Investment

EC502 Macroeconomics
Topic 8

Masao Fukui

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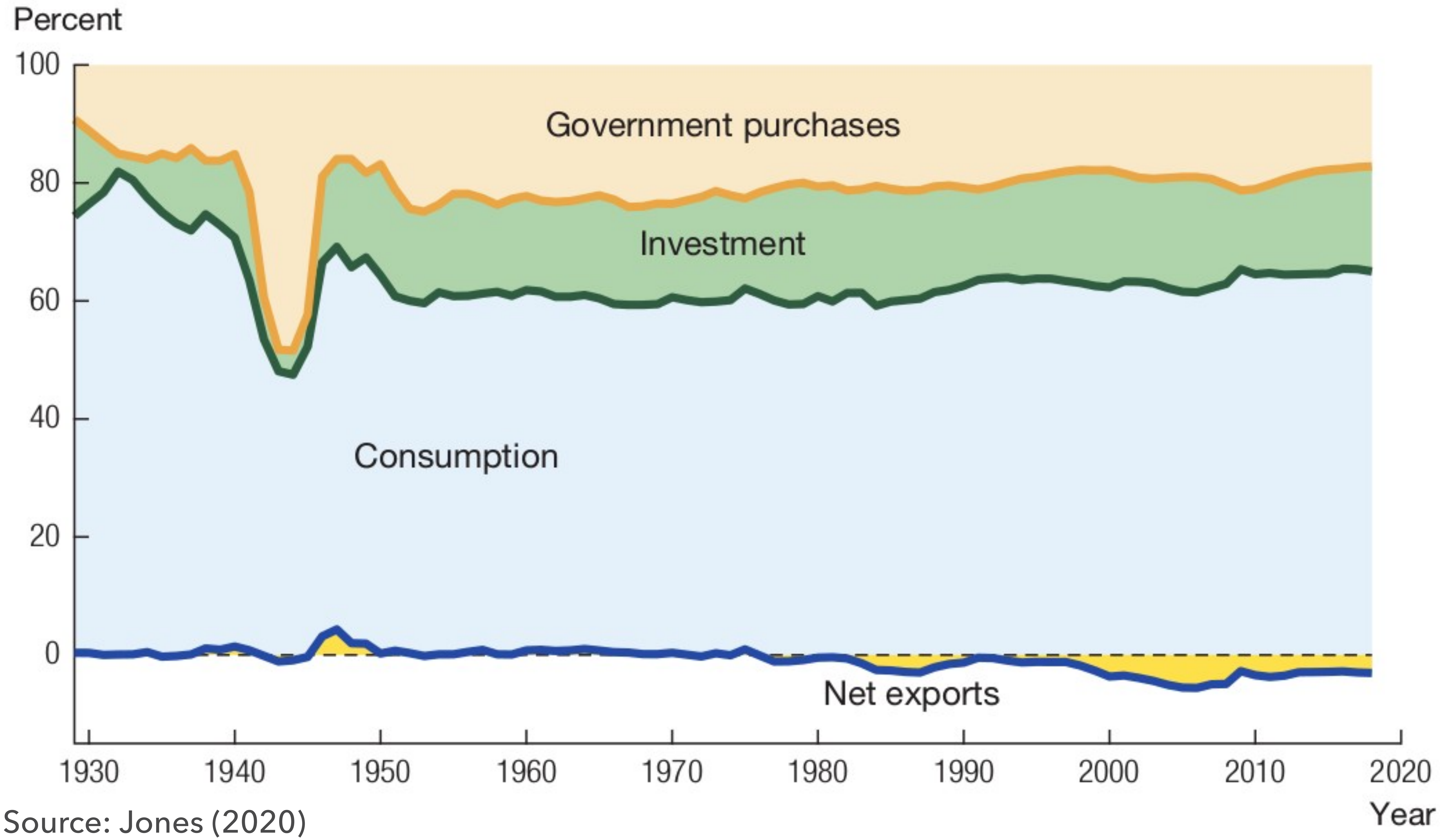
Consumption in GDP



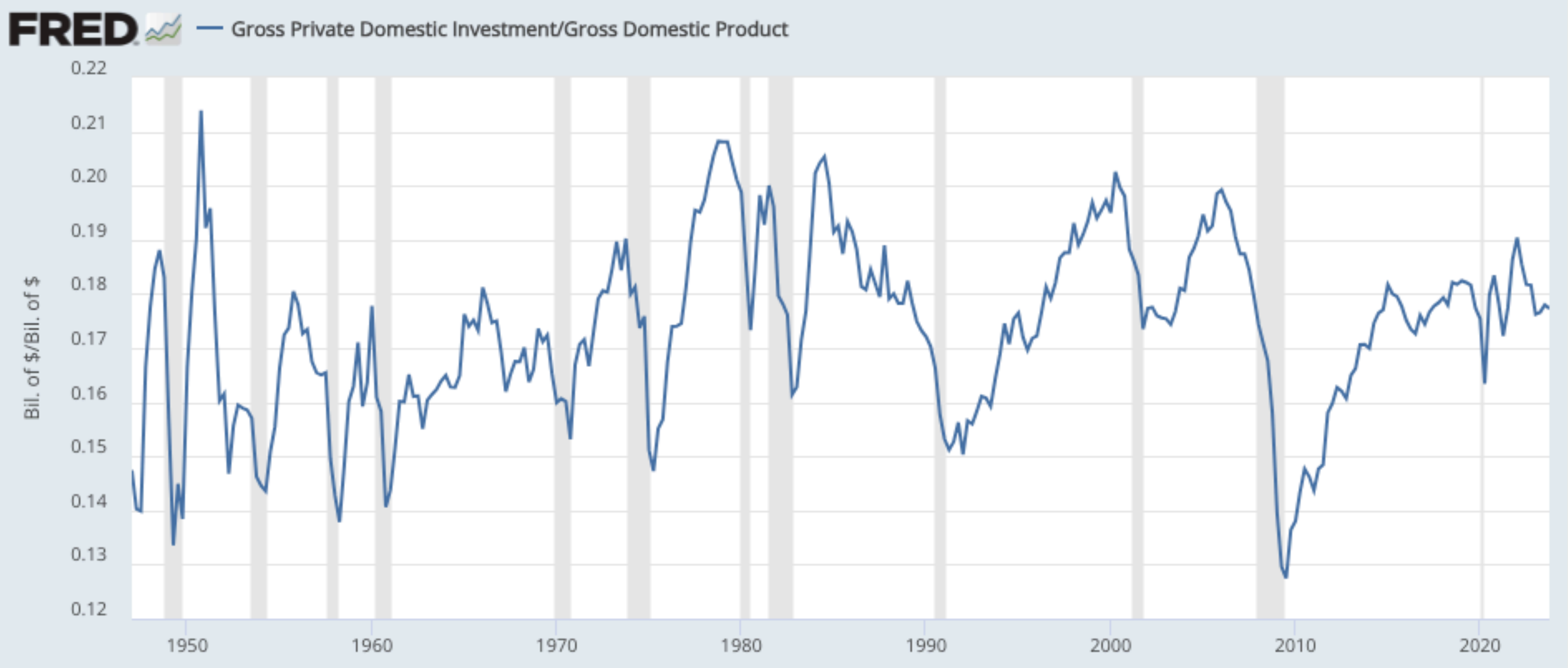
Source: Jones (2020)

Investment


FRED
Real Gross Private Domestic Investment
5,000
4,000
3,000
2,000
1,000
0
Billions of Chained 2017 Dollars
1950
1960
1970
1980
1990
2000
2010
2020

Investment Growth

Investment over GDP



Questions

- Investment constitutes $\approx$ 20% of GDP
- Yet, it is the most volatile component of GDP
- What determines investment?
  - Recall in Solow model, this was mechanical, $I_t = sY_t$
- How can a policy stimulate investment in recessions?

Investment with Two Periods

Setup

- Consider a firm operating the following production function

$$F_t(K_t, L_t) = A_t K_t^{\alpha} L_t^{1-\alpha}$$

- Firms own capital stock K_t and invest with convex adjustment costs $\Phi(I_t, K_t)$

$$K_1 = (1-\delta)K_0 + I_0, \quad \delta : \text{depreciation rate}$$

- Firms hire labor in the competitive labor market with wage w_t
- The firm maximizes the presented discounted value of dividends

$$D_0 + \frac{1}{1+r} D_1$$

where $D_t = F_t(K_t, L_t) - w_t L_t - I_t - \Phi(I_t, K_t)$ is the profit of a firm in period t

Adjustment Costs

- We assume the following adjustment cost function

$$\Phi(I, K) = \frac{\phi}{2}\left(\frac{I}{K}\right)^2 K$$

- This function is increasing and convex in I
  - The additional investment costs more when you are already investing a lot
- This function is constant returns to scale in (I, K)
  - doubling your investment and capital also doubles the cost of investment

Firm's Problem

- Given K_0, a firm solves

$$\max_{L_0, I_1, K_1, L_1} \left[F_0(K_0, L_0) - w_0 L_0 - I_0 - \Phi(I_0, K_0)\right] + \frac{1}{1+r}\left[F_1(K_1, L_1) - w_1 L_1\right]$$

subject to

$$K_1 = K_0(1-\delta) + I_0$$

- The first-order conditions with respect to L_t:

$$\frac{\partial F_t(K_t, L_t)}{\partial L_t} = w_t \tag{1}$$

- The first-order condition with respect to I_0 is

$$1 + \frac{\partial \Phi(I_0, K_0)}{\partial I_0} = \frac{1}{1+r}\frac{\partial F_1(K_1, L_1)}{\partial K_1} \tag{2}$$

LHS: marginal cost of investment, RHS: marginal benefit of investment

Investment Solution

- With our functional forms, we can solve for labor demand using (1):

$$L_t = (1-\alpha)^{1/\alpha} A_t^{1/\alpha} w_t^{-1/\alpha} K_t \tag{3}$$

- Equation (2) is

$$1 + \phi \frac{I_0}{K_0} = \frac{1}{1+r} \alpha A_1 K_1^{\alpha-1} L_1^{1-\alpha} \tag{4}$$

- Combining (3) and (4),

$$\frac{I_0}{K_0} = \frac{1}{\phi}\left[\frac{1}{1+r}\alpha(1-\alpha)^{\frac{1-\alpha}{\alpha}} A_1^{\frac{1}{\alpha}} w_1^{-\frac{1-\alpha}{\alpha}} - 1\right]$$

Comparative Statics

$$\frac{I_0}{K_0} = \frac{1}{\phi}\left[\frac{1}{1+r}\alpha(1-\alpha)^{\frac{1-\alpha}{\alpha}}A_1^{\frac{1}{\alpha}}w_1^{-\frac{1-\alpha}{\alpha}} - 1\right] \tag{5}$$

Investment is higher when

- interest rate, r, is lower
- future productivity, A_1, is higher
- future wage, w_1, is lower

All should be intuitive

Value of Firms

- Let us rewrite firm's investment in a different way
- Define the value of firms (discounted future profits):

$$V_1 = \frac{1}{1+r} D_1$$

  - In principle, V_1 should correspond to the stock price of the firm

- Using the definition, $D_1 = F_1(K_1, L_1) - w_1 L_1$, and labor demand (3),

$$V_1 = \frac{1}{1+r} \alpha A_1^{\frac{1}{\alpha}} (1-\alpha)^{\frac{1-\alpha}{\alpha}} w_1^{-\frac{1-\alpha}{\alpha}} K_1$$

- Define $q_1 \equiv V_1 / K_1$, which we call as "q"

Q-Theory of Investment

- With the definition of "q", we can rewrite investment equation (5) as

$$\frac{I_0}{K_0} = \frac{1}{\phi}\left[q_1 - 1\right]$$

- We often refer to the above expression as "q-theory of investment"
- Investment is positive if and only if $q_1 > 1$
  - The average value of capital is higher than its cost
- Investment is negative if and only if $q_1 < 1$
  - The average value of capital is lower than its cost
- Importantly, q_1 summarizes the impact of r, w_1, A_1 ("sufficient statistics")

Investment with Many Periods

Investment Problem with Many Periods

- We generalize the previous model to many periods, $t = 0, \ldots, T$
- The firm solves

$$\max_{\{I_t, K_{t+1}, D_t, L_t\}} \sum_{t=0}^{T} \frac{1}{\prod_{s=0}^{t-1}(1 + r_s)} D_t$$

subject to

$$D_t = F_t(K_t, L_t) - w_t L_t - I_t - \Phi(I_t, K_t)$$

$$K_{t+1} = (1 - \delta)K_t + I_t$$

Lagrangian

- The Lagrangian is

$$\mathscr{L} = \sum_{t=0}^{T} \frac{1}{\prod_{s=0}^{t-1}(1+r_s)} \left\{ \left[F_t(K_t, L_t) - w_t L_t - I_t - \Phi(I_t, K_t)\right] + q_{t+1}\left[K_{t+1} - (1-\delta)K_t - I_t\right] \right\}$$

- First-order conditions with respect to L_t, I_t, K_t are

$$\frac{\partial F_t(K_t, L_t)}{\partial L_t} = w_t$$

$$1 + \frac{\partial \Phi(I_t, K_t)}{\partial I_t} = q_{t+1}$$

$$q_t = \frac{1}{1 + r_{t-1}} \left[\frac{\partial F_t(K_t, L_t)}{\partial K_t} - \frac{\partial \Phi(I_t, K_t)}{\partial K_t} + (1-\delta) q_{t+1}\right]$$

Optimality Conditions

- With our functional form assumptions, the first two conditions can be written as

$$L_t = (1-\alpha)^{1/\alpha} A_t^{1/\alpha} w_t^{-1/\alpha} K_t \tag{7}$$

$$\frac{I_t}{K_t} = \frac{1}{\phi}\left[q_t - 1\right]$$

- Using the above two, the thrid condition is

$$\begin{aligned} q_t &= \frac{1}{1+r_t}\left[\alpha A_{t+1} K_{t+1}^{\alpha-1} L_{t+1}^{1-\alpha} + \frac{\phi}{2}\left(\frac{I_{t+1}}{K_{t+1}}\right)^2 + (1-\delta) q_{t+1}\right] \\ &= \frac{1}{1+r_t}\left[\alpha (A_t)^{\frac{1}{\alpha}} (1-\alpha)^{\frac{1-\alpha}{\alpha}} w_t^{-\frac{1-\alpha}{\alpha}} - \frac{I_{t+1}}{K_{t+1}} - \frac{\phi}{2}\left(\frac{I_{t+1}}{K_{t+1}}\right)^2 + \left(\frac{I_{t+1}}{K_{t+1}} + (1-\delta)\right) q_{t+1}\right] \end{aligned} \tag{8}$$

Firm's Value

- Define the firm's value as the cumulative discounted sum of future profits

$$
\begin{aligned}
V_t &= \sum_{k=t}^{T} \frac{1}{\prod_{s=t}^{t}(1+r_s)} D_k \\
&= \frac{1}{1+r_t}\left[D_t + \sum_{k=t+1}^{T} \frac{1}{\prod_{s=t+1}^{t}(1+r_s)} D_k\right] \\
&= \frac{1}{1+r_t}\left[F_t(K_t, L_t) - w_t L_t - I_t - \Phi(I_t, K_t) + V_{t+1}\right]
\end{aligned}
$$

Firm's Value per unit Capital = Q

- The firm's value per unit capital is, using (6),

$$\frac{V_t}{K_t} = \frac{1}{1+r_t}\left[\alpha(A_t)^{\frac{1}{\alpha}}(1-\alpha)^{\frac{1-\alpha}{\alpha}}w_t^{-\frac{1-\alpha}{\alpha}} - \frac{I_t}{K_t} - \frac{\phi}{2}\left(\frac{I_t}{K_t}\right)^2 + \frac{K_{t+1}}{K_t}\frac{V_{t+1}}{K_{t+1}}\right]$$

$$= \frac{1}{1+r_t}\left[\alpha(A_t)^{\frac{1}{\alpha}}(1-\alpha)^{\frac{1-\alpha}{\alpha}}w_t^{-\frac{1-\alpha}{\alpha}} - \frac{I_t}{K_t} - \frac{\phi}{2}\left(\frac{I_t}{K_t}\right)^2 + \left(\frac{I_{t+1}}{K_{t+1}} + (1-\delta)\right)\frac{V_{t+1}}{K_{t+1}}\right] \quad (9)$$

- Comparing (8) and (9), we conclude

$$q_t = \frac{V_t}{K_t}$$

Q-Theory of Investment

- Q-theory of investment:

$$\frac{I_t}{K_t} = \frac{1}{\phi}\left[q_t - 1\right]$$

- Investment is positive if and only if $q_t > 1$
  - The average value of capital, "q", is higher than its cost
- "q" is of course a function of parameters:

$$q_t = \frac{1}{1+r_t}\left[\alpha(A_{t+1})^{\frac{1}{\alpha}}(1-\alpha)^{\frac{1-\alpha}{\alpha}} w_{t+1}^{-\frac{1-\alpha}{\alpha}} - \frac{I_{t+1}}{K_{t+1}} - \frac{\phi}{2}\left(\frac{I_{t+1}}{K_{t+1}}\right)^2 + \left(\frac{I_{t+1}}{K_{t+1}} + (1-\delta)\right) q_{t+1}\right]$$

  - q_t is higher if A_t is higher, r_t is lower, and w_t is lower

Stimulating Investment through Temporary Tax Incentives

– Zwick and Mahon (2017)

Background

- Background: weak investment during 00-01 and 07-08 recessions
- In response, Congress passed a bill that allows “bonus depreciation”
- The policies were intended as economic stimulus
- What is “bonus depreciation”?
- Was it successful in stimulating investment?

Tax System

- Consider a firm buying $1 million worth of computers
- The firm owes corporate taxes on income net of business expenses
- Expenses on nondurable items (e.g., wages):
  the firm can immediately deduct the full cost of these items on its tax return
- Expenses on investment:
  the firm split deduction over multiple years (exact schedule differs by investment)

Example (corporate tax rate = 35%)

Year:	0	1	2	3	4	5	Total
Normal depreciation							
Deductions (000s)	200	320	192	115	115	58	1,000
Tax benefit ($\tau = 35$ percent)	70	112	67.2	40.3	40.3	20.2	350

Bonus Depreciation

- Bonus depreciation allows the firms to deduct x% of the investment immediately
- The total amount deducted over time does not change
- Bonus depreciation only accelerates the deductions. Why stimulate investment?

Example of 50% bonus depreciation

Year:	0	1	2	3	4	5	Total
Normal depreciation							
Deductions (000s)	200	320	192	115	115	58	1,000
Tax benefit ($\tau = 35$ percent)	70	112	67.2	40.3	40.3	20.2	350
Bonus depreciation (50 percent)							
Deductions (000s)	600	160	96	57.5	57.5	29	1,000
Tax benefit ($\tau = 35$ percent)	210	56	33.6	20.2	20.2	10	350

Modeling Taxes

- Back to the two-period model
- Let τ be corporate tax rate
- Let z_0 be the percentage of $t = 0$ investment that firms can deduct immediately
- The remaining $z_1 \equiv 1 - z_0$ are deducted at $t = 1$

Investment Problem with Taxes

$$\max_{L_0, I_1, K_1, L_1} \left[(1-\tau)\left[F_0(K_0, L_0) - w_0 L_0\right] - I_0 - \Phi(I_0, K_0) + \tau z_0 \left(I_0 - \Phi(I_0, K_0)\right)\right]$$
$$+ \frac{1}{1+r}\left[(1-\tau)\left(F_1(K_1, L_1) - w_1 L_1\right) + \tau z_1 \left(I_0 + \Phi(I_0, K_0)\right)\right]$$

subject to

$$K_1 = K_0(1-\delta) + I_0$$

Optimality Conditions

- The first-order conditions are

$$L_t = (1-\alpha)^{1/\alpha} A_t^{1/\alpha} w_t^{-1/\alpha} K_t \tag{10}$$

$$\left[1 - \tau z_0 - \frac{1}{1+r}\tau z_1\right]\left(1 + \phi \frac{I_0}{K_0}\right) = \frac{1-\tau}{1+r}\alpha A_1 K_1^{\alpha-1} L_1^{1-\alpha} \tag{11}$$

- Define $z^N \equiv \tau z_0 + \dfrac{1}{1+r}\tau z_1$:
  - The presented discounted value of deductions per unit investment
- Plugging (10) into (11),

$$\frac{I_0}{K_0} = \frac{1}{\phi}\left[\frac{1}{1+r}\frac{1-\tau}{1-z^N}\alpha(1-\alpha)^{\frac{1-\alpha}{\alpha}} A_1^{\frac{1}{\alpha}} w_1^{-\frac{1-\alpha}{\alpha}} - 1\right]$$

Impact of Bonus Depreciation in the Model

$$\frac{I_0}{K_0} = \frac{1}{\phi}\left[\frac{1}{1+r}\frac{1-\tau}{1-z^N}\alpha(1-\alpha)^{\frac{1-\alpha}{\alpha}}A_1^{\frac{1}{\alpha}}w_1^{-\frac{1-\alpha}{\alpha}} - 1\right]$$

- Bonus depreciaation b allows firms to deduct extra $b\,\%$ of z_1 at $t = 0$

$$z^N = \tau(z_0 + bz_1) + \frac{1}{1+r}\tau(z_1 - bz_1)$$

- How does bonus depreciation (an increase in b) affect investment?

$$\frac{d(I_0/K_0)}{dz^N} > 0, \quad \frac{dz^N}{db} = \tau z_1\left(1 - \frac{1}{1+r}\right) > 0$$

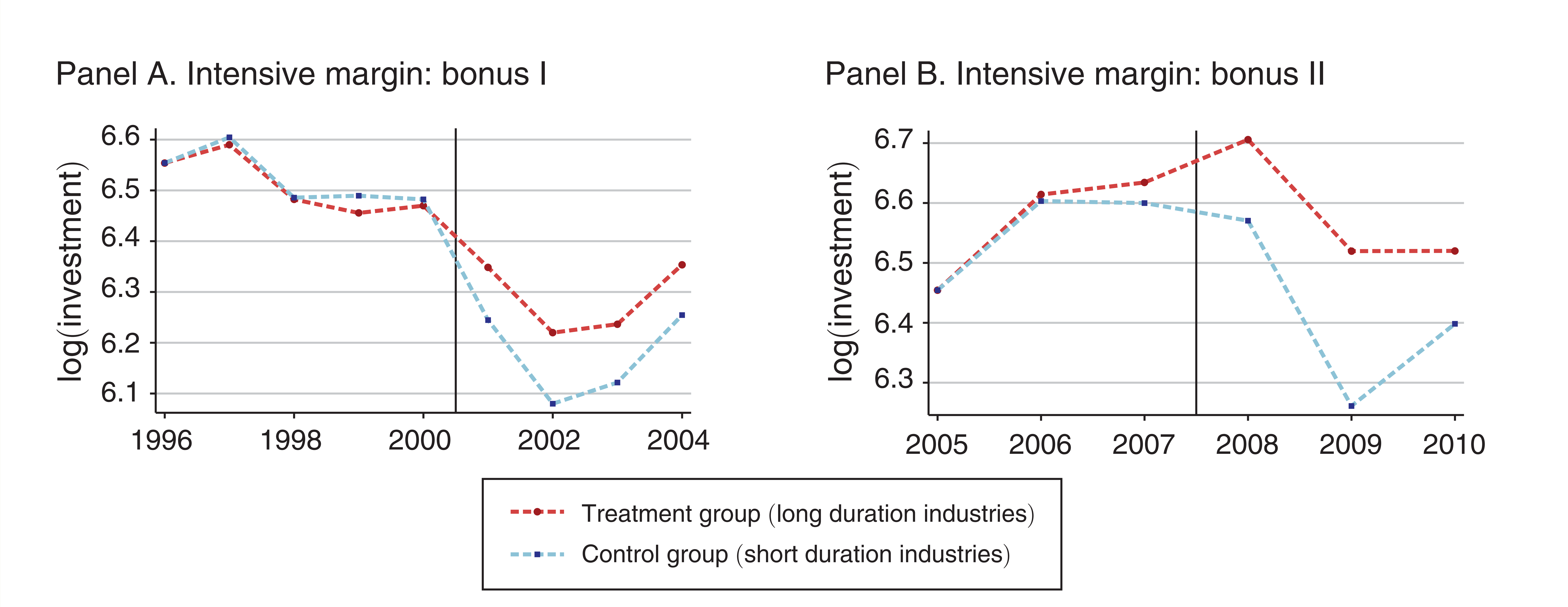
  - Bonus depreciation increases present discounted value of deductions if $r > 0$
  - More deductions lower the effective investment costs and stimulate investment

Empirical Setup

- Bonus depreciation implementation (b in our model):
  - 2001-2003: 30%
  - 2003-2004: 50%
  - 2008-2010: 50%
  - 2009-2010: 100%
- Construct presented discounted value of deductions (z^N in our model) by industry
- Industries had the same b but differed in the original deductions schedule, z_0 & z_1
- If industries have long-duration schedules (higher z_1), the impact of b is higher

$$\frac{dz^N}{db} = \tau z_1 \left(1 - \frac{1}{1+r}\right)$$

Impact of Bonus Derpeciation



- Higher-duration industries increase investment relative to low-duration industries
- Bonus depreciation raised investment of eligible capital by 10-15%

Heterogeneous Responses

TABLE 6—HETEROGENEITY BY EX ANTE CONSTRAINTS

	Sales		Div payer?		Lagged cash	
	Small	Big	No	Yes	Low	High
$z_{N,t}$	6.29	3.22	5.98	3.67	7.21	2.76
	(1.21)	(0.76)	(0.88)	(0.97)	(1.38)	(0.88)
Equality test	$p = 0.030$		$p = 0.079$		$p = 0.000$	
Observations	177,620	255,266	274,809	127,523	176,893	180,933
Clusters (firms)	29,618	29,637	39,195	12,543	45,824	48,936
R^2	0.44	0.76	0.69	0.80	0.81	0.76

- Investment response is larger for
  - smaller firms
  - firms paying no dividends
  - firms with low cash holdings

Why Do Size, Dividend, and Cash Matter?

- Do size, dividend, and cash matter in our model?
- No. Recall:

$$\frac{I_0}{K_0} = \frac{1}{\phi}\left[\frac{1}{1+r}\frac{1-\tau}{1-Z_0}\alpha(1-\alpha)^{\frac{1-\alpha}{\alpha}}A_1^{\frac{1}{\alpha}}w_1^{-\frac{1-\alpha}{\alpha}} - 1\right]$$

- Whether firms are large, pay dividends, or hold cash is irrelevant (on their own)
- Then why do we find such heterogeneity in the data?
- One explanation is financial friction
- The prevalence of financial friction is correlated with size, dividend, and cash
- Constrained firms could react more to bonus depreciation