# Asset Quality Cycles[☆]

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## Abstract

Systemic risk builds up during booms in an economy featuring asymmetric information in asset markets, where investors' hidden effort choices endogenously determine asset quality distribution. Higher asset prices during booms induce more investors to sell their assets, which lowers their incentive to improve quality. This quality deterioration in turn makes the economy vulnerable to future exogenous shocks because market breakdowns become more likely. Private agents do not internalize that their effort choices worsen future adverse selection problems, and thus the planner may improve welfare by taxing trade and thereby lowering asset prices.

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*JEL classification*: D82, E32, E44

# 1. Introduction

Busts always follow booms, and booms are often blamed for being the seed of crises. While existing macroeconomic models focus on leverage as a source of fragility, an alternative but equally widespread view among policymakers emphasizes that deterioration in the quality of assets during boom periods creates subsequent instability.[1] Yet we often lack a formal understanding of the mechanism behind this view and why it warrants policy intervention. This paper presents a simple theory where endogenous deterioration of asset quality during booms inefficiently creates fragility.

The theory builds on the interaction of two frictions in asset markets: asymmetric information about asset quality and entrepreneurs' hidden effort choices that endogenously determine the quality distribution of assets. Entrepreneurs differ in their productivity and thus benefit by trading capital, but asymmetric information about the quality of capital hampers this reallocation. The quality of capital is determined endogenously when entrepreneurs invest. Since improving quality is costly, entrepreneurs who sell capital do not exert effort because they sell at the same price, regardless of the underlying quality.

In this environment, the quality of assets in the economy deteriorates in response to a positive shock that raises asset prices. Such a shock induces the marginal entrepreneurs to sell the assets that they otherwise would have kept, and these entrepreneurs stop exerting effort to improve quality. This quality deterioration naturally increases the fragility of the economy because market breakdowns are more likely in the subsequent periods through the standard Akerlof's (1970) lemons problem. The effort choices of entrepreneurs are socially inefficient as they do not internalize that the creation of lemons worsens future adverse selection problems. I demonstrate a case in which taxing trade and thereby lowering asset prices always improves ex-ante welfare by correcting this externality. This result holds despite the availability of ex-post interventions.

Although the quality of assets in the economy deteriorates, the quality of assets traded in the market improves in response to the positive shock. This is driven by the fact that marginal entrepreneurs stop exerting effort because they now expect to sell high-quality assets. The improvement in market quality in turn amplifies the response of output and asset prices by mitigating adverse selection problems. It is tempting to think that the deterioration of quality in the economy acts to dampen the booms, but this is not the case. An empirical implication of this result is that looking at the quality of assets that circulate in the market tells us little about the quality of assets in the overall economy.

A series of normative analyses sheds light on the nature of inefficiency. In particular, it is not the quality deterioration per se, but its interaction with future adverse selection problems that justifies policy intervention. This point is made through a comparison between a static environment in which entrepreneurs trade only once and the dynamic extension in which entrepreneurs sequentially trade their capital. The hidden effort is not distortionary in the former, but it is in the latter environment.

Although the model in this paper is abstract, it could be useful for understanding numerous boom-bust episodes in the real world. During the dot-com bubble in the United States in the late 1990s, investors looking for "hot" new stocks triggered a period of high demand for tech stocks. Entrepreneurs with weak fundamentals reacted to this by creating lemons because they were expecting to sell off the companies to investors eventually. The 2008 financial crisis offers another example. Securitization, which enabled debt to be sold in secondary markets, became increasingly popular in the run-up to the crisis. It is often argued that this created a decline in the quality of debt and was at least partially responsible for the recent financial crisis.[2] Despite its simplicity, I view my model as the one that formalizes these narratives and points to why they might have been socially inefficient.

## 1.1. Related Literature

This paper builds on the recent studies which show that adverse selection problem is potentially crucial to explaining the sudden collapse of the financial markets (Bigio, 2015, 2016; Guerrieri and Shimer, 2014;

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[1]For example, Kindleberger (2015) writes "Minsky emphasized the 'quality' of debt to gauge the fragility of the credit structure."

[2]Indeed, Keys, Mukherjee, Seru, and Vig (2010) provided causal evidence that securitization led to the lax screening of borrowers by banks.

Kurlat, 2013, 2016). The same motivation has led several papers to explore optimal intervention in the presence of adverse selection (Tirole, 2012; Philippon and Skreta, 2012). In their models, quality distribution is exogenously given.[3] Now asking where the low-quality assets come from is the natural next step. My model takes into account why lemons exist in the first place and shows how the creation of lemons interacts with the business cycles.

Several papers also tackle a question similar to mine. Matsuyama (2013) and Martin (2008) provide models of counter-cyclical credit quality. In their theories, the driving force is the pro-cyclicality of borrowers' net worth. Gorton and Ordoñez (2014) share the same motivation as this paper to explain why busts follow booms. They stress opacity of information as a driver of credit booms and as a source of fragility. In contrast to these papers, pro-cyclical asset prices are the driving force for generating counter-cyclical asset quality in my model.

Eisfeldt (2004), Eisfeldt and Rampini (2006), Cui (2017), and Lanteri (2017) document and explain why capital reallocation is so volatile and pro-cyclical. My analysis takes those aspects as given and studies the implications for endogenous determination of asset quality.

In independent works, Neuhann (2017) and Caramp (2016) pursue an idea similar to the one in this paper.[4] Both papers share a key insight as with this paper that higher asset prices lead to less effort by investors. Among others, the most important difference between this paper is that they both feature financial friction, in addition to adverse selection and moral hazard. I focus on a minimal set of ingredients by abstracting away financial friction. In so doing, my analysis clarifies that financial friction is not necessary to generate endogenous fluctuations in asset quality or to justify policy interventions.

**Layout.** Section 2 presents a simple two-period model to highlight the positive implications of the mechanism. Section 3 extends the model to allow sequential trading and shows that it has novel normative implications. Section 4 concludes. Proofs are collected in the online appendix.

## 2. Mechanisms at Play

This section presents a simplest two-period model, which highlights the forces that endogenously determine the quality distribution of assets in the economy.

### 2.1. Environment

The model is a two-period model ($t = 0, 1$). The economy is populated by a mass of ex-ante identical entrepreneurs. There are two goods: capital goods and perishable consumption goods. There are two types of capital: high- and low-quality capital. They differ in efficiency units when used for production. Capital goods are irreversible: once consumption goods are transformed into capital, they cannot be consumed. Capital is sometimes referred to as assets.

Entrepreneurs live for two periods and are initially endowed with one unit of consumption goods at $t = 0$. Their preferences are given by $\mathbb{E}_0[c_1]$. In words, entrepreneurs only consume in period 1 and are risk neutral. Entrepreneurs can save consumption goods via investment technology at $t = 0$, which converts a unit of consumption good into a unit of capital. As consumption goods are perishable and entrepreneurs do not value consumption at $t = 0$, they optimally choose to transform all endowment into capital.

At the beginning of $t = 1$, entrepreneurs independently draw productivity $z$ from the CDF, $G(z)$, and its PDF is denoted by $g(z)$. The production technology is constant returns to scale in capital and depends on the quality of capital. Specifically, entrepreneurs with productivity $z$ produce an amount $\phi z$ output per unit high-quality capital, where $\phi$ is the aggregate productivity component. Low-quality capital produces nothing, and thus, is assumed be completely useless for simplicity.

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[3]Tirole (2012) provides a brief analysis of an economy where quality distribution is endogenously chosen in adverse selection economy.

[4]Relatedly, Kawai (2014) studies a micro model and shows that moral hazard by sellers can destroy gains from trade. Zryumov (2015) studies a model in which the timing of entry shapes the time-varying market quality.

After observing productivity, but before operating any production technology, entrepreneurs have two decisions to make. First, entrepreneurs can exert effort to improve the fraction of high-quality capital they obtain from ongoing investments. In particular, fraction $\pi \in \{\pi_h, \pi_l\}$ of an investment is realized as high-quality capital, where $0 < \pi_l < \pi_h \leq 1$. This fraction is $\pi_h$ if entrepreneurs exert effort, and it is $\pi_l$ if they do not. Effort incurs fixed cost $\kappa > 0$ per unit of investment. $\kappa$ can also be interpreted as a maintenance cost. I assume $\kappa$ is arbitrarily small—i.e., $\kappa \to 0$. Although it is not essential, this assumption is useful to simplify the expositions. The choice of investment quality is the first key ingredient in the model.

Second, after investment materializes, entrepreneurs can trade capital in a market with asymmetric information where buyers cannot observe the quality of capital. The market is competitive and anonymous. Here, competitiveness implies that entrepreneurs take the price of capital as given. Anonymity ensures that capital is traded at pooling price $p$. An asset market featuring asymmetric information about asset quality is the second key ingredient in the model.

In equilibrium, entrepreneurs with higher productivity become buyers. Buyers observe the quality of the capital after purchase. To install high-quality capital, entrepreneurs need to incur quadratic adjustment costs as follows:
$$\Gamma(\lambda^M d) = \frac{\gamma}{2}(\lambda^M d)^2, \tag{1}$$

where $\gamma > 0$ is a parameter, $d$ is the amount of capital purchased, and $\lambda^M$ is the fraction of high-quality capital as a portion of the purchased capital, which will be endogenously determined. The assumption that the reallocation of capital incurs quadratic adjustment costs closely follows Eisfeldt and Rampini (2006) and ensures that the demand for capital is bounded. After trading capital, entrepreneurs produce, consume output, and die.

**Asset Supply.** The entrepreneurs' decision problems are characterized in a backward manner. Before describing quality-choice decisions, I describe the optimal selling decisions given the type of capital these entrepreneurs own. First, all entrepreneurs find it optimal to sell low-quality capital if price $p$ is strictly positive as it is useless. Second, only entrepreneurs with $z \leq p/\phi$ choose to sell high-quality capital because selling it at price $p$ is more profitable than using it to produce $\phi z$.

Given these decisions, entrepreneurs optimally choose whether to exert effort in their ongoing investment. If an entrepreneur is going to sell both low- and high-quality capital, then she does not have any incentive to incur strictly positive effort cost $\kappa$ because whatever the underlying quality, she sells at the same price, $p$. On the contrary, if an entrepreneur's productivity is high enough to keep high-quality capital, it is optimal to pay the effort cost. Thus entrepreneurs exert effort if and only if they plan to keep high-quality capital, which is the case when $z > p/\phi$. The fraction of high-quality capital in the economy is given by

$$\lambda \equiv \pi_l G(p/\phi) + \pi_h(1 - G(p/\phi)), \tag{2}$$

because those who exert effort yield $\pi_h$ units of high-quality capital, and those who do not yield $\pi_l$ units of high-quality capital. The fraction of high-quality capital in the market is

$$\lambda^M(p/\phi) \equiv \frac{\pi_l G(p/\phi)}{G(p/\phi) + (1 - \pi_h)(1 - G(p/\phi))}. \tag{3}$$

The denominator is the total supply of assets, and the numerator is the supply of high-quality assets because only entrepreneurs with sufficiently low productivity sell high-quality assets, who have only $\pi_l$ units.

For now, let us focus on the supply side of entrepreneurs' decision problems, taking the asset price $p$ as exogenous. The following comparative statics follows straightforwardly from the above expression, and shapes the backbone of the model.

**Proposition 1.** *An increase in the asset price, $p$, (i) decreases the fraction of high-quality capital in the economy, $\lambda$, and (ii) increases the fraction of high-quality capital in the market, $\lambda^M$.*

Note that these results require little structure of the model. Any shocks that lead to higher asset prices will induce marginal entrepreneurs to sell assets they otherwise would have kept, and thus, they stop exerting

effort to maintain quality. Given that asset prices are highly pro-cyclical in the data, the above proposition predicts counter-cyclicality in average asset quality in the economy. However, this does not translate into a decline in market quality. Instead, perhaps surprisingly, market quality is pro-cyclical, although the economy-wide quality is counter-cyclical. This is because entrepreneurs who do not exert effort are the only suppliers of high-quality capital. In response to an increase in asset prices, $p$, fewer entrepreneurs exert effort, resulting in more sales of high-quality assets.

**Asset Demand.** Given the technology described earlier, each entrepreneur chooses how much capital to purchase, which solves

$$\max_{d \geq 0} \left\{ \phi z \lambda^M d - pd - \Gamma(\lambda^M d) \right\}.$$

Together with the specification in equation (1), the demand for capital by an entrepreneur with productivity $z$ is given by

$$d(z; p, \lambda^M) = \max \left\{ \frac{1}{(\lambda^M)^2 \gamma} (\phi z \lambda^M - p), 0 \right\}. \tag{4}$$

Therefore an entrepreneur with productivity $z > p/(\phi \lambda^M)$ will demand a strictly positive amount of capital.

### 2.2. Equilibrium

As Proposition 1 illustrates, the key is the relationship between asset prices and the quality of assets in the economy. Now I endogenize the asset price as an equilibrium object.

It is convenient to normalize prices by aggregate productivity, $\hat{z} \equiv p/\phi$. The aggregate supply function of capital is given by

$$S(\hat{z}) = G(\hat{z}) + (1 - \pi_h)(1 - G(\hat{z})).$$

The first term represents the fact that entrepreneurs with productivity lower than $\hat{z}$ sell all capital they hold. The second term represents the fact that entrepreneurs with high enough productivity will sell only low-quality capital which is $1 - \pi_h$ of their capital holdings because they exert effort.

The aggregate demand for capital can be derived simply by integrating (4):

$$D(\hat{z}) = \int_{\hat{z}/\lambda^M(\hat{z})} \frac{\phi}{(\lambda^M(\hat{z}))^2 \gamma} (z \lambda^M(\hat{z}) - \hat{z}) dG(z), \tag{5}$$

where the market quality, $\lambda^M$, is given by (3). Finally, the proportion of high-quality capital in the economy, $\lambda$, is given by (2).

The definition of equilibrium is as follows:

**Definition 1.** *An equilibrium consists of the price of capital, $p$, the proportion of high-quality capital in the market, $\lambda^M$, the proportion of high-quality capital in the economy, $\lambda$, such that (i) given $p$ and $\lambda^M$, the entrepreneurs choose an effort level and trading volume to maximize consumption, (ii) the market for capital clears: $S(\hat{z}) \geq D(\hat{z})$ with equality whenever $\hat{z} > 0$, where $\hat{z} \equiv p/\phi$, (iii) $\lambda$ and $\lambda^M$ are given by (2) and (3), respectively.*

Figure 1 illustrates the equilibrium characterization by plotting the supply curve, $S(\hat{z})$, and the demand curve, $D(\hat{z})$. The intersections between the two curves are equilibria in this economy. As is common in the literature on adverse selection, there exist multiple equilibria. This comes from the non-monotonicity of the demand curve represented in equation (5). A higher $\hat{z}$ while holding $\lambda^M$ constant decreases the demand for capital, but this is counteracted by an increase in the average quality of the capital traded in the market, $\lambda^M$. For example, there always exists an equilibrium in which the market breaks down—i.e., $p = 0$. Since multiple equilibria are not my interest here, I follow the literature and focus on the highest
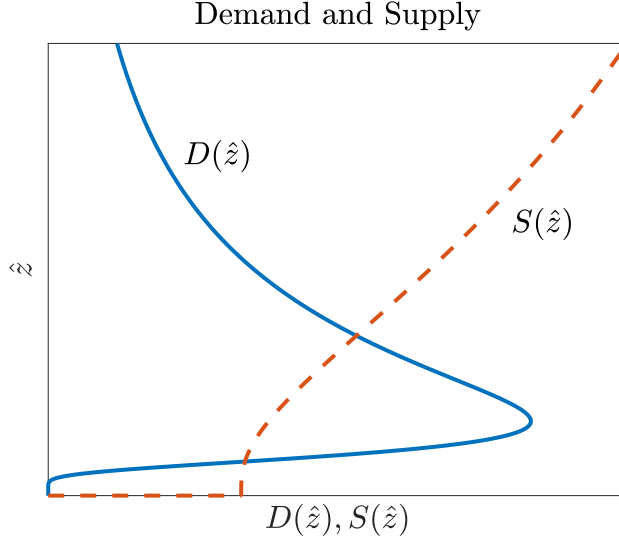
Figure 1: Demand and Supply for Capital. *Notes*: Illustration of demand and supply curves for capital. The intersections of two curves are equilibria as in Definition 1.

price-quantity equilibrium.[5] Wilson (1980) and Stiglitz and Weiss (1981) argue that this might not be a reasonable equilibrium concept when buyers have incentive to raise prices further to attract a better asset pool. Appendix B shows that the sufficient condition for this issue to not arise is

$$\frac{d \log \lambda^M(\hat{z})}{d \log \hat{z}} \leq 1, \tag{6}$$

for all $\hat{z} > \hat{z}^*$, where $\hat{z}^*$ is the price in competitive equilibrium. Intuitively, the quality pool of assets should not react too strongly to higher prices.

In the special case of $\pi_h = 1$, the market quality does not vary with price, $\lambda^M(\hat{z}) = \pi_l$, as long as $\hat{z} > 0$. This implies that the demand function, $D(\hat{z})$, is monotonically decreasing for all $\hat{z} > 0$. Section 3 leverages this observation to prove the existence in an extended setup.

### 2.3. Equivalence to an Economy with Trade Costs and Trade Taxes

I show that the previous economy is isomorphic to a symmetric information economy with iceberg trade costs and trade taxes. As Kurlat (2013) shows, adverse selection is equivalent to having trade taxes. As it will turn out, moral hazard is equivalent to introducing iceberg trade costs in addition to trade taxes.

Consider the previous economy but with symmetric information, where the quality of capital is observable by buyers. In this circumstance, all entrepreneurs exert effort to obtain an amount $\pi_h$ of high-quality capital and an amount $1 - \pi_h$ of low-quality capital. Because low-quality capital is useless, they can be omitted from the subsequent analysis. Assume now that the government imposes an ad valorem tax of $\tau^{sym}$ on the purchase of capital. The total revenue collected from the tax, $T = \tau^{sym} p^{sym} S^{sym}$, is rebated equally to all entrepreneurs, where $p^{sym}$ denotes price and $S^{syn}$ is the total supply of assets in this symmetric information economy, which will be described. The buyers' demand for high-quality capital solves

$$\max_{k^d \geq 0} \left\{ \phi z k^d - p^{sym}(1 + \tau^{sym}) k^d - \Gamma(k^d) \right\}.$$

Solving the above problem and aggregating over $z$ yields total demand for the capital as follows:

$$D^{sym}(\hat{z}^{sym}) = \int_{\hat{z}^{sym}(1+\tau^{sym})} \frac{\phi}{\gamma}(z - (1 + \tau^{sym})\hat{z}^{sym}) dG(z),$$

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[5]See Kurlat (2013) or Bigio (2016), for example. All subsequent results here are true at any stable equilibrium.

with $\hat{z}^{sym} \equiv p^{sym}/\phi$.

On the supply side, in order to deliver a unit of capital to other entrepreneurs, sellers must ship $\chi \geq 1$ units of capital. When using their own capital, they do not incur these iceberg trade costs. Entrepreneurs with $\phi z \geq p^{sym}\frac{1}{\chi}$ optimally keep their capital, and others with $\phi z < p^{sym}\frac{1}{\chi}$ decide to sell. The total supply of assets is

$$S^{sym}(\hat{z}^{sym}) = \pi_h \frac{1}{\chi} G(\hat{z}^{sym}/\chi)$$

because there are $\pi_h$ amount of assets, and each entrepreneur sells $1/\chi$ amount of assets.

Equilibrium is given by the intersection of supply and demand curves, $S^{sym}(\hat{z}^{sym})$ and $D^{sym}(\hat{z}^{sym})$. The following proposition shows that this economy, with a particular choice of $(\tau, \chi)$, delivers an allocation equivalent to that in the asymmetric information economy.

**Proposition 2.** *Suppose $\tau^{sym} = (1 - \lambda^{M*}\chi)/(\lambda^{M*}\chi)$ and $\chi = \pi_h/\pi_l$, where $\lambda^{M*}$ is the equilibrium value of the asymmetric information economy. Then the allocations (consumption and production for each entrepreneur) of the symmetric information economy with trade costs and trade taxes and the asymmetric information economy are equivalent. Prices are related through $\hat{z}^{sym}/\chi = \hat{z}^*$, where $\hat{z}^*$ is the equilibrium price in the asymmetric information economy.*

Because $\lambda^{M*} \leq \pi_l$, it follows that $\tau^{sym} \geq 0$, which implies that the economy faces taxes. Without moral hazard, $\pi_h = \pi_l$, there is no trade cost, $\chi = 1$, which is reminiscent of the equivalence result in Kurlat (2013). With moral hazard, $\pi_h > \pi_l$, the fact that entrepreneurs stop maintaining quality when they decide to sell is like having iceberg trade costs because real resources are lost when assets are sold. The direct implication of this result is that moral hazard changes the technology as opposed to creating distortion. I come back to this issue when discussing the model's normative implications.

## 2.4. Aggregate Shocks

Let me turn to the comparative statics analysis with respect to aggregate productivity, $\phi$. This is meant to capture business cycle variation in asset prices. A few remarks are in order. The analysis focuses on a change in $\phi$ while holding $\gamma$ fixed. As is clear from the expression in equation (5), if $\phi$ and $\gamma$ increase proportionally, there will be no effect on equilibrium. Eisfeldt and Rampini (2006) argue that in order to match the observed data on reallocation, the aggregate productivity (here $\phi$) and the cost of reallocation (here $\gamma$) must negatively co-move along the business cycle. Therefore, changing $\phi$ while holding $\gamma$ fixed is a reasonable way to capture business cycle variation in reallocation. Moreover, as Proposition 1 illustrates, no matter what the underlying shocks are, my main results are robust as long as they produce pro-cyclical fluctuations in asset prices and reallocation. In fact, pro-cyclicality of asset prices and reallocation is one of the most robust empirical regularities.

The following proposition shows how a change in aggregate productivity influences reallocation, asset quality in the market, and asset quality in the economy:

**Proposition 3.** *An increase in aggregate productivity, $\phi$, (i) increases the cut-off productivity below which entrepreneurs sell high-quality capital, $\hat{z} \equiv p/\phi$, and the volume of trade, (ii) reduces the average quality of capital in the economy, $\lambda$, and (iii) increases the average quality of capital traded in the market, $\lambda^M$.*

The first result is straightforward. An increase in $\phi$ increases the demand, $D$, without affecting the supply curve, $S$, and thus equilibrium cut-off, $\hat{z}$, and trading volume must rise to clear the market. The second and third results follow from the combination of the first result and Proposition 1. Given that the cut-off productivity is higher, fewer entrepreneurs are exerting effort, but more entrepreneurs are selling high-quality capital.

The fact that market quality increases in response to increase in aggregate productivity plays the role of amplification. Consider two economies, one with asymmetric information and another with symmetric information with trade costs and constant taxes such that, in the absence of shocks, allocations are identical. The asymmetric information economy features endogenous tax rates, where tax rates decrease in response to an increase in market quality, $\lambda^M$.

**Corollary 1.** *Relative to a symmetric information economy with trade costs and fixed trade taxes, in the economy with asymmetric information, in response to an increase in $\phi$, asset prices, aggregate output and trading volume increase more.*

It is tempting to think that an economy-wide decline in average quality dampens the response to a positive productivity shock, but as the above corollary shows, this is not the case. Instead, an increase in market quality mitigates adverse selection problems and thus amplifies the responses of the economy through fostered capital reallocation.[6]

### 2.5. Normative Implications of a Benchmark Model

What are the normative implications of the model? Before moving to the second-best policy, let me briefly describe the first-best allocation. In the first-best allocation, (i) all entrepreneurs should exert effort because $\kappa \to 0$, and (ii) entrepreneurs with productivity below $z^{FB}$ should not produce, entrepreneurs with productivity $z \geq z^{FB}$ should produce an amount $\phi z(\pi_h + d(z))$ of output with $d(z) = \frac{\phi}{\gamma}(z - z^{FB})$, and $z^{FB}$ satisfies $\pi_h G(z^{FB}) = \int_{z^{FB}} d(z)dG(z)$. It is clear that the first-best allocation is not attainable in equilibrium. Moreover, the volume of trade is lower than the first best, $\hat{z}^* < z^{FB}$, where $\hat{z}^*$ is the cut-off of sellers in equilibrium.

In what follows, I ask whether the equilibrium volume of trade is constrained efficient or not. Consider a planner's problem, where the planner faces the same informational friction as private agents. The planner cannot observe identities of entrepreneurs, types of capital, and effort choice of entrepreneurs. This assumption implies that the planner is only able to manipulate prices and aggregate quantities. I first focus on ex-ante efficiency, where the planner maximizes ex-ante expected welfare. With risk neutrality, this is equivalent to maximizing the total amount of consumption. In this case, Proposition 2 already provides the answer. Recall that Proposition 2 points out that the economy is isomorphic to an economy with trade costs and trade taxes. A natural conjecture from this result is that the planner can undo any distortion coming from trade taxes by subsidizing trade. Let $\tau$ denote the ad valorem tax on asset purchases in an asymmetric information economy. The following proposition shows that this is indeed the case.

**Proposition 4.** *The equilibrium volume of trade is too low relative to the ex-ante constrained efficient allocation. The planner can maximize ex-ante expected welfare by subsidizing trade, $\tau = \lambda^M(\hat{z}^{sym}/\chi)\chi - 1 \leq 0$, where $\hat{z}^{sym*}$ is the asset price in a symmetric information economy without taxes.*

Despite the concern of moral hazard, the planner always finds it optimal to facilitate trade. In so doing, the planner can improve reallocation, which was distorted by adverse selection, at the cost of lowering average quality in the economy. The economics behind this result is easy to grasp if we recall that adverse selection alone is isomorphic to trade taxes, but moral hazard is isomorphic to iceberg trade costs, which is a part of the technology. Both the private agents and the planner understand that some resources will be lost when assets are traded. The planner does not have a superior tool than private agents to deal with moral hazard. Therefore, the planner optimally deals with the adverse selection problems by subsidizing trade. Note that in a special case with $\pi_h = 1$, the equilibrium is constrained efficient ($\tau = 0$). This happens because under $\pi_h = 1$ market quality does not vary with trading volume, $\lambda^M = \pi_l$, and thus, facilitating trade does not alleviate adverse selection problems.

Previous arguments have considered ex-ante welfare, a criterion before the idiosyncratic productivity shocks have been realized. This may not be an appropriate criterion because a policy may benefit some entrepreneurs by hurting others ex-post. Indeed, the subsidization policy described above does not achieve Pareto improvement. Consider an entrepreneur who neither buys nor sells high-quality assets with the subsidization policy, under which the lump-sum tax and revenue from sales of low-quality capital exactly cancel. She would have been better without the policy as she did not need to pay the lump-sum tax.

Is Pareto improvement possible?[7] In answering this question, instead of looking at policies in an asymmetric information economy, it is more convenient to look for a tax policy in a symmetric information

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[6]The amplification mechanism is exactly that described in Kurlat (2013).

[7]Bigelow (1990) derives a sufficient condition for competitive equilibrium to be interim constrained efficient in the setup similar to that of Akerlof (1970).

economy. The following proposition shows that policy in one economy can be mapped to the policy in another economy.

**Proposition 5.** *For any tax policy $\tau$ in an economy with asymmetric information, there exists a tax rate $\tau^{sym}$ in an economy with symmetric information that delivers an equivalent allocation. Conversely, for any tax policy $\tau^{sym}$ in an economy with symmetric information, there exists a tax rate $\tau$ in an economy with asymmetric information that delivers an equivalent allocation.*

This shows that allocation under any tax rate in a symmetric information economy can be achieved with some trade taxes in the asymmetric information economy. This is a useful result because we no longer need to look at the optimal policy in an asymmetric information economy, but instead can focus on a symmetric information economy. Then the standard argument in the theory of optimal taxation implies that a change in the tax rate can bring about Pareto improvement if the tax rate is beyond the peak of the Laffer curve.[8] Hence the equilibrium in the asymmetric information economy is ex-interim constrained efficient if the implied tax rate, $\tau^{sym} = (1 - \lambda^{M*}\chi)/(\lambda^{M*}\chi)$, is not too high for the government to collect more revenue by reducing taxes in the symmetric information economy. For example, when the market is breaking down, the implicit tax rate is infinite, $\tau^{sym} = \infty$. In this case, the government in a symmetric information economy can increase tax revenue by slightly reducing taxes, which achieves Pareto improvement. In the asymmetric information economy, this is equivalent to slightly subsidizing trade. Sellers and buyers of high-quality capital benefit from this policy, but even entrepreneurs who neither sell nor buy high-quality assets are better off. This is because under wrong side of the Laffer curve, the improvement in price for selling low-quality assets more than offsets the need to pay the lump sum tax.[9]

The arguments so far can be made without any reference to moral hazard. How does moral hazard affect these arguments? The moral hazard consideration is likely to bring the economy closer to ex-interim constrained efficient allocation. Conditional on the same severity of adverse selection problems—i.e., the same value of $\lambda^{M*}$—the implicit tax rate, given in Proposition 2, is lower when there is moral hazard, $\chi > 1$, than when there is no moral hazard, $\chi = 1$. This is because moral hazard implies that trading becomes more technologically costly and thus trade is less distorted. In the extreme case with $\pi_h = 1$, the implied tax rate is zero at the highest price-quantity equilibrium, so that the economy is both ex-ante and ex-interim constrained efficient. These results may at first seem surprising because the quality deterioration during booms does not itself justify the policy intervention. However, I will overturn this conclusion when I extend the model to a dynamic framework in the following section.

## 3. Extension: Sequential Trade

This section extends the previous model by allowing capital to be traded sequentially. This consideration will yield novel normative implications. The key idea is that although moral hazard does not distort the current asset market's condition, the creation of low-quality capital will distort the future market.

### 3.1. Environment

The model is now extended to a three-period model ($t = 0, 1, 2$). Entrepreneurs' preferences are modified as $\mathbb{E}_0[c_1 + c_2]$, and hence there is no discounting over time. The first two periods, $t = 0, 1$, are the same as before where agents invest, draw idiosyncratic productivity from distribution function $G_1$, choose effort, and trade assets. In the final period, $t = 2$, agents draw a new idiosyncratic productivity from the distribution function $G_2$ independently from the past and trade the capital again. There is no depreciation of capital nor new investment at $t = 2$.

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[8]See Appendix C for more formal discussion.

[9]In the current setup, the assumption is that the investment projects are diversified so that the entrepreneurs obtain both high- and low-quality capital. If, instead, entrepreneurs obtain either high- or low-quality capital with probability $\pi \in \{\pi_h, \pi_l\}$, then those who obtain high-quality capital and do not trade are always worse off as a result of the subsidization policy.

The cost of installing capital at $t = 2$ is

$$\Gamma(d_2) = \frac{\gamma}{2}(d_2)^2, \tag{7}$$

where $d_2$ is the total capital demanded. Note that unlike at $t = 1$, entrepreneurs have to incur the cost of installing capital which includes both high- and low-quality capital. The assumption here is that entrepreneurs do not observe quality before installation. If the cost of installing capital at $t = 2$ involves only high-quality capital, then improvement in market quality at $t = 2$ has two countervailing effects. On one hand, it increases demand because the purchased assets are more likely to contain high-quality capital. On the other hand, as there is more high-quality capital circulating, its marginal product is lower. This latter effect complicates the analysis while providing few insights. Thus, I abstract away this effect by assuming the composition of capital does not affect its marginal product.[10]

The realization of the aggregate productivity shock at $t = 2$, $\phi_2$, is stochastic and continuously distributed with some distribution function, while $\phi_1$ is deterministic. Finally, $\pi_h = 1$ is assumed throughout this section (i.e., investment always materialize as high-quality capital with effort). This helps to guarantee the existence of equilibrium and to isolate the novel welfare implication, as will be discussed in detail later.

To summarize the new ingredients in this section, (i) the functional form of the reallocation cost at $t = 2$ is given by (7); (ii) investment occurs at $t = 0$ only, and there is no depreciation, investment, or effort choice in the subsequent periods; (iii) entrepreneurs re-draw idiosyncratic productivity at $t = 2$, independently from the past, but not necessarily from an identical distribution; (iv) aggregate productivity at $t = 2$, $\phi_2$, is randomly drawn from a continuous distribution; (v) $\pi_h = 1$.

### 3.2. Equilibrium

The equilibrium analysis consists of two steps, moving backward in time ($t = 2$ and $t = 1$). At period $t = 2$, given the holdings of high- and low-quality capital which are determined at $t = 1$, the agents trade their capital. Entrepreneurs with productivity $z < \hat{z}_2^s$ sell their high-quality capital, and entrepreneurs with $z \geq \hat{z}_2^s$ keep their high-quality capital, where $\hat{z}_2^s \equiv p_2/\phi_2$ is cut-off productivity and $p_t$ is the price of capital at time $t$. These decisions are the same as in the previous model. The assumption that productivity draws at $t = 2$ is independent of their capital holdings allows simple aggregation of capital supply:

$$S_2(\hat{z}_2^s) = (1 - \lambda_2) + \lambda_2 G_2(\hat{z}_2^s),$$

where $\lambda_2$ is the fraction of high-quality capital in the economy at $t = 2$ which is endogenously determined at $t = 1$. The fraction of high-quality capital that circulates in the market at $t = 2$ is given by

$$\lambda_2^M(\hat{z}_2^s) = \frac{\lambda_2 G_2(\hat{z}_2^s)}{(1 - \lambda_2) + \lambda_2 G_2(\hat{z}_2^s)}.$$

The individual demand for capital purchases solve

$$\max_{d_2 \geq 0} \left\{ \lambda_2^M(\hat{z}_2^s)\phi_2 z d_2 - p_2 d_2 - \Gamma(d_2) \right\},$$

and thus the aggregate demand function is

$$D_2(\hat{z}_2^s; \phi_2) = \int_{\hat{z}_2^s/\lambda_2^M(\hat{z}_2, \lambda_2)} \frac{\phi_2}{\gamma}(z\lambda_2^M(\hat{z}_2^s; \lambda_2) - \hat{z}_2^s)dG_2(z). \tag{8}$$

The market clearing condition at $t = 2$ is $S_2(\hat{z}_2^s) \geq D_2(\hat{z}_2^s; \phi_2)$ with equality whenever $\hat{z}_2^s > 0$. Denote $\hat{z}_2^s(\phi_2; \lambda_2)$ as the market clearing price for given values of $\phi_2$ and $\lambda_2$.

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[10] Readers may feel uncomfortable because the information structure at $t = 2$ and $t = 1$ is different. However, I assume $\pi_h = 1$ through this section and thus $\lambda_1^M = \pi_l$, which is a constant. Therefore it is equivalent to assuming that entrepreneurs do not observe quality before installing both at $t = 1$ and $t = 2$, but the cost parameter at $t = 1$ is $\gamma_1 \equiv \gamma(\pi_l)^2$. The assumption that the cost involves only high-quality capital in the benchmark model was important only to establish the equivalence result.

Let us now move on to characterize the equilibrium at $t = 1$. Each agent takes future quality $\lambda_2$ as given, and their decisions endogenously determine $\lambda_2$. The entire equilibrium is a fixed point in terms of $\lambda_2$. Let $v_2^h$ and $v_2^l$ denote the values of holding a unit of high- and low-quality capital at the end of $t = 1$, respectively. They are given by

$$v_2^h(\lambda_2) \equiv \mathbb{E}\left[\phi_2 \max\{z_2, \hat{z}_2^s(\phi_2; \lambda_2)\}\right]$$
$$v_2^l(\lambda_2) \equiv \mathbb{E}[\phi_2 \hat{z}_2^s(\phi_2; \lambda_2)],$$

which capture that the high-quality capital is sold when productivity falls below the cutoff, and the low-quality capital is always sold at $t = 2$. The expectation is taken over both the aggregate shocks, $\phi_2$, and the idiosyncratic shocks, $z_2$. The productivity cut-off at $t = 1$, $\hat{z}_1^s$, at which entrepreneurs are indifferent between selling and holding on to high-quality capital satisfies

$$\phi_1 \hat{z}_1^s + v_2^h(\lambda_2) = p_1, \tag{9}$$

where the left-hand side is the value of holding on to high-quality capital and the right-hand side is the benefit from selling it. Entrepreneurs below this cut-off sell high-quality capital. Low-quality capital is always sold in equilibrium because the equilibrium price, $p_1$, will always be higher than $v_2^l$ (otherwise there will be infinite demand). The individual demand at $t = 1$ solves

$$\max_{d_1 \geq 0} \left\{ \lambda_1^M(\hat{z}_1^s)(\phi_1 z + v_2^h(\lambda_2))d_1 + (1 - \lambda_1^M(\hat{z}_1^s))v_2^l(\lambda_2)d_1 - p_1 d_1 - \Gamma(\lambda_1^M(\hat{z}_1^s)d_1) \right\},$$

and this yields

$$d_1(z; \hat{z}_1^s) = \max\left\{ \frac{\phi_1}{\gamma(\lambda_1^M(\hat{z}_1^s))^2} \left( \lambda_1^M(\hat{z}_1^s)z + (1 - \lambda_1^M(\hat{z}_1^s))\tilde{v}_2(\lambda_2)\frac{1}{\phi_1} - \hat{z}_1^s \right), 0 \right\},$$

where $\hat{z}_1^s$ is defined in equation (9), $\tilde{v}_2(\lambda_2) \equiv v_2^l(\lambda_2) - v_2^h(\lambda_2)$, and $\lambda^M(\hat{z}_1^s)$ is the market proportion of high-quality capital at $t = 1$:

$$\lambda_1^M(\hat{z}_1^s) = \frac{\pi_l G_1(\hat{z}_1^s)}{G_1(\hat{z}_1^s) + (1 - \pi_h)(1 - G_1(\hat{z}_1^s))}.$$

The market clearing condition at $t = 1$ is given by $S_1(\hat{z}_1^s) \geq D_1(\hat{z}_1^s)$ with equality whenever $\hat{z}_1^s > 0$, where

$$S_1(\hat{z}_1^s) \equiv G_1(\hat{z}_1^s) + (1 - \pi_h)(1 - G_1(\hat{z}_1^s)), \tag{10}$$

$$D_1(\hat{z}_1^s) \equiv \int d_1(z; \hat{z}_1^s) dG_1(z). \tag{11}$$

The resulting fraction of high-quality capital at $t = 2$ is

$$\lambda_2 = \pi_l G(\hat{z}_1^s) + \pi_h(1 - G(\hat{z}_1^s)). \tag{12}$$

Equilibrium consists of $(\hat{z}_1^s, \{\hat{z}_2^s\}, \lambda_2)$ such that the market for capital clears in each period, and $\lambda_2$ is consistent with the entrepreneurs' effort choices, (12). As in the previous section, I deal with the issue of multiplicity by focusing on the highest price-quantity equilibrium in each period. This equilibrium can be characterized as follows. First, compute the highest price at $t = 2$ for each aggregate state, $(\phi_2, \lambda_2)$. With this $\hat{z}_2^s(\phi_2; \lambda_2)$ in hand, one can solve for $t = 1$ equilibrium taking $\lambda_2$ as given to compute the highest price and associated cut-off at $t = 1$, $\hat{z}_1^s(\lambda_2)$. The entire equilibrium is given by a fixed point $\psi(\lambda_2) = \lambda_2$ , where $\psi$ is defined as the following mapping:

$$\psi(\lambda_2) \equiv \pi_l G(\hat{z}_1^s(\lambda_2)) + \pi_h(1 - G(\hat{z}_1^s(\lambda_2))). \tag{13}$$

Figure 2 graphically illustrates this mapping. First, $\psi$ is a decreasing function. Intuitively, an increase in future quality improves the value of low-quality assets, $v_2^l$, more than the value of high-quality assets,
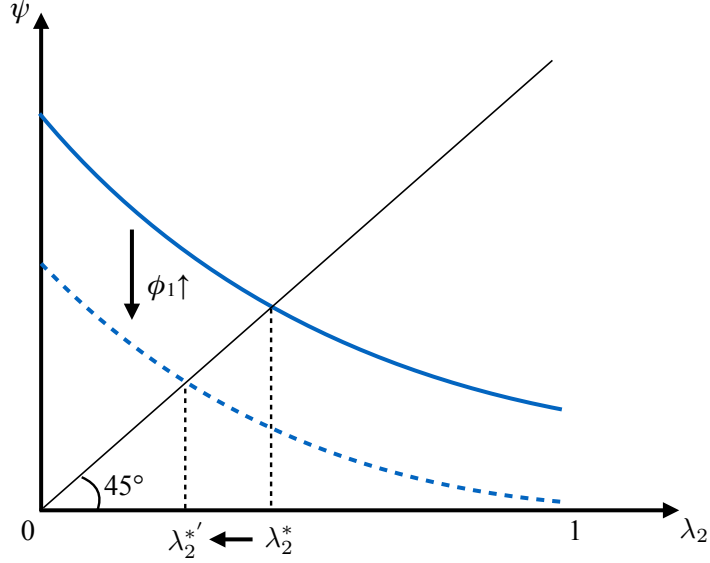
Figure 2: Mapping $\psi$. *Notes*: Illustration of mapping $\psi$ defined in (13). The intersection with the 45 degree line is the equilibrium value of $\lambda_2$. The dashed line shows how the mapping $\psi$ shifts down in response to an increase in $\phi_1$. As a result, the equilibrium $\lambda_2$ will be lower.

$v_2^h$. This is because low-quality assets are always sold at $t = 2$, but high-quality assets are sold only when the productivity at $t = 2$ is sufficiently low. This makes the market at $t = 1$ more liquid by reducing the difference in the value of high- and low-quality assets. As a result, an expectation of higher $\lambda_2$ induces less entrepreneurs to exert effort, which implies that $\psi$ is decreasing. However, $\psi$ may not be continuous. The discontinuity comes from the non-monotonicity of the excess demand function at $t = 1$. Therefore an equilibrium in this extended setting may fail to exist. Here, the assumption that $\pi_h = 1$ comes into in play. This assumption makes the excess demand function at $t = 1$ monotone for all $\hat{z}_1^s > 0$ as explained in the previous section, and this ensures the continuity of $\psi$. Given that $\psi$ is continuous and decreasing, it follows that an equilibrium exists.

**Lemma 1.** *Assume $\pi_h = 1$. An equilibrium with sequential trading exists. That is, there exists a fixed point, $\lambda_2 = \psi(\lambda_2)$. Moreover, the fixed point is unique.*

Now consider a positive shock to $\phi_1$. Since an increase in $\phi_1$ increases demand at $t = 1$ for a given $\lambda_2$, the mapping $\psi$ uniformly shifts down, as illustrated in Figure 2. It follows that the equilibrium asset equality at $t = 2$, $\lambda_2$, deteriorates. Define the fragility of the economy at $t = 2$, $f_2$, as the probability of a market breakdown at $t = 2$:

$$f_2 = \mathbb{E}_\phi \left[ \mathbb{I}(p_2 = 0) \right],$$

where $\mathbb{I}(p_2 = 0)$ is an indicator function that takes the value one when the asset price at $t = 2$ is zero, and zero otherwise. A decrease in $\lambda_2$ decreases demand while increases supply at $t = 2$, and thus $p_2 = 0$ is more likely to be the only market clearing price, leading to more fragility at $t = 2$. The following proposition summarizes the results.

**Proposition 6.** *An increase in aggregate productivity at $t = 1$, $\phi_1$, (i) increases the output, trading volume, and asset prices at $t = 1$, but decreases the output, trading volume, asset prices, market quality, and average quality in the economy at $t = 2$ for all states, and (ii) increases the fragility of the economy at $t = 2$, $f_2$.*

As we saw in Proposition 3, an increase in current productivity creates the current boom. However, the current boom lowers the average quality of assets in the economy and negatively affects future economic

conditions through by worsening adverse selection problems. This provides a novel perspective on why booms can be seeds of future crises.

### 3.3. Normative Implications under Sequential Trading

What are the policy implications in this extended setting? Again, consider a tax on asset purchases for both periods, $\{\tau_1, \tau_2(\phi_2)\}$, and assume the planner maximizes ex-ante expected welfare, which is equal to the total amount of consumption in $t = 1$ and $t = 2$. Note that in writing $\tau_2(\phi_2)$, the planner can freely intervene at $t = 2$ depending on the realization of aggregate productivity shocks. The assumption that $\pi_h = 1$ is maintained, so that $\lambda_1^M \equiv \pi_l$.

The demand function at $t = 1$ under the tax is given by

$$\hat{d}_1(z; \hat{z}_1^s) = \max\left\{ \frac{1}{\gamma(\lambda_1^M)^2}\left(\lambda_1^M(\phi_1 z + v_2^h(\lambda_2)) + (1 - \lambda_1^M)v_2^l(\lambda_2) - (1 + \tau_1)p_1\right), 0\right\},$$

with $p_1 \equiv \phi_1 \hat{z}_1^s + v_2^h(\lambda_2)$ and the demand function at $t = 2$ is

$$\hat{d}_2(z; \hat{z}_2^s, \phi_2) = \max\left\{ \frac{\phi_2}{\gamma}(z\lambda_2^M(\hat{z}_2^s) - (1 + \tau_2(\phi_2))\hat{z}_2^s), 0\right\}.$$

An equilibrium with tax consists of $(\hat{z}_1^s, \{\hat{z}_2^s(\phi_2)\}, \lambda_2)$ such that the capital market clears in each period

$$S_1(\hat{z}_1^s) \geq \int \hat{d}_1(z; \hat{z}_1^s)dG_1(z) \quad \text{with equality whenver } \hat{z}_1^s > 0, \tag{14}$$

$$S_2(\hat{z}_2^s) \geq \int \hat{d}_2(z; \hat{z}_2^s, \phi_2)dG_2(z) \quad \text{with equality whenver } \hat{z}_2^s > 0, \tag{15}$$

and $\lambda_2$ is given by (12).

Define the production from reallocated capital (net of installing costs) at $t = 1$ and $t = 2$ as

$$X_1^r(\hat{z}_1^s) \equiv \int \left[ \phi_1 \lambda_1^M z\hat{d}_1(z; \hat{z}_1^s) - \Gamma(\lambda_1^M \hat{d}_1(z; \hat{z}_1^s)) \right]dG_1(z)$$

$$X_2^r(\hat{z}_2^s, \phi_2) \equiv \int \left[ \phi_2 z\lambda_2^M(\hat{z}_2^s)\hat{d}_2(z; \hat{z}_2^s, \phi_2) - \Gamma(\hat{d}_2(z; \hat{z}_2^s, \phi_2)) \right]dG_2(z),$$

respectively. The planner's problem is to maximize total expected consumption subject to the equilibrium conditions:[11]

$$\max_{\tau_1, \hat{z}_1^s, \{\hat{z}_2^s(\phi_2), \tau_2(\phi_2)\}, \lambda_2} \int_{\hat{z}_1^s} \phi_1 z\pi_h dG_1(z) + X_1^r(\hat{z}_1^s) + \mathbb{E}_\phi\left[ \int_{\hat{z}_2^s(\phi_2)} \phi_2 z\lambda_2 dG_2(z) + X_2^r(\hat{z}_2^s(\phi_2), \phi_2) \right]$$

$$\text{subject to} \quad (12), (14), (15).$$

The first two terms in the objective function are total consumption at $t = 1$, which include the production from capital not sold and the production from reallocated capital. The last two terms are total consumption at $t = 2$, which are analogous to the previous two terms. The following lemma characterizes the solution.

**Lemma 2.** *Assume $\pi_h = 1$. The optimal intervention, $\{\tau_1, \tau_2(\phi_2)\}$, satisfies the following two conditions.*

$$\tau_1 = -\frac{1}{p_1 g_1(\hat{z}_1^s)} \frac{\partial \lambda_2}{\partial \hat{z}_s^1} \mathbb{E}_\phi\left[ \frac{\partial \lambda_2^M}{\partial \lambda_2} x_2^r(\phi_2) - \phi_2 \hat{z}_2^s(\phi_2)\tau_2(\phi_2)(1 - G_2(\hat{z}_2^s(\phi_2))) \right], \tag{16}$$

$$\tau_2(\phi_2) = -\frac{1}{\phi_2 \hat{z}_2^s(\phi_2)g_2(\hat{z}_2^s(\phi_2))\lambda_2} \frac{\partial \lambda_2^M}{\partial \hat{z}_2^s} x_2^r(\phi_2), \tag{17}$$

*where $x_2^r(\phi_2) \equiv \int \phi_2 z\hat{d}_2(z; \hat{z}_2^s(\phi_2), \phi_2)dG_2(z) \geq 0$.*

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[11]Ex-ante optimal policy is not necessarily time consistent. I ignore this problem by assuming that the planner is able to commit to the future sequence of taxes and subsidies.

The right-hand side of the condition (17) is negative because $\frac{\partial \lambda_2^M}{\partial \hat{z}_2} \geq 0$. Therefore it follows that $\tau_2(\phi_2) \leq 0$ for all $\phi_2$. This result echoes the previous two-period arguments: as the economy at $t = 2$ is a static adverse selection economy for a given $\lambda_2$, the planner will find it optimal to subsidize capital purchases to improve the quality of the pool of assets that trade. Now consider the condition (16). First, note that $\frac{\partial \lambda_2}{\partial \hat{z}_1^s} = -g_1(\hat{z}_1^s)(\pi_h - \pi_l) < 0$. In words, the higher asset prices at $t = 1$ imply lower quality of assets in the next period. Second, $\frac{\partial \lambda_2^M}{\partial \lambda_2} > 0$ because higher quality in the economy translates into higher market quality. Combined with the previous observation that $\tau_2(\phi_2) \leq 0$, it follows that the right-hand side is positive. This directly implies that $\tau_1 \geq 0$. These arguments lead to the following proposition.

**Proposition 7.** *Assume $\pi_h = 1$. The equilibrium volume of trade at $t = 1$ is too high relative to the ex-ante constrained efficient allocation. The planner can maximize ex-ante welfare by taxing trade at $t = 1$.*

This result is in sharp contrast to Proposition 4. Proposition 4 states that equilibrium volume of trade was always too low, but now we reach the opposite conclusion. Why is that? In a sequential trading setting, the planner wants to mitigate adverse problems not only at $t = 1$, but also at $t = 2$. Although the planner has access to ex-post policy at $t = 2$, this cannot completely resolve the adverse selection problems. However, the planner has the first-best way to mitigate the adverse selection problems at $t = 2$; taxing trade at $t = 1$ to improve the quality of assets in the economy. Of course, this may worsen the adverse selection problems at $t = 1$ if $\pi_h < 1$. Therefore the planner faces a trade-off in choosing tax at $t = 1$: (i) enhance reallocation to improve current market conditions; (ii) restrict reallocation to improve future market conditions. In general, the planner strikes a balance between these two effects in choosing whether to tax or subsidize asset purchases. Under $\pi_h = 1$, however, the first consideration is necessarily absent because the market quality is a constant, $\lambda_1^M = \pi_l$. This leads to an unambiguous result, where the planner always finds it optimal to tax asset purchases rather than to subsidize them.

In the benchmark model in Section 2, the equivalence result states that moral hazard acts as if it is changing technology rather than creating distortion. Here this is no longer the case. Moral hazard distorts the economy because it worsens future adverse selection problems. Private agents do not realize that if they demand fewer assets and lower asset prices, it will alleviate future adverse selection problems. The planner corrects this externality by taxing trade. The result identifies a rationale for why quality deterioration during boom periods is an inefficient outcome and why it warrants policy intervention, as commonly argued by policymakers.

Although the focus so far has been on ex-ante efficiency, it is sometimes possible for the ex-ante optimal policy to bring about Pareto improvement, even after the realization of idiosyncratic productivity at $t = 1$. In Appendix D, I show a numerical example of such a case. Taxing asset purchases harms buyers and sellers at $t = 1$. However, they all benefit from better market conditions at $t = 2$, where the entrepreneurs draw new productivity. Of course, such Pareto improvement is not always guaranteed, but it is a reminder that a policy leaning against the wind may achieve its goal without necessarily creating winners and losers.

## 4. Conclusion

This paper has presented a simple theory in which endogenous deterioration of quality during booms inefficiently creates fragility in the economy. Existing macroeconomic theories emphasize the role of leverage as a source of systemic risk. Here, I provided an alternative view through the lens of the model. Although many practitioners and policymakers have been wary of inefficient quality deterioration during booms, there has been little work that formalizes these views. My model precisely speaks to why quality deteriorates during booms, why this creates fragility, and why it is inefficient. Although I demonstrate the idea using a fairly abstract framework, the main insights of this paper apply to a wide variety of economic circumstances where adverse selection is a potential source of market failure.

The key prediction of the model concerns the relationship between asset prices and the underlying moral hazard problems. This opens up several avenues for future research. Empirically, the model delivers a testable prediction of the relationship between asset prices and asset quality. Theoretically, introducing

asset price bubbles would be an interesting extension of the current model. The mechanism should be further pronounced when asset price bubbles are present.

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# Online Appendix to "Asset Quality Cycles"

## Appendix A. Proofs

*Appendix A.1. Proof of Proposition 1*

It is straightforward to see

$$\frac{d\lambda(p/\phi)}{dp} = -(\pi_h - \pi_l)\frac{1}{\phi}g(p/\phi)$$
$$< 0$$

and

$$\frac{d\lambda^M(p/\phi)}{dp} = \frac{\frac{1}{\phi}\pi_l g(p/\phi)(1-\pi_h)}{(G(p/\phi) + (1-\pi_h)(1-G(p/\phi)))^2} \geq 0,$$

where the inequality is strict if $\pi_h < 1$.

*Appendix A.2. Proof of Proposition 2*

Set $\tau^{sym} = (1 - \lambda^{M*}\chi)/(\lambda^{M*}\chi)$ and $\chi = \pi_h/\pi_l$, and guess that $\hat{z}^{sym} = \chi\hat{z}^*$. I will verify that allocations of symmetric information and asymmetric information are identical. The individual demand function for high-quality capital in a symmetric information economy can be rewritten as

$$k^{d,sym}(z) = \max\left\{\frac{\phi}{\gamma}(z - \frac{1}{\lambda^{M*}}\hat{z}^*), 0\right\},$$

which is equivalent to the amount of high-quality capital that entrepreneurs obtain in the asymmetric information economy, $\lambda^{M*}d(z) = \max\left\{\frac{\phi}{\gamma}(z - \frac{1}{\lambda^{M*}}\hat{z}^*), 0\right\}$. Because the supply function in the symmetric information economy is given by $S^{sym}(\hat{z}^{sym}) = \pi_l G(\hat{z}^*)$, the supply side of high-quality capital is also same as that for an asymmetric information economy. The lump-sum transfer is

$$T = \tau^{sym}\phi\hat{z}^{sym}S^{sym}(\hat{z}^{sym})$$
$$= \frac{(1 - \lambda^{M*}\chi)}{(\lambda^{M*}\chi)}\phi\chi\hat{z}^*\lambda^{M*}S(\hat{z}^*)$$
$$= (1 - \lambda^{M*}\frac{\pi_h}{\pi_l})\phi\hat{z}^*S(\hat{z}^*)$$
$$= \left(1 - \frac{\pi_h G(\hat{z}^*)}{S(\hat{z}^*)}\right)\phi\hat{z}^*S(\hat{z}^*)$$
$$= (1 - \pi_h)\phi\hat{z}^*,$$

which is equivalent to the revenue from sales of lemons for keepers in an asymmetric information economy. Therefore the consumption of keepers $(z \geq \hat{z}^*)$ in a symmetric information economy and in an asymmetric information economy coincide. The consumption for sellers $(z < \hat{z}^{sym}/\chi)$ in a symmetric information economy is

$$p^{sym}\pi_h\frac{1}{\chi} + T = \phi\chi\hat{z}^*\pi_l + (1 - \pi_h)\phi\hat{z}^*$$
$$= \phi\hat{z}^*,$$

which is the total sales revenue in a symmetric information economy. The above arguments prove that consumption and production for each entrepreneur in the two economies are identical, and prices are related through $\hat{z}^{sym}/\chi = \hat{z}^*$.

*Appendix A.3. Proof of Proposition 3*

From the market clearing condition, $\frac{\partial \hat{z}}{\partial \phi} \geq 0$ holds at the highest price equilibrium. Therefore both price and trading volumes increase. The rest follows from Proposition 1.

*Appendix A.4. Proof of Proposition 4*

This is a special case of Proposition 5.

*Appendix A.5. Proof of Proposition 5*

The proof consists of mimicking the same procedure as was applied in Proposition 2. Start from an asymmetric information economy with tax rate $\tau$. Set $\tau^{sym} = \frac{1+\tau-\lambda^{M*}\chi}{\lambda^{M*}\chi}$ and $\chi = \pi_h/\pi_l$, and guess that $\hat{z}^* = \hat{z}^{sym}/\chi$ holds. Then the individual demand function for high-quality capital in a symmetric information economy can be rewritten as

$$k^{d,sym}(z) = \max\left\{\frac{\phi}{\gamma}(z - \frac{1}{\lambda^{M*}}(1+\tau)\hat{z}^*), 0\right\},$$

which is equivalent to the amount of high-quality capital that entrepreneurs obtain in the asymmetric information economy, $\lambda^{M*}d(z) = \max\left\{\frac{\phi}{\gamma}(z - \frac{1}{\lambda^{M*}}(1+\tau)\hat{z}^*), 0\right\}$. The supply function in the asymmetric information economy is given by $S(\hat{z}^{sym}) = \pi_l G(\hat{z}^*)$, which is again the same as the supply of high-quality capital in the asymmetric information economy. The lump-sum transfer is

$$\begin{aligned}
T &= \tau^{sym}p^{sym}S(p^{sym}/\phi)\\
&= \frac{(1+\tau-\lambda^{M*}\chi)}{(\lambda^{M*}\chi)}\phi\chi\hat{z}^*\lambda^{M*}S(\hat{z}^*)\\
&= (1+\tau-\lambda^{M*}\frac{\pi_h}{\pi_l})\phi\hat{z}^*S(\hat{z}^*)\\
&= (1-\pi_h)\phi\hat{z}^* + \tau\phi\hat{z}^*S(\hat{z}^*),
\end{aligned}$$

which is the amount that keepers of high-quality capital receive from the sales of low-quality assets and lump-sum transfers. The total consumption for sellers ($z < \hat{z}^{sym}/\chi$) is

$$\begin{aligned}
p^{sym}\pi_h\frac{1}{\chi} + T &= \phi\chi\hat{z}^*\pi_l + (1-\pi_h)\phi\hat{z}^* + \tau\phi\hat{z}^*S(\hat{z}^*),\\
&= \phi\hat{z}^* + \tau\phi\hat{z}^*S(\hat{z}^*),
\end{aligned}$$

which includes the sales of all the capital and lump-sum transfers in the asymmetric information economy. Therefore allocations are equivalent.

I will prove the converse. Start from a symmetric information economy with tax rate $\tau^{sym}$ and $\chi = \pi_h/\pi_l$. Guess that $\hat{z}^* = \hat{z}^{sym}/\chi$. Set $\tau = \lambda^M(\hat{z}^{sym}/\chi)\chi\tau^{sym} + \lambda^M(\hat{z}^{sym}/\chi)\chi - 1$ in the asymmetric information economy. Then the individual demand function for capital in an asymmetric information economy can be written as

$$\begin{aligned}
k^d(z) &= \max\left\{\frac{\phi}{\gamma(\lambda^{M*})^2}(z\lambda^{M*} - \lambda^{M*}\chi(1+\tau^{sym})\hat{z}^*), 0\right\}\\
&= \max\left\{\frac{\phi}{\gamma\lambda^{M*}}(z - (1+\tau^{sym})\hat{z}^{sym}), 0\right\}.
\end{aligned}$$

Thus the effective demand for high-quality capital is $\lambda^{M*}k^d(z) = \max\left\{\frac{\phi}{\gamma}(z - (1+\tau^{sym})\hat{z}^{sym}), 0\right\}$, which is the same as the demand for high-quality capital in the symmetric information economy. The sum of the

sale of low-quality capital and lump-sum transfers in the asymmetric information economy is

$$
\begin{aligned}
\phi\hat{z}^*(1-\pi_h) + \tau\phi\hat{z}^*S(\hat{z}^*) &= \phi\hat{z}^{sym}/\chi\left[(1-\pi_h) + \tau S(\hat{z}^*)\right] \\
&= \phi\hat{z}^{sym}/\chi\left[(1-\pi_h) + (\lambda^M/\chi(1+\tau^{sym})-1)S(\hat{z}^*)\right] \\
&= \phi\hat{z}^{sym}/\chi\left[(1-\pi_h) - S(\hat{z}^*) + \pi_h G(\hat{z}^*)(1+\tau^{sym})\right] \\
&= \phi\hat{z}^{sym}\pi_l G(\hat{z}^*)\tau^{sym} \\
&= \phi\hat{z}^{sym}S^{sym}(\hat{z}^{sym})\tau^{sym},
\end{aligned}
$$

which is same as the lump-sum transfer each agent receives in an asymmetric information economy. The sales revenue from high-quality assets is also same because $\phi\hat{z}^* = \phi\hat{z}^{sym}/\chi$. Therefore we confirm that the allocations are identical.

### Appendix A.6. Proof of Lemma 1

First, I argue that $\psi$ is continuous in $\lambda_2$. Although the price at $t=2$, $\hat{z}_2^s(\phi_2)$, might be discontinuous in $\lambda_2$ for finite numbers of $\phi_2$, once expectations over $\phi_2$ are taken, $v_2^h$ and $v_2^l$ are continuous in $\lambda_2$ because such discontinuous points have measures of zero. Given $\pi_h = 1$, $\hat{z}_1^s$ is also continuous in $\lambda_2$ because the excess demand function is monotone. This establishes that $\psi$ is continuous. Next, I argue that $\psi$ is decreasing. Note that the supply function at $t=1$, $S_1(\hat{z}_1^s)$, does not depend on $\lambda_2$. The demand function, $D_1(\hat{z}_1^s; \lambda_2)$, is increasing in $\lambda_2$ because

$$
\begin{aligned}
\frac{d}{d\lambda_2}(v_2^l(\hat{z}_2^s) - v_2^h(\hat{z}_2^s)) &= \mathbb{E}_\phi\phi_2\frac{d\hat{z}_2^s}{d\lambda_2}\frac{d}{d\hat{z}_2^s}\left[\hat{z}_2^s - \int_{\hat{z}_2^s}zdG_2(z) - (1-G(\hat{z}_2^s))\hat{z}_2^s\right] \\
&= \mathbb{E}_\phi\phi_2\frac{d\hat{z}_2^s}{d\lambda_2}G(\hat{z}_2^s).
\end{aligned}
$$

Because the demand function at $t=2$ is increasing in $\lambda_2$, and the supply function is decreasing in $\lambda_2$, it follows that the price at $t=2$ is increasing in $\lambda_2$, $\frac{d\hat{z}_2}{d\lambda_2} > 0$. Therefore the above expression is positive. This implies that an increase in $\lambda_2$ increases the demand curve at $t=1$, but keeps the supply curve at $t=1$ unchanged. As a result, the equilibrium cut-off, $\hat{z}_1^s$, increases, which in turn lowers average quality in the economy. Thus $\psi$ is decreasing in $\lambda_2$. These arguments show that there exists a unique fixed point $\lambda_2 = \psi(\lambda_2)$.

### Appendix A.7. Proof of Proposition 6

As argued in the main text, an increase in $\phi_1$ monotonically decreases $\psi$ for all $\lambda_2$. Therefore the equilibrium value of $\lambda_2$ goes down. This not only reduces the production possibility frontier at $t=2$, but also decreases demand and increases supply at $t=2$, and thus lowering asset prices and trading volume. Increased supply and decreased demand at $t=2$ imply that the market at $t=2$ is more likely to break down.

### Appendix A.8. Proof of Lemma 2

Define the value of future high- and low-quality capital as a function of prices in each state:

$$
\begin{aligned}
\tilde{v}_2^h(\{\hat{z}_2^s\}) &\equiv \mathbb{E}\left[\phi_2\max\{z_2, \hat{z}_2^s(\lambda_2, \phi_2)\}\right] \\
\tilde{v}_2^l(\{\hat{z}_2^s\}) &\equiv \mathbb{E}[\phi_2\hat{z}_2^s(\lambda_2, \phi_2)].
\end{aligned}
$$

First, I rewrite the planner's problem as follows.

$$\max_{\tau_1, p_1, \{\hat{z}_2^s, \tau_2\}, \lambda_2} \int_{\hat{z}_1^s(p_1, \{\hat{z}_2^s\})} (\phi_1 z + \tilde{v}_2^h)\pi_h dG_1(z) + \int_{\hat{z}_1^b(p_1, \{\hat{z}_2^s\}; \tau_1)} \left[\phi_1 \lambda_1^M V \hat{d}_1 - \Gamma(\lambda_1^M \hat{d}_1)\right] dG_1(z)$$

$$+ \mathbb{E}_\phi \left[ -\phi_2 \hat{z}_2^s S_2(\hat{z}_2^s, \lambda_2) + \int_{\hat{z}_2^b(\hat{z}_2^s; \tau_2)} \left[\phi_2 z \lambda_2^M \hat{d}_2 - \Gamma(\hat{d}_2)\right] dG_2(z) \right]$$

$$\text{s.t.} \quad S_1(\hat{z}_1^s(p_1, \{\hat{z}_2^s\})) \geq \hat{D}_1(p_1, \{\hat{z}_2^s\}; \tau_1), \quad S_2(\hat{z}_2^s, \lambda_2) \geq \hat{D}_2(\hat{z}_2^s; \tau_2, \lambda_2, \phi_2),$$

$$\lambda_2 = \pi_l G(\hat{z}_1^s) + \pi_h(1 - G(\hat{z}_1^s)),$$

where $V = V(z, \{\hat{z}_2^s\}) \equiv \frac{1}{\lambda_1^M(\hat{z}_1^s)} \left( \lambda_1^M(z + \frac{1}{\phi}\tilde{v}_h^2(\{\hat{z}_2^s\})) + (1 - \lambda_1^M)\frac{1}{\phi}\tilde{v}_l^2(\{\hat{z}_2^s\}) \right)$, and I omitted the arguments for $\lambda_2^M$, $\hat{z}_2^s$, $\hat{d}_1$, $\hat{d}_2$, $\tau_2$ and $\tilde{v}_2^h$ to ease the notation.

I shall focus on the interior solutions. After substituting the definition of $\hat{d}_1$ and $\hat{d}_2$, taking the first order conditions for $\tau_1$ and $\tau_2(\phi_2)$ gives

$$\eta_1 = \phi_1 p_1(1 + \tau_1),$$
$$\eta_2(\phi_2) = h(\phi_2)\phi_2 \hat{z}_2^s(1 + \tau_2),$$

where $\eta_1$ and $\eta_2(\phi_2)$ are Lagrangian multipliers on the constraints for market clearing at $t = 1$ and $t = 2$, respectively and $h(\phi_2)$ is the density function for the realization of $\phi_2$. The FOC w.r.t. $p_1$ is

$$\frac{\partial \hat{z}_1^s}{\partial p_1} \tau_1(\phi_1 \hat{z}_1^s + v_2^h) g_1(\hat{z}_1^s)\pi_h$$

$$+ \frac{\partial \hat{z}_1^s}{\partial p_1}\frac{\partial \lambda_2}{\partial \hat{z}_s^1} \mathbb{E}_\phi \left[ \frac{\partial \lambda_2^M}{\partial \lambda_2} \int_{\hat{z}_2^b} \left[ \phi_2 z(\frac{\phi_2}{\gamma}(z\lambda_2^M - \hat{z}_2^s(1 + \tau_2))) \right] dG_2(z) - \phi_2 \hat{z}_2^s \tau_2(1 - G_2(\hat{z}_2)) \right] = 0,$$

which can be rearranged as

$$\tau_1(\phi_1 \hat{z}^s + v_2^h)g_1(\hat{z}_1^s)\pi_h + \frac{\partial \lambda_2}{\partial \hat{z}_s^1}\mathbb{E}_\phi \left[ \frac{\partial \lambda_2^M}{\partial \lambda_2} \int_{\hat{z}_2^b} \phi_2 z d_2(z, \hat{z}_2^s)dG_2(z) - \phi_2 \hat{z}_2^s \tau_2(1 - G_2(\hat{z}_2)) \right] = 0, \qquad \text{(A1)}$$

which is (16). The first order condition for $\hat{z}_2^s(\phi_2)$ is

$$(1 - G(\hat{z}_1^s))\pi_h \frac{\partial v_2^h}{\partial \hat{z}_2} + \frac{(\phi_1)^2}{\gamma(\lambda_1^M)^2} \int_{z^b} \frac{\partial V}{\partial \hat{z}_2}\lambda_1^M(\lambda_1^M V - (1 + \tau_1)p_1)dG_1(z) - h(\phi_2)\phi_2 S(\hat{z}_2^s)$$

$$+ h(\phi_2)g_2(\hat{z}_2)\lambda_2 \phi_2 \hat{z}_2 \tau_2 + h(\phi_2)\frac{d\lambda_2^M(\hat{z}_2^s)}{d\hat{z}_2^s}\frac{(\phi_2)^2}{\gamma} \int_{\hat{z}_2^s} z(z\lambda_2^M - \hat{z}_2^s(1 + \tau_2))dG_2(z) = 0.$$

By using the market clearing condition, The first three terms can be rewritten as

$$(1 - G(\hat{z}_1^s))\pi_h \frac{\partial \tilde{v}_2^h}{\partial \hat{z}_2^s} + \frac{\partial(\phi_1 \lambda_1^M V)}{\partial \hat{z}_2^s} \underbrace{\int_{z_1^b} \hat{d}_1(z; \hat{z}_1^s)dG_1(z)}_{\equiv D_1} - h(\phi_2)\phi_2 S_2(\hat{z}_2)$$

$$= (1 - G(\hat{z}_1^s))\pi_h \frac{\partial \tilde{v}_2^h}{\partial \hat{z}_2^s} + \lambda_1^M D_1 \frac{\partial \tilde{v}_2^h}{\partial \hat{z}_2^s} + (1 - \lambda_1^M)D_1 \frac{\partial \tilde{v}_2^l}{\partial \hat{z}_2^s} - h(\phi_2)\phi_2 S_2(\hat{z}_2)$$

$$= \lambda_2 \frac{\partial \tilde{v}_2^h}{\partial \hat{z}_2^s} + (1 - \lambda_2)\frac{\partial \tilde{v}_2^l}{\partial \hat{z}_2^s} - h(\phi_2)\phi_2 S_2(\hat{z}_2^s)$$

$$= h(\phi_2)\phi_2 \left[ \lambda_2 G(\hat{z}_2^s) + (1 - \lambda_2) - S_2(\hat{z}_2^s) \right]$$

$$= 0.$$

4

Intuitively speaking, the direct effect on prices should not affect total welfare because change in prices are just transfers. Change in prices affect welfare only through improving allocation. Thus the condition for $\tau_2(\phi_2)$ reduces to

$$g_2(\hat{z}_2)\lambda_2\phi_2\hat{z}_2\tau_2 + \frac{d\lambda_2^M(\hat{z}_2)}{d\hat{z}_2}\frac{(\phi_2)^2}{\gamma}\int_{\hat{z}_2^b} z(z\lambda_2^M - \hat{z}_2^s(1+\tau_2))dG_2(z) = 0,$$

which can be further rearranged as

$$g_2(\hat{z}_2)\lambda_2\phi_2\hat{z}_2\tau_2 + \frac{d\lambda_2^M(\hat{z}_2)}{d\hat{z}_2}\int_{\hat{z}_2^b} \phi_2 z d_2(z, \hat{z}_2^s)dG_2(z) = 0, \tag{A2}$$

This is (17).

Finally, I take into account the possibilities of corner solutions. It is never be optimal to have market collapse at $t = 2$, as it only harms reallocation without any benefit. Therefore $p_2 > 0$ always. It might be optimal to have market collapse at $t = 1$ as it will improve the quality. In this case, we need to replace (A3) to

$$\tau_1(\phi_1\hat{z}_1^s + v_2^h)g_1(\hat{z}_1^s)\pi_h + \frac{\partial\lambda_2}{\partial\hat{z}_s^1}\mathbb{E}_\phi\left[\frac{\partial\lambda_2^M}{\partial\lambda_2}\int_{\hat{z}_2^b} \phi_2 z d_2(z, \hat{z}_2^s)dG_2(z) - \phi_2\hat{z}_2^s\tau_2(1 - G_2(\hat{z}_2))\right] \geq 0. \tag{A3}$$

In this case, we have $\tau_1 \geq 0$, which does not change any of the conclusions drawn in Proposition 7.

*Appendix A.9. Proof of Proposition 7*

Provided in the main text.

## Appendix B. Walrasian Equilibrium and Buyers' Equilibrium

The goal is to derive Equation (6). The buyer's problem for a given price $\hat{z} \equiv p/\phi$ is

$$V^b(\hat{z}) \equiv \max_{k^d \geq 0}\left\{z\lambda^M(\hat{z})k^d - \hat{z}k^d - \frac{1}{\phi}\Gamma(\lambda^M(\hat{z})k^d)\right\}.$$

Buyers do not have any incentive to raise prices beyond competitive equilibrium price, $\hat{z}^*$, if

$$\frac{dV^b(\hat{z})}{d\hat{z}} \leq 0 \tag{B1}$$

for $\hat{z} > \hat{z}^*$. The envelope theorem implies

$$\frac{dV^b(\hat{z})}{d\hat{z}} = z\frac{d\lambda^M(\hat{z})}{d\hat{z}}k^{d*} - k^{d*} - \frac{\gamma}{\phi}\lambda^M(\hat{z})\frac{d\lambda^M(\hat{z})}{d\hat{z}}(k^{d*})^2.$$

Noting that $k^{d*} = \frac{\phi}{(\lambda^M)^2\gamma}(z\lambda^M - \hat{z})$, condition (B1) can be rewritten as

$$\frac{d\lambda^M(\hat{z})}{d\hat{z}}\frac{1}{\lambda^M}\hat{z} - 1 \leq 0,$$

which is equivalent to equation (6). For example, when $G$ is given by standard log-normal and $\pi_h = 0.9$ and $\pi_l = 0.1$, this condition holds as long as $\hat{z}$ is greater than the 9th percentile.

In the case where buyers do not observe quality before installation of capital, the buyers' value for a given price is

$$V^b(\hat{z}) \equiv \max_{k^d \geq 0}\left\{z\lambda^M(\hat{z})k^d - \hat{z}k^d - \frac{1}{\phi}\Gamma(k^d)\right\}.$$

Applying the same argument as above, the buyers do not have any incentive to raise prices if

$$z^{\max}\frac{d\lambda^M(\hat{z})}{d\hat{z}} \leq 1,$$

where $z^{max}$ is the upper bound of $z$.

## Appendix C. Ex-interim Efficiency in the Benchmark Model

Proposition 5 shows that it is sufficient to look for $\tau^{sym}$ instead of looking at tax policy in an economy with asymmetric information. Therefore I ask that starting from a symmetric information economy with tax rate $\tau^{sym*} = \frac{1+\tau-\lambda^{M*}\chi}{\lambda^{M*}\chi}$, whether one can achieve Pareto improvement by changing the tax rate. Obviously, increasing the tax rate does not achieve this goal because it increases the distortion in the economy. In order to bring about Pareto improvement by reducing taxes, it is necessary and sufficient to improve welfare of the entrepreneurs who neither sell nor buy because both buyers and sellers will gain additional benefit. This implies that the equilibrium with asymmetric information is constrained efficient if it is not possible to increase tax revenue by reducing taxes. Letting $T \equiv \tau^{sym}\phi\hat{z}^{sym}\pi_l G(\hat{z}^{sym}/\chi)$ be the total revenue in a symmetric information economy, the sufficient condition for this to be true is

$$\frac{dT}{d\tau^{sym}} \geq 0$$

for all $\tau^{sym} \in [0, \tau^{sym*}]$. Note that this is the exactly the same as the argument behind the Laffer curve in the optimal taxation literature. The condition can be rewritten as

$$(1 + \frac{g(\hat{z}^{sym}/\chi)(\hat{z}^{sym}/\chi)}{G(\hat{z}^{sym}/\chi)})\frac{d\log\hat{z}^{sym}}{d\log\tau^{sym}} \geq -1,$$

where $\frac{d\log\hat{z}^{sym}}{d\log\tau^{sym}} \leq 0$ is the price elasticity with respect to taxes.[12] Conditional on $\frac{d\log\hat{z}^{sym}}{d\log\tau^{sym}}$, the condition is likely to hold when there are smaller masses of entrepreneurs around the cut-off to sell ($g(\hat{z}^{sym}/\chi)$ low), or there is a greater volume of trade (high $G(\hat{z}^{sym}/\chi)$).

## Appendix D. Ex-interim Efficiency in the Extended Model

In this section, I study the ex-interim efficiency (efficiency from the viewpoint of $t = 1$) in the extended model through numerical examples. I ask ex-ante optimal tax policy described in Section 3.3 can bring about Pareto improvement. The following numerical examples show that the answer is sometimes yes.

In this numerical examples, I assume that that realization of $\phi_2$ is deterministic. The distribution of idiosyncratic productivity is assumed be uniform in $[0, 1]$ for both periods. I set $\gamma = 1$, $\pi_l = 0.5$, $\pi_h = 1$, $\phi_1 = 100$, and then I varied $\phi_2$. Figure D.3 shows the welfare of entrepreneurs for different $z$ at $t = 1$. The left panel shows the case of $\phi_2 = 200$. Under this parameterization, the benefit to alleviate adverse selection problems at $t = 2$ is large enough for the planner to find it optimal to ban trade at $t = 1$, while in laissez-faire entrepreneurs trade at $t = 1$ and creates low-quality assets. As is clear from the figure, the ex-ante optimal policy also achieves Pareto improvement. In contrast, the right panel shows the case for $\phi_2 = 50$. In this situation, the benefit of improving market conditions at $t = 2$ is smaller than the previous case. The planner imposes tax rate of $(\tau_1, \tau_2) = (0.33, -0.17)$ to maximize ex-ante welfare, but in this case, it does not achieve the Pareto improvement. These exercises are suggestive that when the gains to alleviate future adverse selection problems are larger, ex-ante optimal policy is more likely to achieve Pareto improvement.

Even when ex-ante optimal policy does not achieve Pareto improvement, it is often possible to find a policy that achieves Pareto improvement. Moreover, tax at $t = 1$ alone might be sufficient to achieve Pareto improvement. Figure D.4 shows an example of such a case. Here, I set $\phi_1 = 100$. The left panel compares welfare under the ex-ante optimal tax with the welfare under laissez-faire. In this case, the ex-ante optimal tax does not achieve Pareto improvement. However, the right panel shows that the small tax rate at $t = 1$ alone achieves Pareto improvement.

---

[12]The expression for $\frac{d\log\hat{z}^{sym}}{d\log\tau^{sym}}$ is

$$\frac{d\log\hat{z}^{sym}}{d\log\tau^{sym}} = \frac{-(1 - G(\hat{z}^{sym}(1+\tau^{sym}))\frac{\phi}{\gamma}\hat{\tau}^{sym}}{(1 - G(\hat{z}^{sym}(1+\tau))\frac{\phi}{\gamma}(1+\tau^{sym}) + \pi_h\frac{1}{\chi^2}g(\hat{z}^{sym}/\chi)}$$
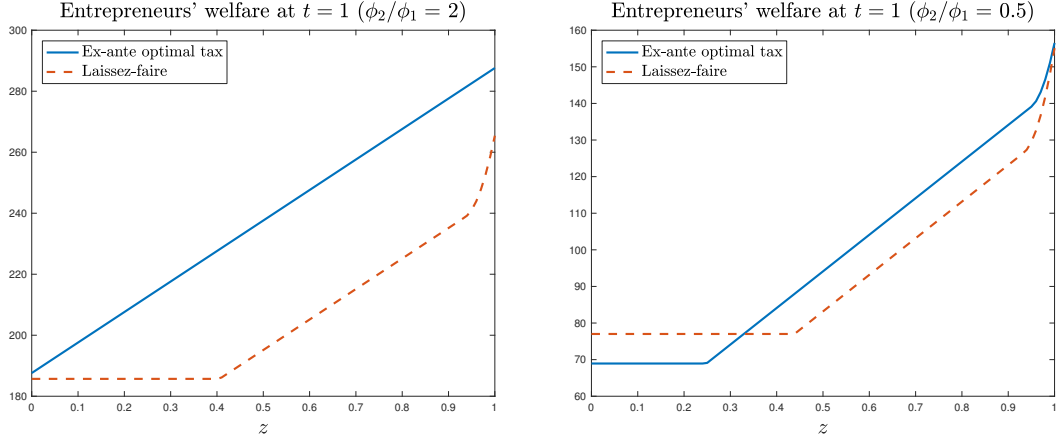
Figure D.3: Welfare with and without taxes

*Note:* Each panel shows the welfare of entrepreneur for each $z$ at $t = 1$ under the optimal trade tax (blue solid line) and under laissez-faire (red dashed line). The left panel is the case for high $\phi_2$, and the resulting optimal tax is high enough to ban trade at $t = 1$ with the optimal tax rate $\tau_1 = 67\%$ and $\tau_2 = 0\%$. The right panel shows the case for low $\phi_2$, and the resulting optimal taxes are $\tau_1 = 33\%$ and $\tau_2 = -17\%$
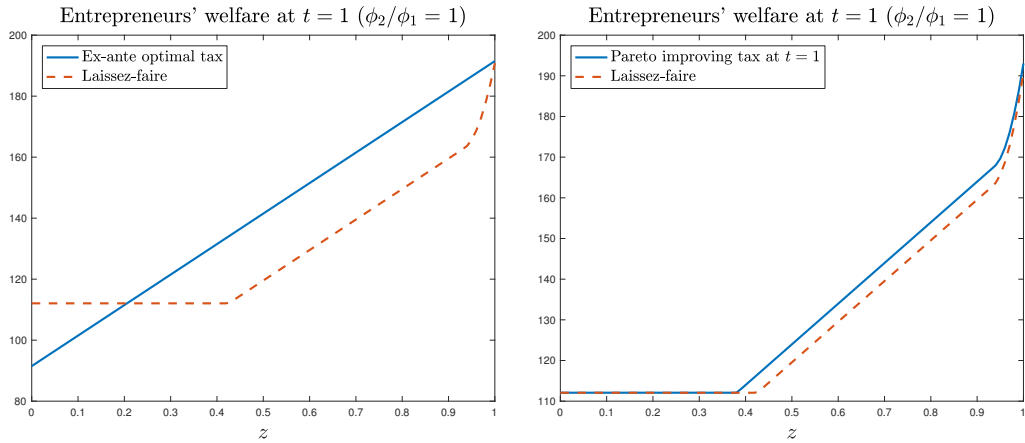


Figure D.4: Ex-ante Optimal Tax and Pareto Improving Tax

*Note:* The left panel compares welfare under the ex-ante optimal tax with welfare under laissez-faire. The tax rates are $\tau_1 = 97\%$ and $\tau_2 = 0\%$. The right panel shows that the Pareto improvement is possible only with small tax at $t = 1$ ($\tau_1 = 5\%, \tau_2 = 0\%$).