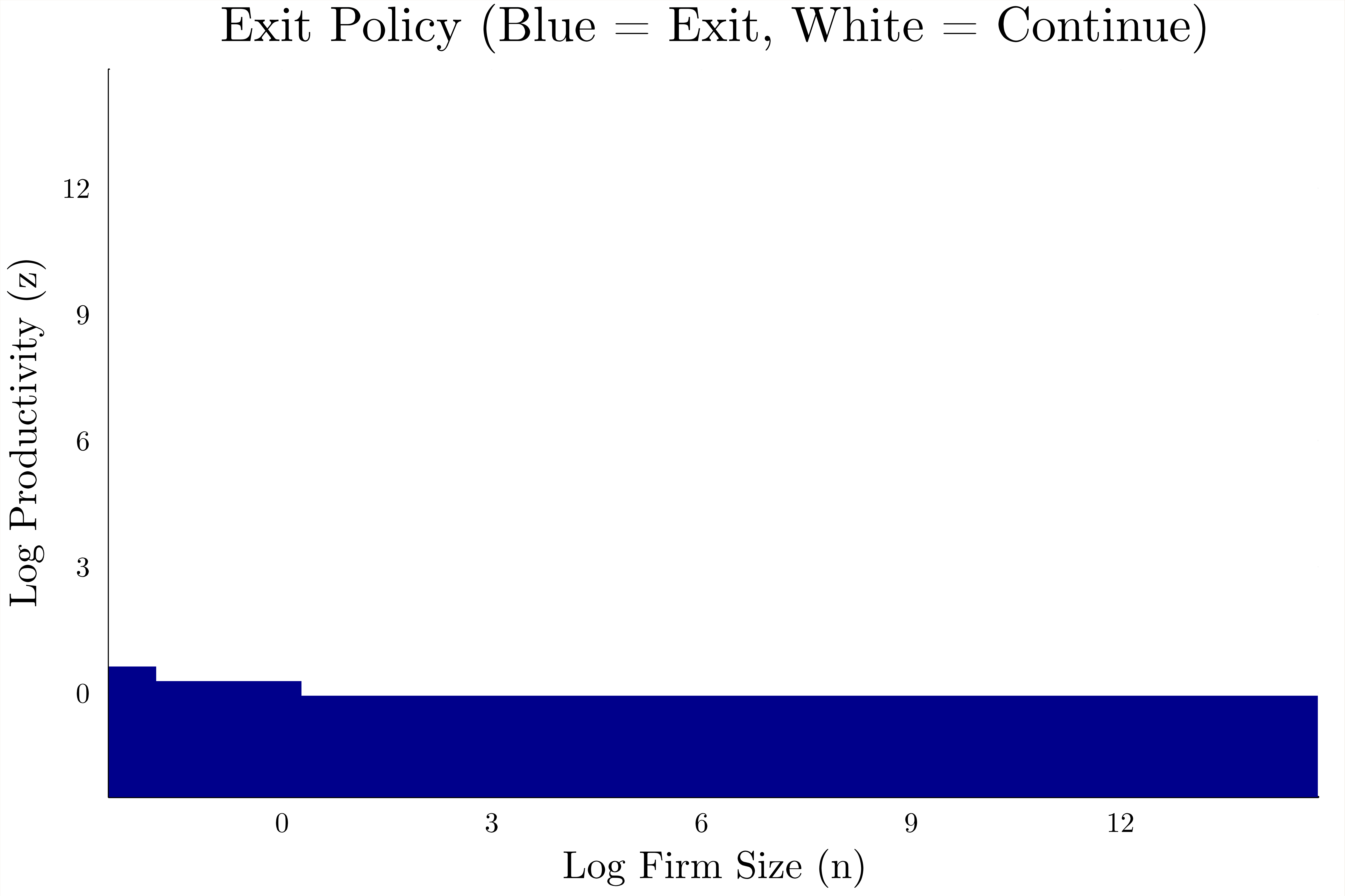
- Same parameter values for those that appear in the previous lecture note
- Set

$$\Phi(h, n_{-1}) = \begin{cases} \frac{\phi_+}{2} h^2 n_{-1} & \text{if } h > 0 \\ \frac{\phi_-}{2} h^2 n_{-1} & \text{if } h < 0 \end{cases}$$

  and assume $\phi_- = 0$ and $\phi_+ = 5$

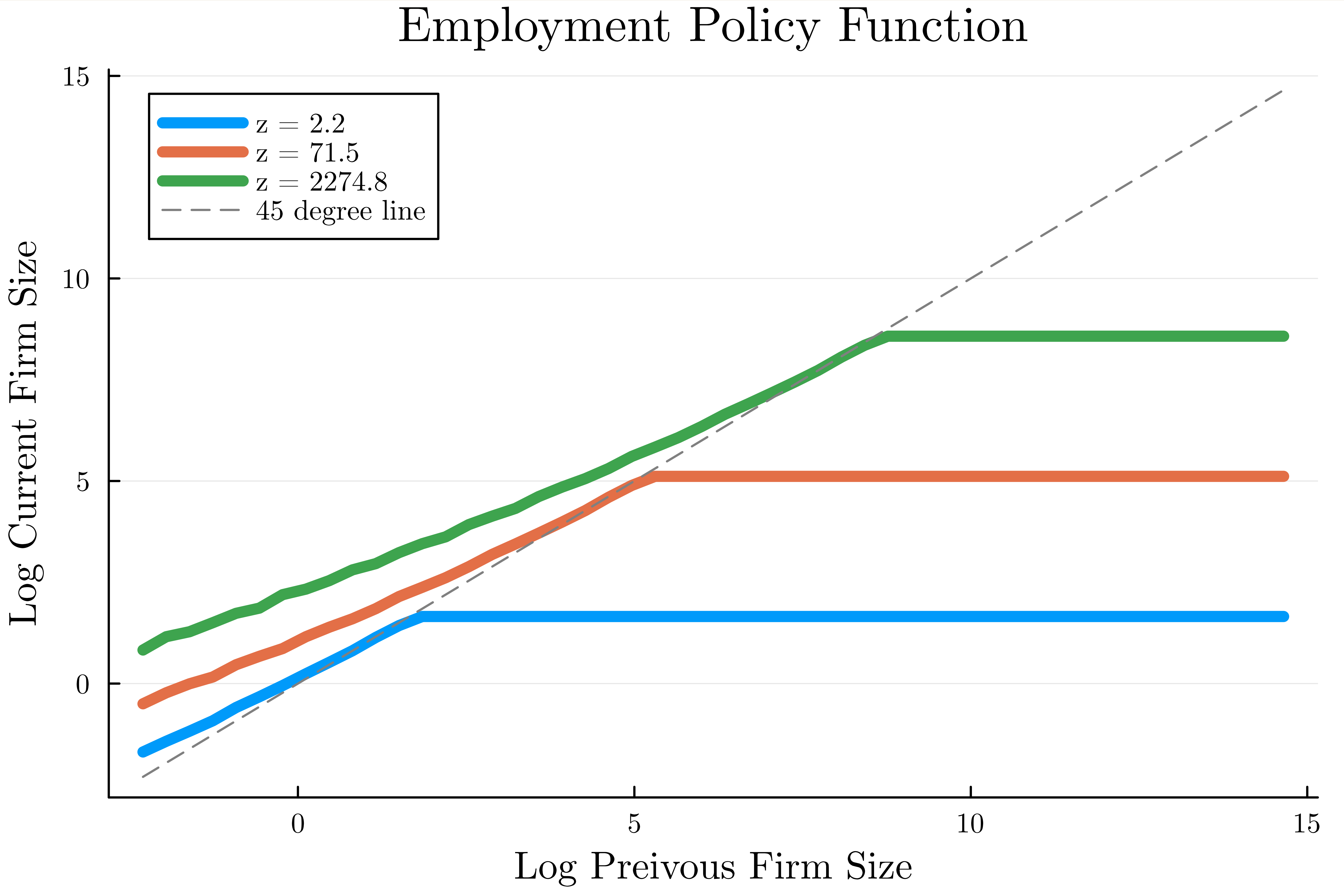
- Set $\delta = 0.1$
- Assume $n_0 = 5$ to roughly match the initial firm size

Exit Policy

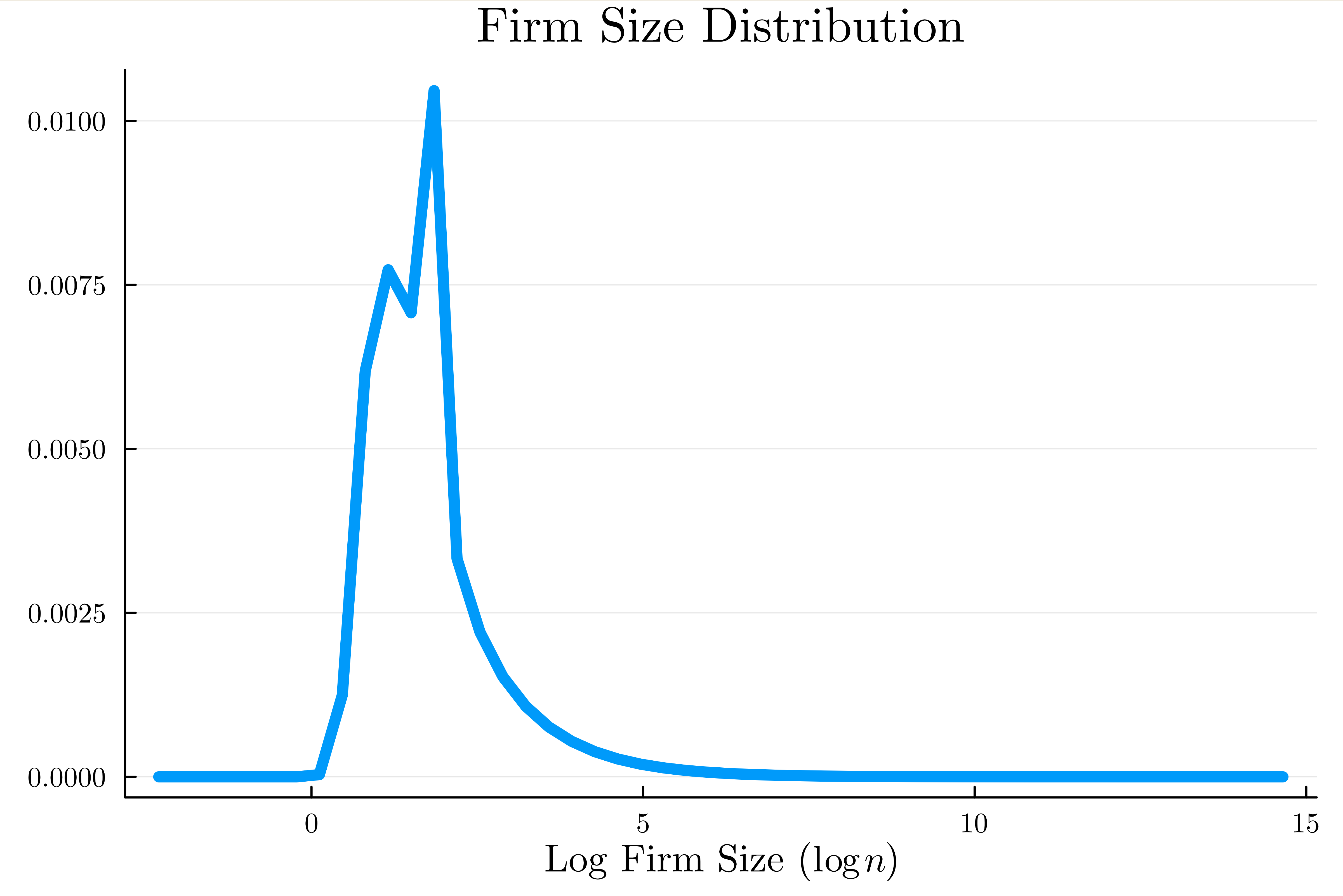


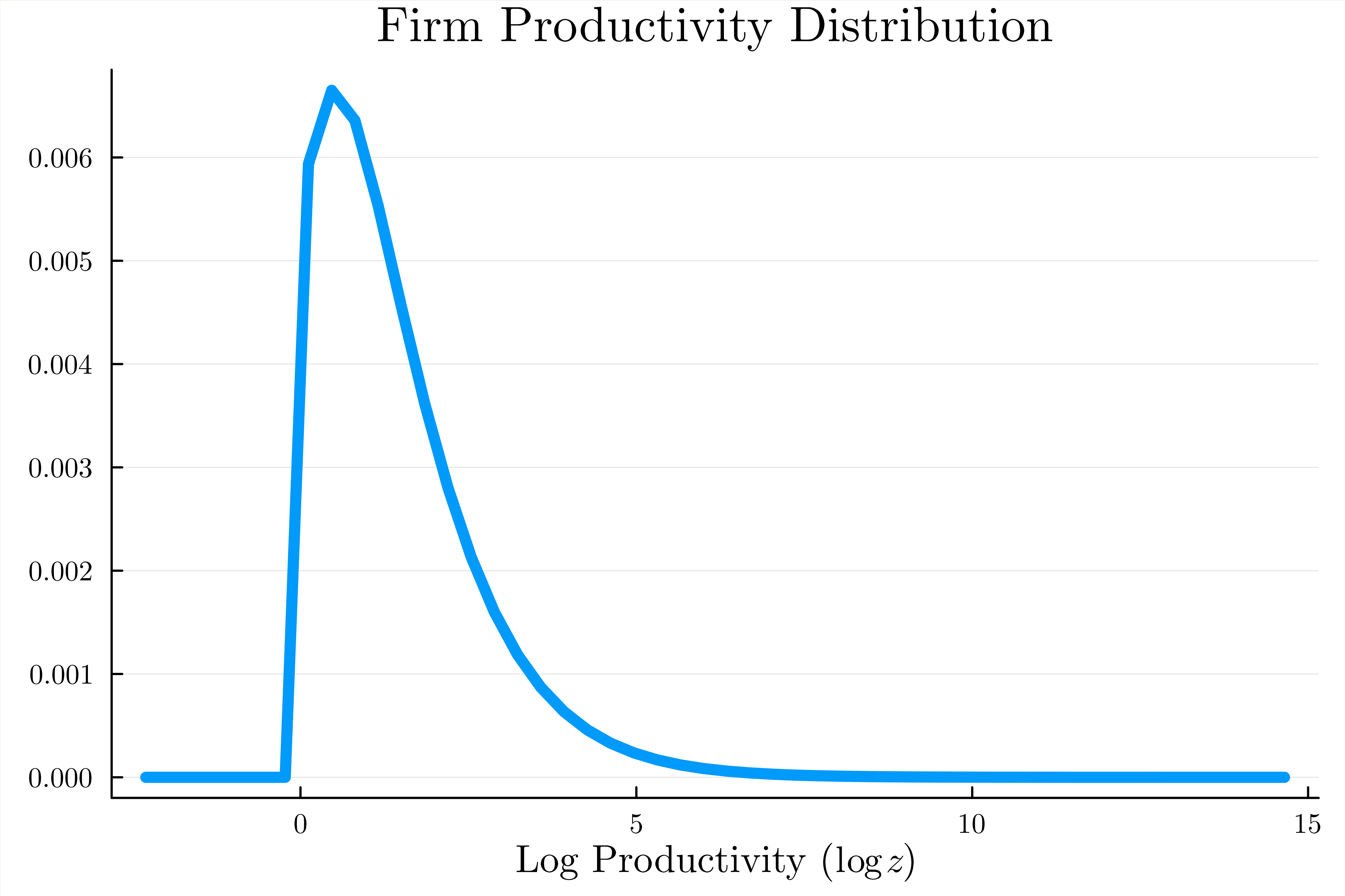
Exit Policy (Blue = Exit, White = Continue)
Log Productivity (z)
12
9
6
3
0
0
3
6
9
12
Log Firm Size (n)

Employment Policy

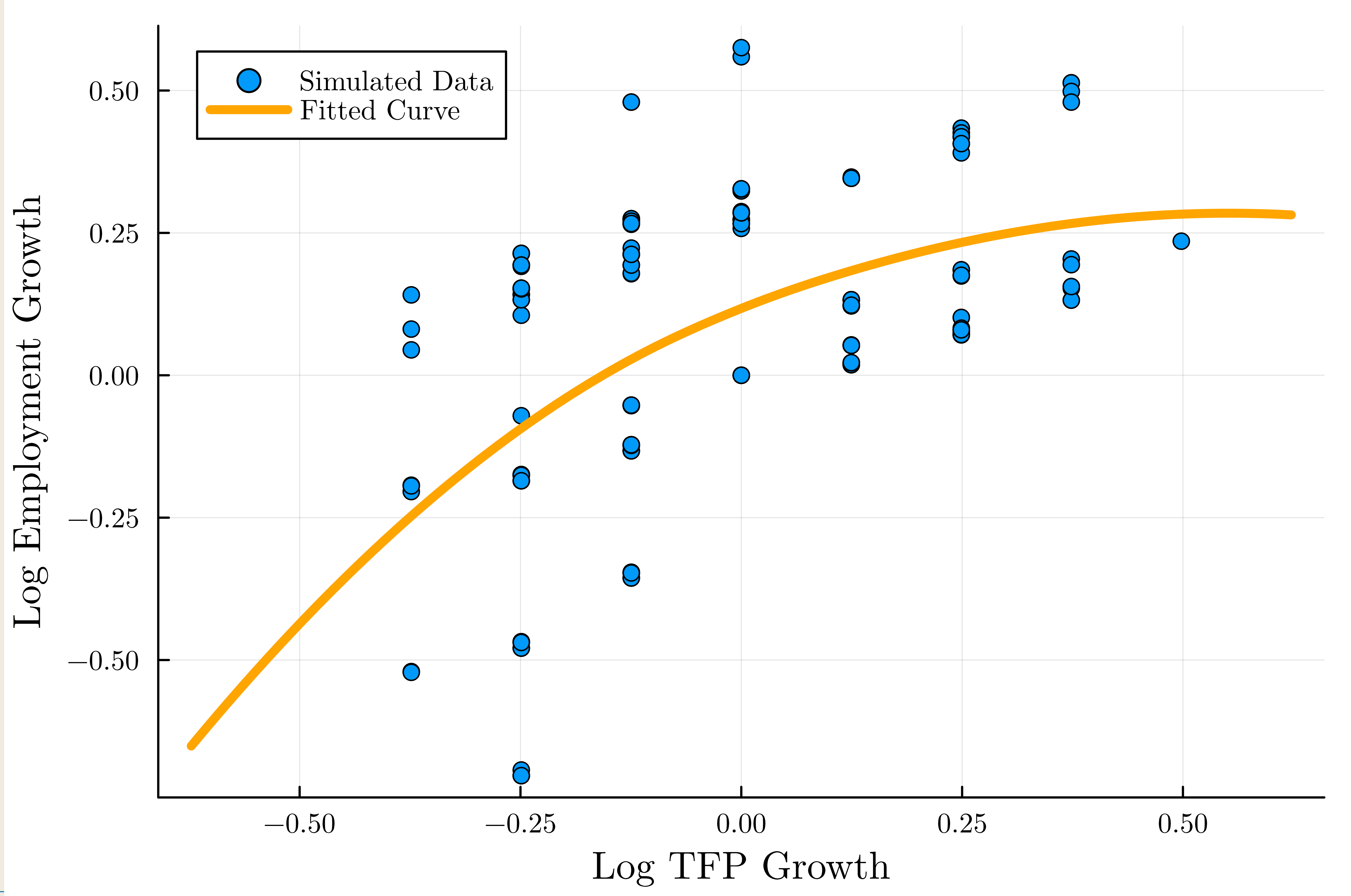


Distribution

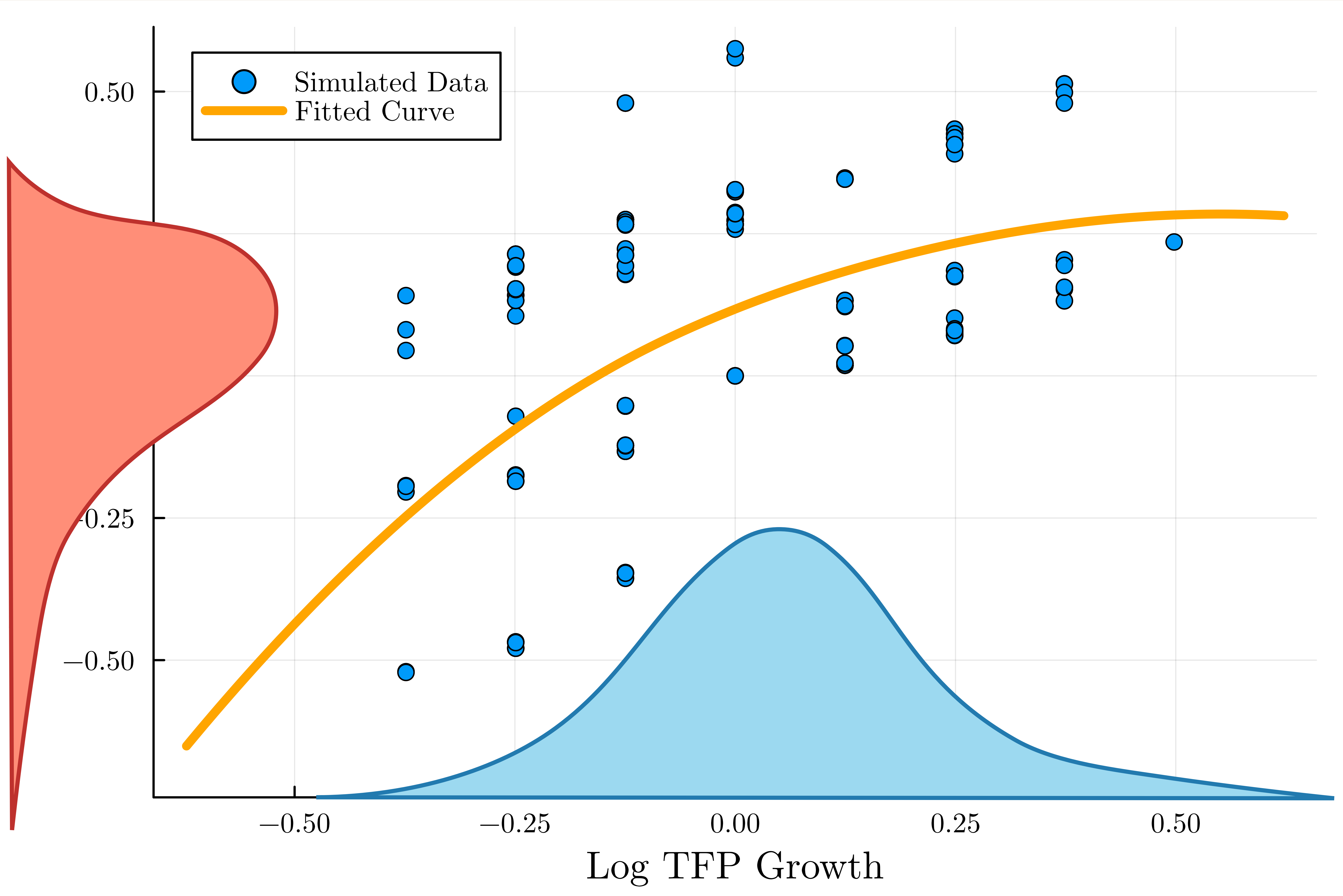




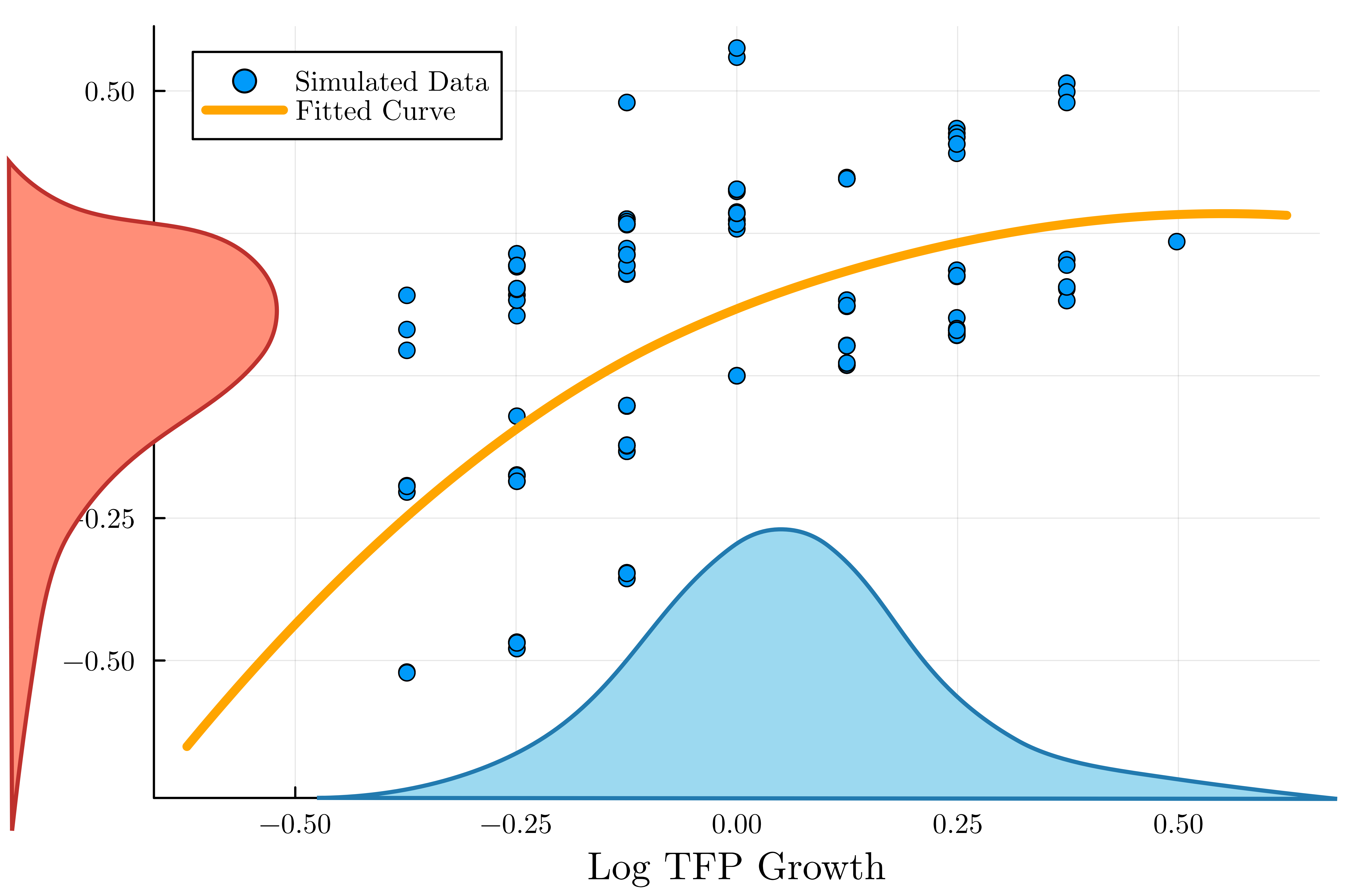
Concave Hiring Rule



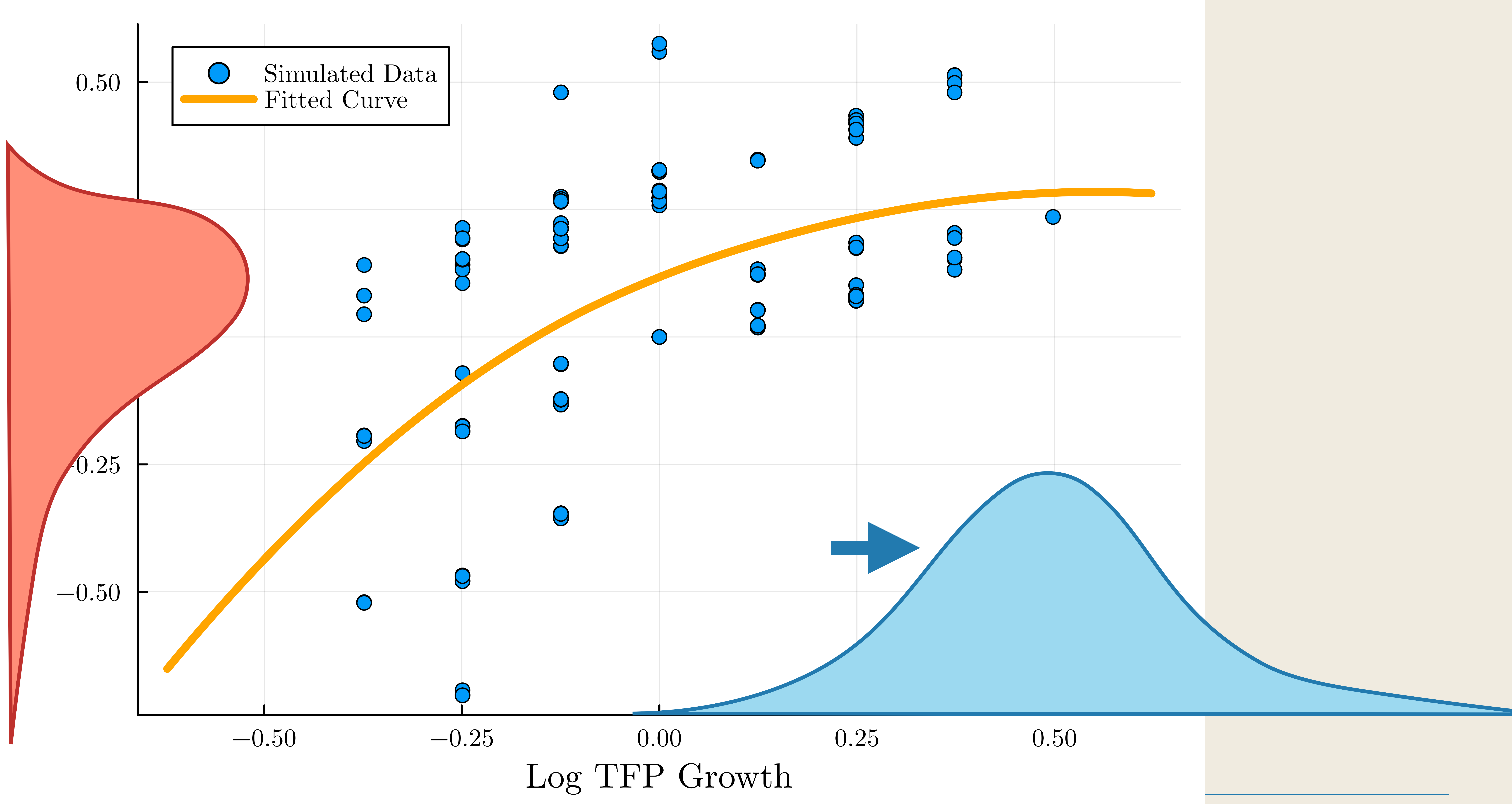
Distribution



Positive Aggregate Shock



Positive Aggregate Shock



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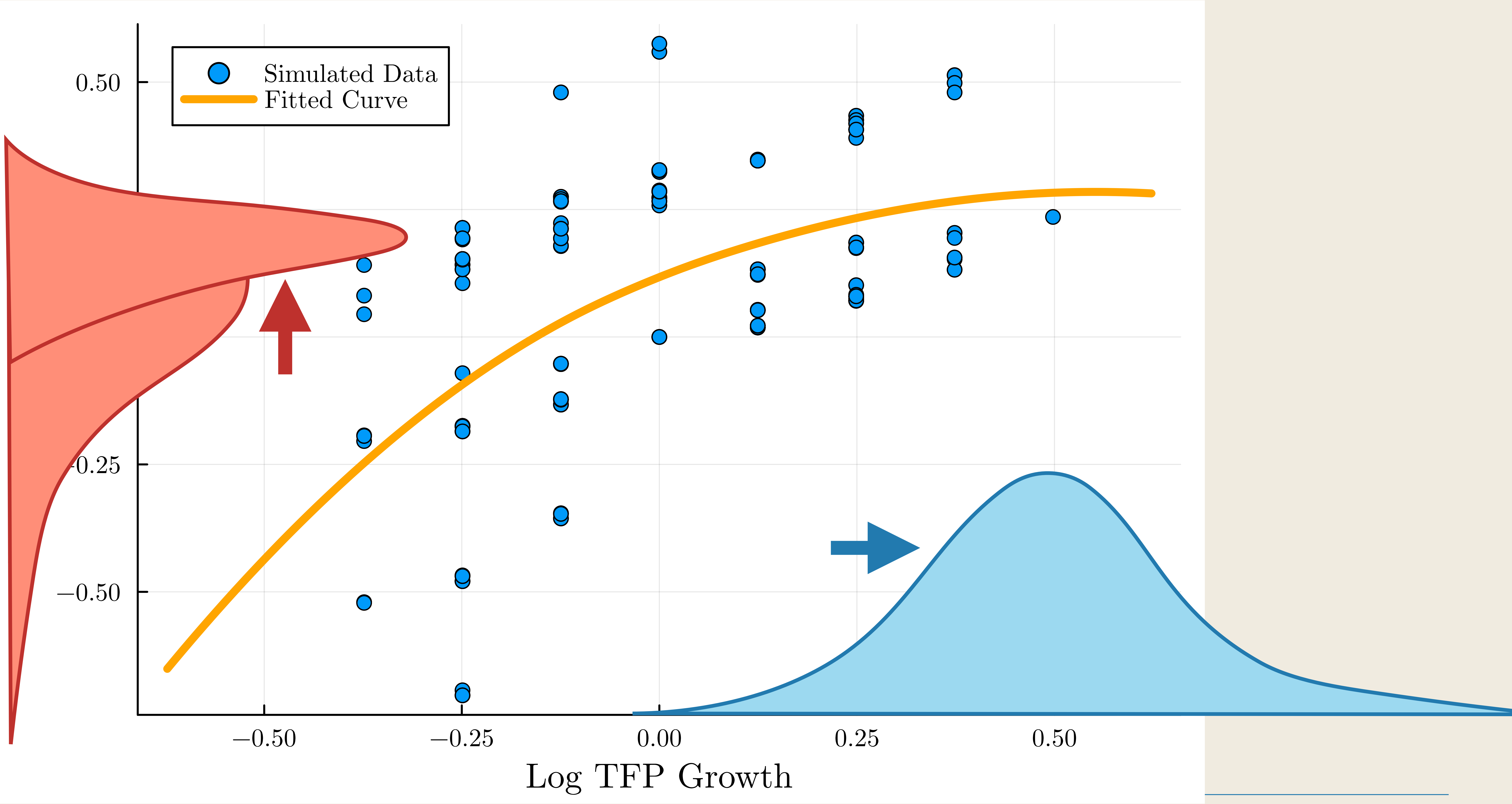
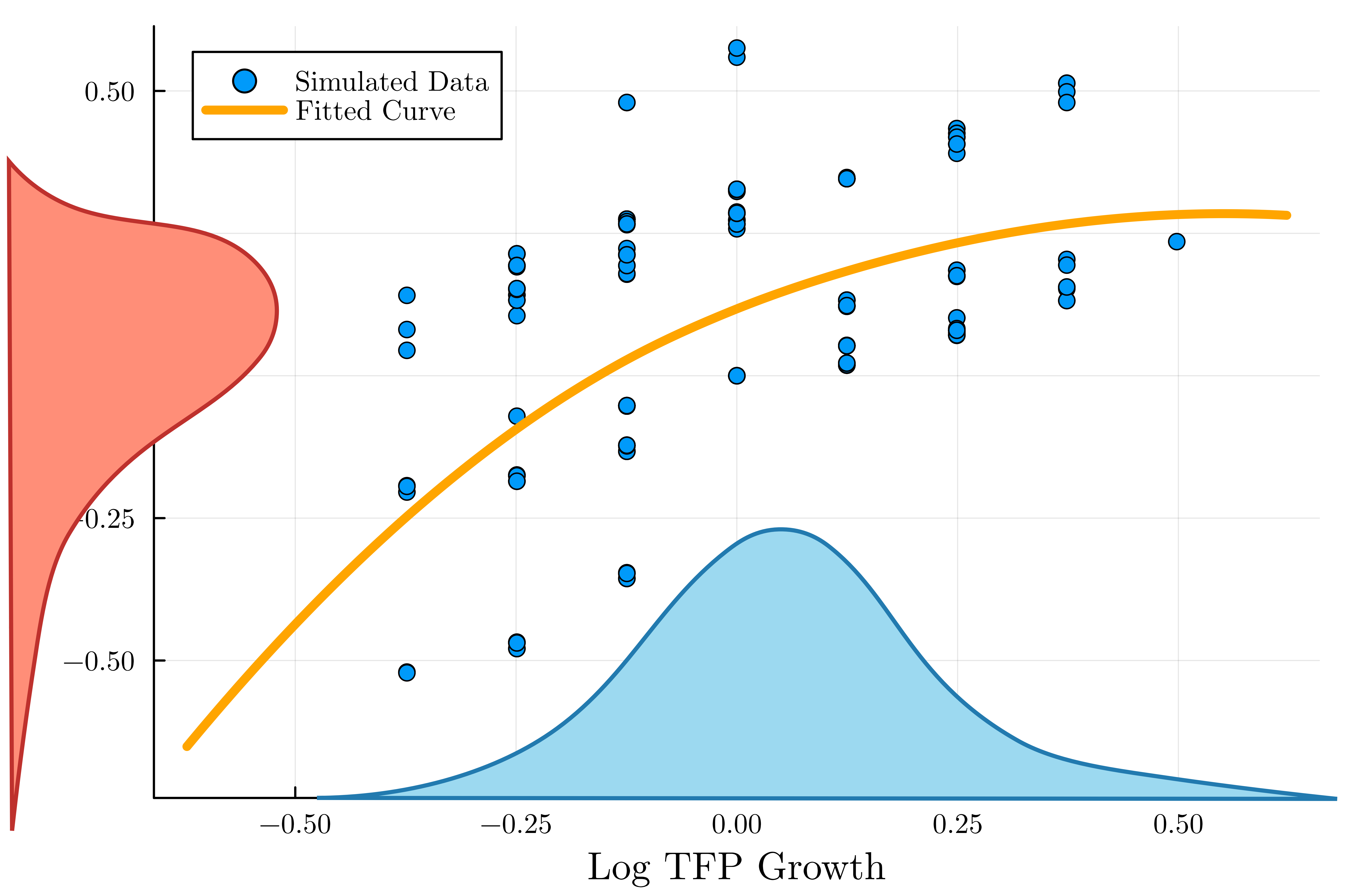
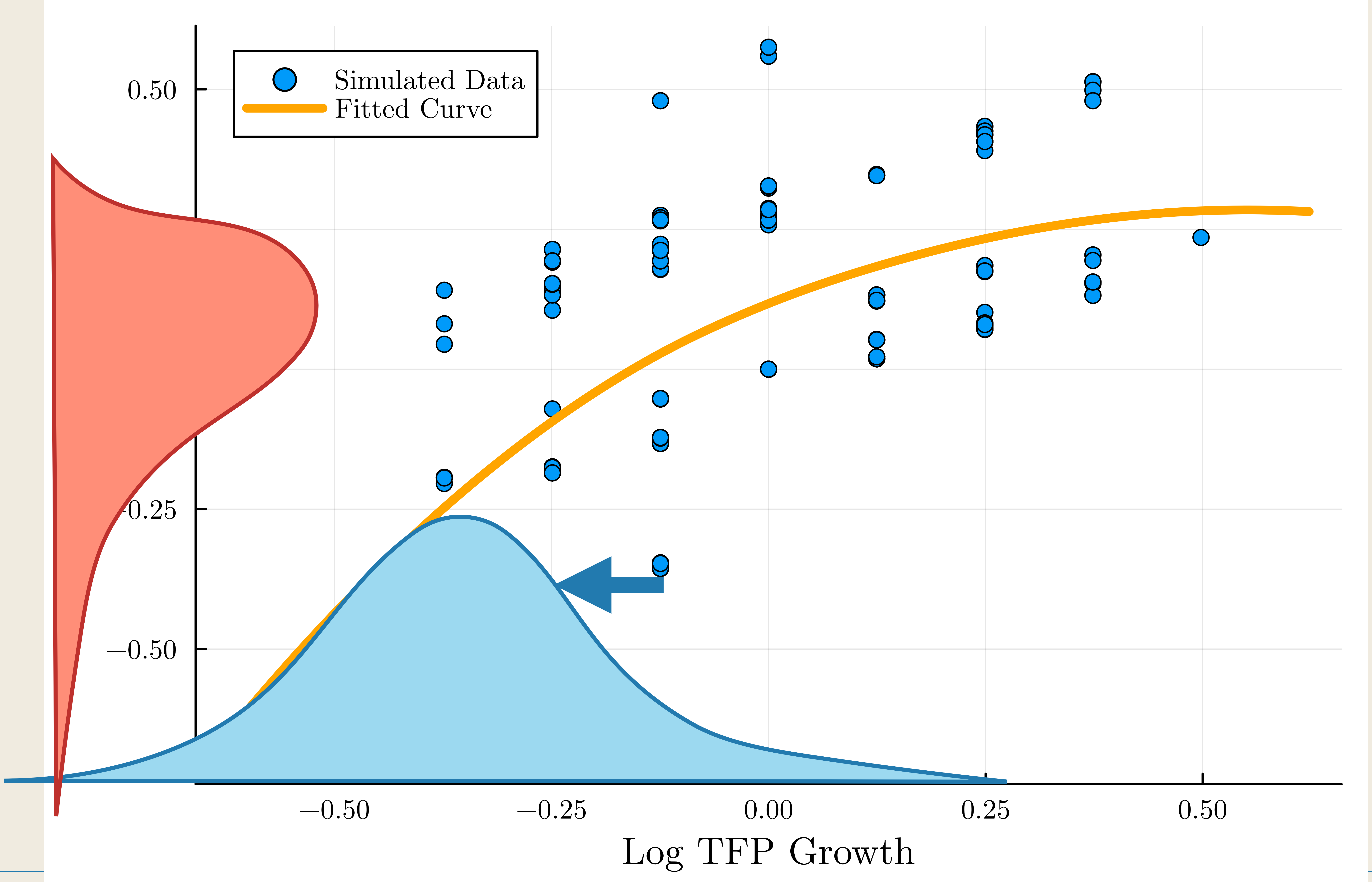


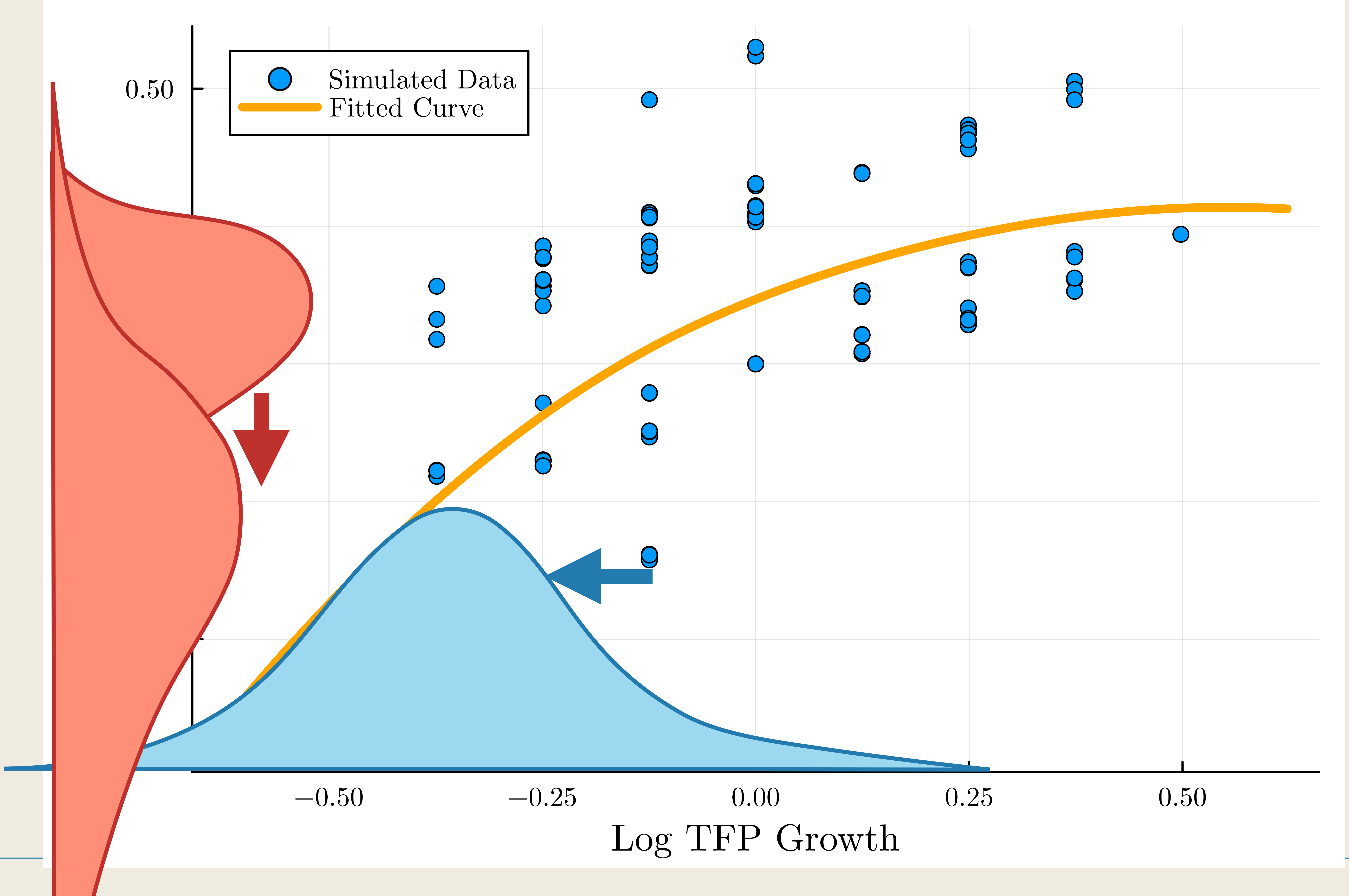
Negative Aggregate Shock



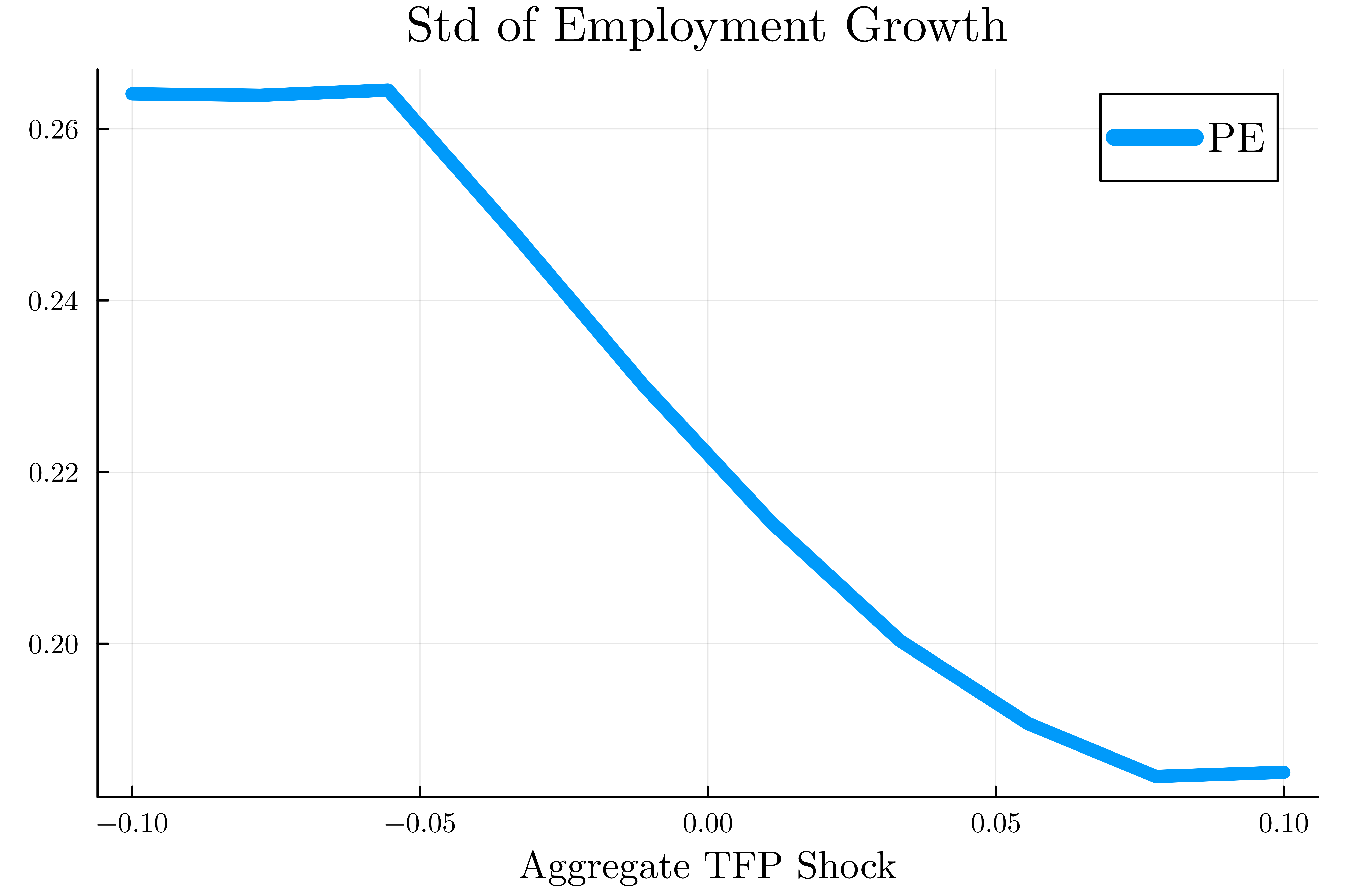
Negative Aggregate Shock



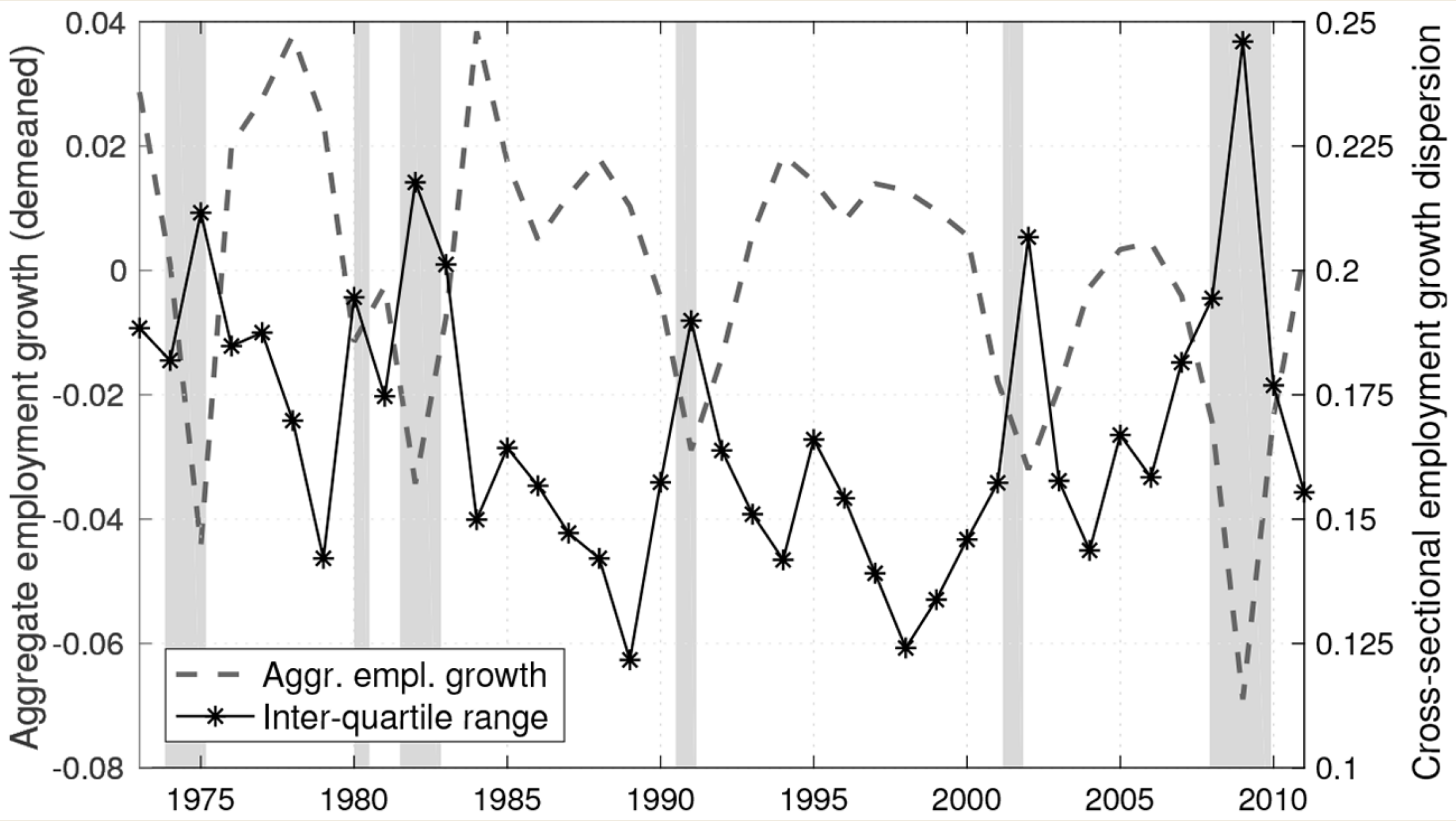
Negative Aggregate Shock



Partial Equilibrium

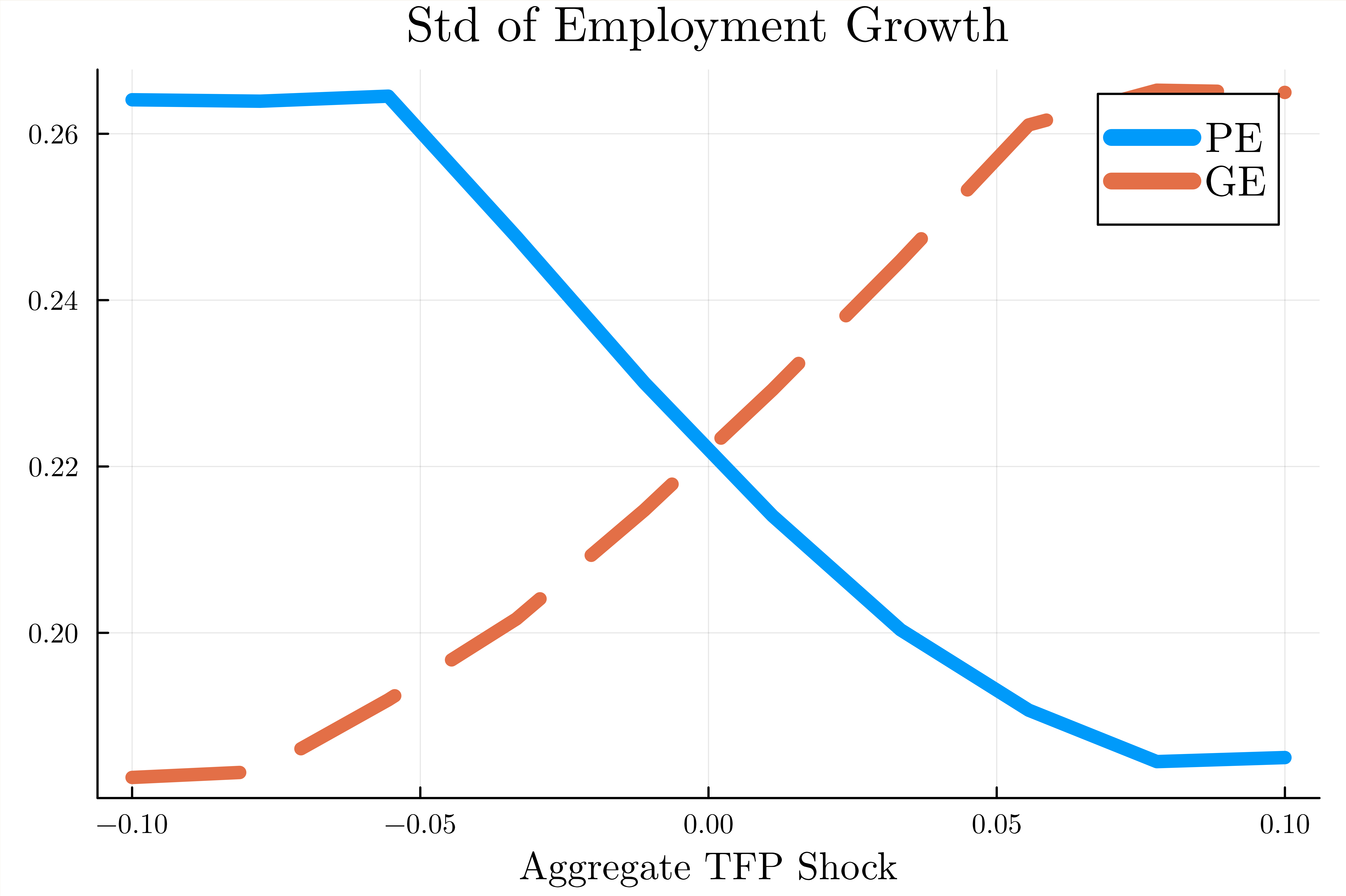


Countercyclical Volatility

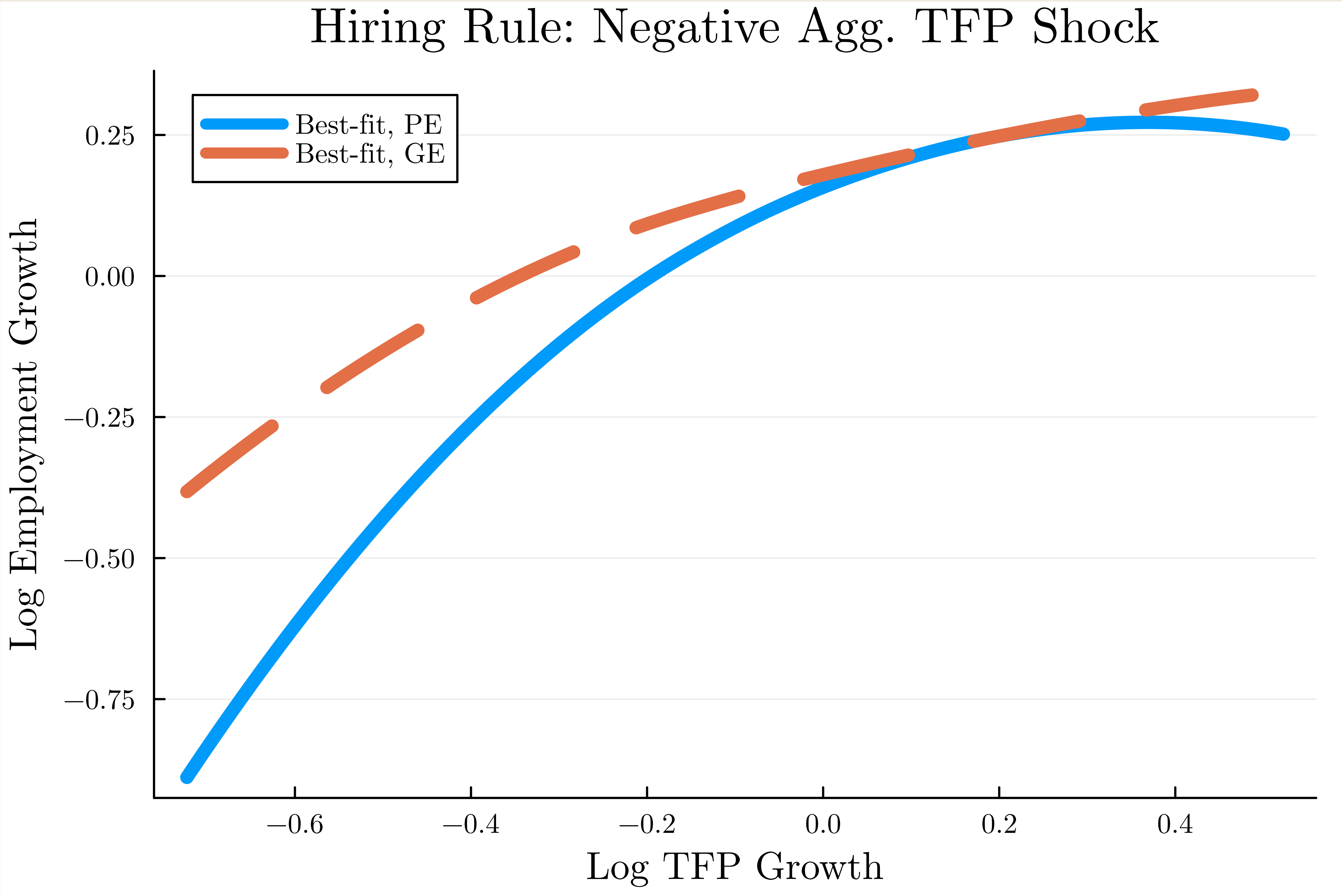


Source: Ilut, Kehrig & Schneider (2018)

Results Flip in GE...



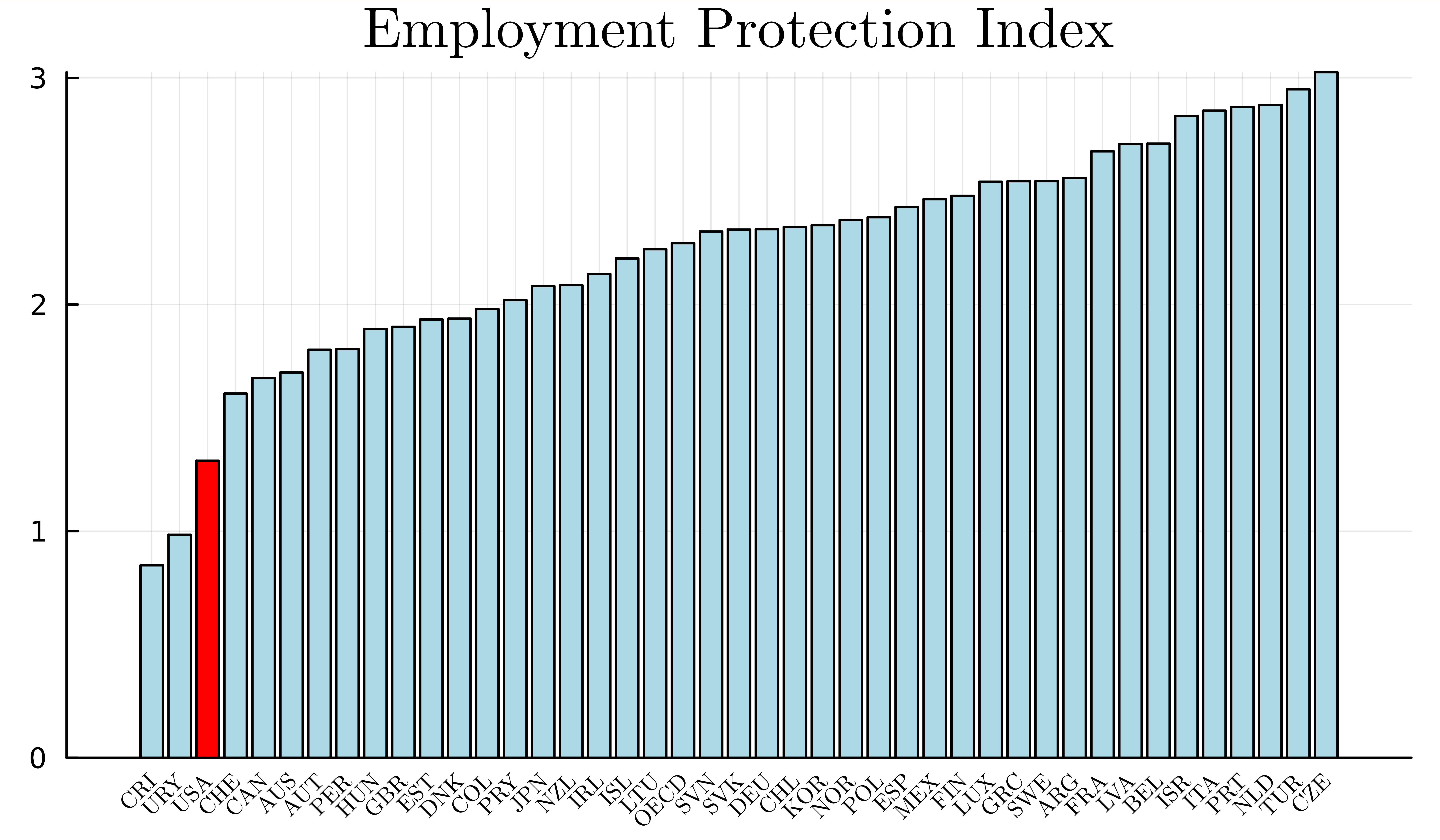
...because Hiring Rule Changes



- GE adjustment in wages:
  1. The flat part of the hiring rule is steeper in booms
  2. The steep part of the hiring rule is flatter in recessions

Firing Cost and Misallocation – Hopenhayn & Rogerson (1993)

Employment Protection Index



Data source: OECD

Question

- What is the cost of strict firing regulations?
- Suppose that in order to fire a worker, firms have to pay $\tau \times$ annual wage salary
  - US: $\tau = 0$
  - Europe: high τ
- Firing costs take the form of taxes
  - Distinct from adjsutment cost Φ, which is a part of the technology
- The collected tax revenue is rebated back to households as lump-sum transfers

Bellman Equation

- Bellman equation:

$$v(n_{-1}, z) = \max \left\{ v^*(n_{-1}, z),\ -\Phi(-(1-\delta), n_{-1}) - \tau w(1-\delta)n \right\}$$

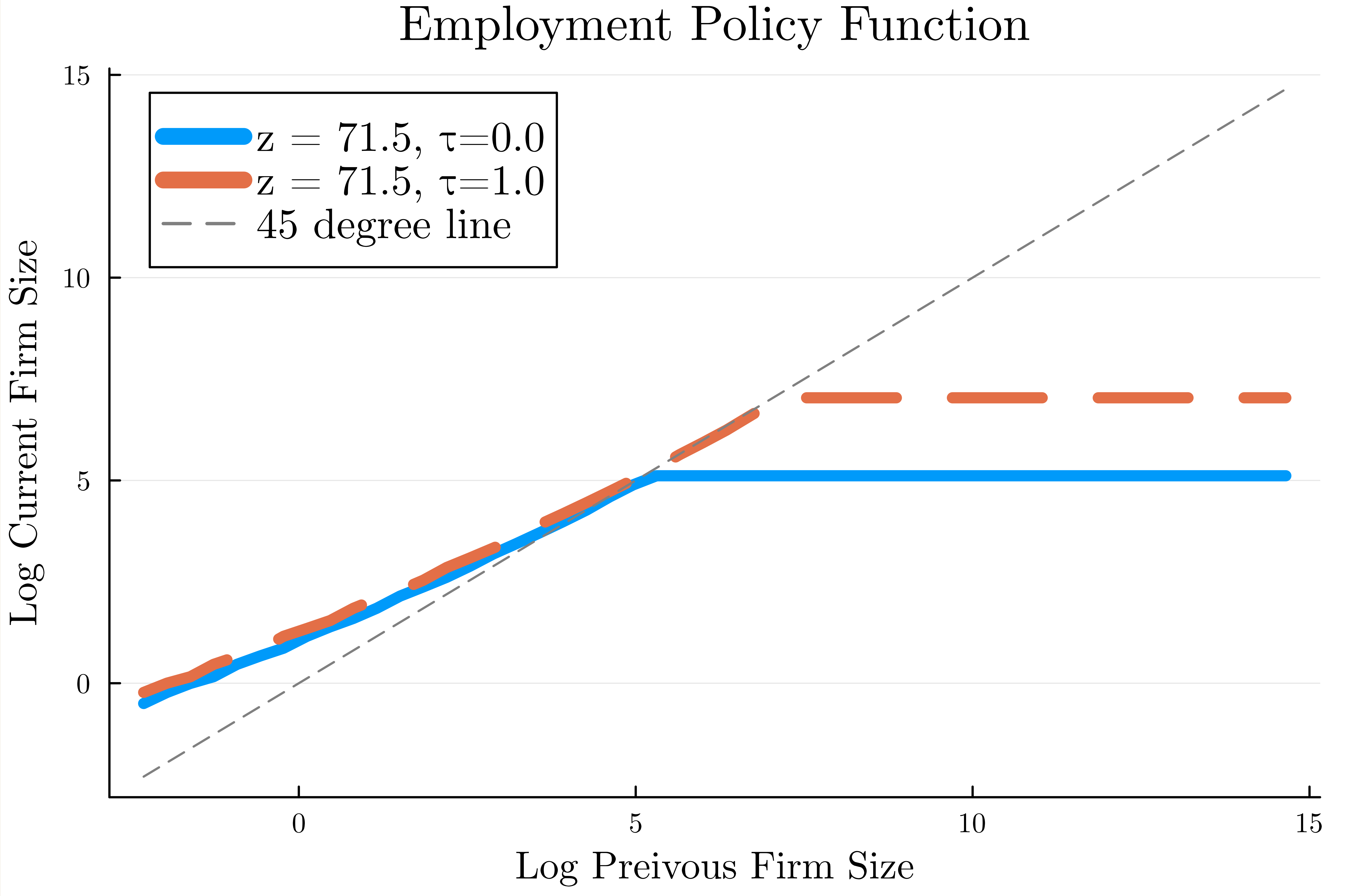
  where v^* is the continuation value

$$v^*(n_{-1}, z) = \max_{h,n}\ f(n, z) - wn - c_f - \Phi(h, n_{-1}) + \tau wnh\mathbb{1}[h < 0]\ + \beta\mathbb{E}v(n, z')$$

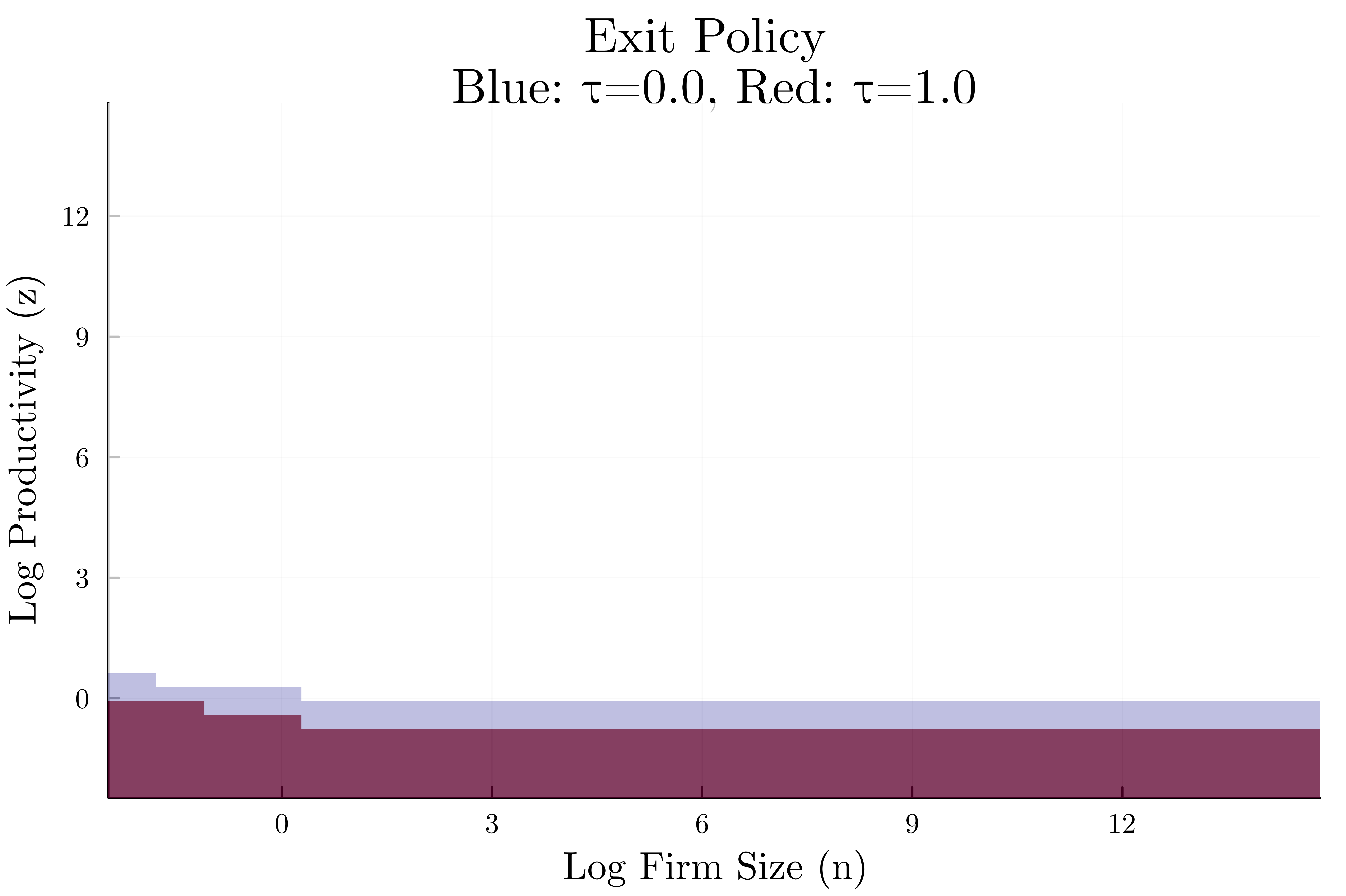
$$\text{s.t.} \quad n = n_{-1}(1 - \delta + h)$$

- The rest of the model is unchanged

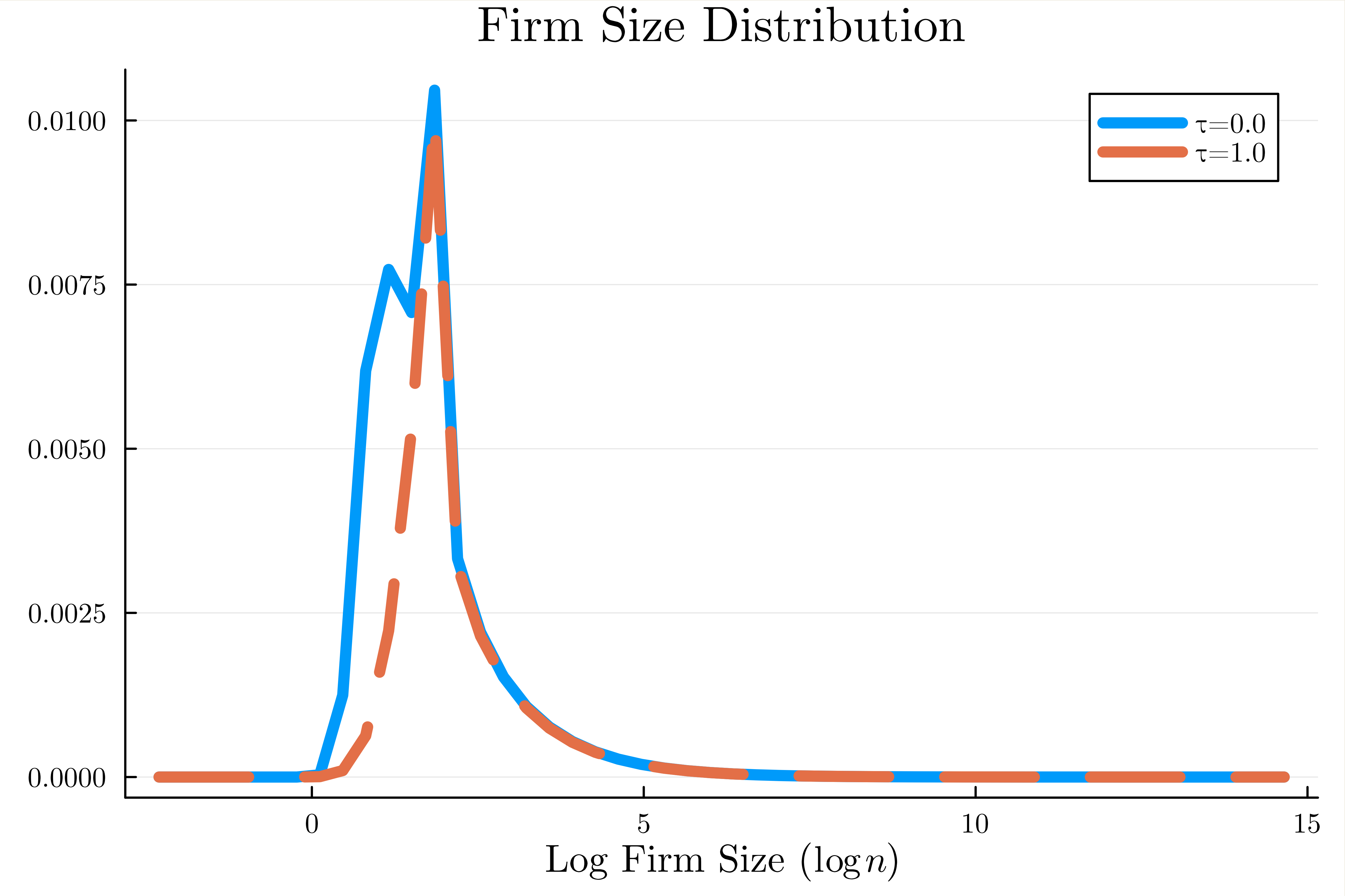
Firms Fire Less

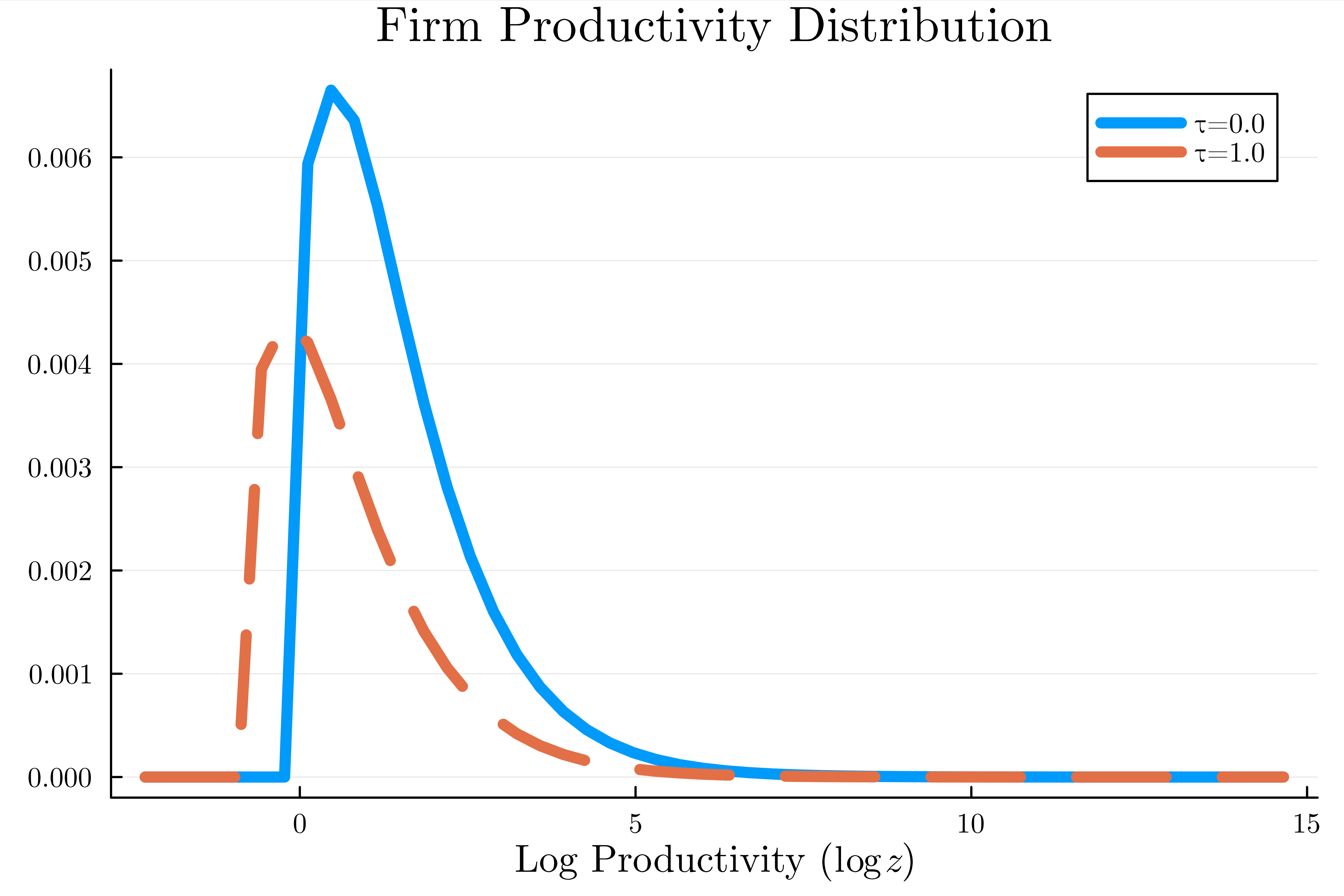


Firms Exit Less

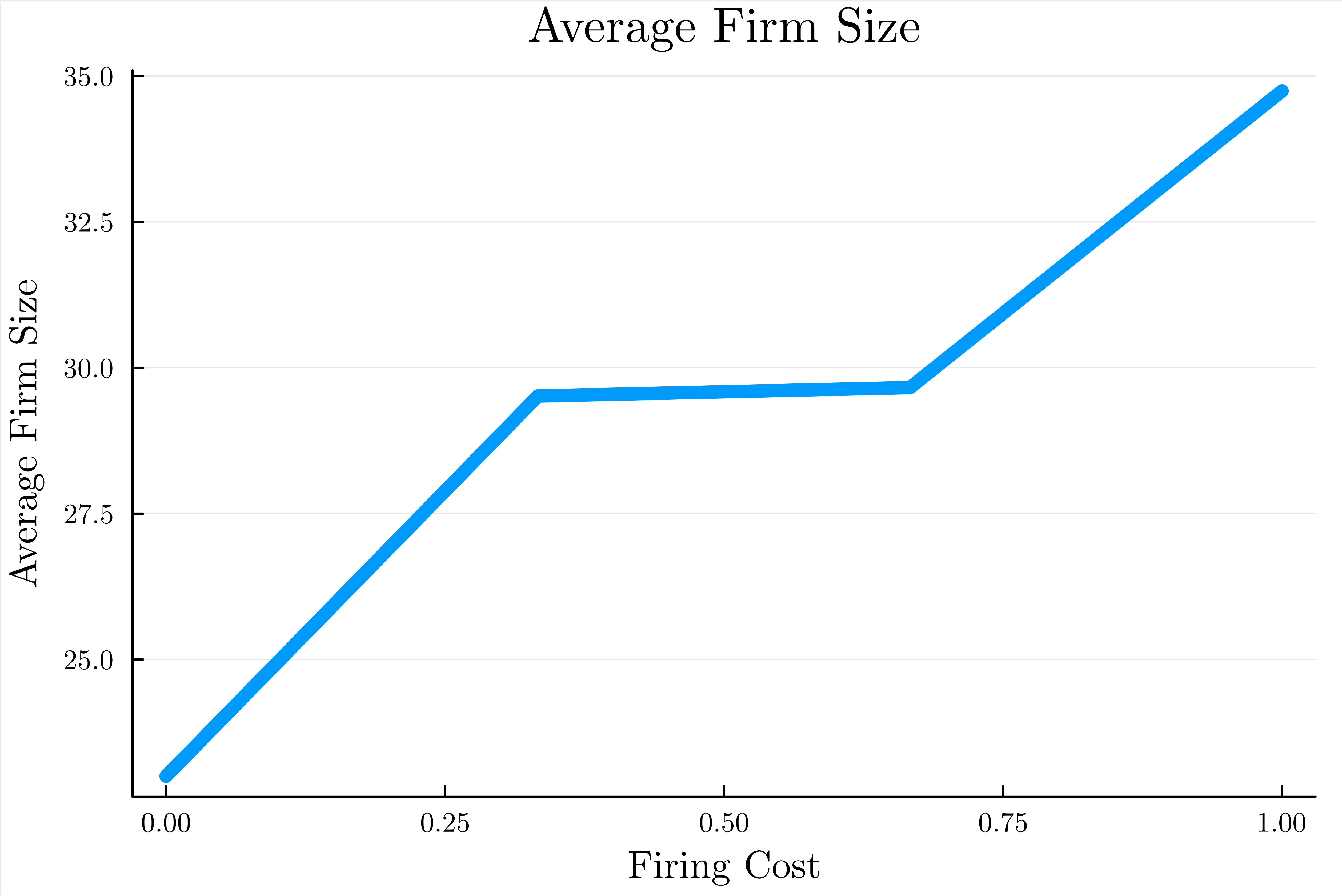


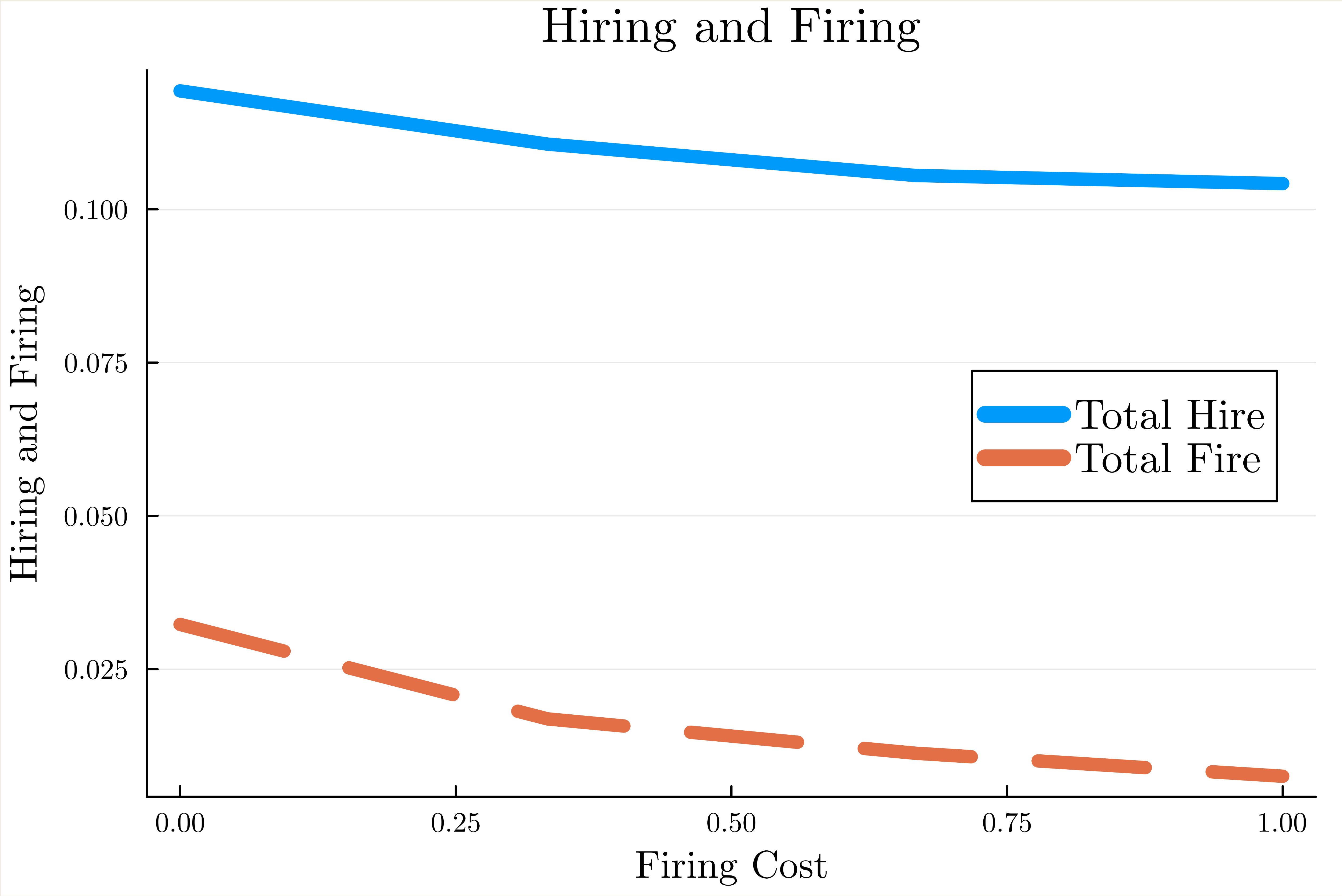
Firms Are Larger and Less Productive



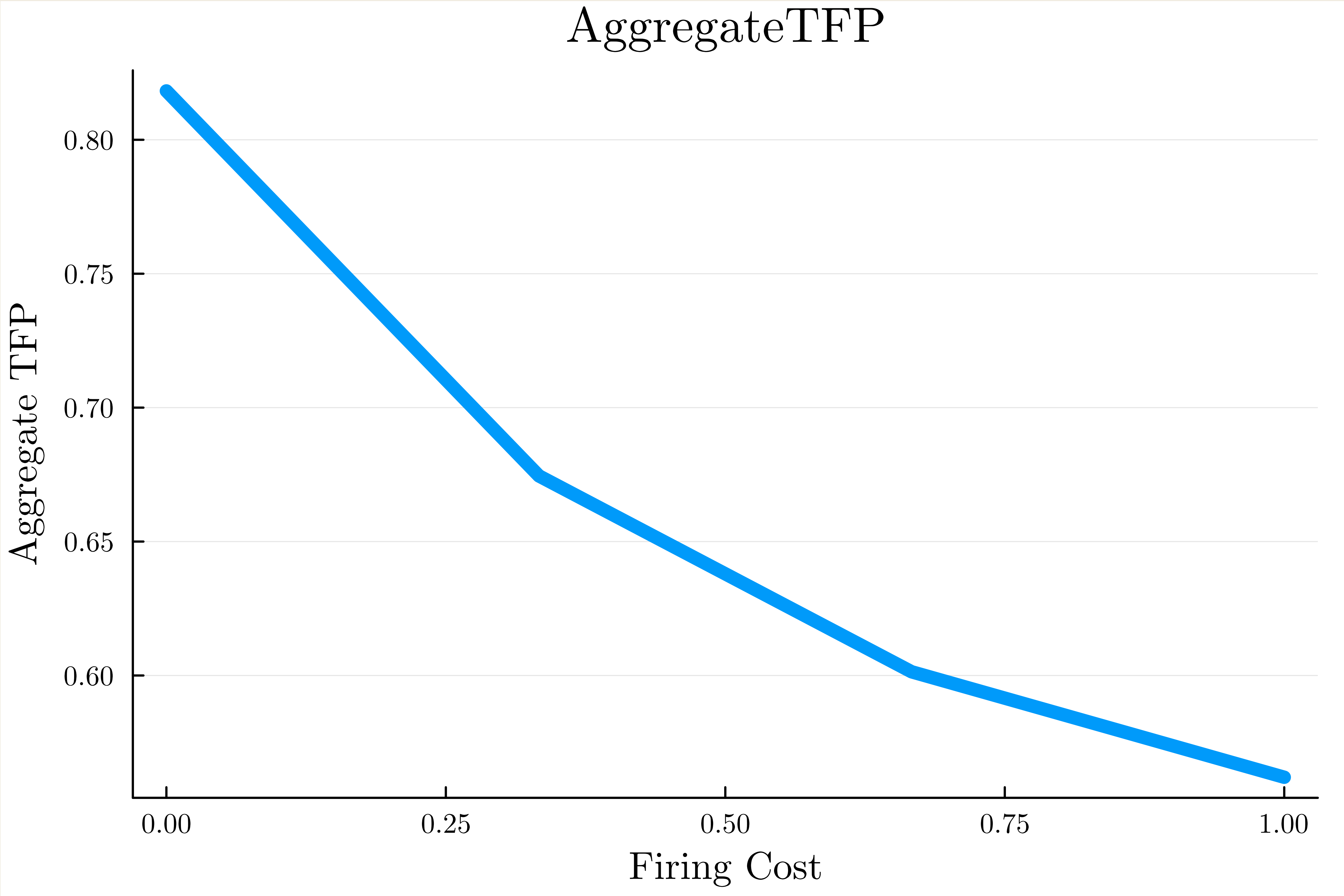


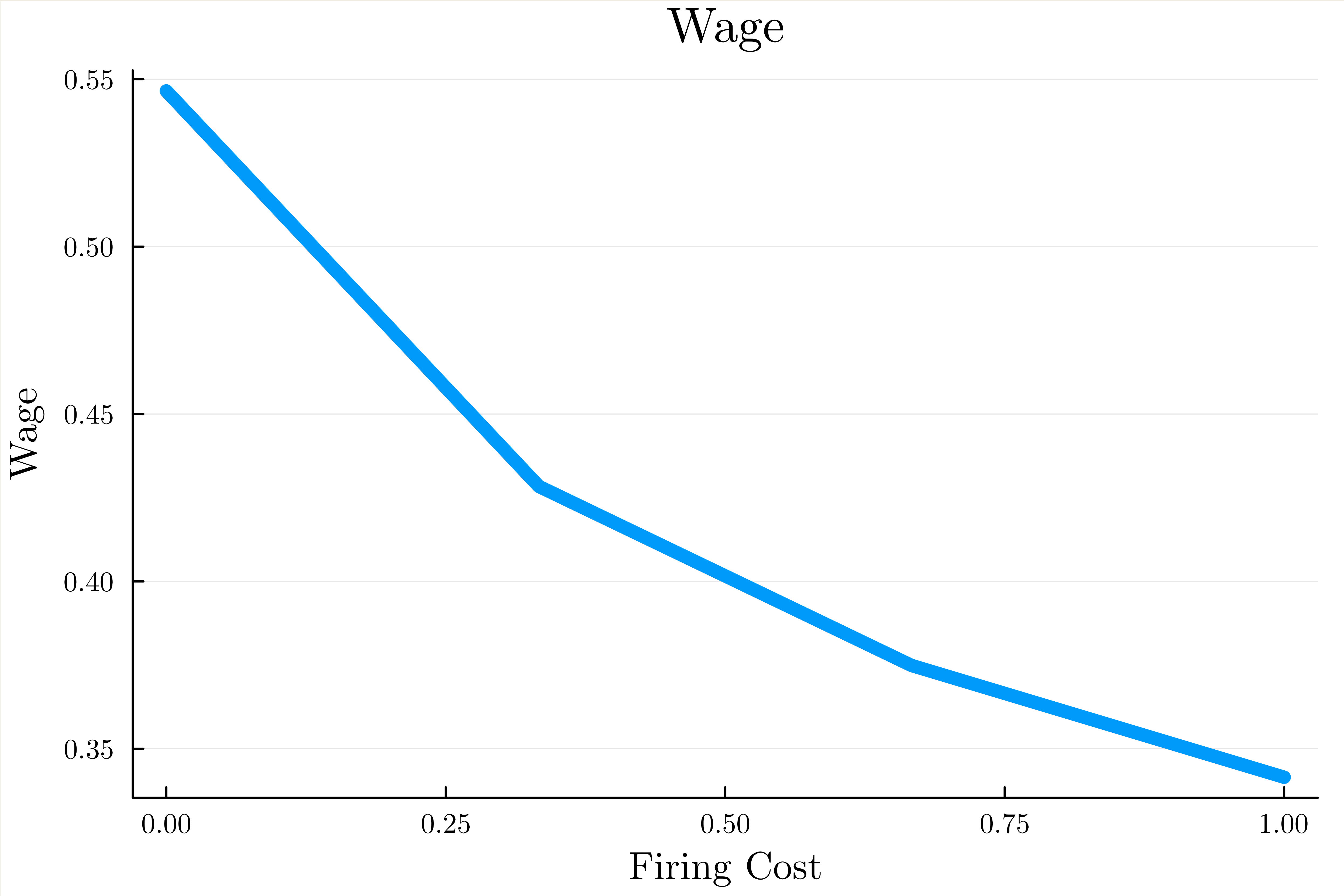
Firing Cost ↑ ⇒ Labor Reallocation↓



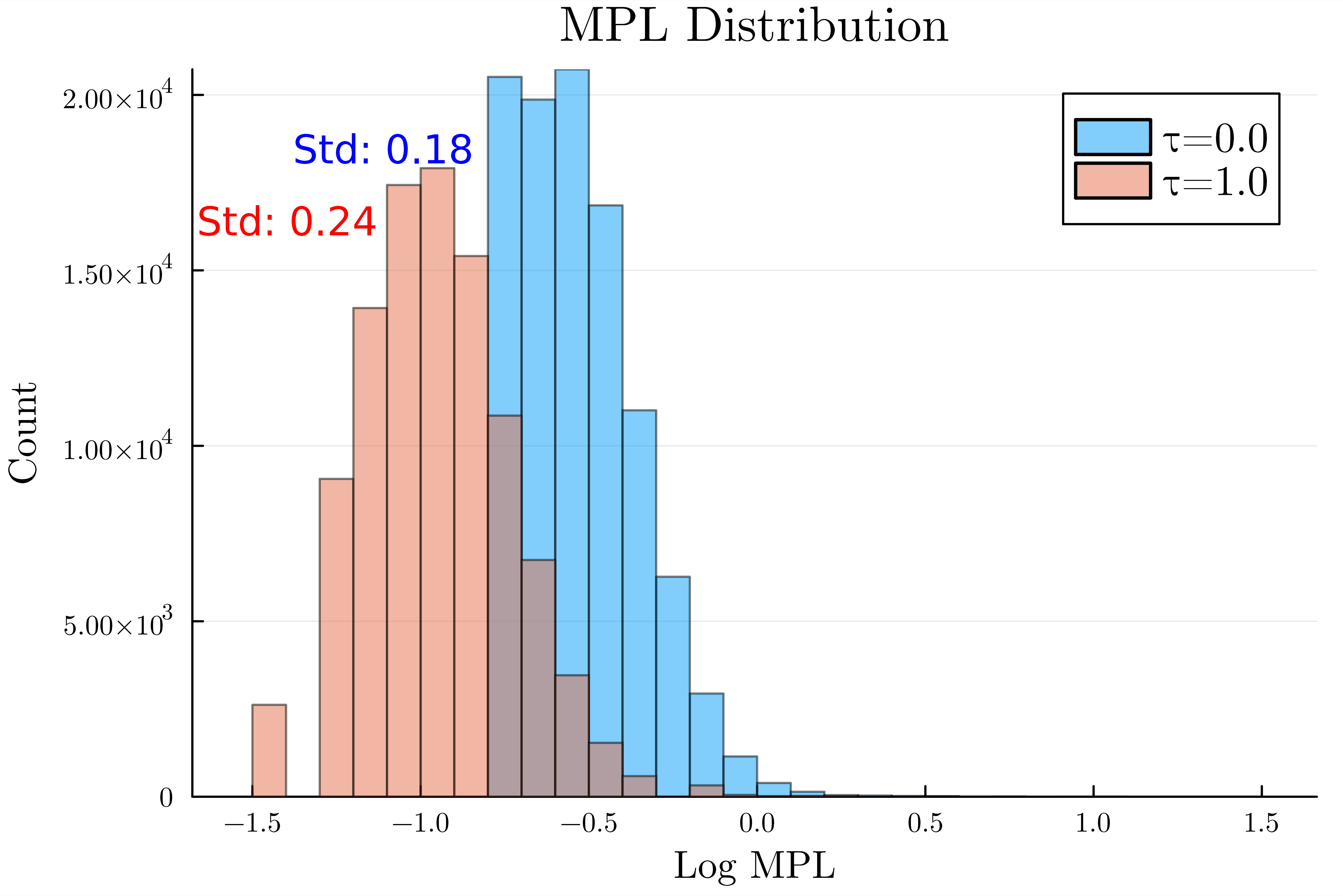


Firing Cost ↑ ⇒ TFP↓ & Wage↓





More Misallocation



Question

- Why, then, do so many countries regulate firing?
- An interesting idea is that firing cost could be a commitment device for firms (Karabay & McLaren, 2011; Créchet, 2024; Souchier, 2023)
- Normative aspects of labor market institutions seem underexplored

Broader Questions on Hopenhayn-Rogerson

- What is z?
  - Many argue z relates to the customer base
    (e.g., Einav, Klenow, Levin & Murciano-Goroff, 2022; Foster, Haltiwanger, Syverson, 2015; Argente, Fitzgerald, Moreira & Priolo, 2021)
- Is the entry really free?
  - Cagetti and De Nardi (2006): a model of entrepreneurship with financial friction
- Does the model get the age distribution right?
  - The average age of Walmart/Amazon size class in the model is 100 years
  - Walmart is 60 years old, and Amazon is 30 years old
- In the model, large firms are large just by luck (ex-ante homogenous). Are they?
  - Hurst & Pugsley (2011) and Pugsley, Sedláček & Sterk (2020) argue not

Non-Parametric Identification of Misallocation

– Carrillo, Donaldson, Pomeranz, & Singhal (2023)

What is the Cost of Misallocation?

- How large is the cost of misallocation in the data?
- Let us step back and consider a **static** model with a **fixed mass** of firms
- Each firm i produces using

$$y_i = f_i(n_i) \tag{7}$$

- The efficient allocation solves

$$Y^* \equiv \max_{\{n_i\}} \int f_i(n_i) di$$

$$\text{s.t.} \quad \int n_i di = L$$

- The solution features equalization of MPL:

$$f_i'(n_i) = w \quad \text{for all } i$$

Variance of MPL as a Sufficient Statistics

- Take arbitrary allocation $\{n_i\}$. Up to a second order around the efficient allocation

$$\log Y - \log Y^* \approx -\frac{1}{2}\mathrm{Var}_{\lambda/\epsilon}\left[\log(MPL_i)\right]$$

where $\mathrm{Var}_{\lambda/\epsilon}[X_i] = \sum_{i=1}^{N} \frac{\lambda_i^*}{\epsilon_i^*}\left(X_i - \mathbb{E}_{\lambda/\epsilon}[X_i]\right)^2$, $\mathbb{E}_{\lambda/\epsilon}[X_i] = \sum_{i=1}^{N} \frac{\lambda_i^*}{\epsilon_i^*} X_i$, $MPL_i = f_i'(n_i)$,
$\lambda_i = w_i^* n_i^* / Y^*$, and $\epsilon_i \equiv -\frac{d \log MPL_i}{d \log n_i}$.

- (Weighted) variance of MPL is the key moment for the cost of misallocation

- Testing the presence of misallocation $\Leftrightarrow$ testing $\mathrm{Var}(MPL_i) = 0$

- How do we get the distribution of MPL?

1. Assume $f_i(n_i) = Z_i n_i^{\alpha}$, and then $MPL_i = \alpha \frac{y_i}{n_i}$ (Hsieh & Klenow, 2009)
2. Nonparametrically identify the distribution of MPL (Carrillo et al. 2023)

Nonparametric Identification

- Taking the first-order approximation of equation (7),

$$\Delta y_i = \beta_i \Delta n_i + \varepsilon_i$$

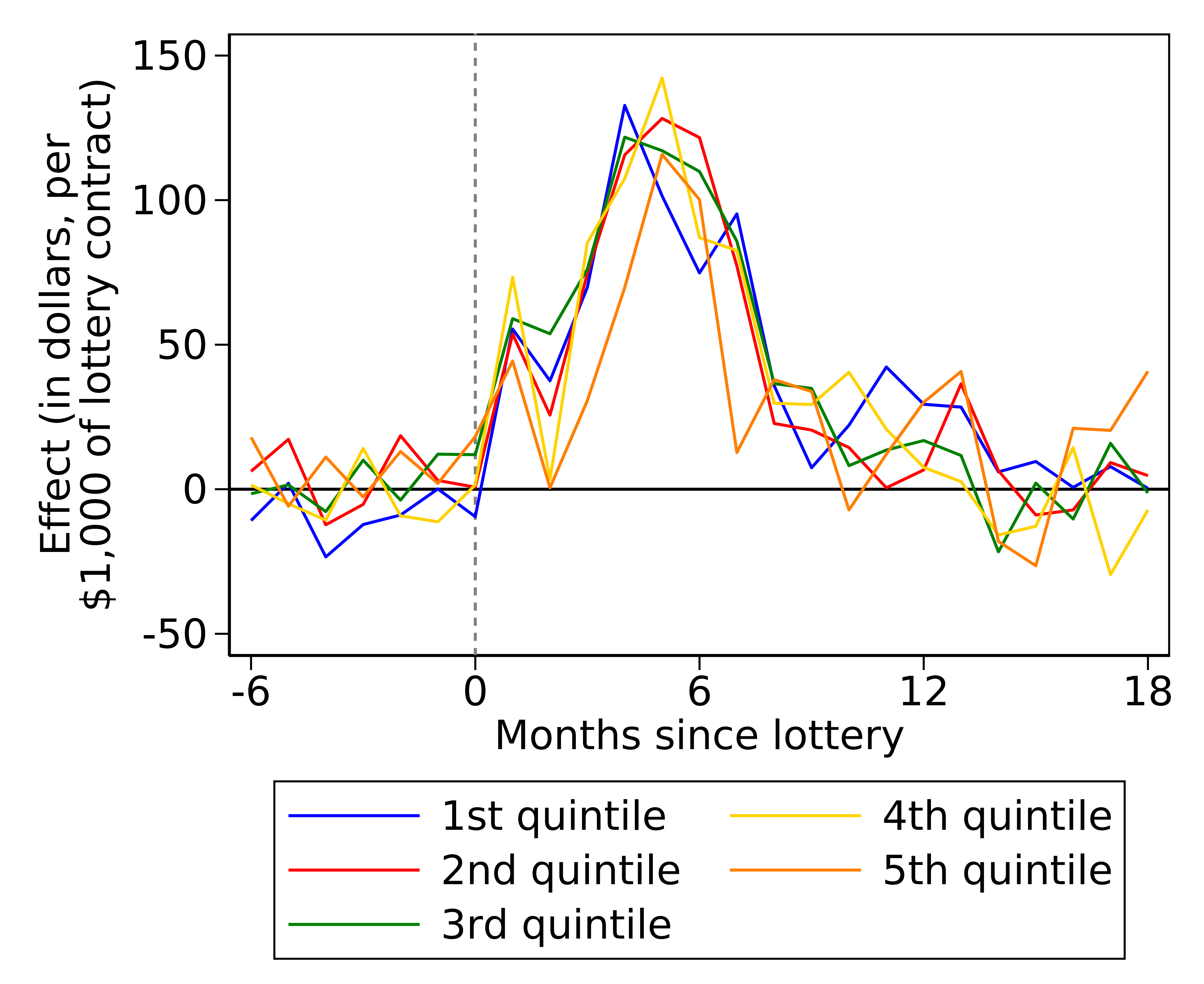
  - ε_i: technology shocks (i.e., changes in $f_i(\cdot)$)
  - $\beta_i = f_i'(n_i) = MPL_i$: treatment effect of exogenously increasing n_i on y_i
- With suitable instruments Z_i that exogenously shift n_i, $\mathbb{E}[\beta_i^k]$ ($k = 1,2,\ldots$) are identified (Masten & Torgovitsky, 2016)

Empirical Implementation

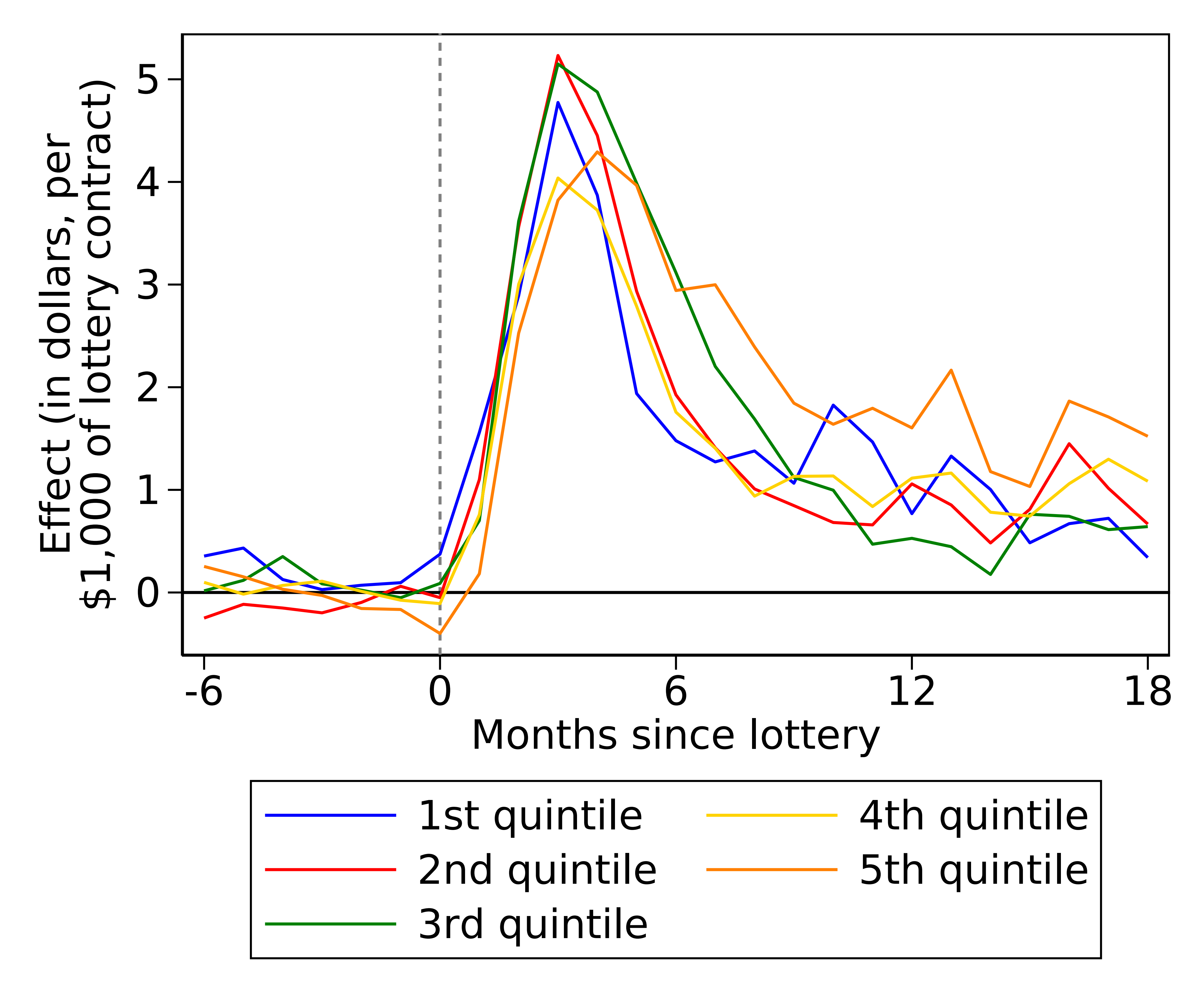
- Construction sector in Ecuador, 2009-2014
- Public construction projects were allocated through a randomized lottery
- Lottery serves as an ideal instrument
  - **exogeneity**: orthogonal to technology shocks ε_i or MPL_i
  - **relevance**: winning a lottery does shift n_i

Heterogenous Treatment Effects by Firm Size?

Sales



Labor Inputs



Small Cost of Misallocation

Table 4: Estimated Cost of Misallocation

	$\mathbb{E}_{\bar{\lambda}}[\bar{\mu}]$ (1)	$\mathbb{V}ar_{\bar{\lambda}}[\bar{\mu}]$ (2)	$\frac{\Delta W}{W}$ (3)
Panel (a): IVCRC estimates			
Baseline	1.126	0.014	0.016
	[1.093, 1.161]	[0, 0.341]	[0, 0.261]
Panel (b): Alternative procedure assuming common scale elasticities			
Constant returns-to-scale ($\gamma = 1$)	1.240	0.611	0.479
	[1.223, 1.257]	[0.544, 0.730]	[0.427, 0.572]

- Assume $\epsilon_i = 3$ for all i
- The welfare cost of misallocation is 1.6%
- Hsieh-Klenow type calculation implies 48% of welfare loss in the same dataset

Question

Does this approach capture the full story of misallocation in Hopenhayn-Rogerson?

– No, for two reasons:

1. Entry & exit
2. Dynamics

Two Open Questions

1. Entry & exit

- In HR, distortion ⇒ fewer firms enter
  - This reduces the mean of *MPL* (not just about variance!)
- In HR, exit threshold changes ⇒ the selection of the active firms change
  - This again shifts the mean of *MPL*

2. Dynamics

- Laissez-faire of Hopenhayn-Rogersion with labor adjustment costs is efficient
- But, *MPL* is not equalized in a static sense
- Firms hire workers until
  (present discounted value of hiring a worker) = (hiring cost today)
- Hiring a worker is an investment

How do we incorporate these two issues without imposing strong assumptions?

Hopenhayn-Rogerson with Search Frictions

– Based on McCrary (2022)

Search Friction

- Only unemployed workers search for a job
- Firms need to post vacancies to hire workers
- Assume firing is costless
  - Can easily extend to the case with costly firing
- Each vacancy meets an unemployed worker with probability λ^F
- Let U be the unemployed's value function

Value and Policy Functions

- Policy functions:
  $\chi(n, z) \in \{0,1\}$: exit, $w(n, z)$: wage, $s(n, z)$: firing, $v(n, z)$: vacancy

- The value function of the firm:

$$J(n, z) = (1 - \chi(n, z))J^*(n, z)$$

$$J^*(n, z) = f(n, z) - w(n, z)n - c_f - \Phi(v(n, z), n) + \beta \mathbb{E} J(n', z')$$

$$\text{s.t.} \quad n' = (1 - \delta - s(n, z))n + \lambda^F v(n, z)n$$

- The value function of the workers:

$$W(n, z) = (1 - \chi(n, z))\left[w(n, z) + \beta(1 - \delta - s(n, z))\mathbb{E}W(n', z') + \beta(\delta + s(n, z))U\right] + \chi(n, z)U$$

Wage Barganing

- In each period, a coalition of workers and a firm bargains to determine w, v, s, χ

- We assume Nash bargaining with worker bargaining power γ:

$$\max_{\chi(n,z),v(n,z),w(n,z),s(n,z)} (W(n,z)n - Un)^{\gamma} J(n,z)^{1-\gamma} \tag{1}$$

- Noting $\dfrac{\partial W(n,z)}{\partial w} = -\dfrac{\partial J(n,z)}{\partial w}$, FOC w.r.t. w gives

$$\big(W(n,z)n - Un\big) = \gamma S(n,z), \quad J(n,z) = (1-\gamma)S(n,z) \tag{2}$$

where $S(n,z) \equiv J(n,z) + (W(n,z) - U)n$ is the joint match surplus

- Substituting (2) back into (1), we have

$$\max_{\chi(n,z),v(n,z),s(n,z)} \gamma^{\gamma}(1-\gamma)^{1-\gamma} S(n,z)$$

- **Result**: vacancy, firing, and exit policies maximize joint match surplus

Bellman Equation for Surplus

- The joint match surplus solves

$$S(n, z) = \max\{S^*(n, z), 0\} \tag{3}$$

where

$$S^*(n, z) = \max_{v,s,n'} f(n, z) - c_f - \Phi(v, n) - (1 - \beta)Un + \beta\left(1 - \frac{\gamma\lambda^F v}{(1 - \delta - s) + \lambda^F v}\right)\mathbb{E}S(n', z')$$

$$\text{s.t.} \quad n' = (1 - \delta - s)n + \lambda^F vn$$

- Why is there extra discounting $1 - \frac{\gamma\lambda^F v}{(1 - \delta - s) + \lambda^F v}$?
    - New hires will "steal" a portion of next-period surplus $S(n', z')$

Mathing

- The total of number of mathces are dictated by a CRS matching function:

$$\mathcal{M}(u, v)$$

- The meeting prob. are

$$\lambda^U = \frac{\mathcal{M}(u, v)}{u} \equiv \mathcal{M}(1,\theta), \quad \lambda^F = \frac{\mathcal{M}(u, v)}{v} \equiv \mathcal{M}(1/\theta,1) \tag{4}$$

where $\theta \equiv v/u$ is the market tightness and

$$u = 1 - \int\int ng(n, z)dndz$$

$$v = \int\int v(n, z)g(n, z)dndz$$

Rest of the Model

- The value of unemployment is

$$U = b + \beta\lambda^U \int\int \frac{v(n,z)}{v} \left[\gamma S(n'(n,z), z') \frac{1}{n'(n,z)} + U \right] g(n,z) dndz \tag{5}$$

where b is the UI benfit, and $g(n,z)$ is the mass of firms with (n,z)

- The free-entry condition is

$$\int (1-\gamma) S(n_0, z) \psi_0(z) dz = c_e \tag{6}$$

- The steady-state distribution satisfies

$$g(n', z') = \int\int (1 - \chi(n,z)) \Pi(z' \mid z) \mathbb{I}[n'(n,z) = n'] g(n,z) dndz + m\mathbb{I}[n' = n_0] \psi_0(z') \tag{7}$$

Equilibrium Definition

A steady-state recursive equilibrium consists of

- value and policy functions: $\{S(n,z), n'(n,z), v(n,z), s(n,z), \chi(n,z)\}$
- the market tightness and meeting probabilities: $\{\theta, \lambda^U, \lambda^F\}$
- the value of unemployment: U
- the steady state distribution: $g(n,z)$

such that

1. value and policy functions solve the Bellman equation (3)
2. the market tightness and the meeting probabilities satisfy (4)-(5)
3. the value of unemployment satisfies (6)
4. the steady state distribution satisfies (7)

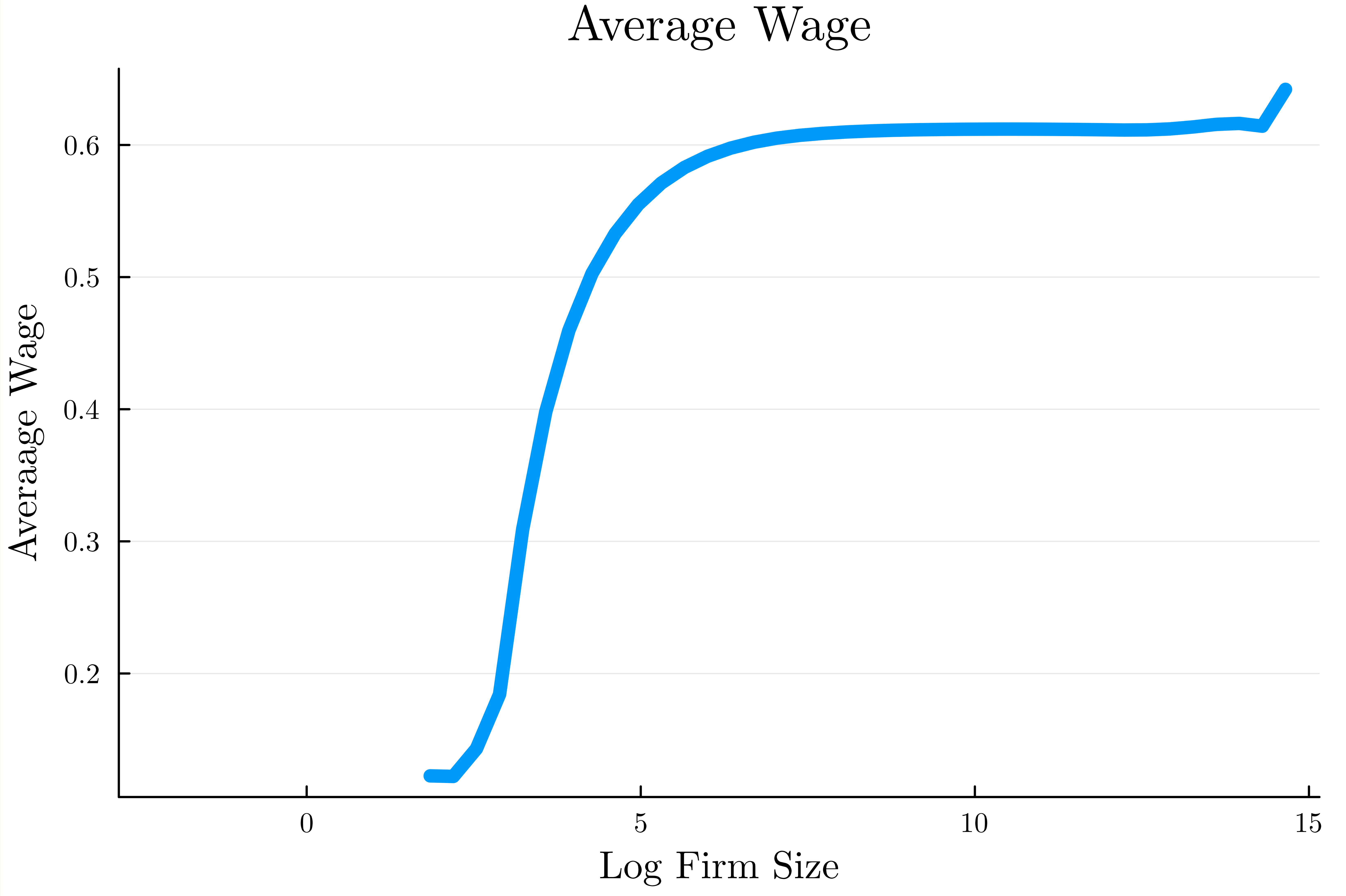
Implications for Firm Wage

- Search frictions allow us to talk about the firm wage
  - This wasn't possible in Hopenhayn-Rogerson
- The wage of firm (n, z) is given by

$$w(n,z) = \gamma \frac{1}{n}\left\{f(n,z) - c_f - \Phi(v,n)\right\} + \beta(1-\gamma)\frac{\gamma\lambda^F v}{n'}\mathbb{E}S(n',z') + (1-\gamma)(1-\beta)U$$

- Recall, in Burdett-Mortensen, wage and firm size were related monotonically
- Is it possible to break it?

Low Adjustment Cost



High Adjustment Cost