Business Cycles

EC502 Macroeconomics
Topic 9

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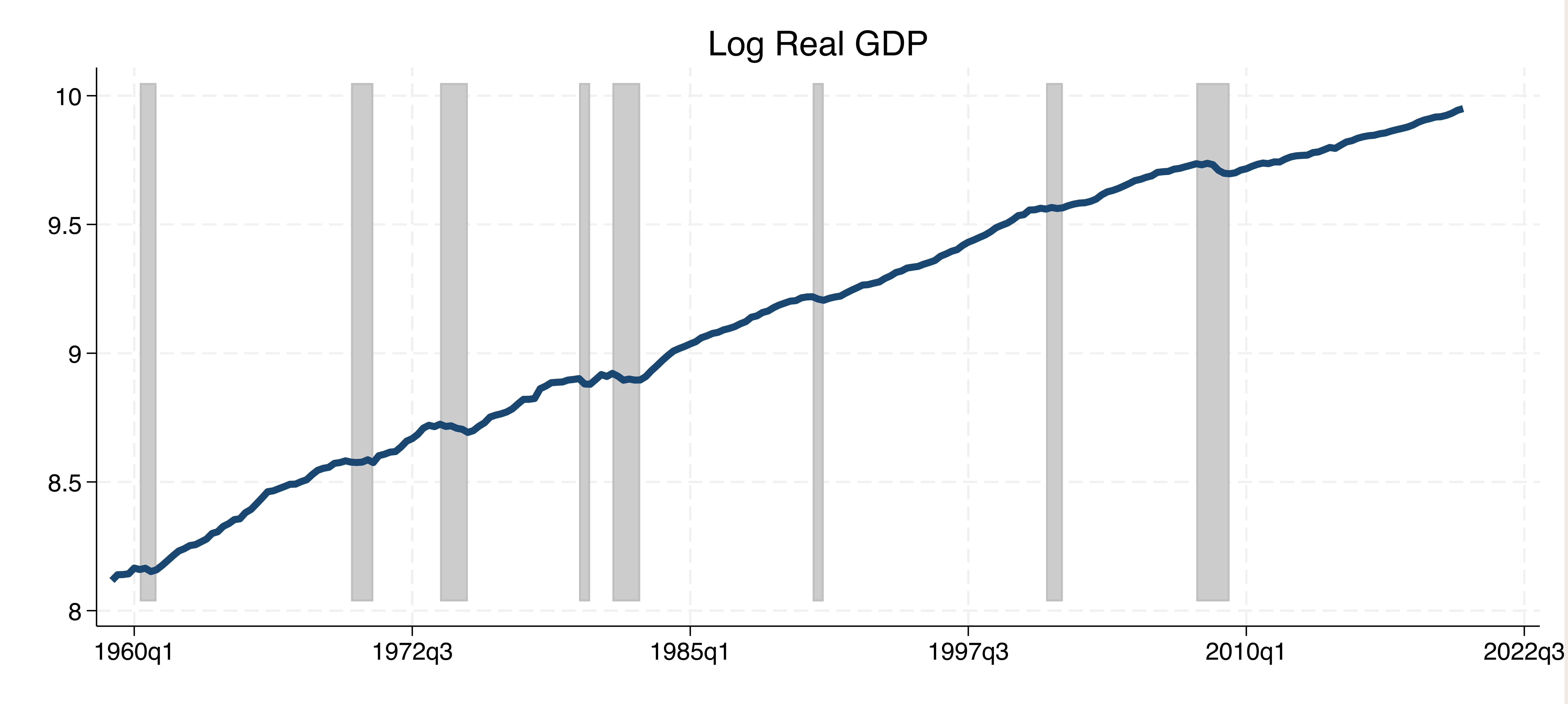
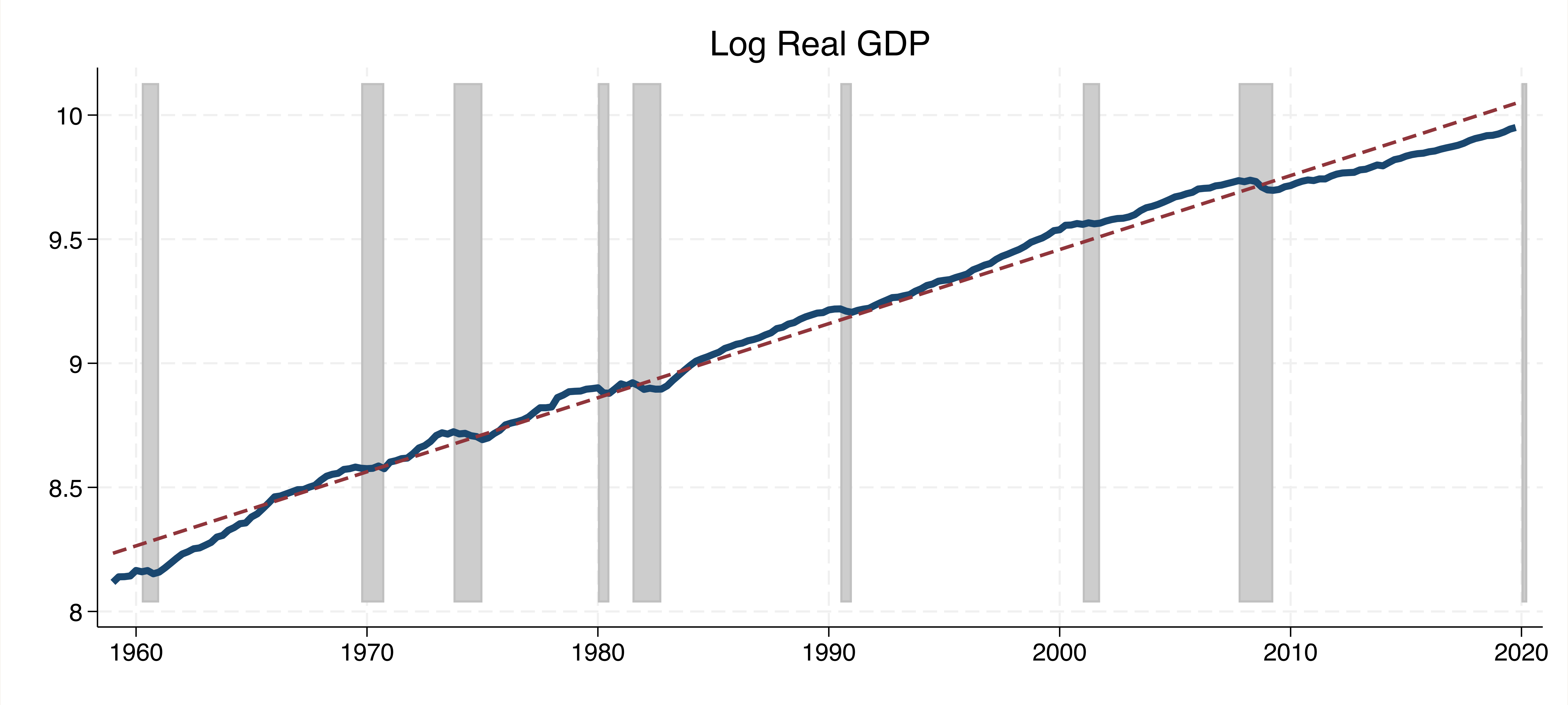
2024 Spring

Business Cycles



- Business cycles: joint movements of economic activity at **medium** frequency
- How do we extract medium frequency from the data?

Linear Trend

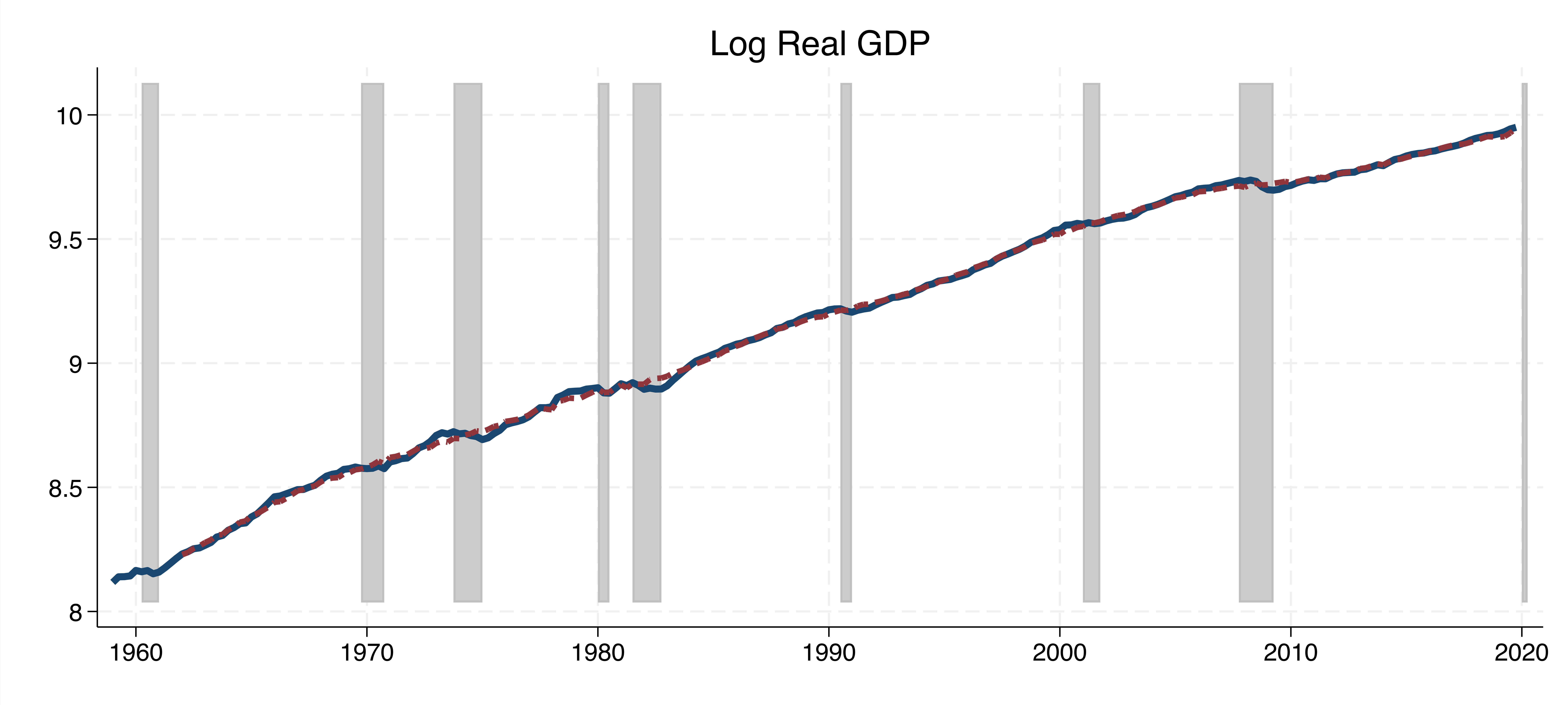


- One option is to detrend using a linear trend
- One may argue this is not very sensible

Baxter-King Bandpass Filter

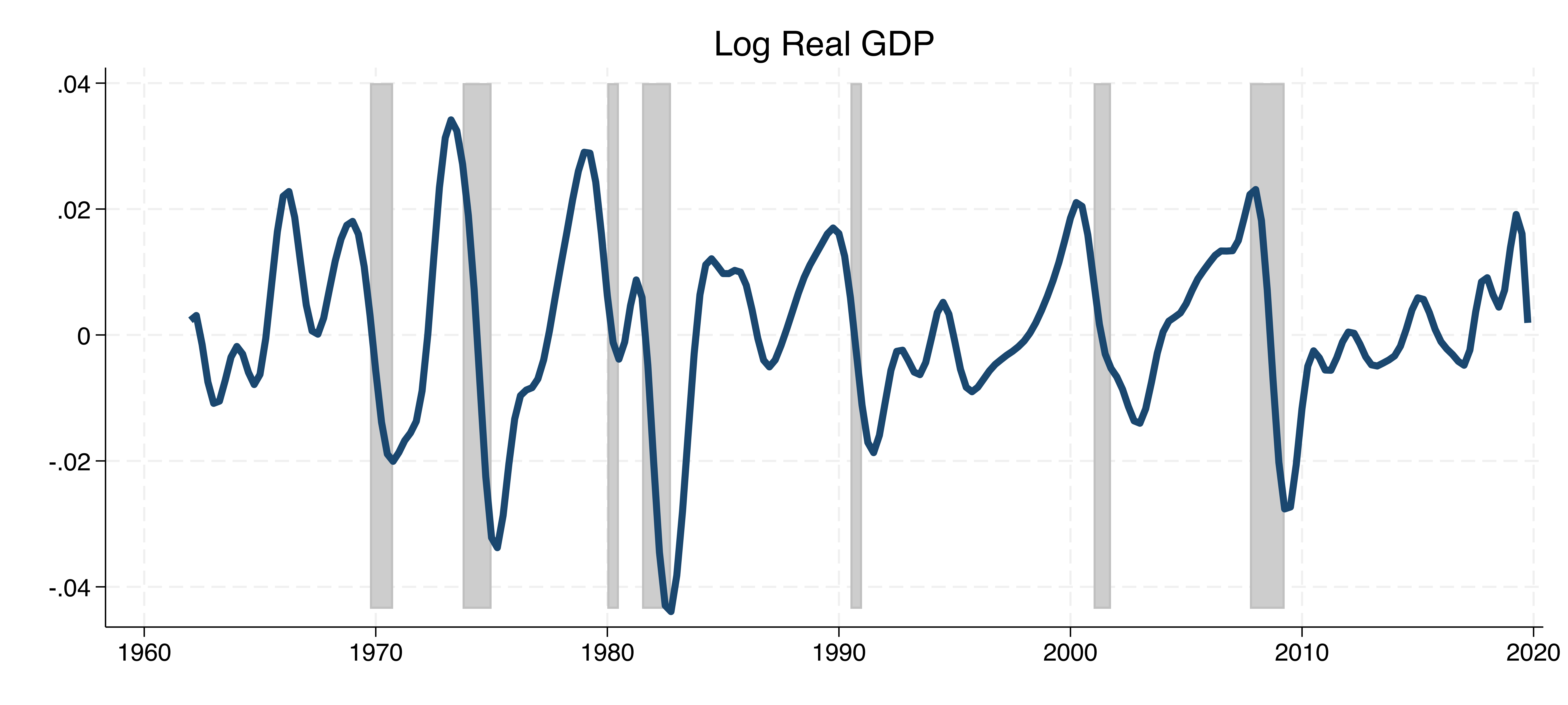
- There are various ways to isolate medium-frequency movements
- We will focus on Baxter-King bandpass filter (because we are at BU!)
- Statistical procedure to distinguish medium- and low-frequency movements
- Popular alternative: Hodrick-Prescott filter

Trend using Baxter-King Filter



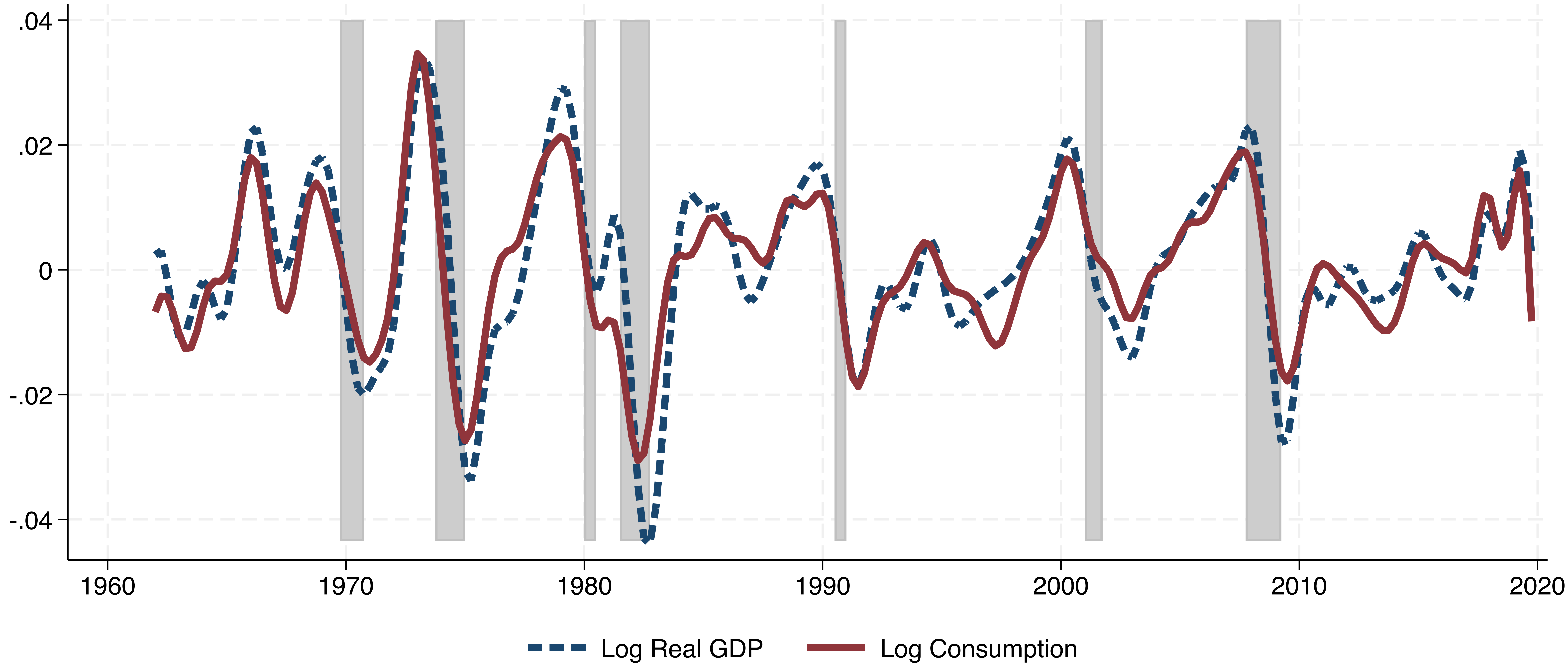
- Use Baxter-King filter to extract low-frequency (more than 8 years) components

Business Cyclical Components of GDP

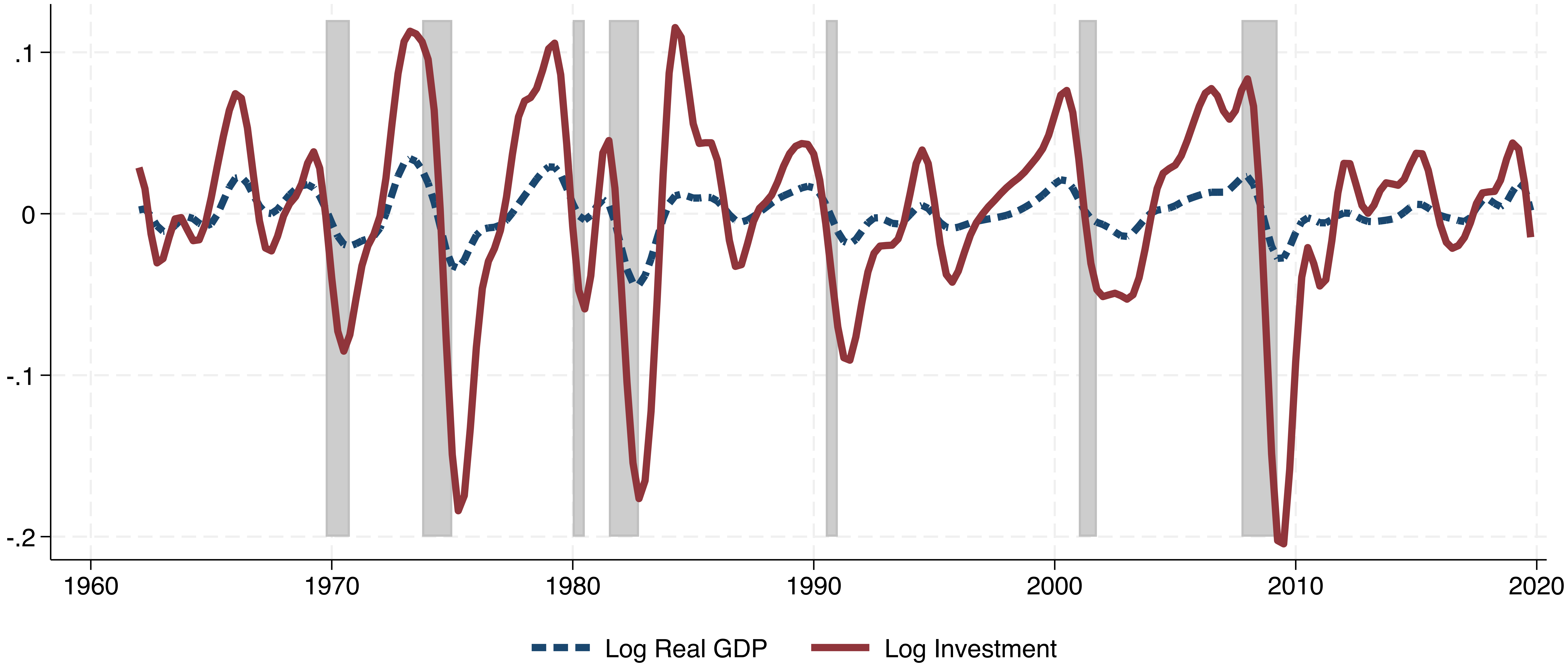


- Baxter-King filter to extract medium-frequency movements (1.5-8 years)

Consumption

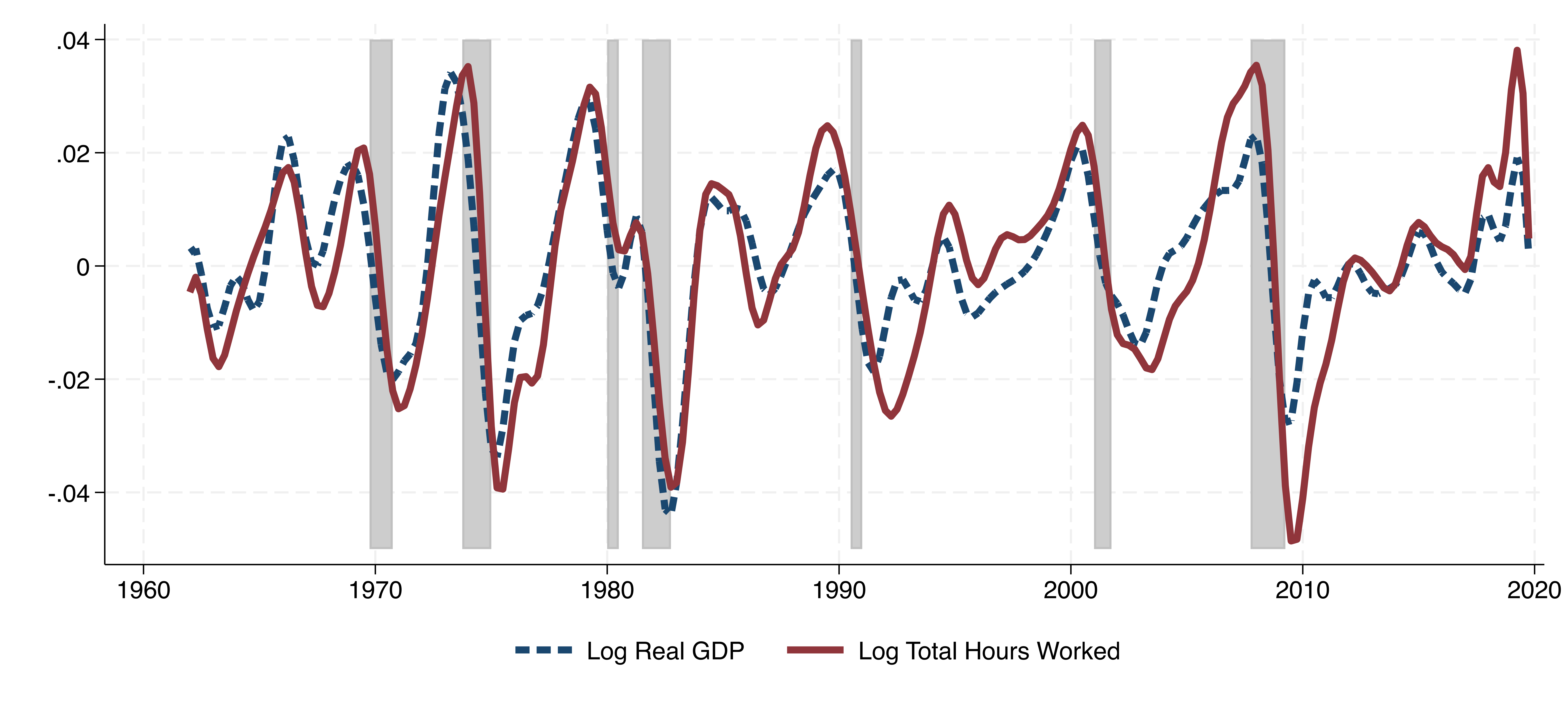


Investment

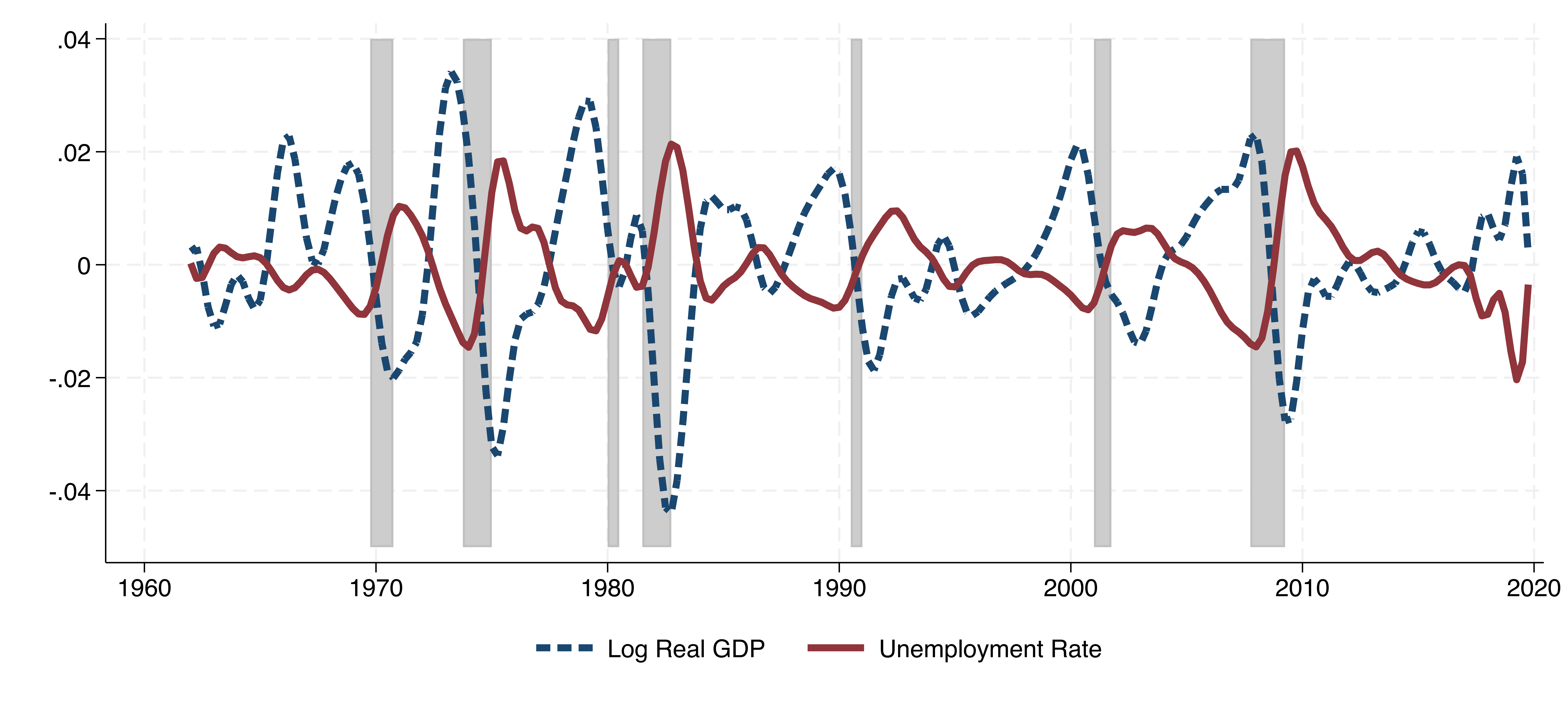


.1
0
-.1
-.2
1960
1970
1980
1990
2000
2010
2020
Log Real GDP
Log Investment

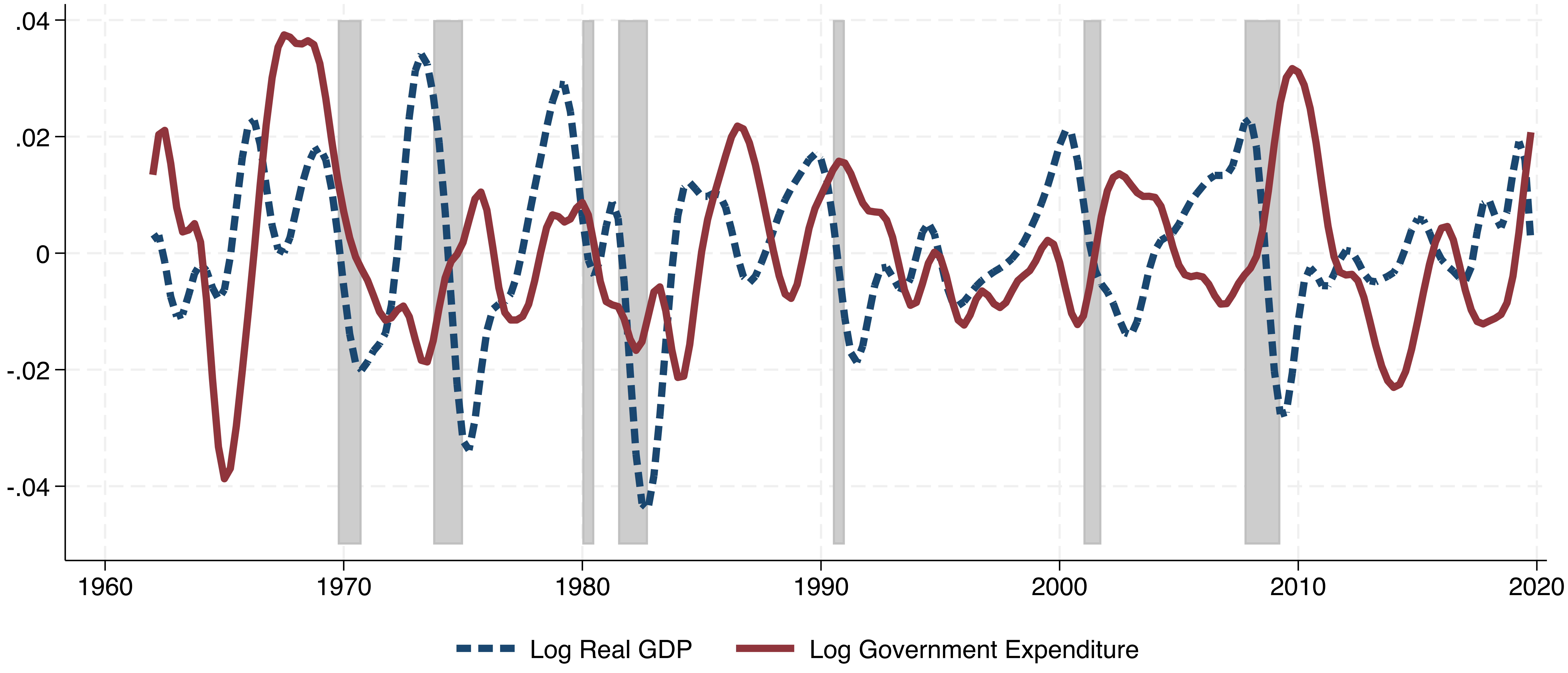
Hours Worked



Unemployment Rate



Log Government Expenditure



In Search of Unified Explanation

One is led by the facts to conclude that, with respect to the qualitative behavior of comovements among series, business cycles are all alike.

To theoretically inclined economists, this conclusion should be attractive and challenging, for it suggests the possibility of a unified explanation of business cycles.

— Robert Lucas (1977)

Goal

Search for a theory that explains

1. Positive comovements between Y, C, I, L

2. $\text{std}(\log I) \gg \text{std}(\log Y) \approx \text{std}(\log C) \approx \text{std}(\log L)$

Methodology

- We will build a model of the macroeconomy that endogeneizes
  1. labor supply
  2. consumption
  3. investment
- At this point, we already have all the tools. We learned theories of
  1. labor supply
  2. consumption
  3. investment
- Now we put everything together in one model!

Real Business Cycle Theory with Two-Periods

Setup

- Two-periods, $t = 0,1$
- The economy is populated by
  1. a continuum of identical households: consume, save, & supply labor
  2. a continuum of identical firms: hire labor & invest

Households

- Households earn labor income and profits income (households own firms)
- We assume labor supply at $t = 1$ is exogenous (simplification)
- Households have the following preferences

$$u(C_0) - v(l_0) + \beta u(C_1) \tag{1}$$

- The budget constraints are

$$C_0 + a_0 = w_0 l_0 + D_0 \tag{2}$$

$$C_1 = (1 + r)a_0 + w_1 l_1 + D_1 \tag{3}$$

- Given (r, w_0, w_1), households choose $\{C_0, C_1, l_0, a_0\}$ to maximize (1) s.t. (2)-(3)

Firms

- The firms solve the same problem as in the previous lecture note

$$\max_{L_0, I_1, K_1, L_1} D_0 + \frac{1}{1+r} D_1$$

subject to

$$D_0 = F_0(K_0, L_0) - w_0 L_0 - I_0 - \Phi(I_0, K_0)$$

$$D_1 = F_1(K_1, L_1) - w_1 L_1$$

$$K_1 = (1 - \delta) K_0 + I_0$$

Market Clearing Conditions

- Unlike before, now all prices, (r, w_0, w_1) are endogenous
- How are they pinned down? demand = supply
- Market clearing conditions:

$$C_0 + I_0 + \Phi(I_0, K_0) = F_0(K_0, L_0)$$

$$C_1 = F_1(K_1, L_1)$$

$$l_0 = L_0$$

$$l_1 = L_1$$

- When all prices are endogenous, we call it as “general equilibrium model”

Equilibrium Definition

1. Given $\{r, w_0, w_1\}$, households optimally choose $\{C_0, C_1, l_0, l_1, a_0\}$
2. Given $\{r, w_0, w_1\}$, firms optimally choose $\{I_0, L_0, L_1, K_1\}$
3. Markets clear

Functional Form Assumptions

- We will impose the following familiar functional form assumptions

$$u(C) = \frac{C^{1-\sigma}}{1-\sigma}$$

$$v(l) = \bar{v}\frac{l^{1+\nu}}{1+\nu}$$

$$F_t(K, L) = A_t K_t^{\alpha} L_t^{1-\alpha}$$

$$\Phi(I, K) = \frac{\phi}{2}\left(\frac{I}{K}\right)^2 K$$

Characterizing Equilibrium

Optimality Conditions

- The housheolds' optimal choice of labor supply imlies

$$\bar{v}l_0^{\nu} = w_0 C_0^{-\sigma} \tag{4}$$

- The households' optimal consumption-saving decision implies

$$C_0^{-\sigma} = \beta(1+r)C_1^{-\sigma} \tag{5}$$

- The firm's optimal labor demand:

$$w_t = (1-\alpha)A_t K_t^{\alpha} L_t^{-\alpha} \tag{6}$$

- The firm's optimal investment:

$$1 + \phi\frac{I_0}{K_0} = \frac{1}{1+r}\alpha K_1^{\alpha-1} L_1^{1-\alpha} \tag{7}$$

- Impose $l_0 = L_0$ and substitute (6) into (4) to obtain

$$\bar{v} L_0^{\alpha+\nu} = (1-\alpha) A_0 K_0^{\alpha} C_0^{-\sigma} \tag{8}$$

- Solve (7) for $1 + r$ and substitute it into (5) to obtain

$$C_0^{-\sigma} = \beta \frac{\alpha A_1 K_1^{\alpha-1} L_1^{1-\alpha}}{1 + \phi I_0 / K_0} C_1^{-\sigma} \tag{9}$$

- Recall the goods market clearing conditions and the evolution of capital stock are

$$C_0 + I_0 + \frac{\phi}{2} \frac{I_0^2}{K_0} = Y_0 \tag{10}$$

$$Y_0 = A_0 K_0^{\alpha} L_0^{1-\alpha} \tag{11}$$

$$C_1 = A_1 K_1^{\alpha} L_1^{1-\alpha} \tag{12}$$

$$K_1 = (1-\delta) K_0 + I_0 \tag{13}$$

- $\{C_0, C_1, I_0, L_0, K_1, Y_0\}$ solve (8)-(13)

Four Equations with Four Unknowns (C_0, I_0, Y_0, L_0)

- Plugging (12) & (13) into (9) gives a relationship btwn C_0 & I_0, which we write as $\hat{C}_0(I_0)$:

$$C_0^{-\sigma} = \beta \frac{\alpha A_1 \left[K_0(1-\delta)+I_0\right]^{\alpha-1} L_1^{1-\alpha}}{1+\phi I_0/K_0} \left(A_1 \left[K_0(1-\delta)+I_0\right]^{\alpha} L_1^{1-\alpha}\right)^{-\sigma} \tag{13}$$

- Eq. (8) is giving a relationship between c_0 and L_0:

$$\bar{v} L_0^{\alpha+\nu} = (1-\alpha) A_0 K_0^{\alpha} C_0^{-\sigma} \tag{8}$$

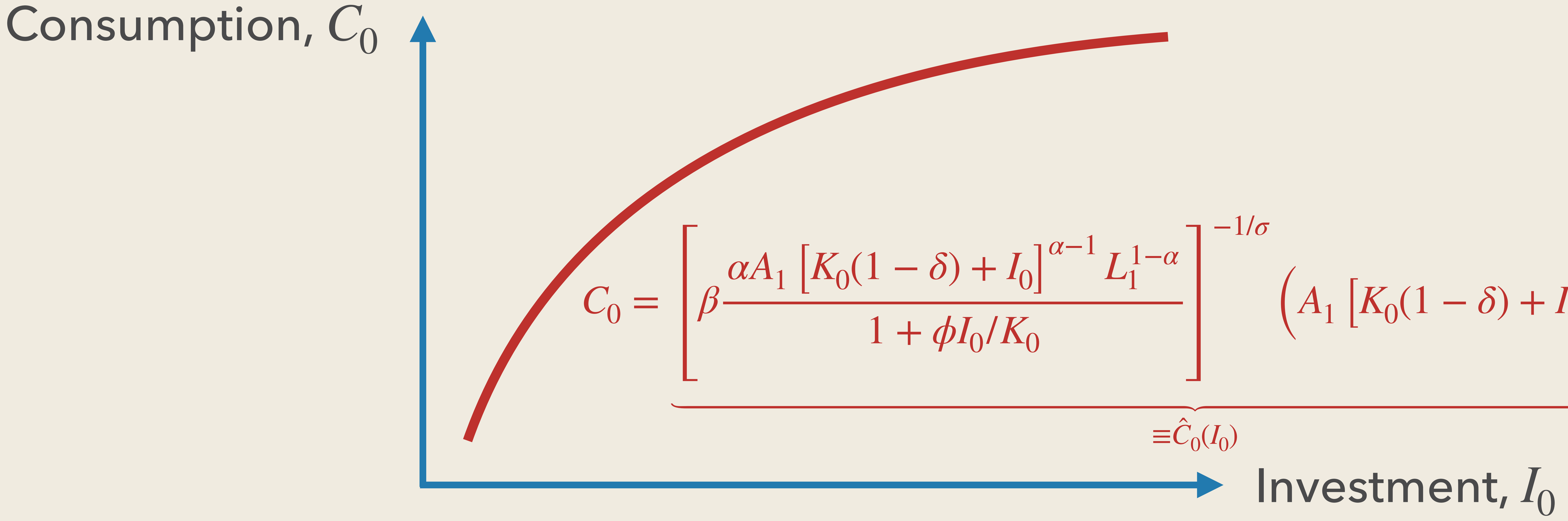
- Use (13) to rewrite (10), which gives a relationship between Y_0 and I_0:

$$\hat{C}_0(I_0) + I_0 + \frac{\phi}{2}\frac{I_0^2}{K_0} = Y_0 \tag{14}$$

- Eq. (11) gives a relationship between Y_0 and L_0:

$$Y_0 = A_0 K_0^{\alpha} L_0^{1-\alpha} \tag{11}$$

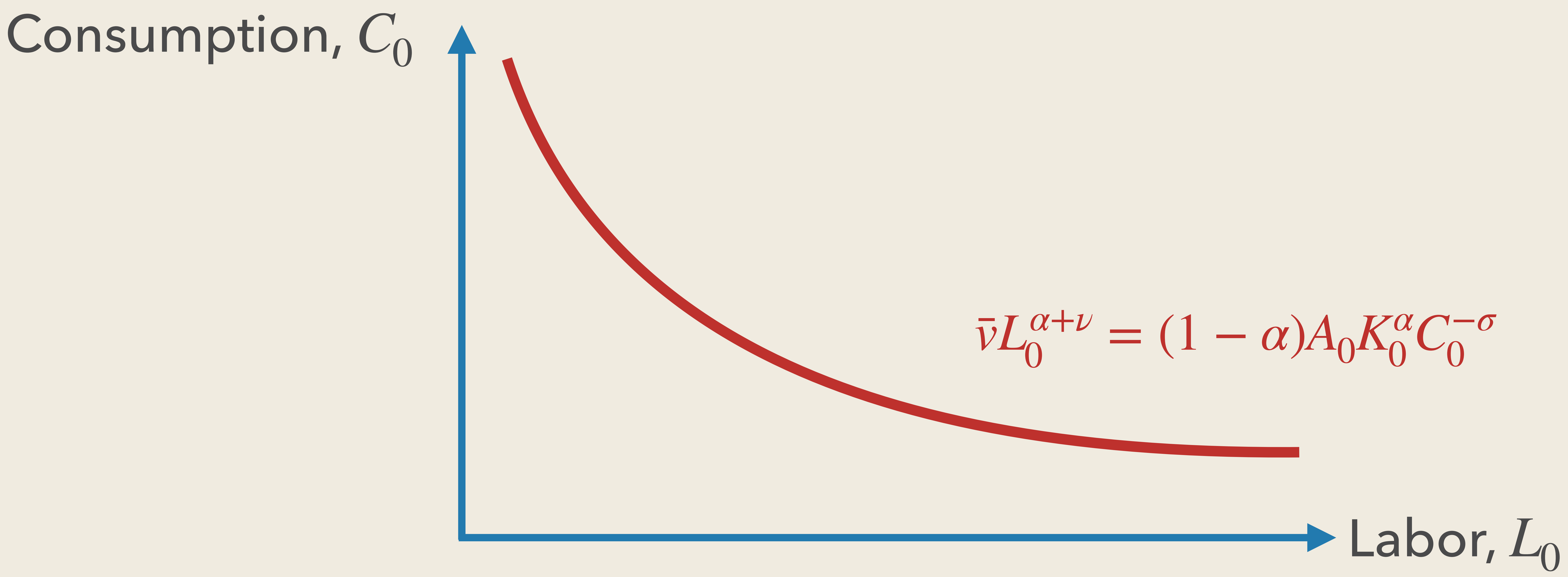
Euler Equation



$$C_0 = \underbrace{\left[\beta \frac{\alpha A_1 \left[K_0(1-\delta)+I_0\right]^{\alpha-1} L_1^{1-\alpha}}{1+\phi I_0/K_0}\right]^{-1/\sigma} \left(A_1 \left[K_0(1-\delta)+I_0\right]^{\alpha} L_1^{1-\alpha}\right)}_{\equiv \hat{C}_0(I_0)}$$

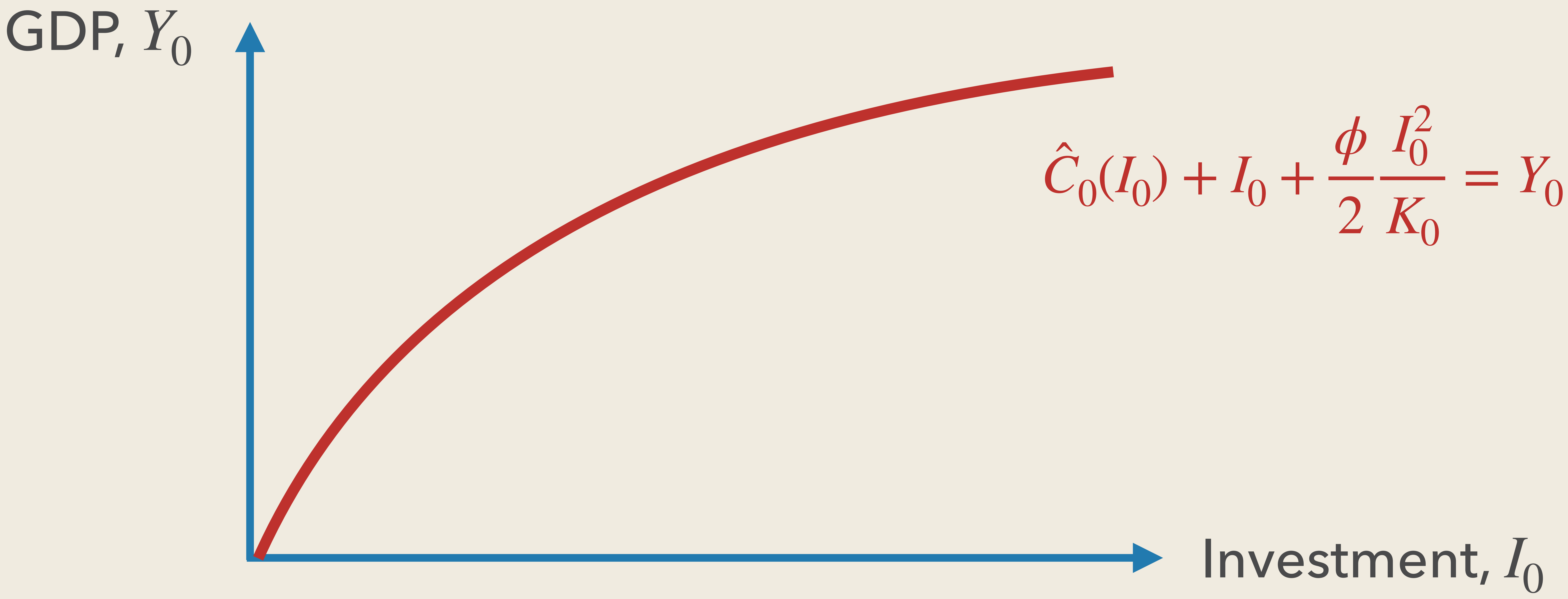
- Eq. (13) defines an increasing relationship between C_0 and I_0
  - If firms invest more, future consumption will be higher
  - Consumption smoothing implies today's consumption will be also higher

Labor Supply



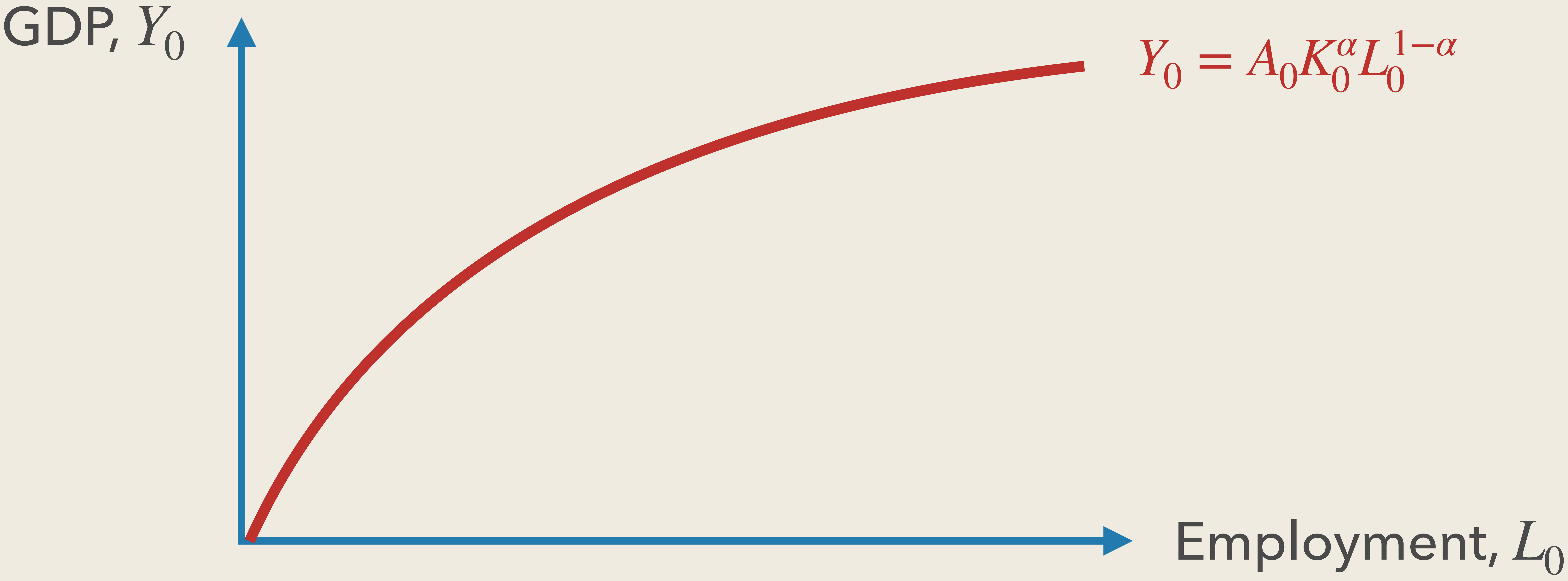
- Eq. (8) defines a decreasing relationship between C_0 and L_0
  - As households consume more, marginal utility of consumption declines
  - This discourages households to work (income effect)

Market Clearing



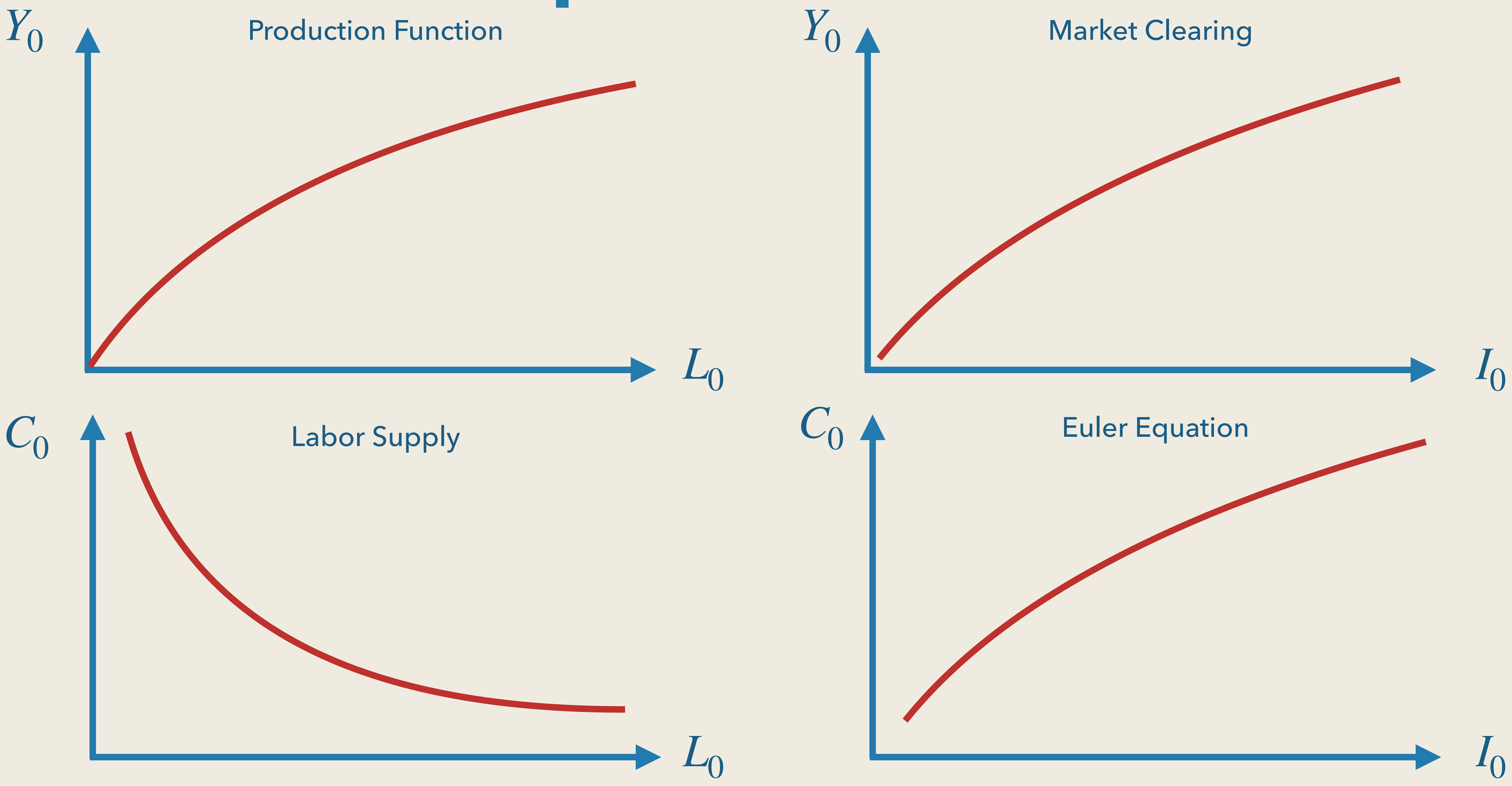
- Eq. (14) defines an increasing relationship between Y_0 and I_0
  - More investment leads to higher consumption today
  - In order to sustain higher consumption and investment, output needs to be higher

Production Function

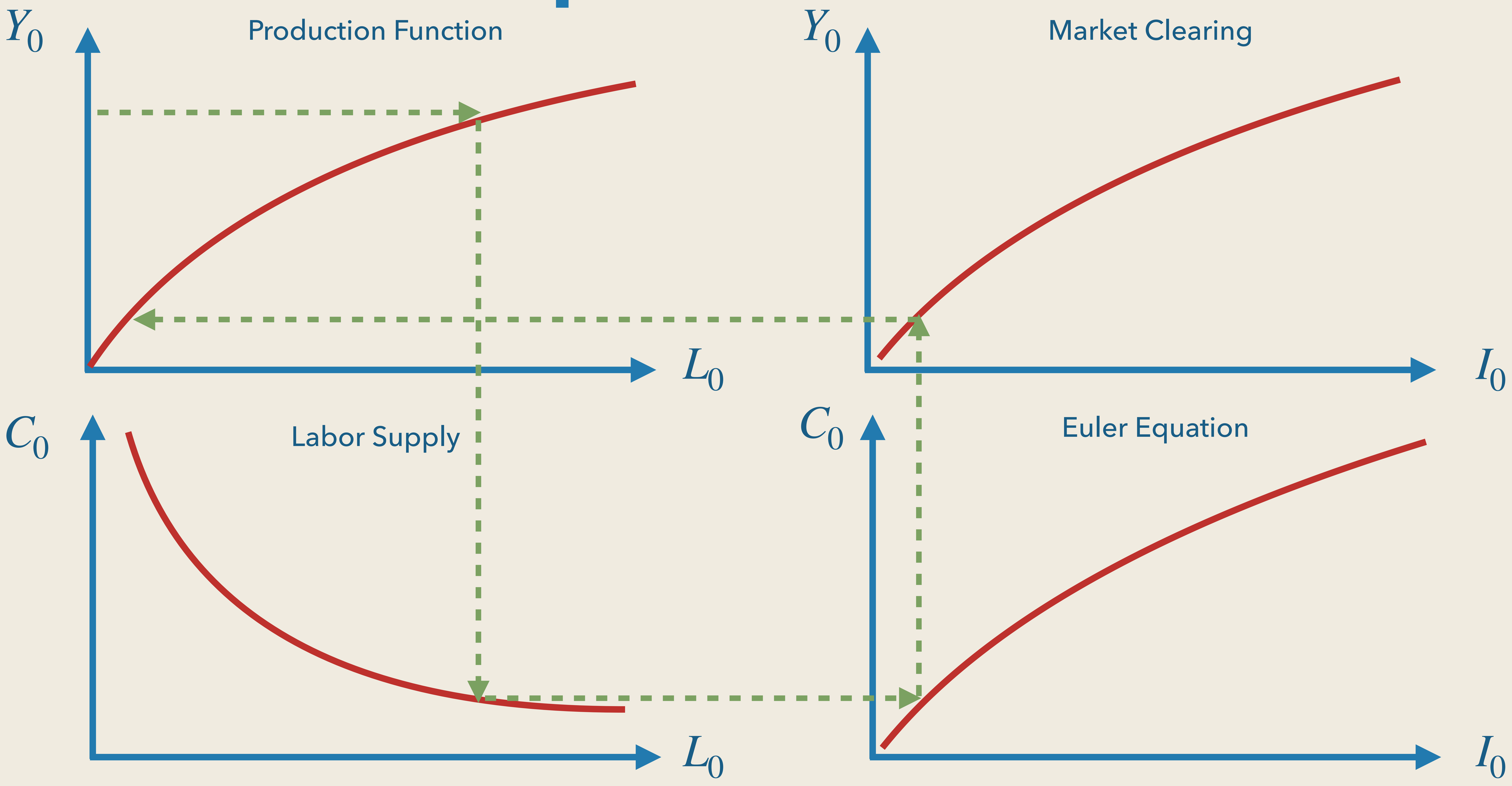


- Eq. (11) defines an increasing relationship between Y_0 and L_0
    - More employment leads to more production

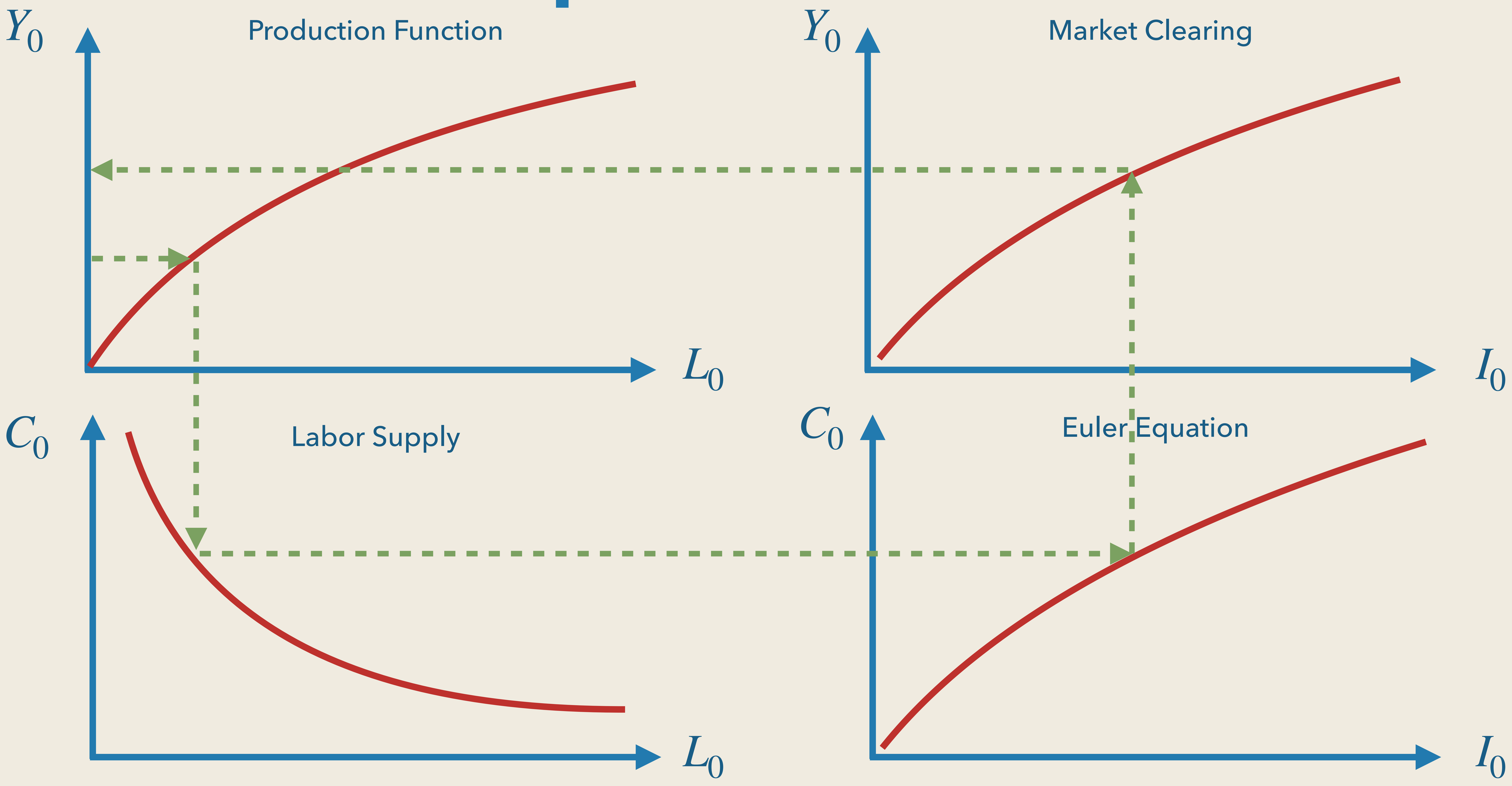
Graphical Solution



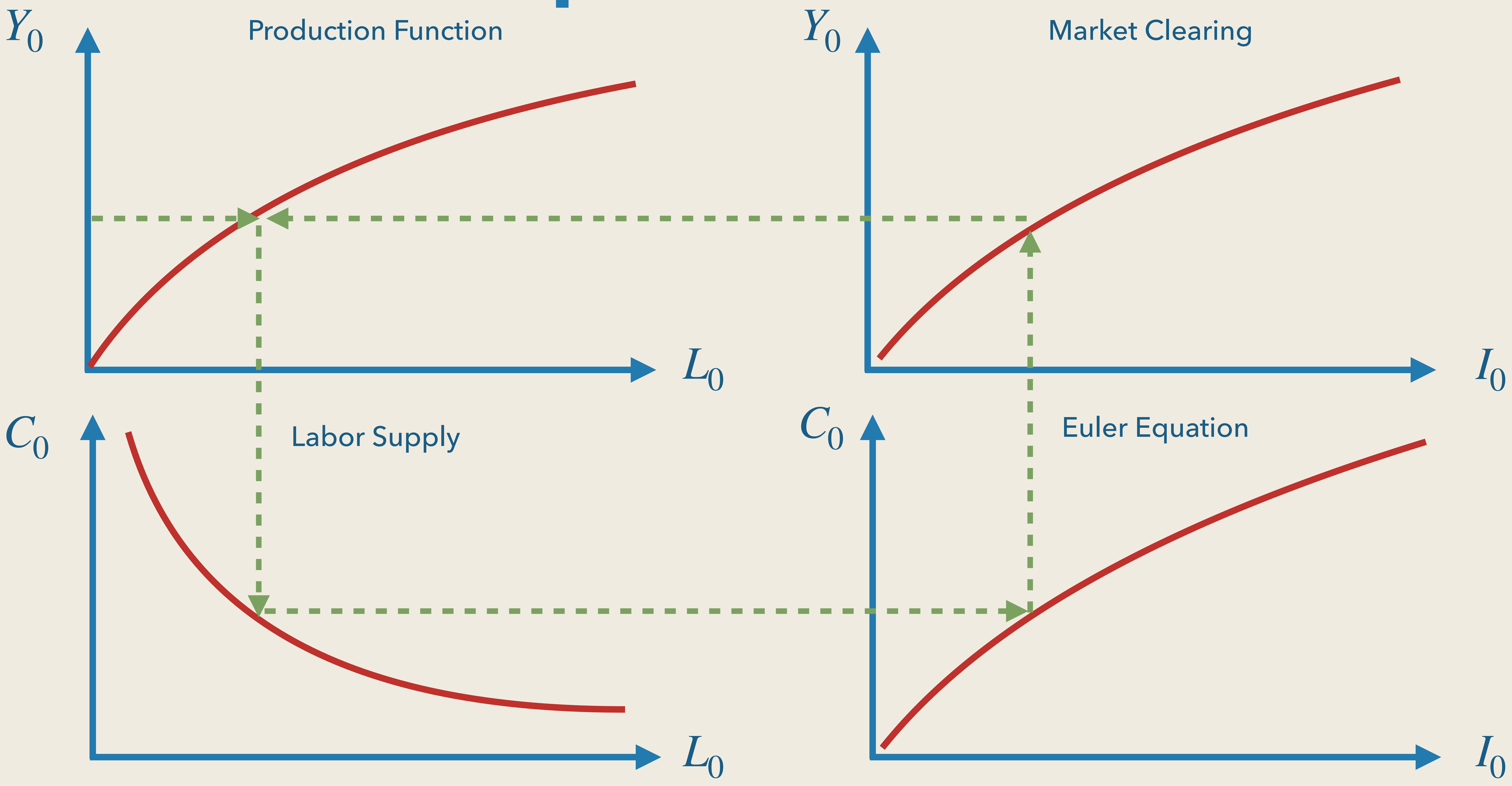
Graphical Solution



Graphical Solution



Graphical Solution



What Drives Business Cycles?

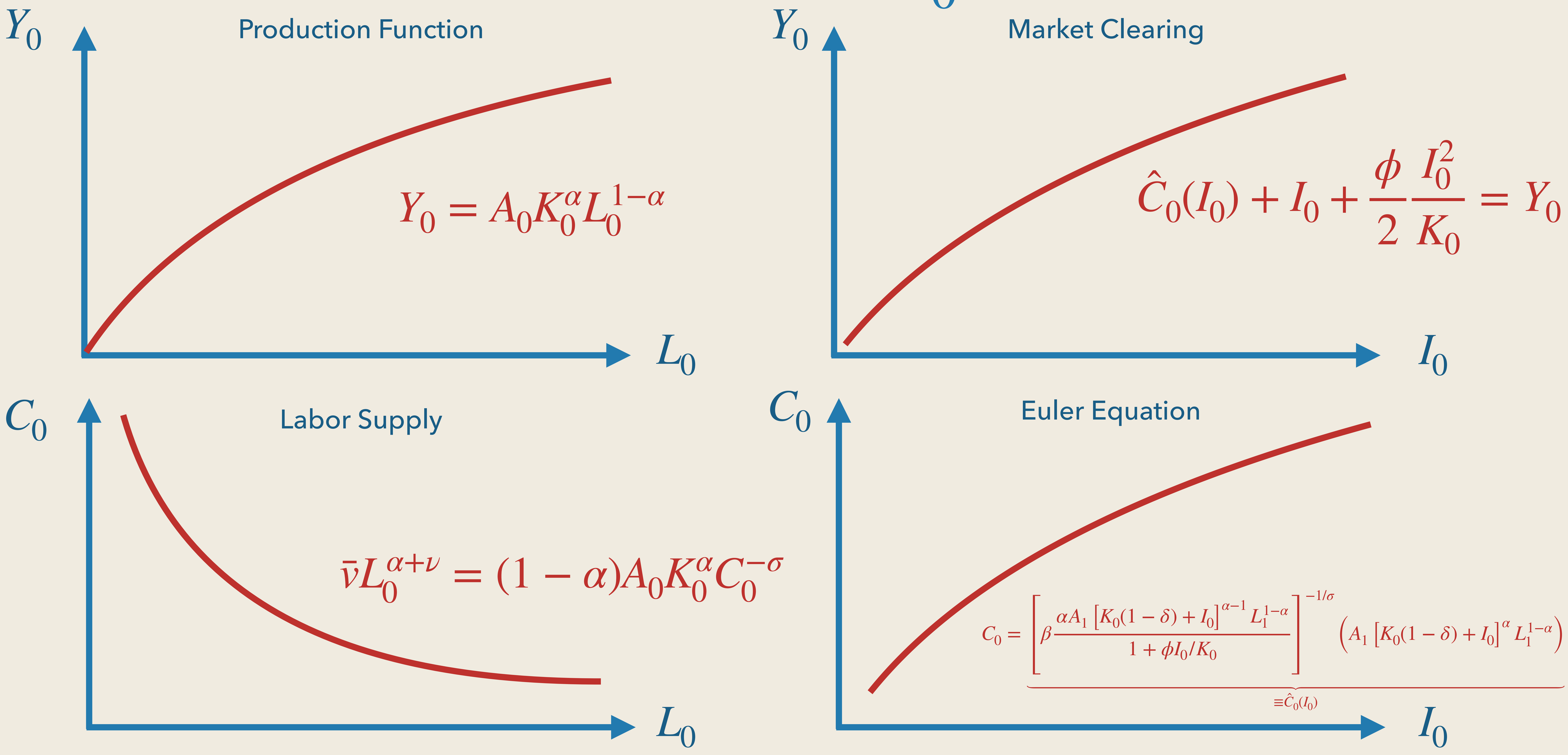
Exercise

- We consider various "shocks" to our economy
  - Shocks: exogenous changes in some aspects of the economy
- We then study how (C_0, I_0, L_0, Y_0) endogenously respond to the shocks
- Ask: do the endogenous responses look like business cycles?

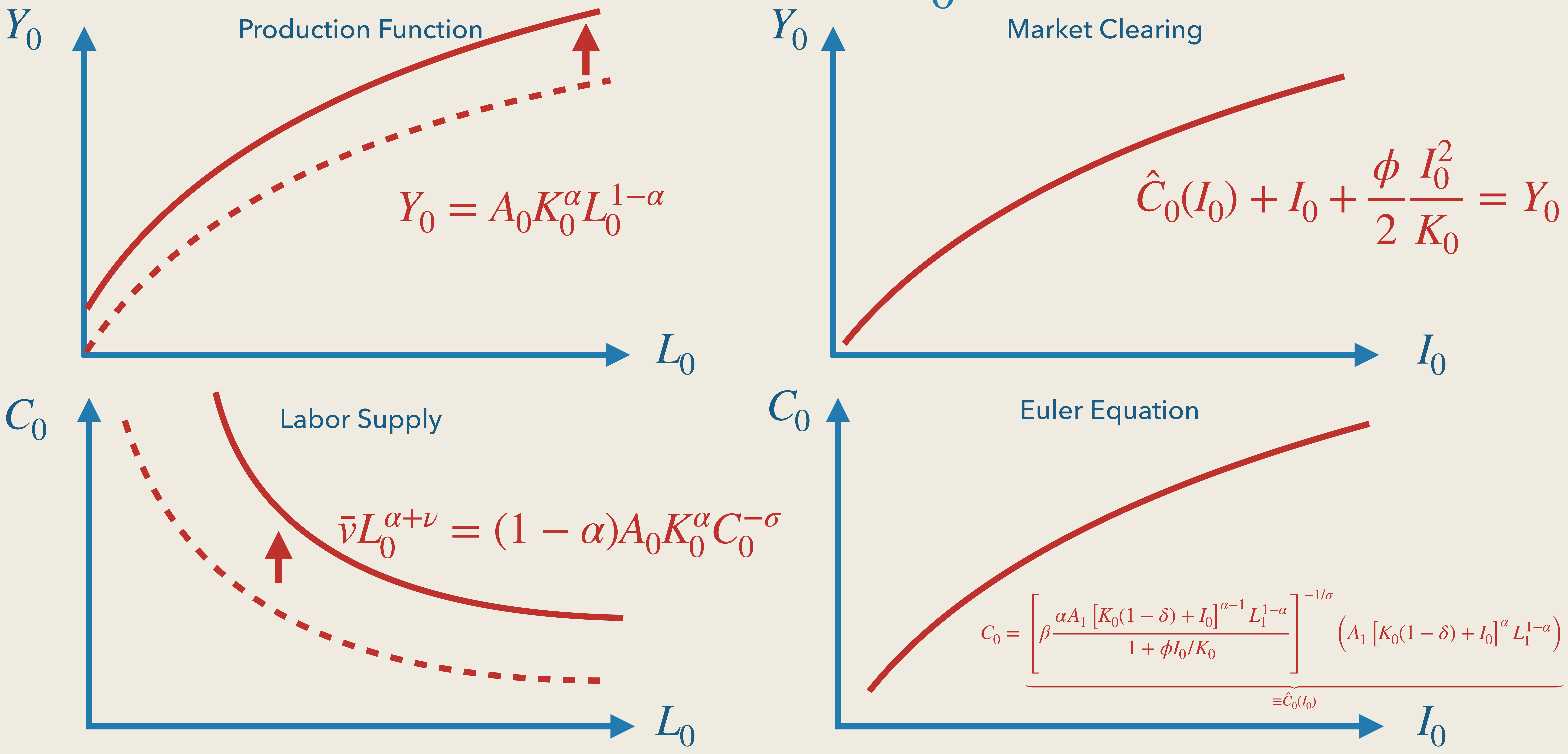
Shocks

- What shocks should we study?
- Let us explore various possibilities
  1. Changes in productivity today, A_0
  2. Changes in productivity in the future, A_1
  3. Changes in households' discount factor, β
  4. Changes in firms' desire to invest, ϕ
  5. Changes in households' incentive to work, $\bar{v}$

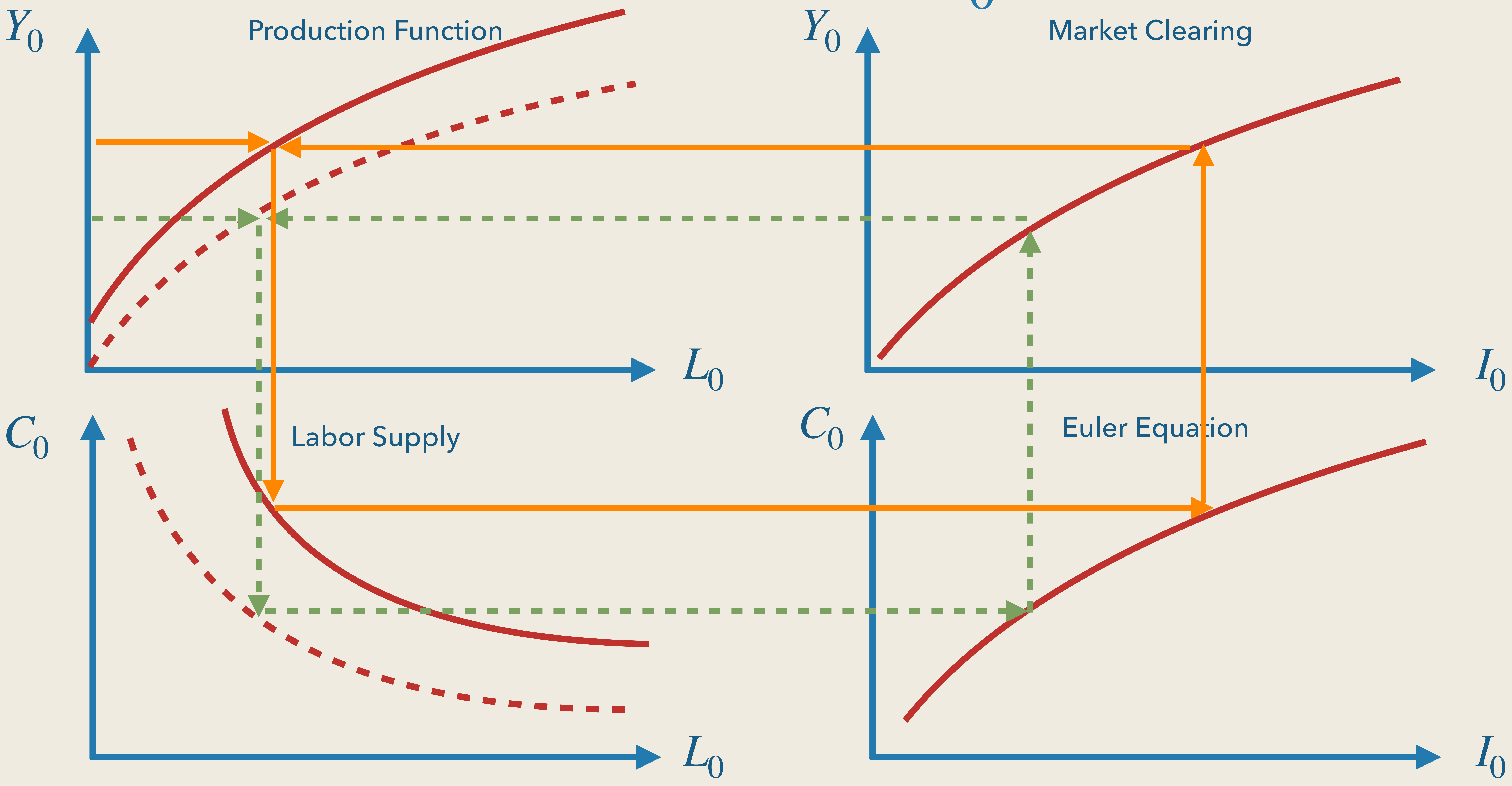
Increase in A_0



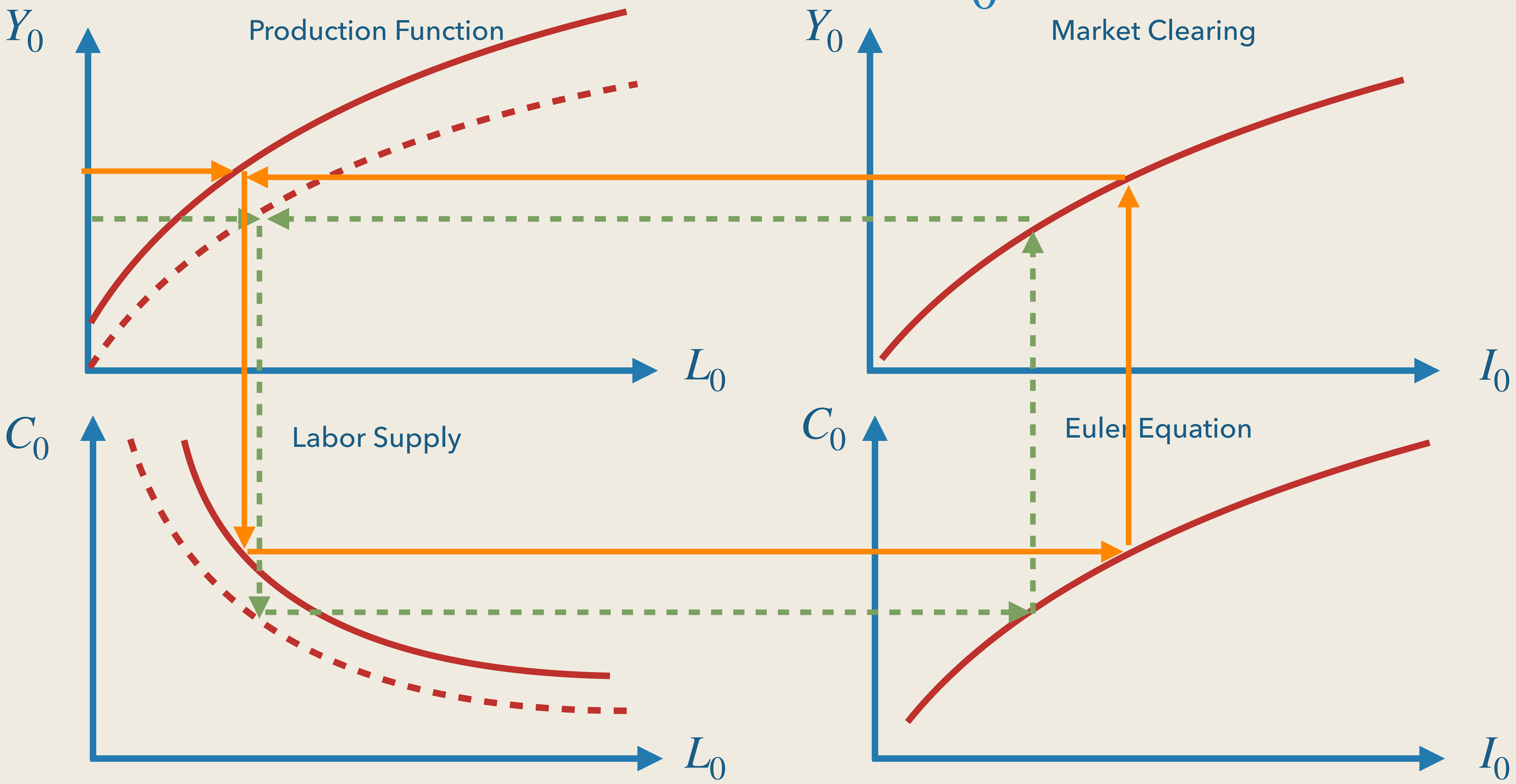
Increase in A_0



Increase in A_0



Increase in A_0



Changes in Today's Productivity

- An increase in A_0 increases GDP, consumption, and investment
- The impact of hours worked is generally ambiguous
- What is the mechanism?
  1. An increase in $A_0 \Rightarrow$ increases w_0 and $D_0 \Rightarrow$ Households are wealthier
  2. Households increase consumption both at time 0 and 1 (consumption smoothing)
  3. In order to increase consumption at $t = 1$, households need to save: $Y - C$ go up
  4. In order $Y - C$ to go up, I must go up because $Y - C = I + \Phi$
  5. Because w_0 increases, substitution effect increases labor supply
  6. Because (w_0, D_0) increase, income effect decreases labor supply
- As long as σ is not too large, we can show that 5 dominates 6
  $\Rightarrow$ hours worked increases

Real Business Cycle

- Therefore, an increase in A_0 produces something that looks like business cycles!
  - Booms are the time when productivity is high
  - Recessions are the time when productivity is low
- This is called the Real Business Cycle model
  - Kydland and Prescott won Nobel Prize for developing this in 2004

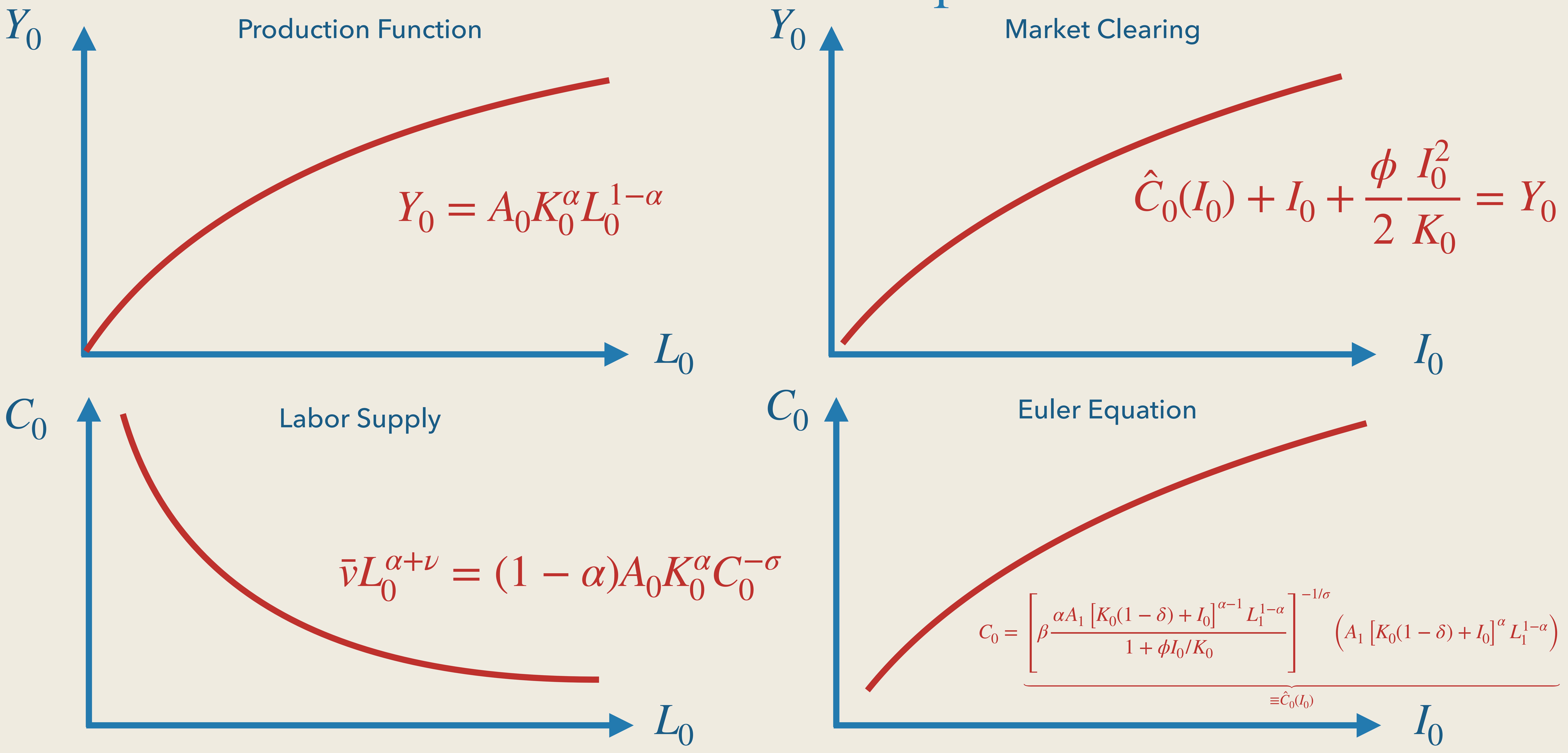
Summary

	Y	C	I	L
$A_0 \uparrow$	$\uparrow$	$\uparrow$	$\uparrow$	$\uparrow$
$A_1 \uparrow$				
$\beta \uparrow$				
$\phi \uparrow$				
$\bar{v} \uparrow$				

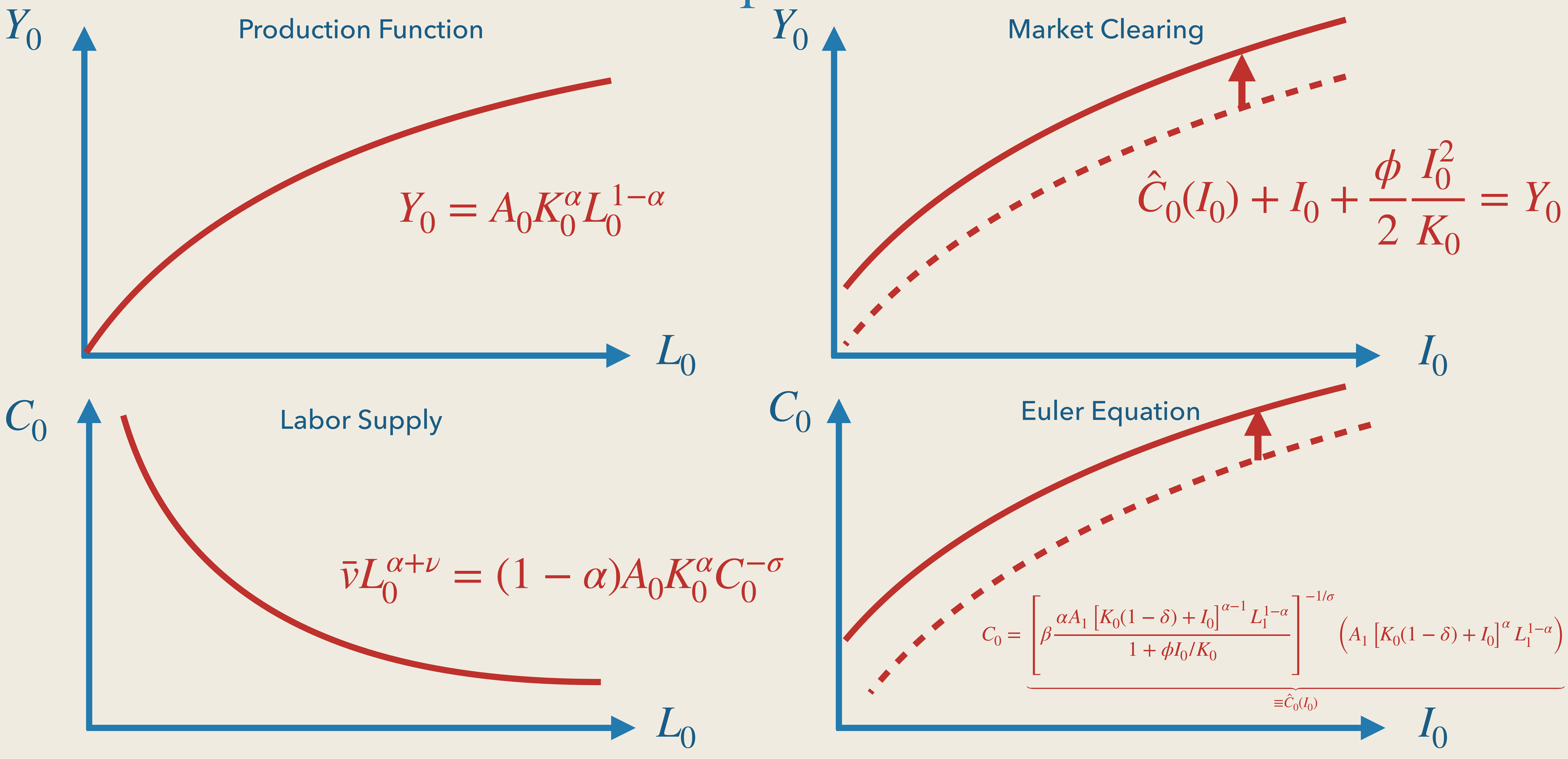
Optimism and Pessimism

- What about other shocks?
- What if we shock future productivity A_1?
  - Booms are the time when people expect future to be bright (optimistic)
  - Recessions are the time when people are pessimistic

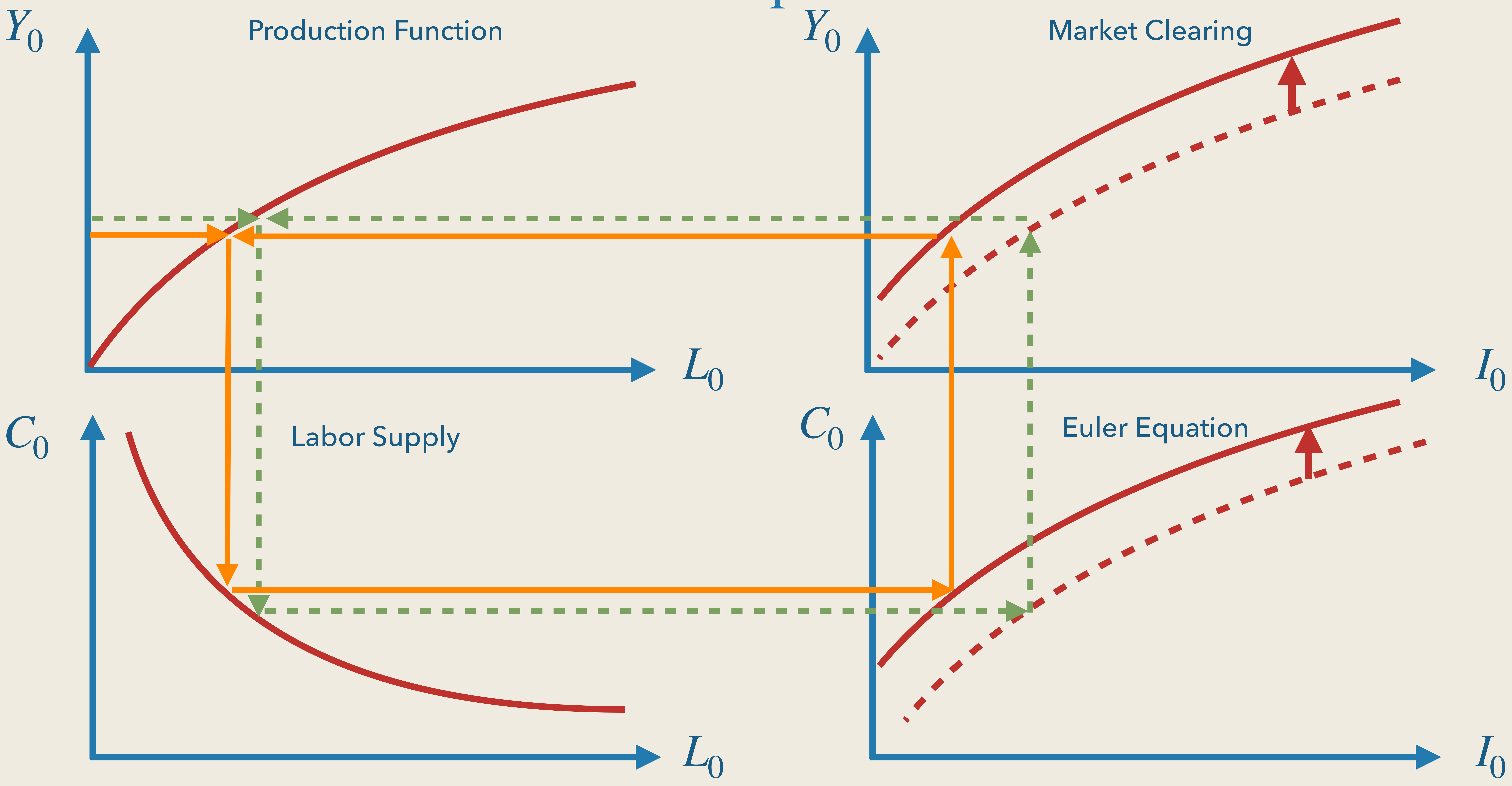
Increase in A_1



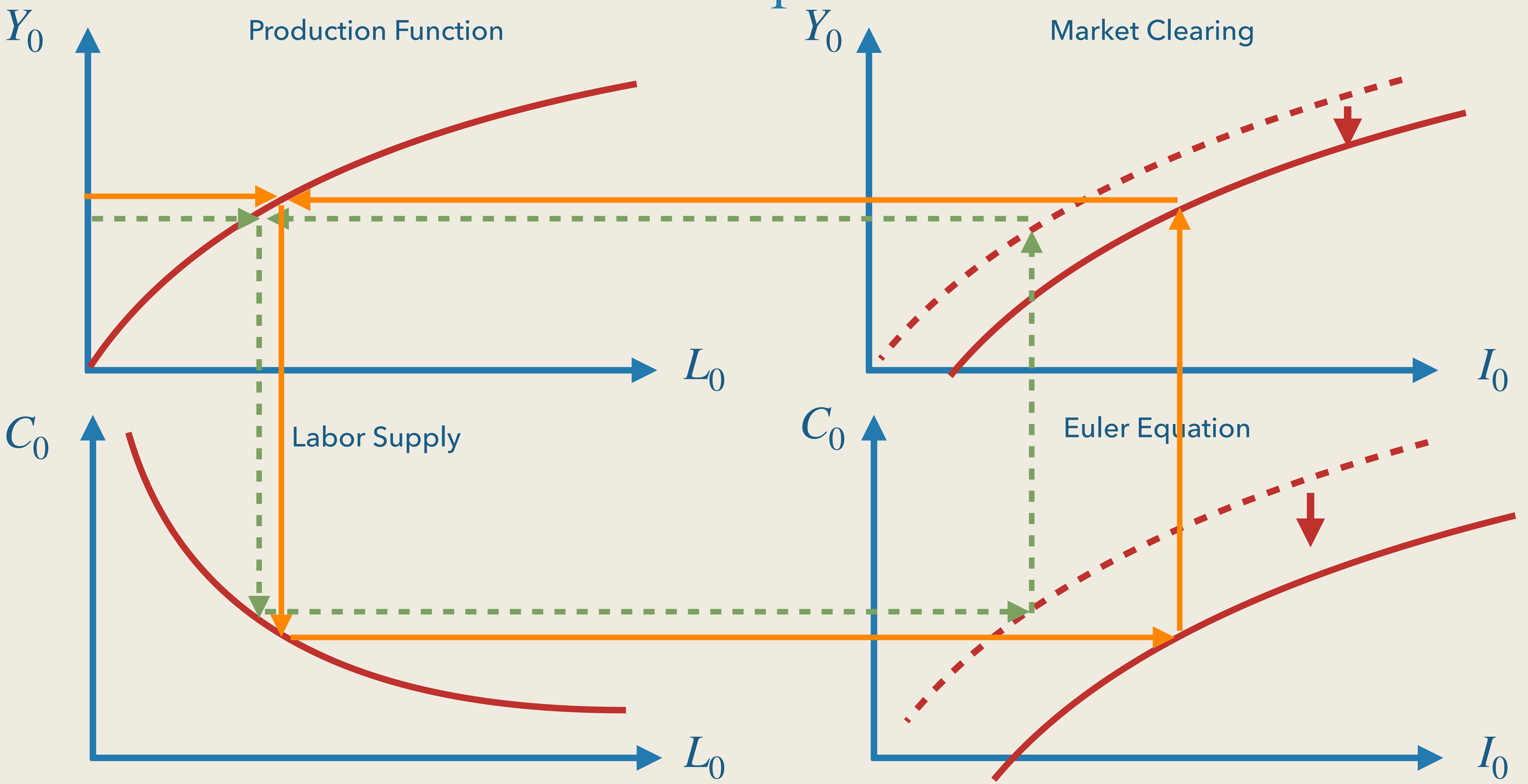
Increase in A_1 when $\sigma > 1$



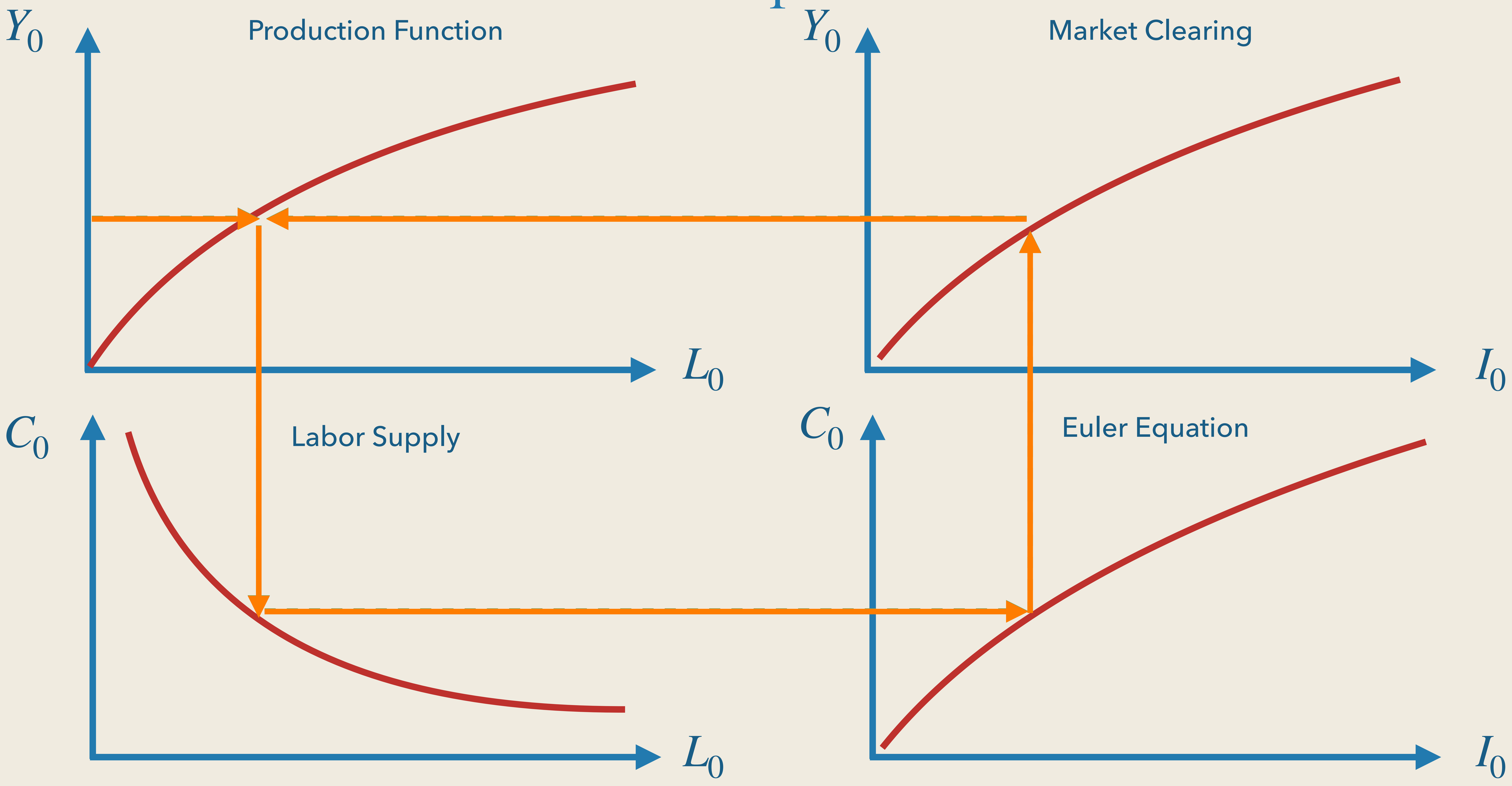
Increase in A_1 when $\sigma > 1$



Increase in A_1 when $\sigma < 1$



Increase in A_1 when $\sigma = 1$



Can Optimism Generate Business Cycles?

- An increase in A_1 increases D_1 and investment and thereby r
  - If r is higher, households would like to save more through substitution effect
  - If (r, D_1) are higher, would like to consume more today through income effect
- When $\sigma > 1$, the latter dominates and C_0 increases
  - Then L_0 decreases through income effect, and Y_0 goes down
  - As a result, $I_0 = Y_0 - C_0$ decreases as well.
- When $\sigma < 1$, the former dominates and C_0 decreases
  - Then L_0 increases through income effect, and Y_0 goes up
  - As a result, $I_0 = Y_0 - C_0$ increases
- When $\sigma = 1$, two effects cancel and nothing happens
- Does this look like a business cycle? – No.

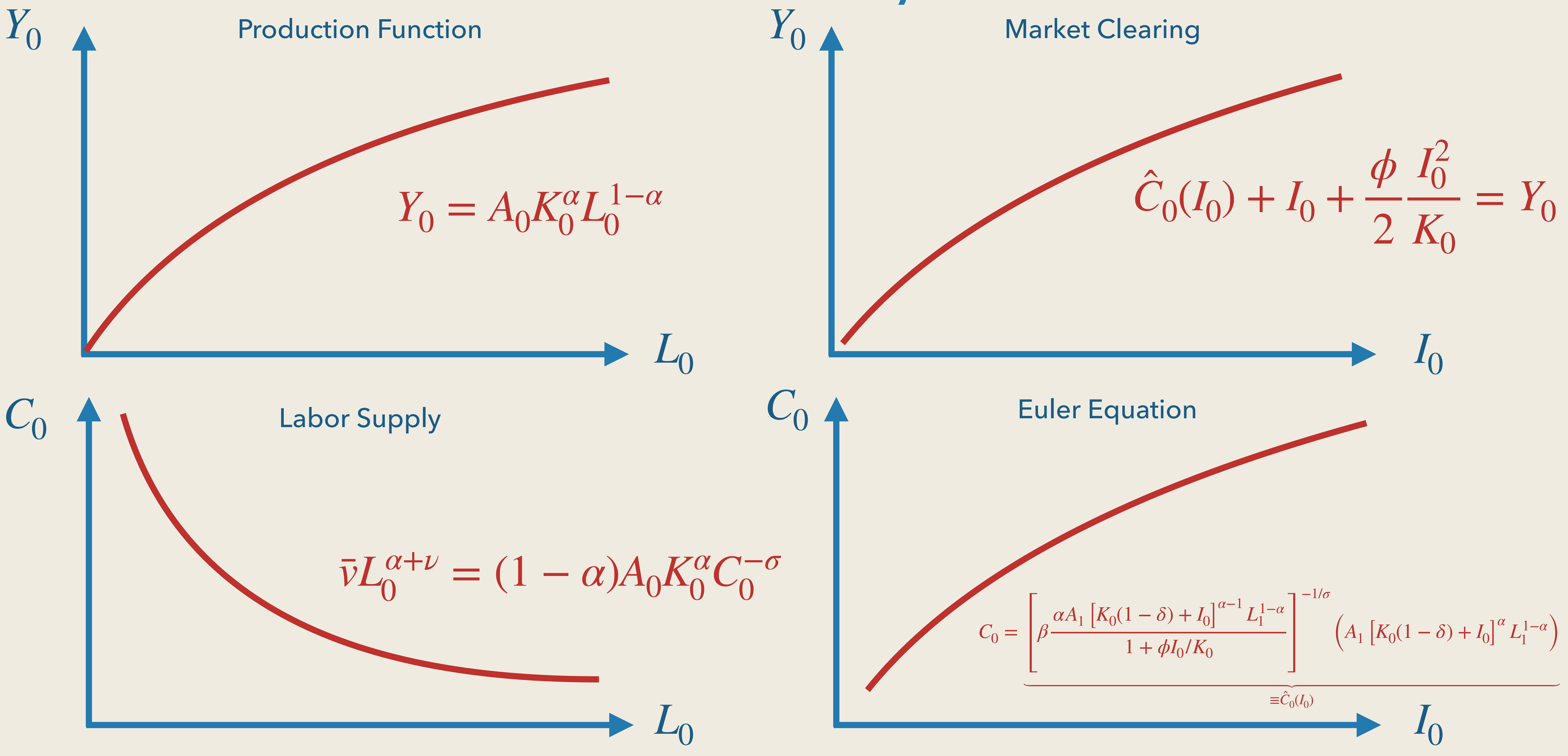
Summary

	Y	C	I	L
$A_0 \uparrow$	$\uparrow$	$\uparrow$	$\uparrow$	$\uparrow$
$A_1 \uparrow (\sigma > 1)$	$\downarrow$	$\uparrow$	$\downarrow$	$\downarrow$
$A_1 \uparrow (\sigma < 1)$	$\uparrow$	$\downarrow$	$\uparrow$	$\uparrow$
$\beta \uparrow$				
$\phi \uparrow$				
$\bar{\nu} \uparrow$				

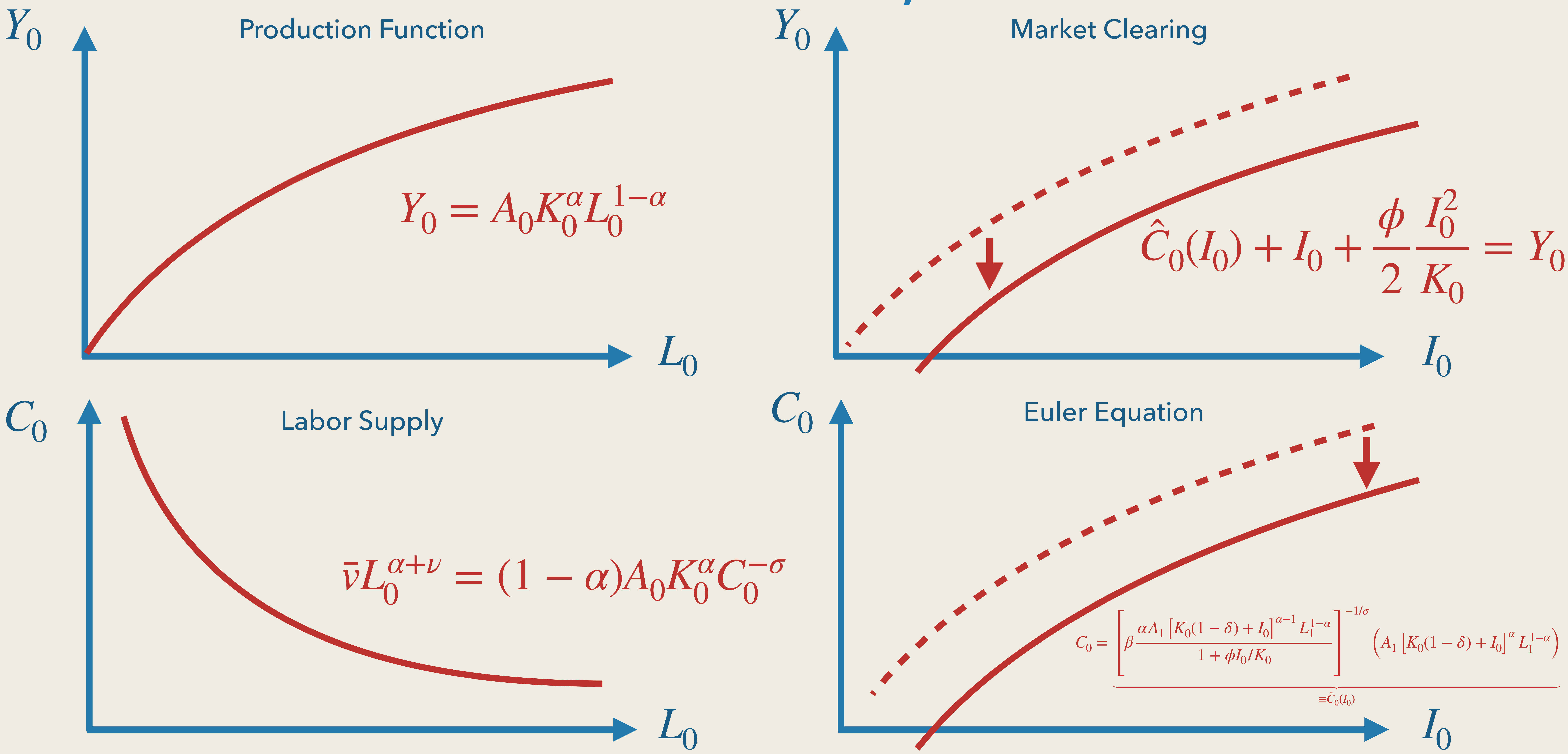
Changes in Discount Factor

- How about changes in β?
  - Booms are the times when households would like to consume more today
  - Recessions are the times when households would like to consume less today

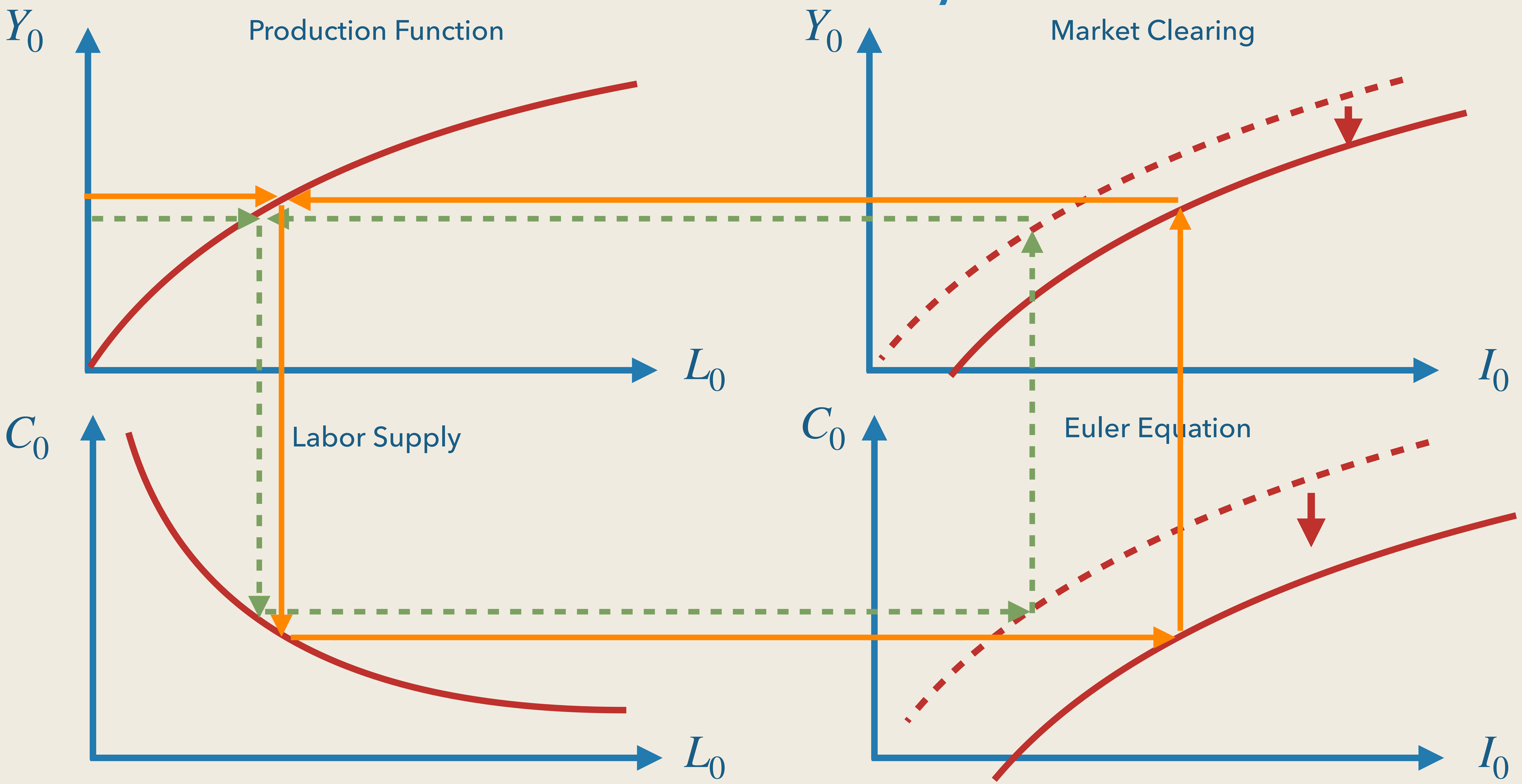
Increase in β



Increase in β



Increase in β



Postponing Consumption

- An increase in β decreases C_0
  - This increases L_0 through income effect
  - Investment increases because $I_0 = Y_0 - C_0$
- Does this look like a business cycle? – No.

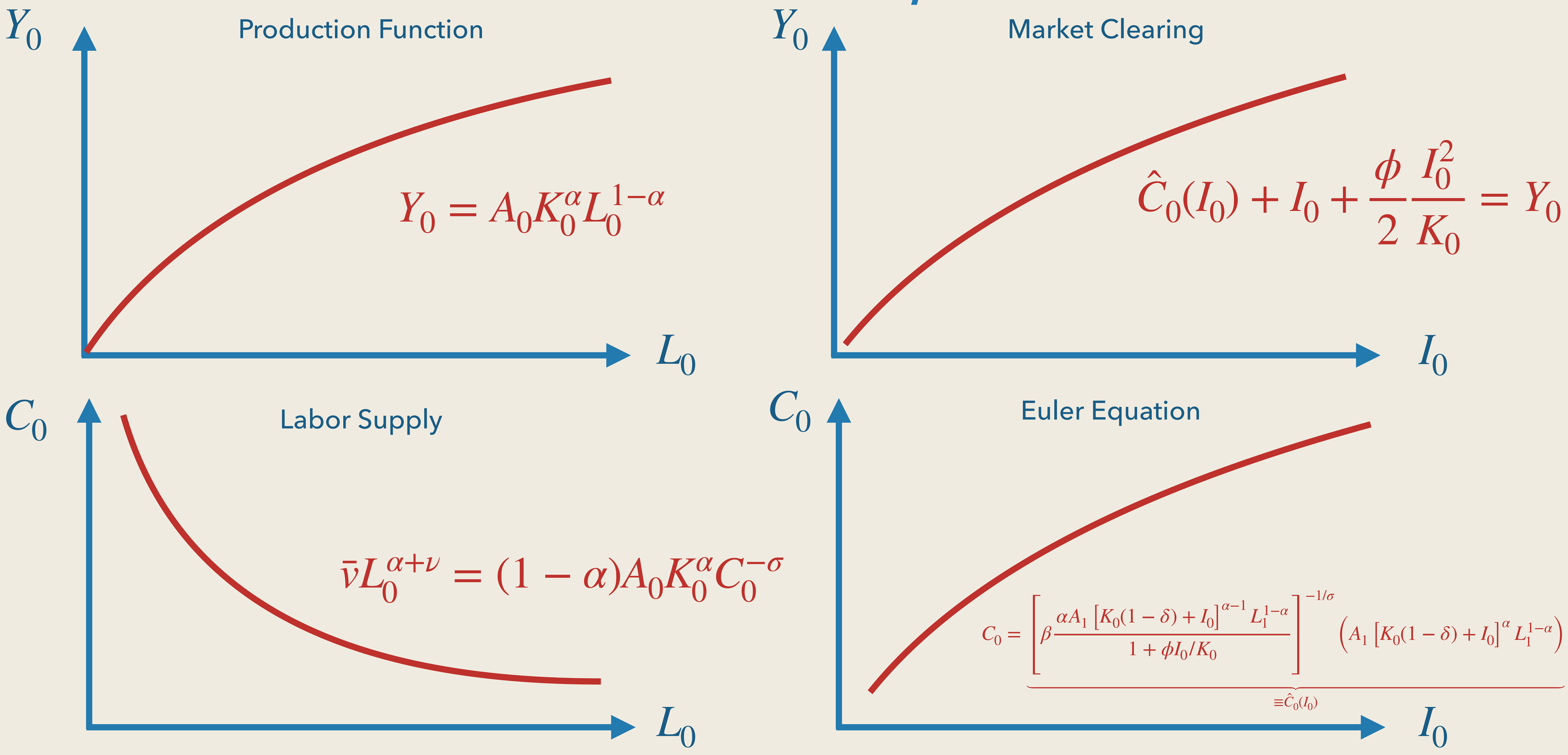
Summary

	Y	C	I	L
$A_0 \uparrow$	$\uparrow$	$\uparrow$	$\uparrow$	$\uparrow$
$A_1 \uparrow (\sigma > 1)$	$\downarrow$	$\uparrow$	$\downarrow$	$\downarrow$
$A_1 \uparrow (\sigma < 1)$	$\uparrow$	$\downarrow$	$\uparrow$	$\uparrow$
$\beta \uparrow$	$\uparrow$	$\downarrow$	$\uparrow$	$\uparrow$
$\phi \uparrow$				
$\bar{v} \uparrow$				

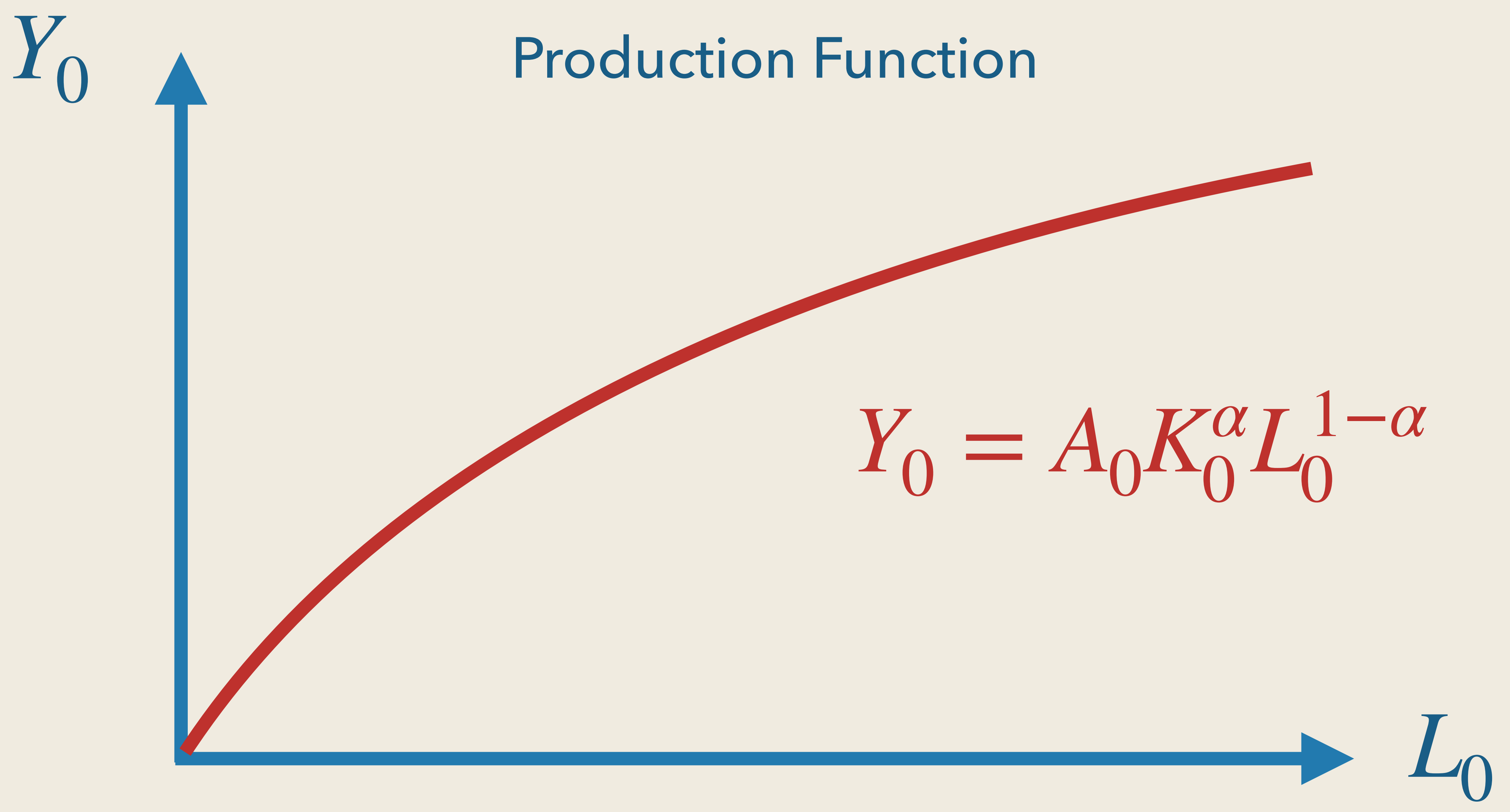
Investment Shock

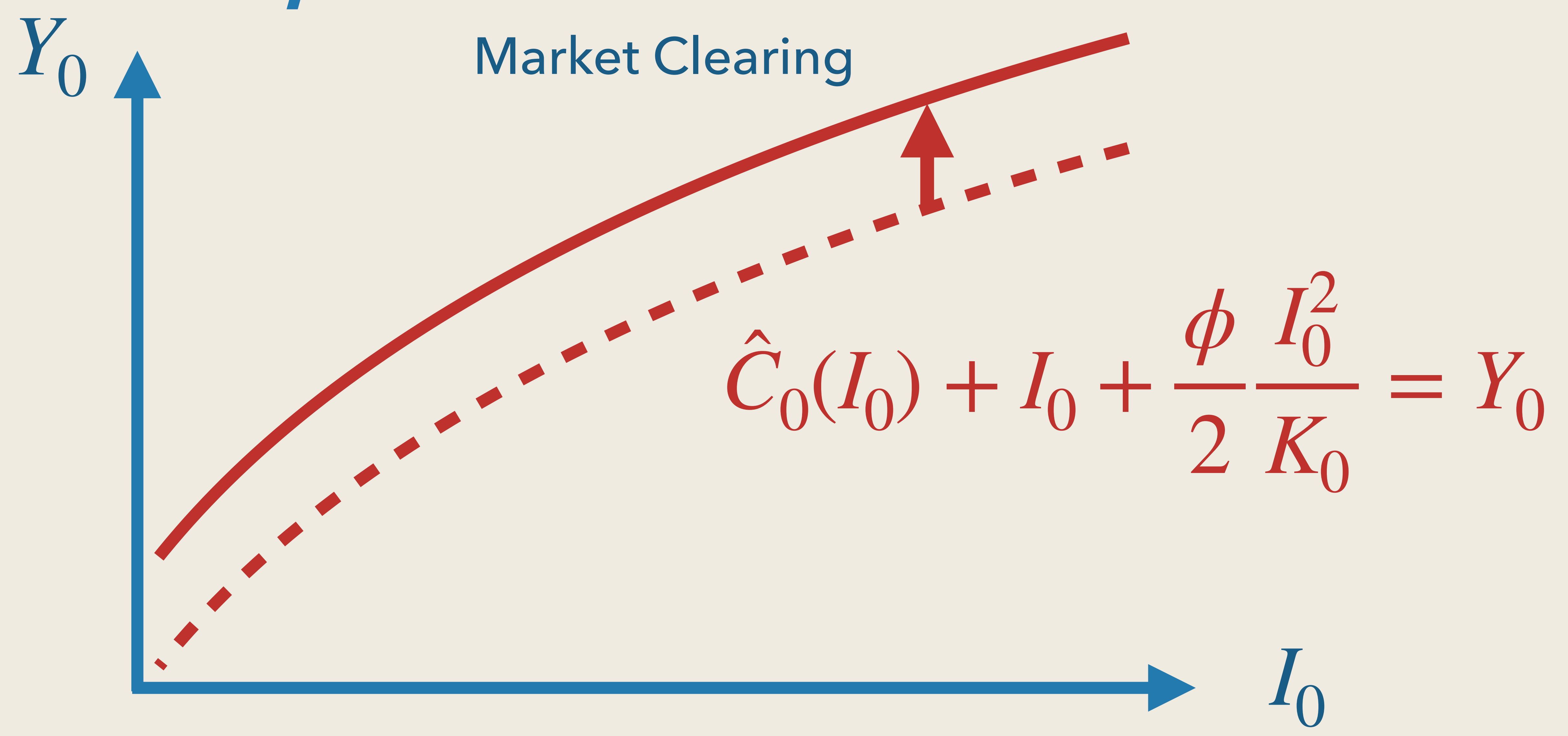
- Let now us consider the shock to investment cost, ϕ
  - Booms are the time when firms find it easy to invest
  - Recessions are the time when firms find it difficult to invest

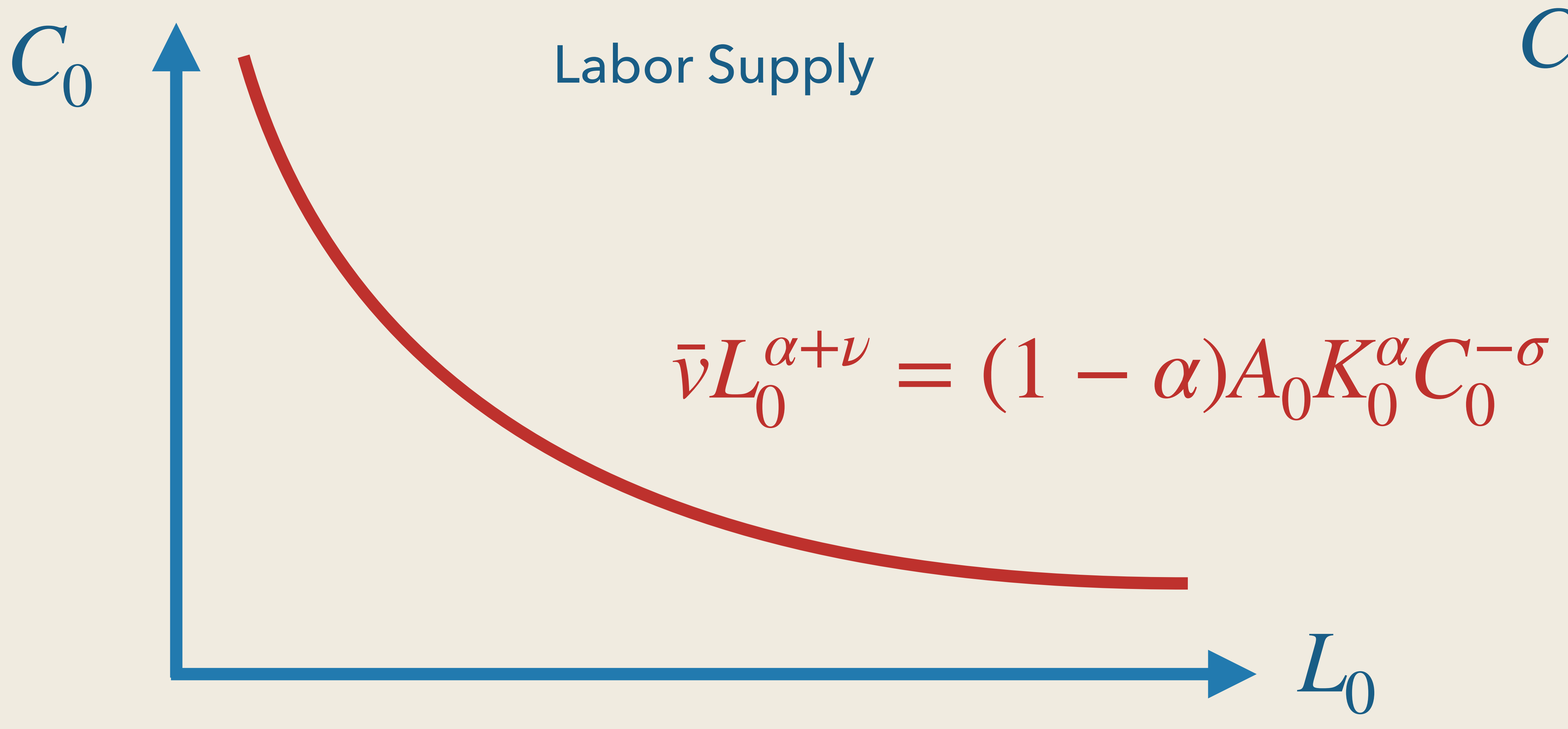
Increase in ϕ

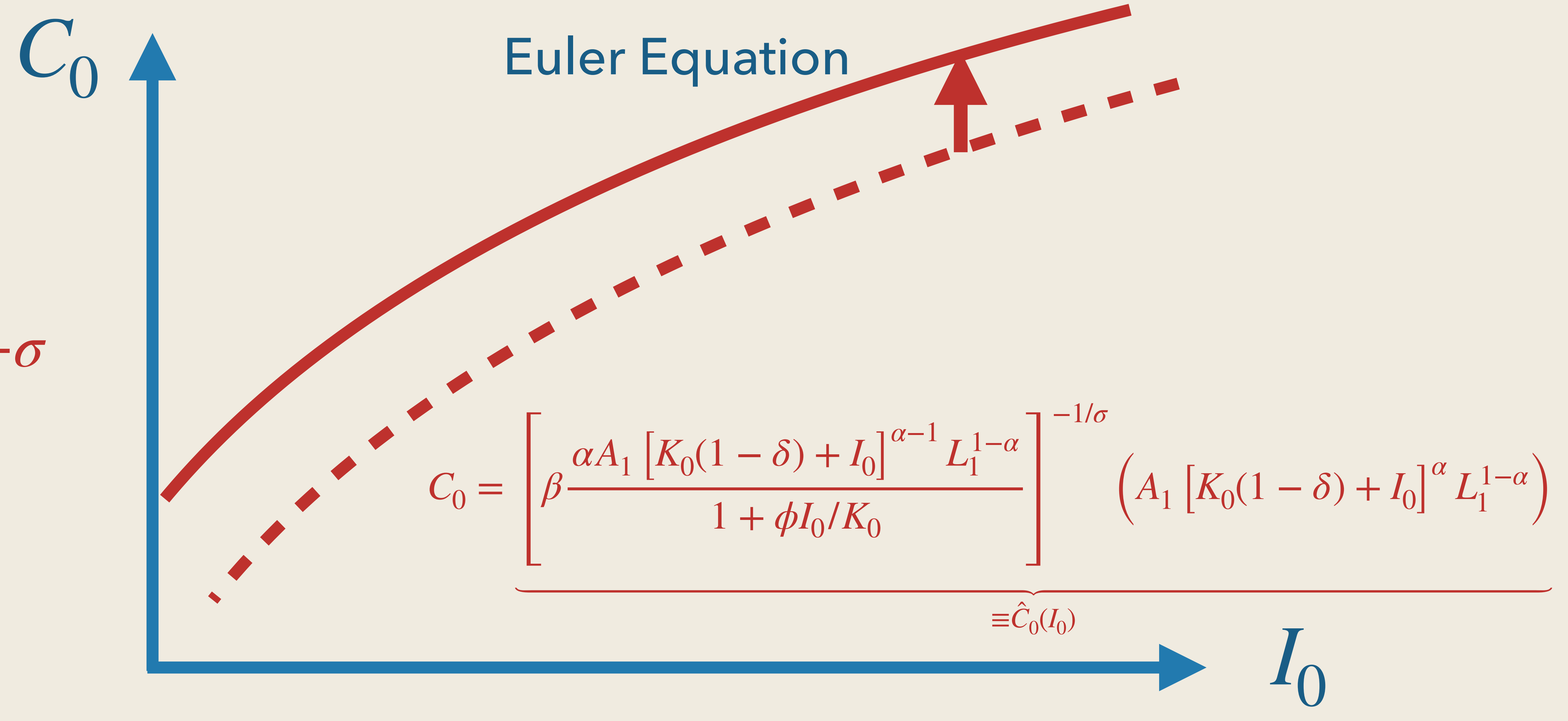


Increase in ϕ

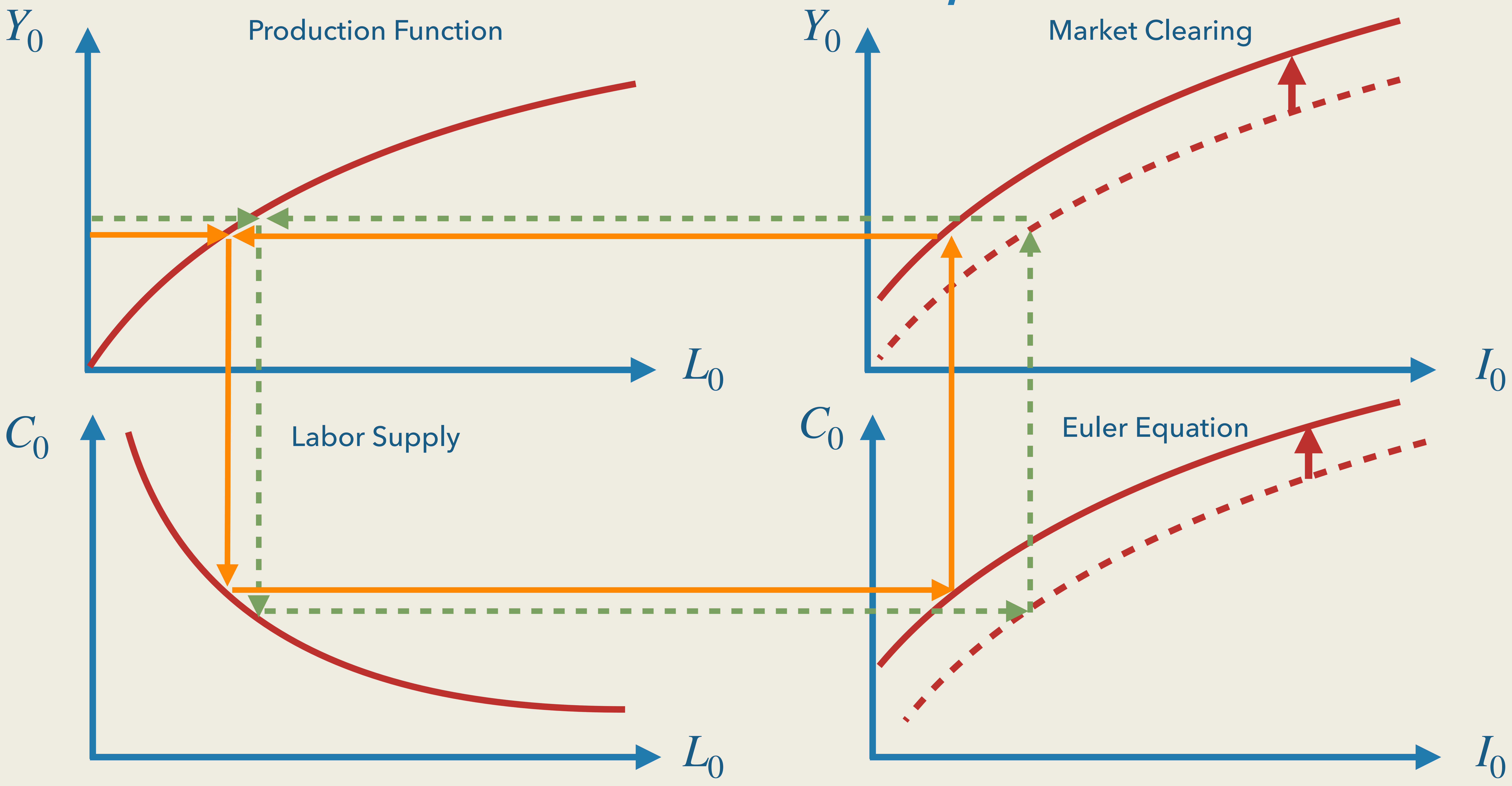








Increase in ϕ



Investment Shock

- An increase in ϕ decreases I_0
  - Since $C_0 = Y_0 - I_0 - \Phi$, consumption increases
  - L_0 decreases through income effect, and Y_0 also decreases
- Does this look like a business cycle? – No.

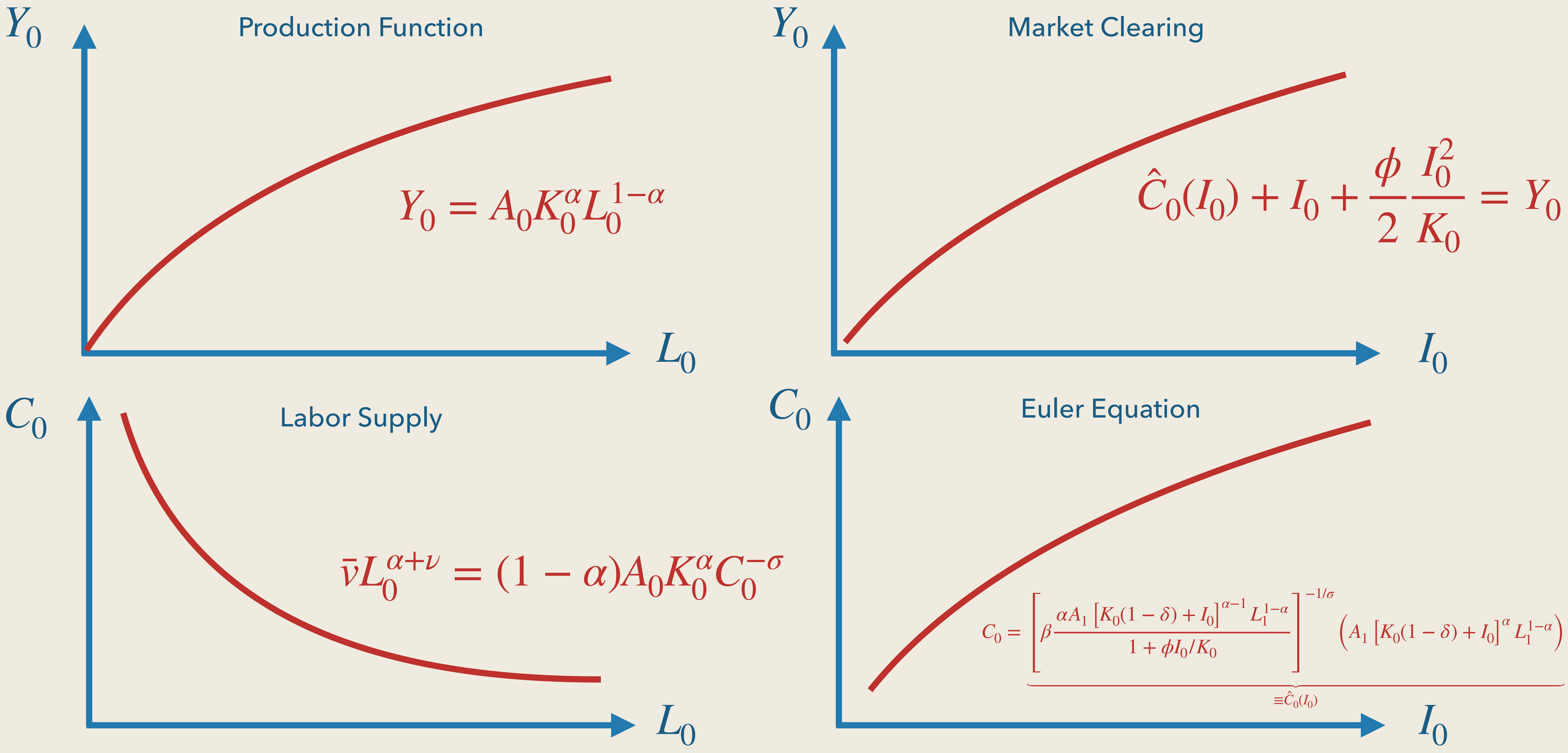
Summary

	Y	C	I	L
$A_0 \uparrow$	$\uparrow$	$\uparrow$	$\uparrow$	$\uparrow$
$A_1 \uparrow (\sigma > 1)$	$\downarrow$	$\uparrow$	$\downarrow$	$\downarrow$
$A_1 \uparrow (\sigma < 1)$	$\uparrow$	$\downarrow$	$\uparrow$	$\uparrow$
$\beta \uparrow$	$\uparrow$	$\downarrow$	$\uparrow$	$\uparrow$
$\phi \uparrow$	$\downarrow$	$\uparrow$	$\downarrow$	$\downarrow$
$\bar{v} \uparrow$				

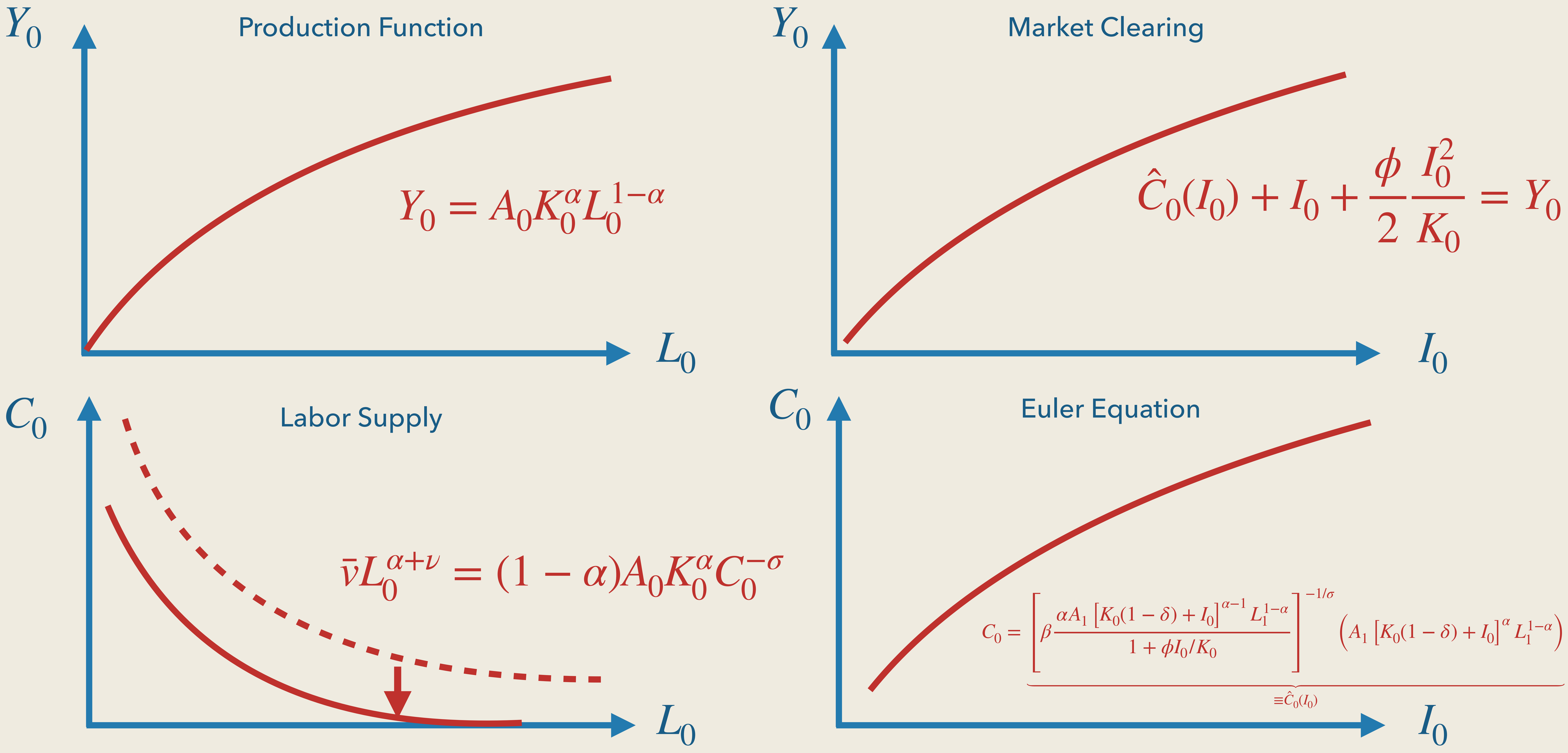
Labor Disutility Shock

- What about changes in $\bar{v}$? Literal interpretation:
  - Booms are the times when households want to or can work more
  - Recessions are the times when households do not or cannot work enough

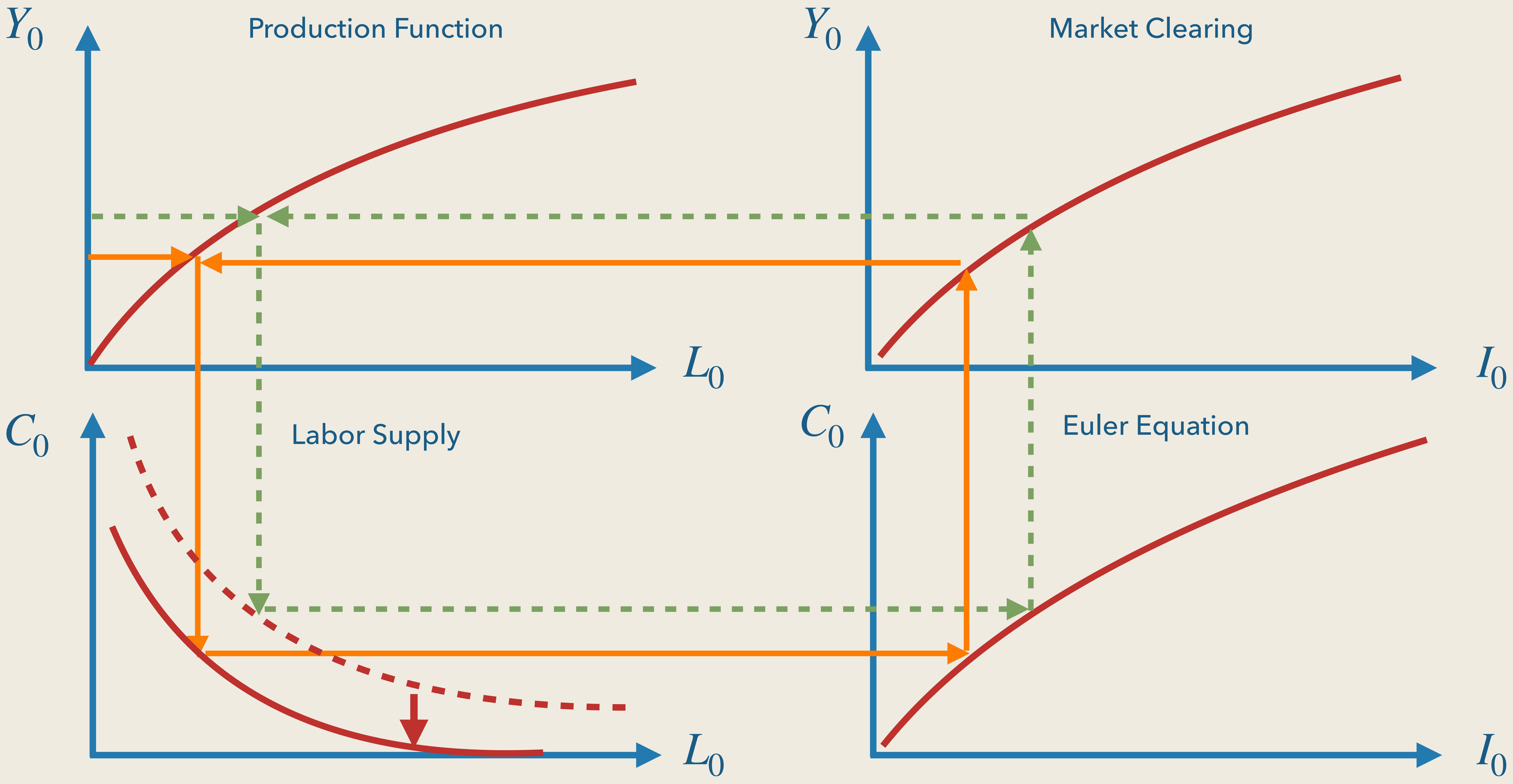
Increase in $\bar{v}$



Increase in $\bar{v}$



Increase in $\bar{v}$



Labor Disutility Shock

- An increase in $\bar{v}$ decreases L_0 , which decreases the output, Y_0
  - Then r needs to rise to lower C_0 & I_0 and clear the market
- Does this look like a business cycle? – Yes.

Summary

	Y	C	I	L
$A_0\uparrow$	$\uparrow$	$\uparrow$	$\uparrow$	$\uparrow$
$A_1\uparrow(\sigma>1)$	$\downarrow$	$\uparrow$	$\downarrow$	$\downarrow$
$A_1\uparrow(\sigma<1)$	$\uparrow$	$\downarrow$	$\uparrow$	$\uparrow$
$\beta\uparrow$	$\uparrow$	$\downarrow$	$\uparrow$	$\uparrow$
$\phi\uparrow$	$\downarrow$	$\uparrow$	$\downarrow$	$\downarrow$
$\bar{v}\uparrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$

Real Business Cycle with Infinite Horizon

Quantiative Model

- We now extend the previous model to conduct *quantitative* analaysis
- We will assume the time horizon is infinite, $t = 0, \ldots, \infty$
- Can our model replicate business cycles quantitatively?

Households and Firms

- Households solve

$$\max_{\{C_t, l_t, a_t\}} \sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{1-\sigma}}{1-\sigma} - \bar{\nu} \frac{l_t^{1+\nu}}{1+\nu} \right]$$

subject to

$$C_t + a_t = (1 + r_{t-1}) a_{t-1} + w_t l_t + D_t$$

- Firms solve

$$\max_{\{I_t, K_{t+1}, D_t, L_t\}} \sum_{t=0}^{\infty} \frac{1}{\prod_{s=0}^{t-1}(1+r_s)} D_t$$

subject to

$$D_t = A K_t^{\alpha} L_t^{1-\alpha} - w_t L_t - I_t - \frac{\phi}{2} \left(\frac{I_t}{K_t} \right)^2 K_t$$

$$K_{t+1} = (1-\delta) K_t + I_t$$

Equilibrium Definition

Equilibrium consists of $\{C_t, l_t, I_t, L_t, K_{t+1}\}$ and $\{w_t, r_t\}$ such that

1. Given $\{w_t, r_t\}$, households optimally choose $\{C_t, l_t, a_t\}$
2. Given $\{w_t, r_t\}$, firms optimally choose $\{I_t, K_{t+1}, L_t\}$
3. Markets clear

$$C_t + I_t + \Phi(I_t, K_t) = F_t(K_t, L_t)$$

$$l_t = L_t$$

Equilibrium Conditions: $\{C_t, L_t, I_t, K_{t+1}, q_t, w_t, r_t\}$

1. Euler equation: $$u'(C_t) = \beta(1 + r_t)u'(C_{t+1})$$

2. Labor supply: $$w_t u'(C_t) = v'(L_t)$$

3. Labor demand: $$\frac{\partial F_t(K_t, L_t)}{\partial L_t} = w_t$$

4. Investment: $$\frac{I_t}{K_t} = \frac{1}{\phi}\left[q_t - 1\right]$$

$$q_t = \frac{1}{1 + r_t}\left[\frac{\partial F_{t+1}(L_{t+1}, K_{t+1})}{\partial K_{t+1}} - \frac{I_{t+1}}{K_{t+1}} - \frac{\phi}{2}\left(\frac{I_{t+1}}{K_{t+1}}\right)^2 + \left(\frac{I_{t+1}}{K_{t+1}} + (1 - \delta)\right) q_{t+1}\right]$$

5. Capital stock evolution: $$K_{t+1} = (1 - \delta)K_t + I_t$$

6. Goods market clearing: $$C_t + I_t + \Phi(I_t, K_t) = F_t(K_t, L_t)$$

Procedure

- We set the parameter values to reasonable values (“calibration”)
- We then compute the steady state, where all the variables are constant over time
- Next, we simulate the model in response to a sudden shock
- The shock process is assumed to be AR(1). For example, in the case of productivity,

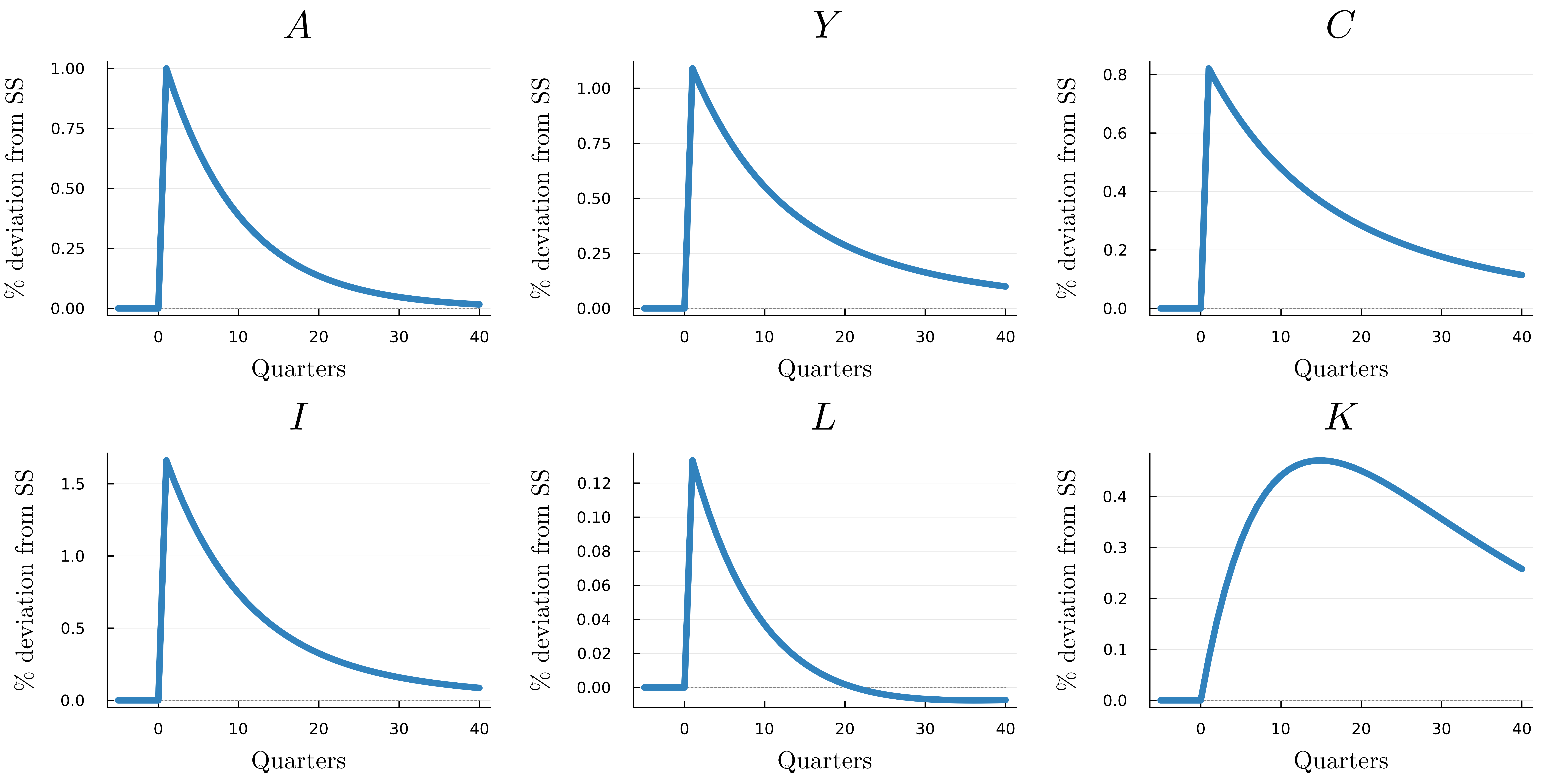
$$(\log A_t - \log A) = \rho(\log A_{t-1} - \log A) + \epsilon_t^A$$

with $\rho \in [0,1)$ and $\epsilon_t^A \sim N(0,\sigma_A^2)$

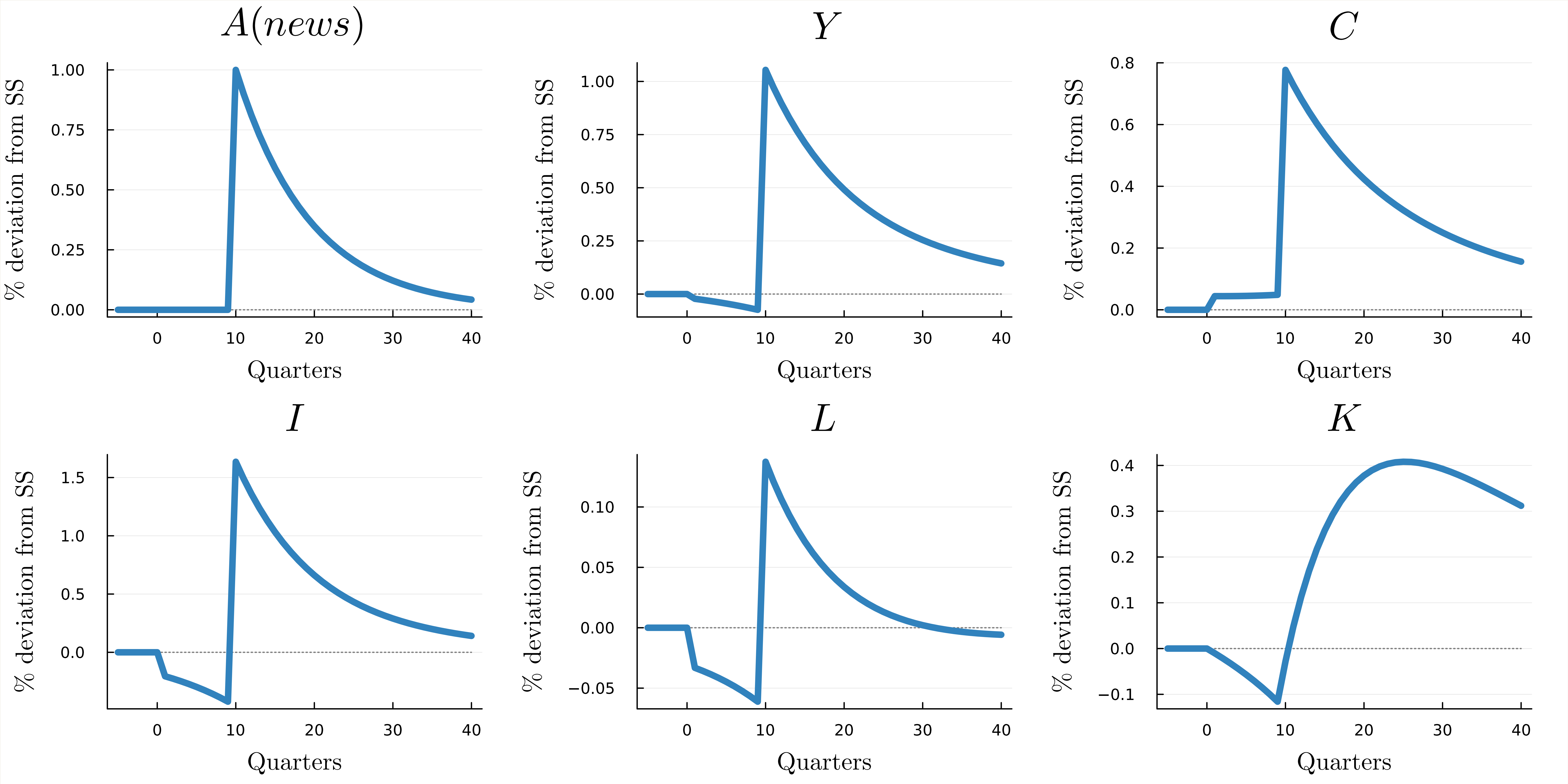
Parameterization

- One period is a quarter
- Set $\alpha = 1/3$ to match labor share
- Set $\sigma = 1$ to be consistent with (roughly) constant hours worked in the long-run
  - This implies a *permanent* change in A does not change L
- Set $\beta = 0.96^{1/4}$ to match 4% interest rate
- Set $\nu = 1$ to be (upper-end of) micro-level labor supply elasticity estimates
- Set $\delta = 5\ \%$ to match $K/Y \approx 3.5$
- Set $\phi = 10$ (match estimates of Zwick-Mahon (2017))
- We assume all shocks have the same persistence of $\rho = 0.9$

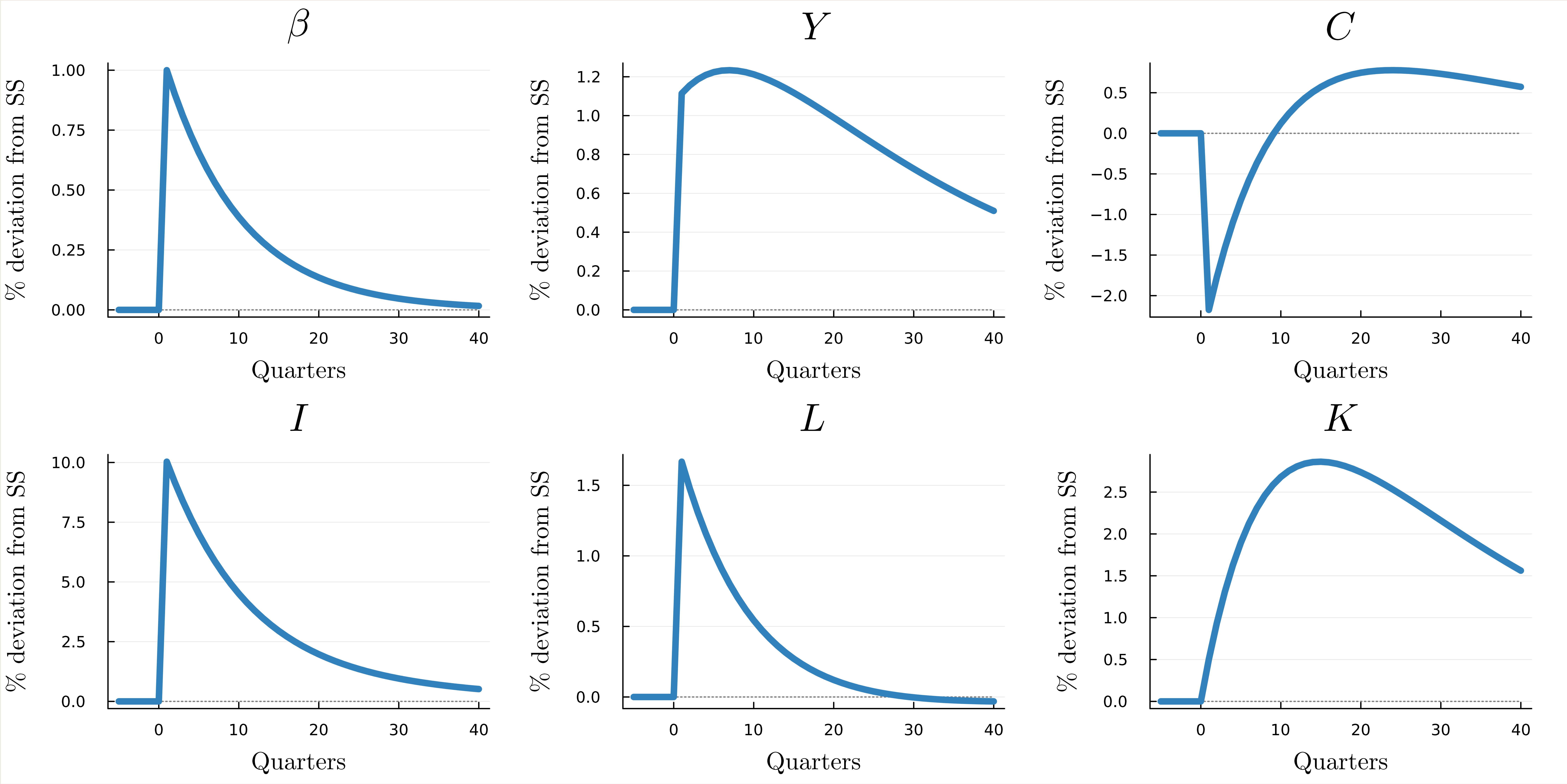
Impulse Response to Productivity Shock



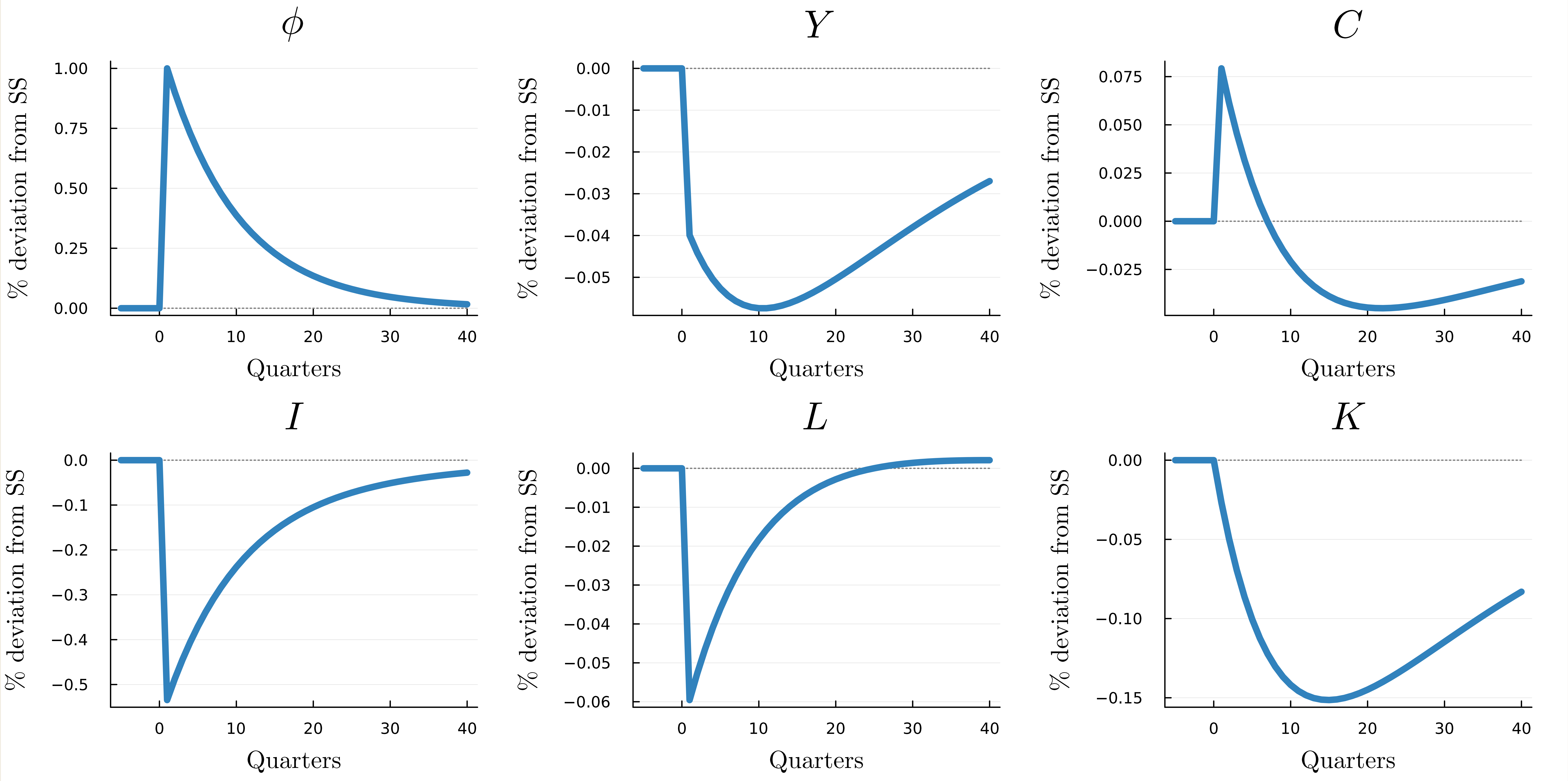
Future Productivity Shock



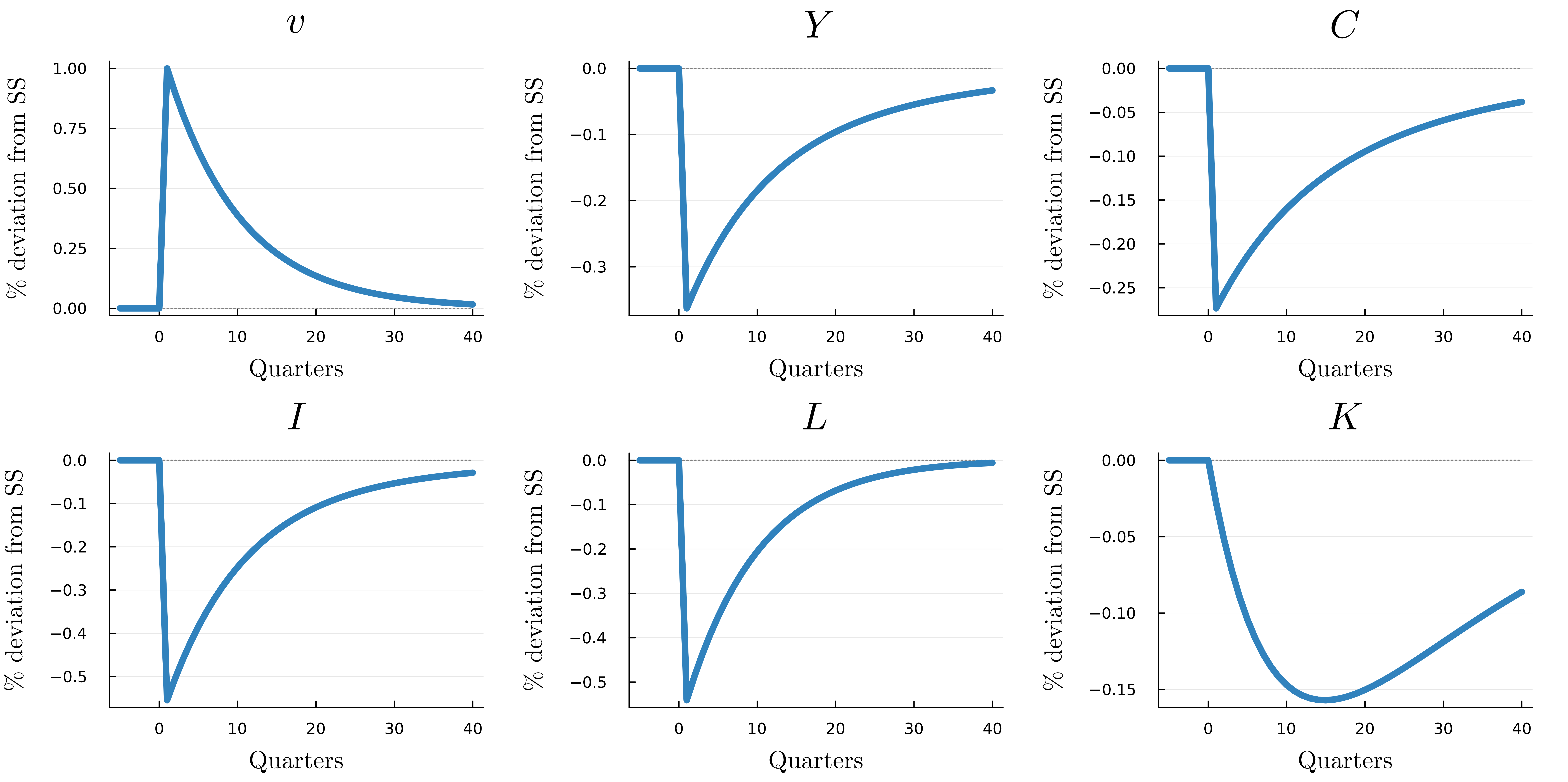
Patience Shock



Investment Cost Shock



Labor Disutility Shock



Correlation

	Data	Model			
		A	β	ϕ	v
corr(ΔY,ΔC)	0.65	0.99	-0.91	-0.80	0.99
corr(ΔY,ΔI)	0.81	0.99	0.97	0.93	0.99
corr(ΔY,ΔL)	0.66	0.99	0.96	0.91	0.99

Volatility

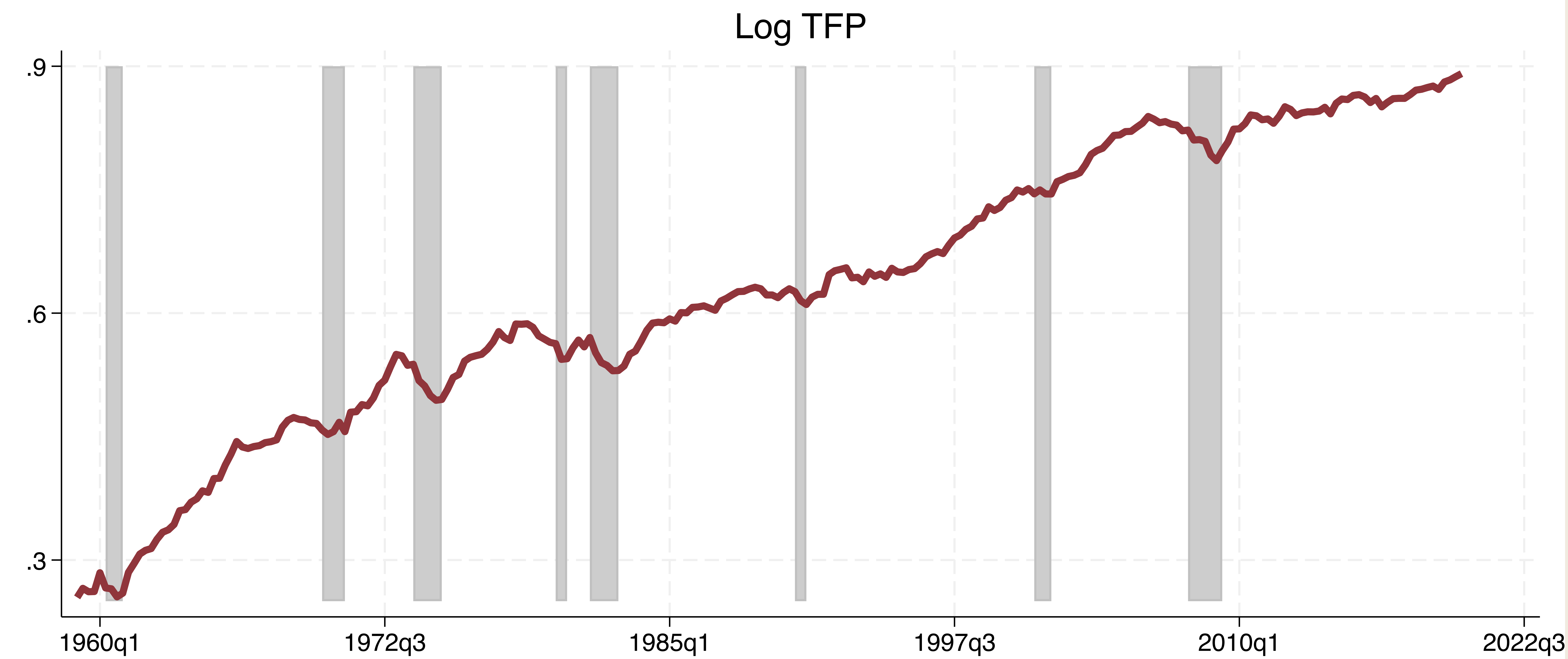
	Data	Model			
		A	β	ϕ	v
std(ΔC)/std(ΔY)	0.80	0.75	2.1	2.1	0.75
std(ΔI)/std(ΔY)	4.9	1.53	9.1	13.3	1.5
std(Δh)/std(ΔY)	0.98	0.12	1.52	1.5	1.5

Takeaway

- Two shocks are fairly successful in accounting for business cycles
  1. Productivity shock
  2. Labor disutility shock
- The former is what Kydland & Prescott (1982) argued for
- Why? – because A_t is directly measurable as Solow residual:

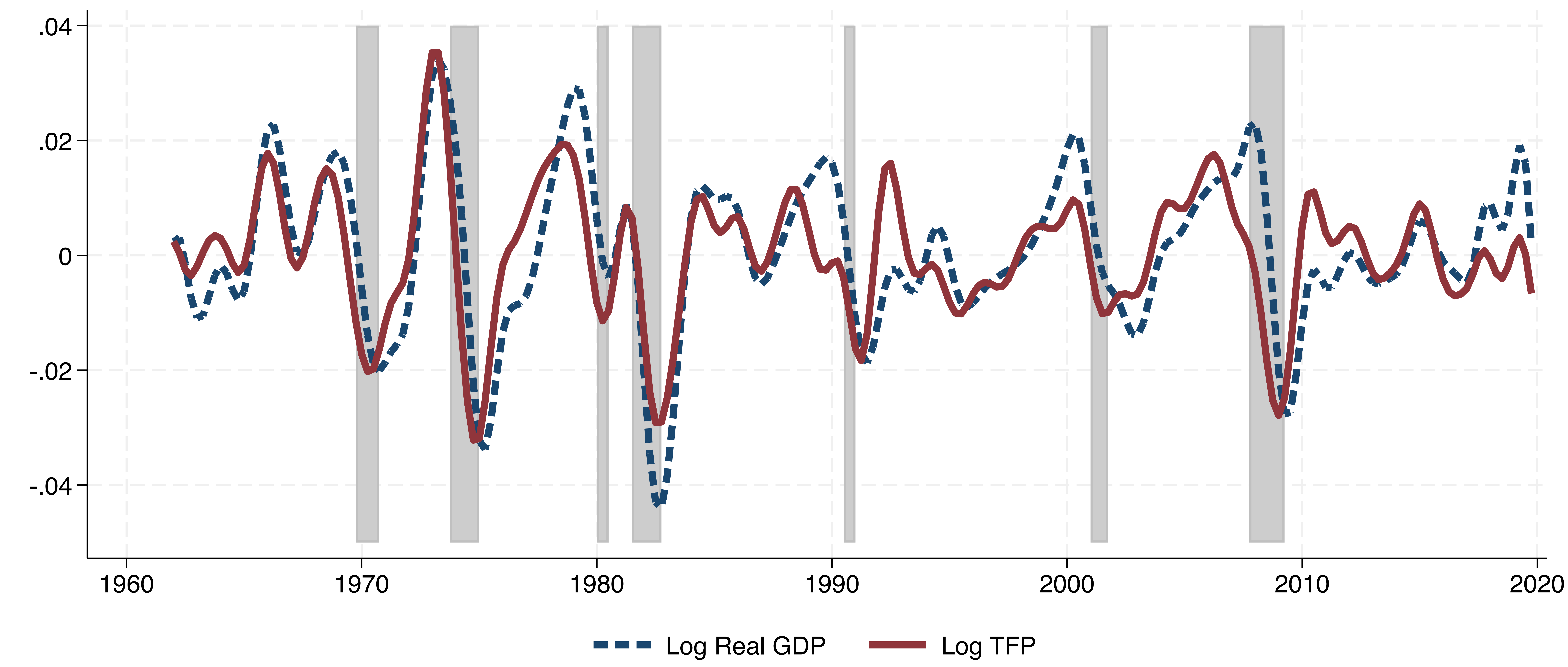
$$\log A_t = \log Y_t - (\alpha \log K_t + (1-\alpha)\log L_t)$$

TFP in the Data



Source: Fernald (2014)

Detrended Log TFP



TFP in the Data and in the Model

	Data	Model			
		A	β	ϕ	v
corr(ΔY,ΔA)	0.80	0.99	0	0	0
std(ΔA)/std(ΔY)	0.58	0.92	0	0	0

Criticisms of the RBC Model and Where We Are

Cheap Criticisms of the RBC

1. Not plausible (most common and non-scientific criticism)
    - Changes in A_t due to technological progress is plausible
    - But this should be lower frequency than business cycles
    - Technological *regress* does not make sense

2. TFP is endogenous (unconstructive criticism)
    - Changes in A_t cannot be treated as exogenous "shock"
    - Changes in A_t could be a result of innovation or misallocation

Both of the above criticisms apply to labor disutility shocks as well

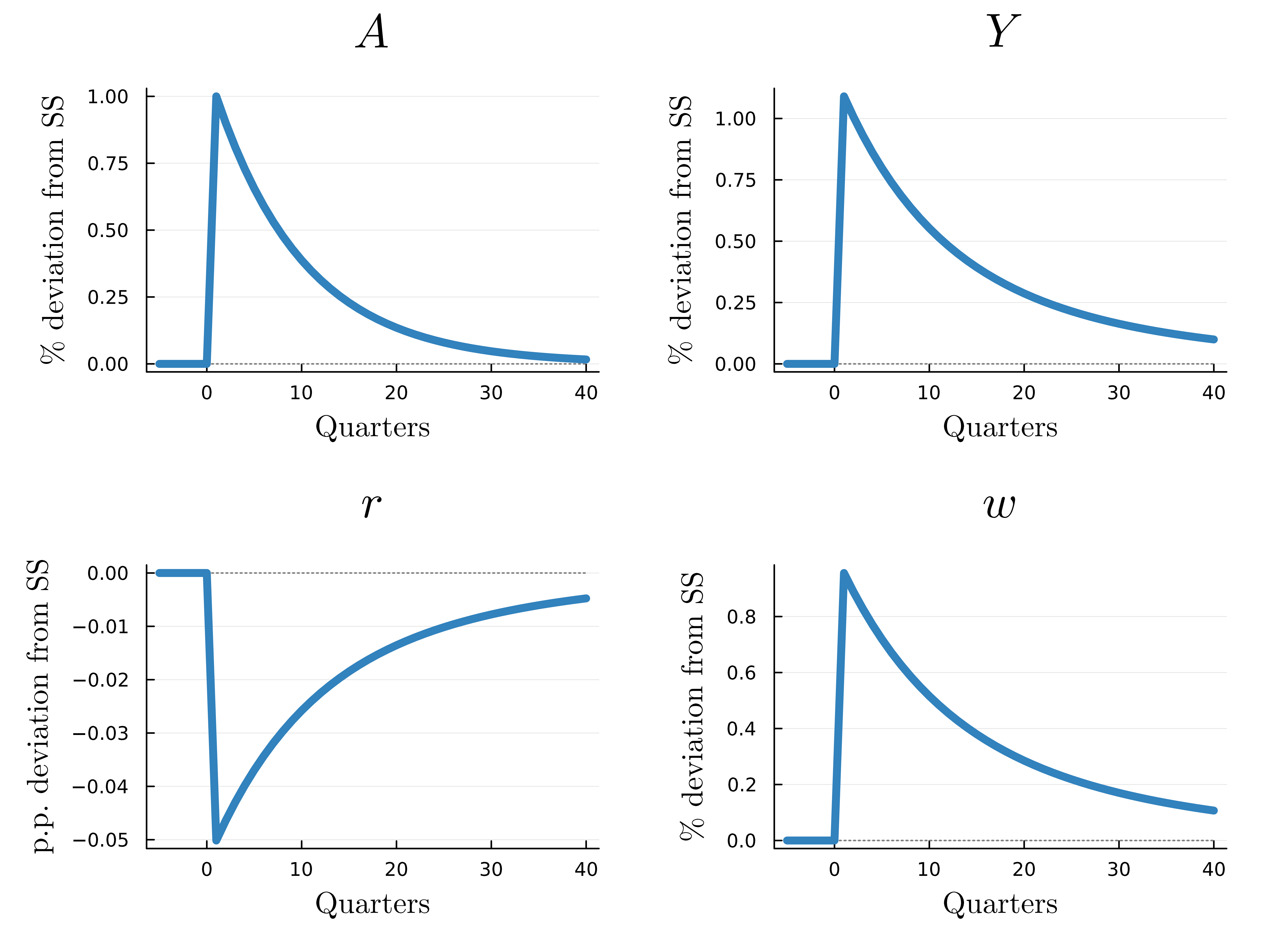
Deeper Critisms of the RBC Model

	Data	Model			
		A	β	ϕ	v
std(ΔC)/std(ΔY)	0.80	0.75	2.1	2.1	0.75
std(ΔI)/std(ΔY)	4.9	1.53	9.1	13.3	1.5
std(Δh)/std(ΔY)	0.98	0.12	1.52	1.5	1.5

3. The model generates too little volatility in L
    - This is a valid point. RBC mechanism lacks forces to generate volatile L
    - This led many researchers to focus on shocks that look like $\bar{v}$ shocks

Deeper Critisms of the RBC Model

4. The model fails to replicate the behavior of prices, (r, w), in the data



Corr(ΔY, Δw) = 0.99

Corr(ΔY, Δr) = -0.99

std(Δw)/std(ΔY) = 0.87

std(Δr)/std(ΔY) = 0.05

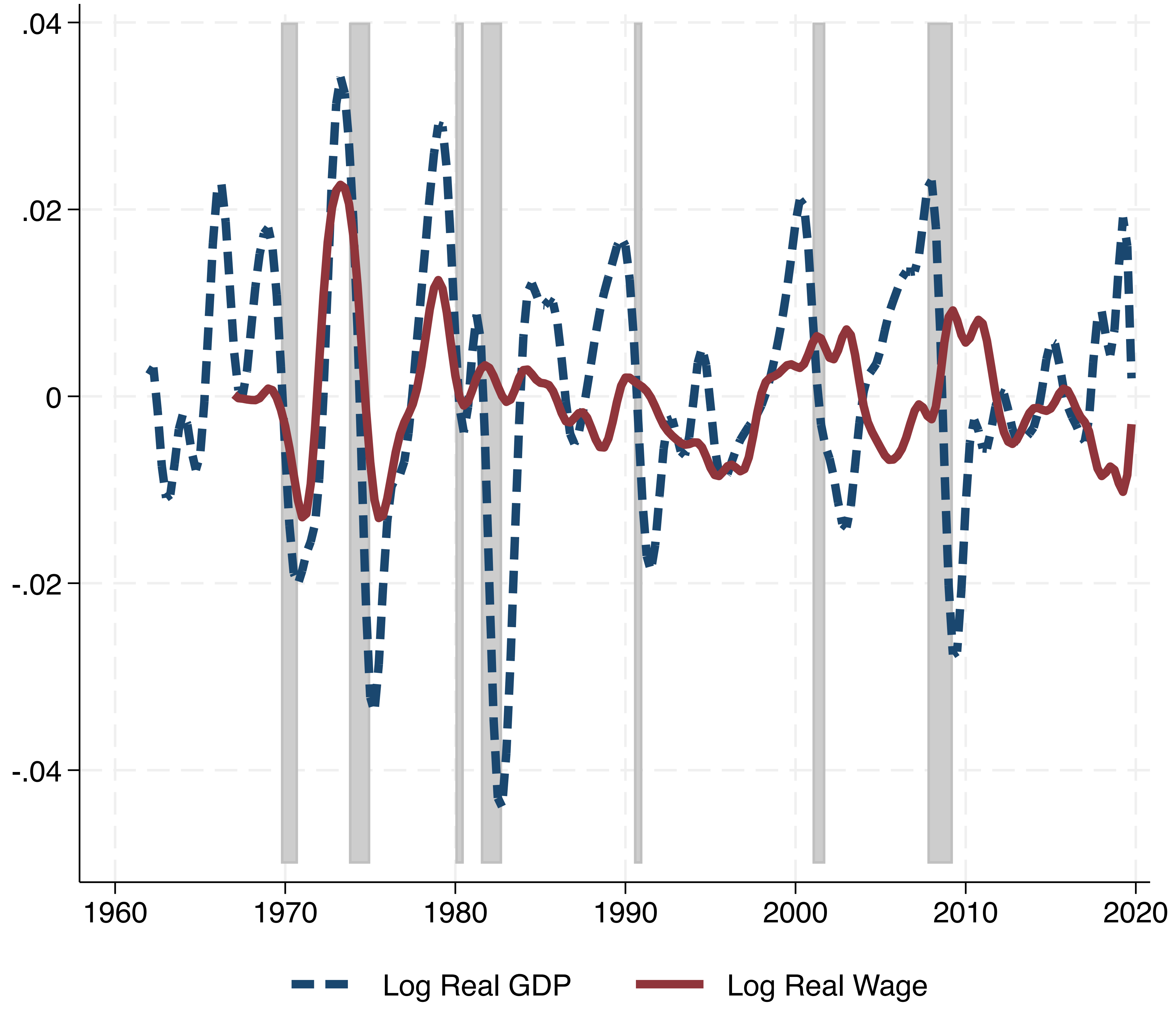
Prices in the Data

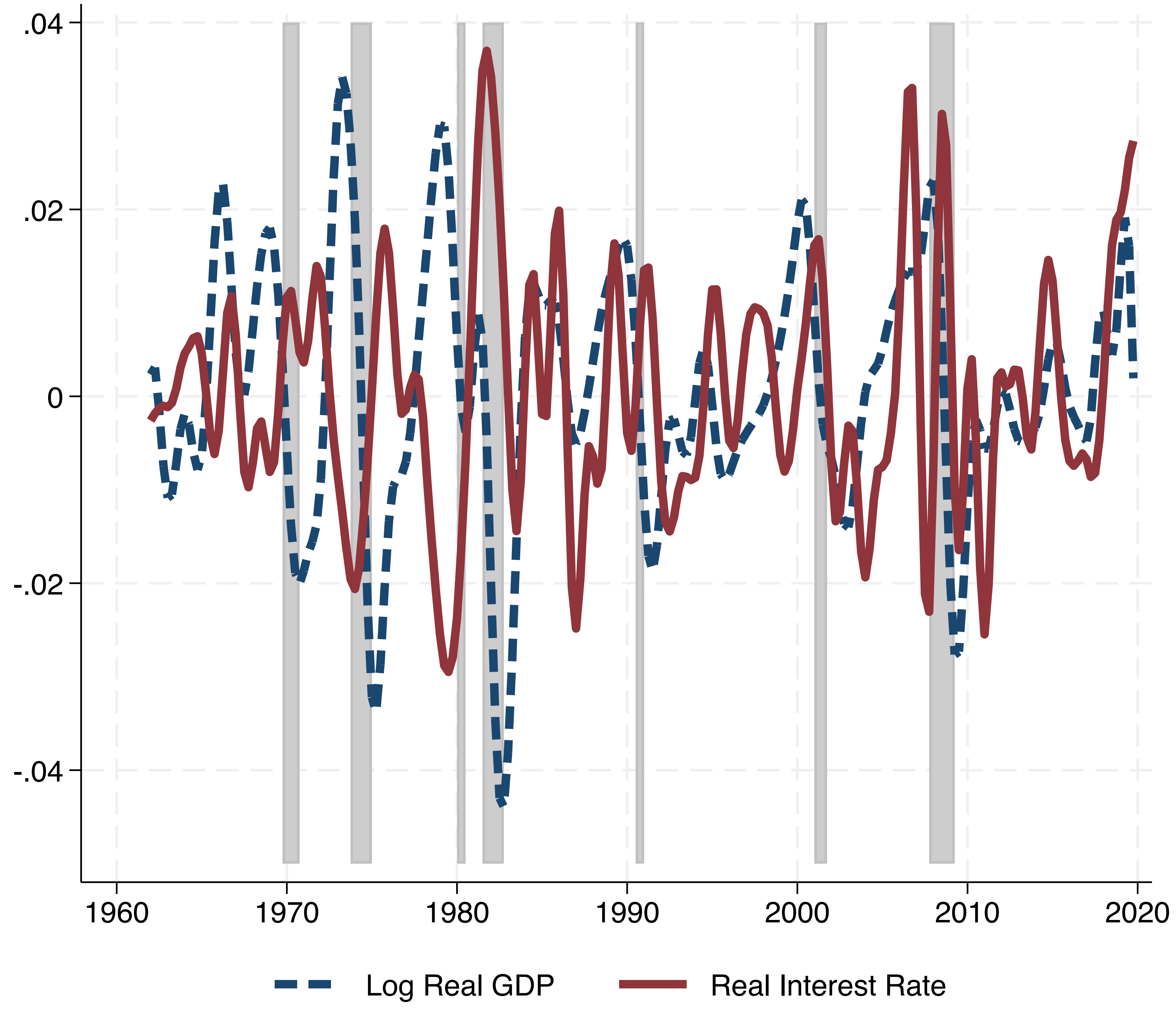
$\text{Corr}(\Delta Y, \Delta w) = 0.22$

$\text{std}(\Delta w)/\text{std}(\Delta Y) = 0.45$

$\text{Corr}(\Delta Y, \Delta r) = 0.002$

$\text{std}(\Delta r)/\text{std}(\Delta Y) = 2.5$





Deeper Critisms of the RBC Model

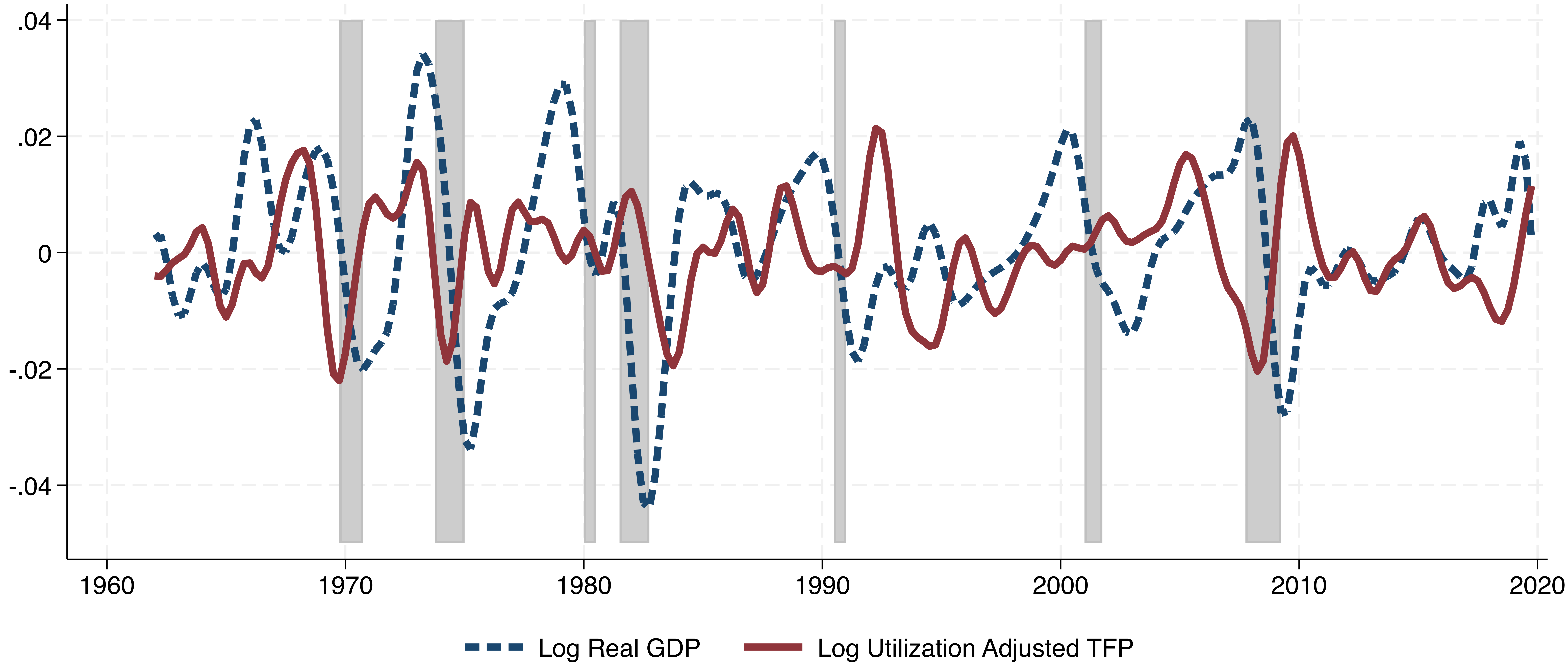
5. Once we measure TFP accurately, the correlation between TFP and GDP is weak:
   - Suppose that

     $$F_t(K_t, L_t) = A_t(u_t K_t)^{\alpha} L_t^{1-\alpha}$$

     u_t: capital utilization rate.
   - Many factories or machines are not utilized in recessions
   - Correct measure of TFP is

     $$\log A_t = \log Y_t - (\alpha(\log u_t + \log K_t) + (1-\alpha)\log L_t)$$

Utilization Adjusted TFP



Where We Are

- Did macroeconomists find a unified explanation of business cycles? – Perhaps not
- Most economists do not accept RBC as the final answer
- But RBC is an extremely useful benchmark model
- Ironically, all the attempts to criticize RBC are still based on RBC
- So in the end, what drives business cycles? Some recently suggested alternatives:
    - Risk/Uncertainty
    - Financial frictions