# **Development and Growth Accounting**

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EC502 Macroeconomics Topic 1





### Why are Some Countries Richer than Others?







### **Cross-Country Income Differences**

#### United States today are

- 1. 5 times richer than people in China
- 2. 10 times richer than people in India
- 3. more than 40 times richer than people in Haiti

What drives these enormous differences in standards of living across countries?



### **Role of Models**

All theory depends on assumptions which are not quite true. That is what makes it theory. The art of successful theorizing is to make the inevitable simplifying assumptions in such a way that the final results are not very sensitive.

-Robert Solow



### **Production Function**

- Suppose the output of a country is produced using
  - 1. Labor, L
  - 2. Physical capital (machines, building, etc), K
- A production function tells us how much we can produce output given L and K:



- We say F(K, L) features
  - constant returns to scale if  $F(\lambda L, \lambda K) = \lambda F(L, K)$
  - decreasing returns to scale if  $F(\lambda L, \lambda K) < \lambda F(L, K)$
  - increasing returns to scale if  $F(\lambda L, \lambda K) > \lambda F(L, K)$

# Y = F(K, L)



# **Cobb-Douglas Production Function** A popular functional form is Cobb-Douglas production function $Y = F(K, L) = AK^{\alpha}L^{\beta}$

- A: the level of technology
- $\alpha, \beta \in [0,1]$ : importance of each factor

Using the previous definition,

- $\alpha + \beta = 1 \Rightarrow$  constant returns to scale
- $\alpha + \beta < 1 \Rightarrow$  decreasing returns to scale
- $\alpha + \beta > 1 \Rightarrow$  increasing returns to scale

We will assume constant returns to scale. Why? **Replication argument:** If all the inputs double, output should double



# **Important Distinction** $F(K, L) = AK^{\alpha}L^{1-\alpha}$

- Here, F(K, L) is constant returns to scale to all inputs
- But, F(K, L) features diminishing returns to a particular input
  - If we only double K, output less than doubles:  $F(2V I) = 2^{\alpha}F(V I)$
  - Equivalently, *F*(*K*, *L*) is concave in both arguments:
    - $F_{KK}(K,L) < 0, \quad F_{LL}(K,L) < 0$

 $F(2K,L) = 2^{\alpha}F(K,L) < 2F(K,L)$ 



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### Development Accounting





# **Decomposing GDP per Capita** $Y_i = A_i K_i^{\alpha} L_i^{1-\alpha}$

- *i*: country
- Divide both sides by population size, N<sub>i</sub>, and taking log:

$$\log(Y_i/N_i) = \log A_i + a$$

 $\alpha \log(K_i/N_i) + (1-\alpha)\log(L_i/N_i)$ 



- *i*: country
- Divide both sides by population size, N<sub>i</sub>, and taking log:



**Decomposing GDP per Capita**  $Y_i = A_i K_i^{\alpha} L_i^{1-\alpha}$ 

 $= \log A_{i} + \alpha \log(K_{i}/N_{i}) + (1 - \alpha)\log(L_{i}/N_{i})$ **Employment per capita** 



- *i*: country
- Divide both sides by population size,  $N_i$ , and taking log:



- How much of differences in GDP per capita due to
  - 1. capital
  - 2. labor
  - 3. technology (which we don't directly observe)

**Decomposing GDP per Capita**  $Y_i = A_i K_i^{\alpha} L_i^{1-\alpha}$ 

$$L\log(K_i/N_i)$$
 +  $(1 - \alpha)\log(L_i/N_i)$   
Employment per  $(1 - \alpha)\log(L_i/N_i)$ 



### **Development Accounting**

- This exercise called development accounting
  - It is accounting because we do not theorize how each component is determined
- Nevertheless, it helps us to guide what theoretical model we should write down
- In order to implement development accounting, we need to take a stand on  $\alpha$
- What value should we use for  $\alpha$ ?

 $\log(Y_i/N_i) = \log A_i + \alpha \log(K_i/N_i) + (1 - \alpha)\log(L_i/N_i)$ 





#### **Factor Shares**

- Factor shares: what fraction of GDP is paid to each factor?
- Suppose firms need pay w to hire workers and r to rent machines
- Firms take (w, r) as given (competitive market) and choose (L, K):
  - K,L
  - Taking the first-order condition with respect to L  $(1 - \alpha)AK^{\alpha}L^{-\alpha} = w$ The firm equalizes the marginal product of labor to wages
- Multiplying both sides of (1) by  $L_{i}$

 $\max AK^{\alpha}L^{1-\alpha} - wL - rK$ 

#### $\frac{wL}{V} = (1 - \alpha) \implies \text{Labor share of GDP is } 1 - \alpha$



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#### **Stable Labor Share**



### **Technology as Residual**

- Labor share  $\approx 2/3$  and stable over time, so we assume  $\alpha = 1/3$
- With the assumed value of  $\alpha$ , we can construct a measure of "technology"  $\log A_i = \log(Y_i/N_i) - \alpha \log(K_i/N_i) - (1 - \alpha)\log(L_i/N_i)$ 
  - Also referred to as "total factor productivity (TFP)" or "Solow residual" •  $\log A_i$  captures differences in GDP not captured by K/N or L/N

  - Measure of our ignorance



### First Look at the Data 2019

|       | Y/N | K / N | L / N | A   |
|-------|-----|-------|-------|-----|
| U.S.  | 100 | 100   | 100   | 100 |
| China | 22  | 33    | 116   | 30  |
| India | 10  | 12    | 76    | 26  |
| Haiti | 2.5 | 7     | 84    | 7   |

Data: Penn World Table 2019

Large differences in K/N and A

Little difference in L/N (employment per person)



#### Variance Decomposition

We can explore more systematically

 $\operatorname{Var}\left(\log Y_i/N_i\right) = \operatorname{Cov}\left(\log(Y_i/N_i), \alpha \log K_i/N_i\right)$ +Cov  $(\log Y_i/N_i, (1 - \alpha)\log L_i/N_i)$ +Cov  $(\log Y_i/N_i, \log A_i)$ 

Therefore, 
$$\frac{\text{Cov}(\log Y_i/N_i, \log X_i)}{\text{Var}(\log Y_i/N_i)} \text{ corre}$$

This can be obtained as a regression coefficient  $\beta_X$  of

If  $\beta_X = 1$ , differences in GDP per capita entirely due to X

- Variance in GDP due to K/N Variance in GDP due to L/NVariance in GDP due to A
- esponds to the share explained by a factor X
- $\log X_i = \beta_X \log(Y_i/N_i) + \gamma + \epsilon_i$







Cross-country income differences due to K/N: 37%, L/N: 7%, A: 56%

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### What Did We Miss?

- Nontirival fraction of income differences due differences in capital
  - This motivates us to build a theory that determines capital
- However, more than half of the differences due to TFP
- Disappointing because more than half attributed to something we don't observe Observable country characteristics explain less than half of income differences Are you convinced? What did we potentially miss?

![](_page_18_Picture_7.jpeg)

![](_page_18_Picture_22.jpeg)

Before, we assumed all workers worked for the same hours in all countries

If  $h_i$  is higher for richer countries, this may help explain income differences

#### 1. Hours Worked

 $Y_i = A_i K_i^{\alpha} (h_i L_i)^{1-\alpha}$ 

- *h*<sub>*i*</sub>: hours worked per worker

![](_page_19_Picture_9.jpeg)

### Hours Worked Declines with GDP

![](_page_20_Figure_1.jpeg)

![](_page_20_Picture_2.jpeg)

### **Development Accounting with Hours Worked**

#### $\alpha \log(K/N)$

![](_page_21_Figure_2.jpeg)

Even more important role of A once we allow hours worked to vary

#### $(1 - \alpha)\log(hL/N)$

 $\log A$ 

![](_page_21_Picture_7.jpeg)

#### 2. Human Capital We have assumed that workers in rich countries and poor countries a Our World in Data same

- Is this plausible? Perhaps not

![](_page_22_Figure_3.jpeg)

![](_page_22_Picture_4.jpeg)

### How Do We Measure Human Capital?

Now we construct the human capita index:

- $L_i^s$ : number of workers with schooling year s
- $\phi^s$ : relative efficiency of workers with schooling year s
- We normalize  $\phi^0 = 1$
- How do we obtain  $\phi^s$ ?

$$L_i = \sum_{s=0}^{S} \phi^s L_i^s$$

ng year *s* th schooling year *s* 

![](_page_23_Picture_8.jpeg)

### Inferring Human Capital from Wages

- Suppose workers with different schooling years are paid different wages
- The profit maximization is now

$$\max_{K,L_i^s} AK^{\alpha} \left( \sum_{s} \phi^s L_i^s \right)^{1-\alpha} - \sum_{s} w_i^s L_i^s - rK$$

Taking the first-order condition with re

 $(1 - \alpha)\phi^{s}AK^{\alpha}$ 

espect to 
$$L_i^s$$
,

$$\left(\sum_{s} \phi^{s} L_{i}^{s}\right)^{-\alpha} = w_{i}^{s}$$

 $\frac{\phi^s}{\phi^0} = \frac{w_i^s}{w_i^0} \implies \text{relative wages informative about } \phi^s$ 

![](_page_24_Picture_11.jpeg)

- Many estimates of  $\{w_i^s\}$  in the labor economics literature
  - How wages vary depending on education
  - Let's talk more about this in a few slides
- Now we plug estimates of  $\phi^s$  and construct our human capital index:

 $L_i =$ 

• With new  $L_i$ , let us re-do development accounting

![](_page_25_Picture_7.jpeg)

$$= \sum_{s=0}^{S} \phi^{s} L_{i}^{s}$$

![](_page_25_Picture_10.jpeg)

### **Differences in Human Capital**

|       | Y/N | K / N | L / N<br>employment | L/N<br>human capital | A<br>human capital |
|-------|-----|-------|---------------------|----------------------|--------------------|
| U.S.  | 100 | 100   | 100                 | 100                  | 100                |
| China | 22  | 33    | 116                 | 83                   | 37                 |
| India | 10  | 12    | 76                  | 44                   | 38                 |
| Haiti | 2.5 | 7     | 84                  | 38                   | 12                 |

Data: Penn World Table 2019

More differences in L/N, but not quite as much as A or K/N

![](_page_26_Picture_5.jpeg)

# Development Accounting with Human Capital $\alpha \log(K/N)$ $(1 - \alpha)\log(L/N)$ $\log A$

![](_page_27_Figure_1.jpeg)

Cross-country income differences due to K/N: 37%, L/N: 18%, A: 45%

![](_page_27_Picture_3.jpeg)

## **Ongoing Debate**

- Human capital explains 18% of cross-country income differences
- This reduces the contribution of our measure of ignorance to less than half
- Lots of debate on the role of human capital:
  - 1. Functional form:  $L_i = G(\{L_i^s\}_{s=0}^S)$  rather than  $L_i = \sum_{s=0}^S \phi^s L_i^s$
  - 2.  $\phi^s$  could be different across countries
  - 3. Schooling is not the only source of human capital (e.g., experience)
- Some argue human capital can explain almost all cross-country differences

![](_page_28_Picture_8.jpeg)

![](_page_28_Picture_17.jpeg)

#### Detour: How Do We Estimate Returns to Education?

![](_page_29_Picture_1.jpeg)

## **How Do Wages Change with Schooling?**

- How should we obtain estimates of returns to education,  $\phi^s$ ?
- How wages vary depending on education:
  - log wage<sub>i</sub> =  $\beta \times$  (Years of Schooling)<sub>i</sub> +  $\epsilon_i$
- Natural to expect that Cov(Years of schooling,  $\epsilon_i$ )  $\neq 0$ 
  - Maybe more talented individuals are more likely to go to schools
- **Cannot run OLS to estimate**  $\beta$ 
  - More educated people are highly paid not necessarily because of education!

Determinants of wage other than schooling

![](_page_30_Picture_9.jpeg)

![](_page_30_Picture_10.jpeg)

#### THE SVERIGES RIKSBANK PRIZE IN ECONOMIC SCIENCES IN MEMORY OF ALFRED NOBEL 2021

![](_page_31_Picture_1.jpeg)

#### David Card

"for his empirical contributions to labour economics"

THE ROYAL SWEDISH ACADEMY OF SCIENCES

#### Guido Joshua D. Angrist W. Imbens

"for their methodological contributions to the analysis of causal relationships"

![](_page_31_Picture_8.jpeg)

# Schooling and Timing of Birth

![](_page_32_Figure_1.jpeg)

![](_page_32_Picture_2.jpeg)

## **Earnings and Timing of Birth**

![](_page_33_Figure_1.jpeg)

![](_page_33_Picture_2.jpeg)

### What are Returns to Education?

- One additional year of schooling  $\Rightarrow \approx 10\%$  increase in earnings
- Why do earnings increase?
  - Maybe classes are useless, but degrees are

![](_page_34_Picture_4.jpeg)

The effect of human capital on earnings: Evidence from a reform at Colombia's top university  $\stackrel{\diamond}{}$ 

Carolina Arteaga

Department of Economics, UCLA, United States

#### A R T I C L E I N F O

JEL classifications. I23 I25 I26 J24 J31 Keywords: Education Human capital Signaling

#### ABSTRACT

In this paper I test whether the return to college education is the result of human capital accumulation or instead reflects the fact that attending college signals higher ability to employers. I exploit a reform at Universidad de Los Andes, which in 2006 reduced the amount of coursework required to earn degrees in economics and business by 20% and 14%, respectively, but did not change the quality of incoming or graduating students. The size of the entering class, their average high school exit exam scores, and graduation rates were not affected by the reform, indicating that selection of students into the degrees remained the same. Using administrative data on wages and college attendance, I estimate that wages fell by approximately 16% in economics and 13% in business. These results suggest that human capital plays an important role in the determination of wages and reject a pure signaling model. Surveying employers, I find that the reduction in wages may have resulted from a decline in performance during the recruitment process, which led students to be placed in lower-quality firms. Using data from the recruitment process for economists at the Central Bank of Colombia, I find that the reform reduced the probability of Los Andes graduates' being hired by 17 percentage points.

![](_page_34_Picture_17.jpeg)

Check for

![](_page_34_Picture_18.jpeg)

# **Development Accounting using Immigrants**

![](_page_35_Picture_1.jpeg)

![](_page_35_Picture_2.jpeg)

### What Do Immigrants Tell Us?

- Let us tackle the problem from a different angle (Hendricks and Schoellman, 2018)
- Focus on immigrants to the US
- How much wage gains do immigrants experience upon arrival to the US?
- Immigrants bring their human capital (L) but do not bring A or K of home country A
  - Instead, they can now use technology or physical capital in the US
- If A or K very important, their wages rise one-for-one with GDP gap
- If A or K not important, their wages should not change

![](_page_36_Picture_8.jpeg)

![](_page_36_Picture_9.jpeg)

## Wage Gains from Immigration

![](_page_37_Figure_1.jpeg)

#### Pre- and Post-Migration Wages

Source: Hendricks and Schoellman (2018)

![](_page_37_Figure_4.jpeg)

Wage Gains at Migration

![](_page_37_Picture_6.jpeg)

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### How Do Wage Gains Compare to GDP Gap?

Group

Hourly Wage Pre-Mig. Post-Mig.

Panel A: NIS Sample by GDP per wor

| < 1/16     | \$2.82  | \$8.91  |
|------------|---------|---------|
| 1/16 - 1/8 | \$4.19  | \$11.83 |
| 1/8 - 1/4  | \$4.95  | \$9.48  |
| 1/4 - 1/2  | \$5.05  | \$9.11  |
| 1/2 - 1    | \$12.64 | \$15.18 |

Source: Hendricks and Schoellman (2018)

- Wage gains are typically much smaller than GDP gap
- Differences in TFP or physical cannot be the whole story

| Development Accounting |         |         |               |
|------------------------|---------|---------|---------------|
| Wage Gain              | GDP Gap | h share | 95% C.I.      |
| rker categor           | у       |         |               |
| 3.2                    | 31.8    | 0.66    | (0.60, 0.73)  |
| 2.8                    | 11.9    | 0.58    | (0.54, 0.62)  |
| 1.9                    | 5.6     | 0.63    | (0.55, 0.71)  |
| 1.8                    | 3.0     | 0.48    | (0.34, 0.62)  |
| 1.2                    | 1.3     | 0.48    | (-0.23, 1.19) |

This implies that human capital is an important component of income differences

![](_page_38_Picture_13.jpeg)

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### Growth Accounting

![](_page_39_Picture_1.jpeg)

![](_page_39_Picture_2.jpeg)

![](_page_39_Picture_3.jpeg)

![](_page_40_Figure_1.jpeg)

![](_page_40_Picture_2.jpeg)

### **Growth Accounting**

- Why do countries grow?
- The growth rate of the economy between t and t + T:  $\Delta_T \log(Y_t/N_t) \equiv \log(Y_t/N_t)$
- With  $Y_t = A_t K_t^{\alpha} L_t^{1-\alpha}$ , we can decompose growth into:  $\Delta_T \log(Y_t/N_t) = \alpha \Delta_T \log(K_t/N_t)$  $+(1-\alpha)\Delta_T\log(1-\alpha)$  $+\Delta_T \log(A_t)$ 

  - Growth accounting: decomposition over time-series • **Development accounting**: decomposition over cross-section

$$g(Y_{t+T}/N_{t+T}) - \log(Y_t/N_t)$$

|             | Growth due to K |
|-------------|-----------------|
| $(L_t/N_t)$ | Growth due to L |
|             | Growth due to A |

![](_page_41_Picture_8.jpeg)

## **Growth Accounting: US**

![](_page_42_Figure_1.jpeg)

![](_page_42_Picture_2.jpeg)

### **Growth Accounting: Asia**

![](_page_43_Figure_1.jpeg)

![](_page_43_Figure_2.jpeg)

![](_page_43_Figure_3.jpeg)

![](_page_43_Picture_4.jpeg)

### **Growth Accounting: Europe**

![](_page_44_Figure_1.jpeg)

![](_page_44_Figure_2.jpeg)

![](_page_44_Figure_3.jpeg)

![](_page_44_Figure_4.jpeg)

![](_page_44_Picture_5.jpeg)

![](_page_45_Figure_0.jpeg)

![](_page_45_Picture_1.jpeg)

![](_page_46_Picture_0.jpeg)

- In almost all countries, the predominant driver of growth is TFP
- Capital is also important
- Labor seems to matter less

### Takeaway from Growth Accounting

![](_page_46_Picture_6.jpeg)

- We have learned two accounting tools
- Development accounting: Cross-sectional decomposition of difference in GDP per capital
- **Growth accounting:** Time-series decomposition of growth in GDP per capital
- Both exercises suggest that
  - 1. important role of *K*
  - 2. even more important role of A
- Next lectures develop theories that determine K and A

![](_page_47_Picture_8.jpeg)

![](_page_47_Picture_13.jpeg)