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# Labor Supply Across Countries and Over Time

EC502 Macroeconomics  
Topic 6

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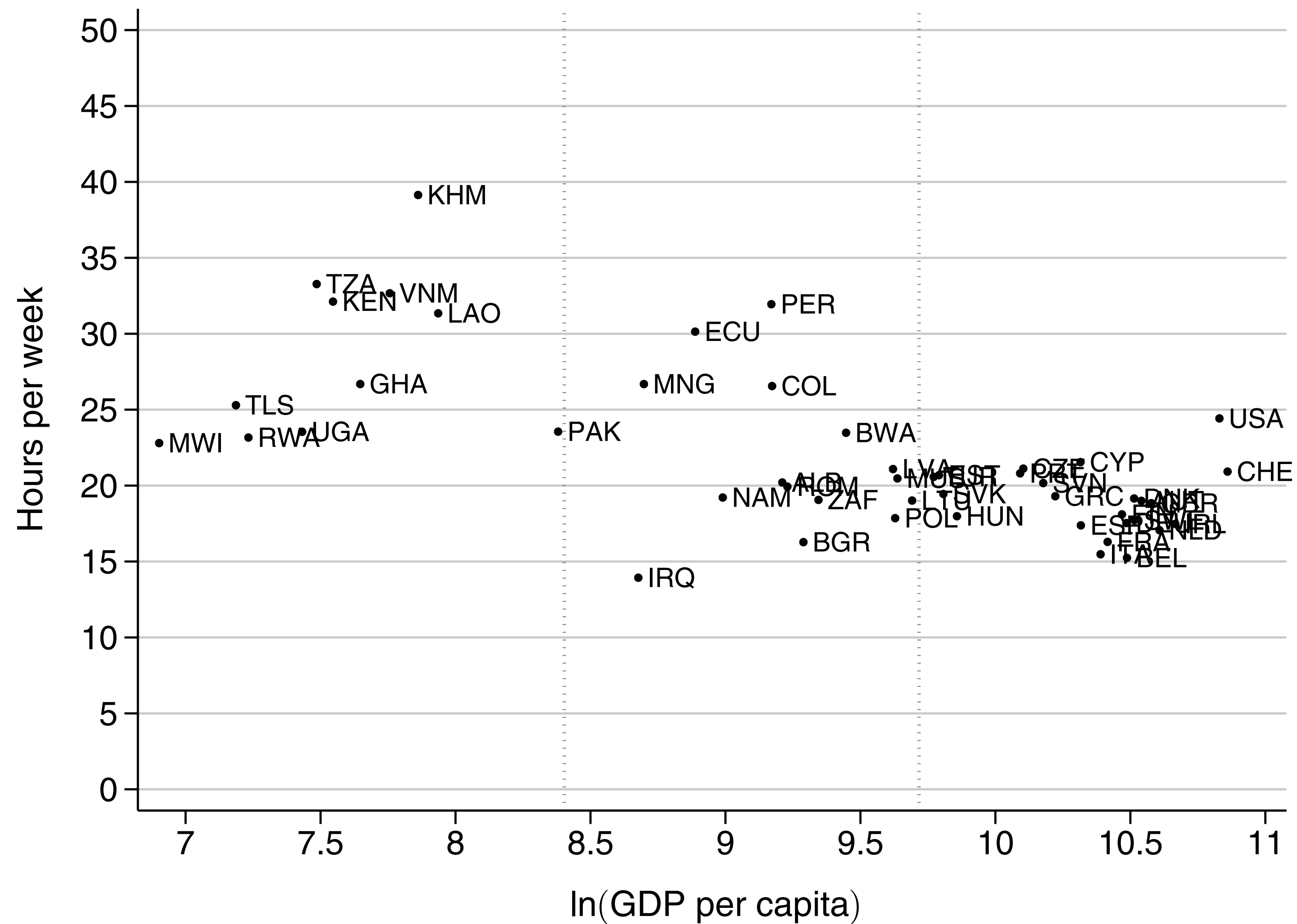
2025 Spring

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# Hours Worked

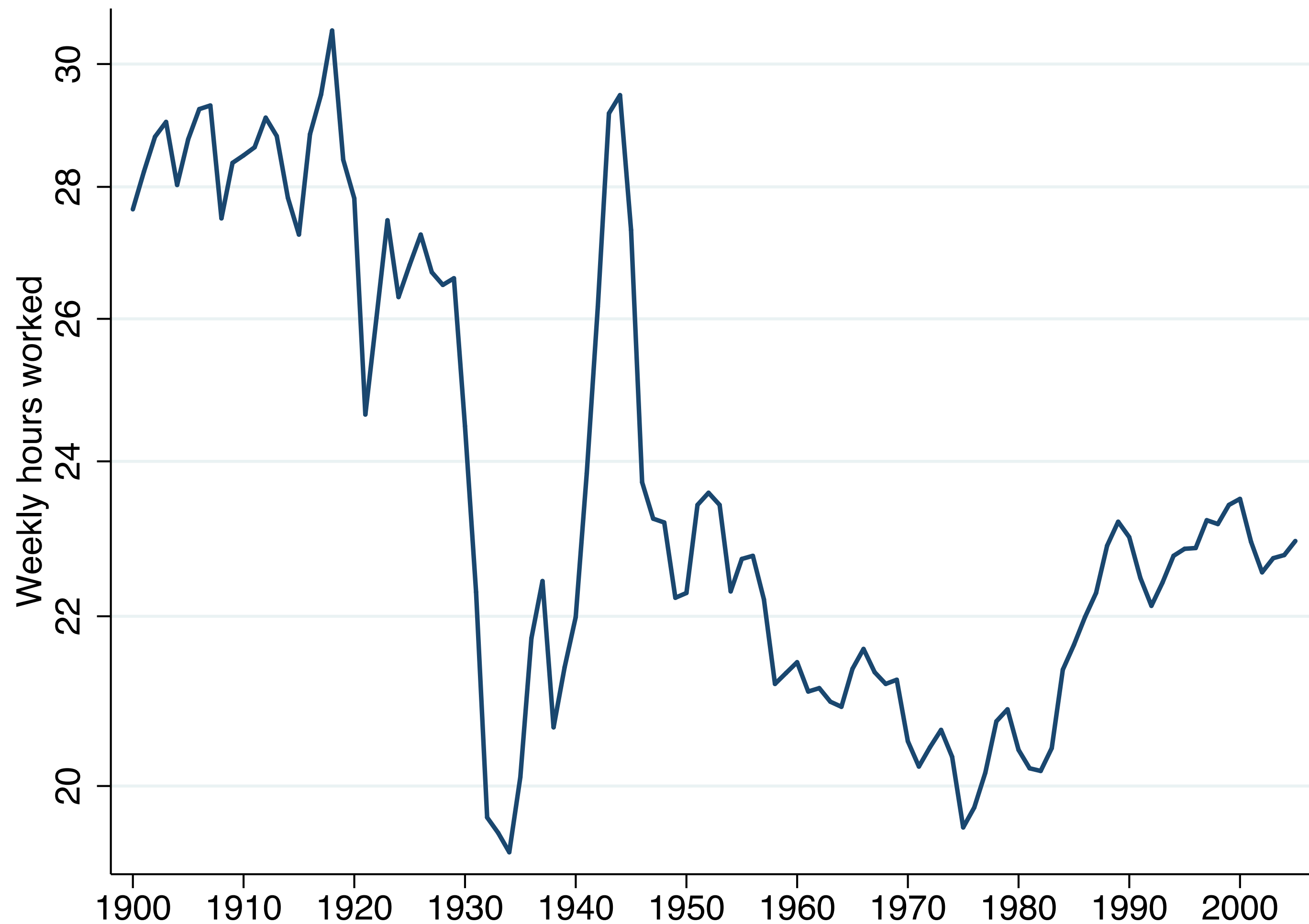
- We took hours worked per person as exogenous so far
- We assumed that everyone supplies a fixed amount of labor
- Is this true?

# Cross-Section in 2005



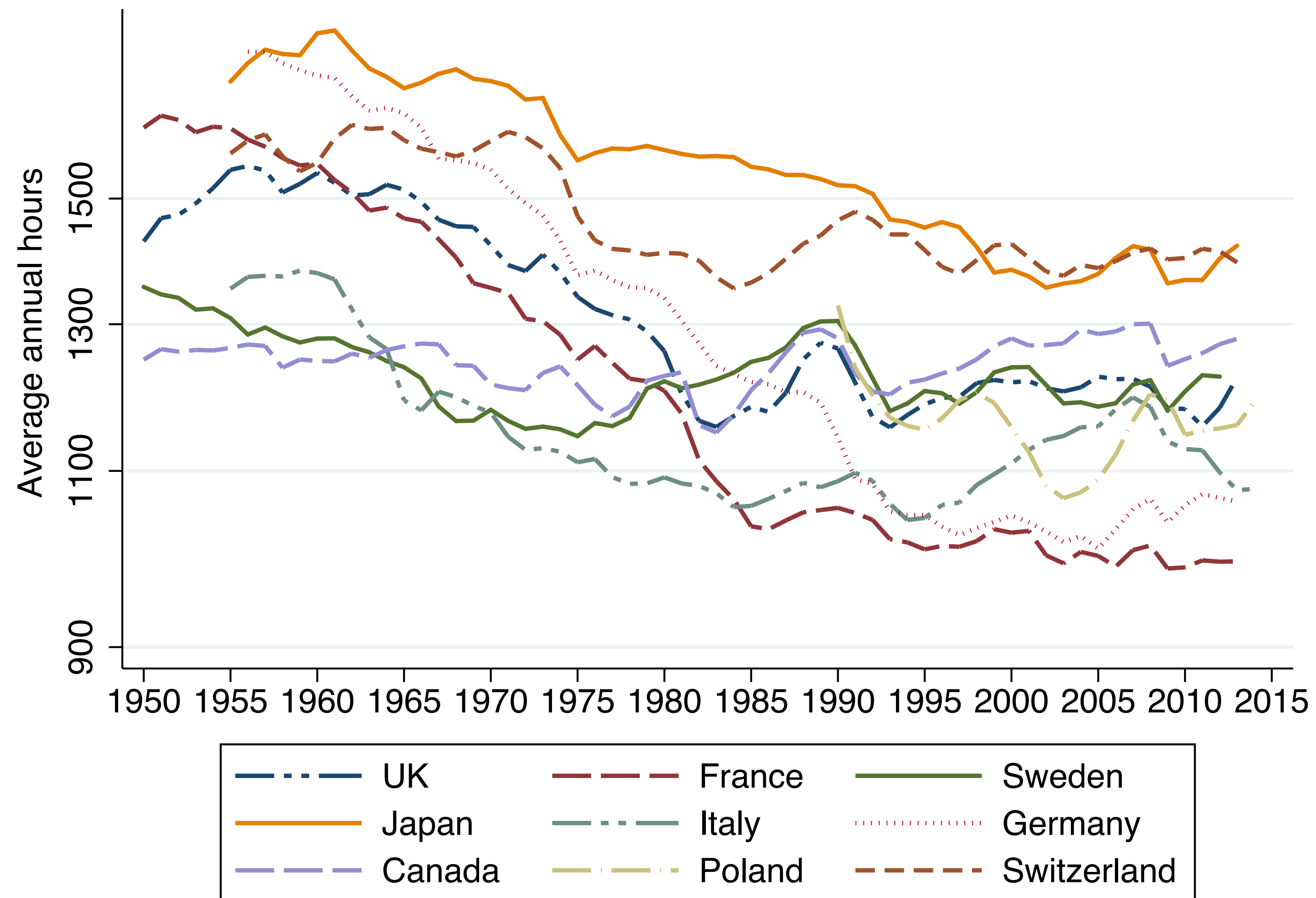
Source: Bick, Fuchs-Schündeln, Lagakos (2018)

# The U.S. Time-Series



Source: Boppart and Krusell (2020)

# Post-WWII Advanced Economies



Source: Boppart and Krusell (2020)

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# Question

- In the cross-section, richer countries tend to work less
- Over the time-series, as a country gets richer, people work less
- Why?

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# A Simple Model of Labor Supply

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# A Model of Labor Supply

- What is the benefit of working more? – earn a higher income
- What is the cost of working more? – pain to work longer hours
- We introduce a minimal model that captures these trade-offs



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# Preferences

- Households have the following utility functions:

$$u(c) - v(l)$$

- We  $u$  is concave and  $v$  is convex:

- $u'(c) > 0$ : households are happier if consumption is higher
- $u''(c) < 0$ : additional consumption is less pleasant if already consuming a lot
- $v'(l) > 0$ : households are less happy if they work more
- $v''(l) > 0$ : additional hours of work are more painful if already working a lot

- The households face the following budget constraint

$$c = wl$$

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# Optimality Condition

- The households decide  $(c, l)$  subject to the budget constraint:

$$\max_{c,l} u(c) - v(l)$$

$$\text{s.t. } c = wl$$

- First-order condition:

$$u'(c)w = v'(l)$$

- LHS: marginal benefit of work
- RHS: marginal cost of work

# Functional Form Assumptions

- For simplicity, we assume

$$u(c) = \frac{c^{1-\sigma} - 1}{1-\sigma}, \quad v(l) = \bar{v} \frac{l^{1+\nu}}{1+\nu}$$

with  $\sigma > 0$  and  $\nu > 0$

- One can check:

- $u'(c) = c^{-\sigma} > 0$ ,  $u''(c) = -\sigma c^{-\sigma-1} < 0$
- $v'(l) = \bar{v} l^{\nu} > 0$ ,  $v''(l) = \bar{v} \nu l^{\nu-1} > 0$

- For  $\sigma = 1$ ,

$$u(c) = \log c \quad \text{when } \sigma = 1$$

- Obtained as a limit of  $\sigma \rightarrow 1$  (apply L'hospital's rule)

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# Optimal Labor Supply Solutions

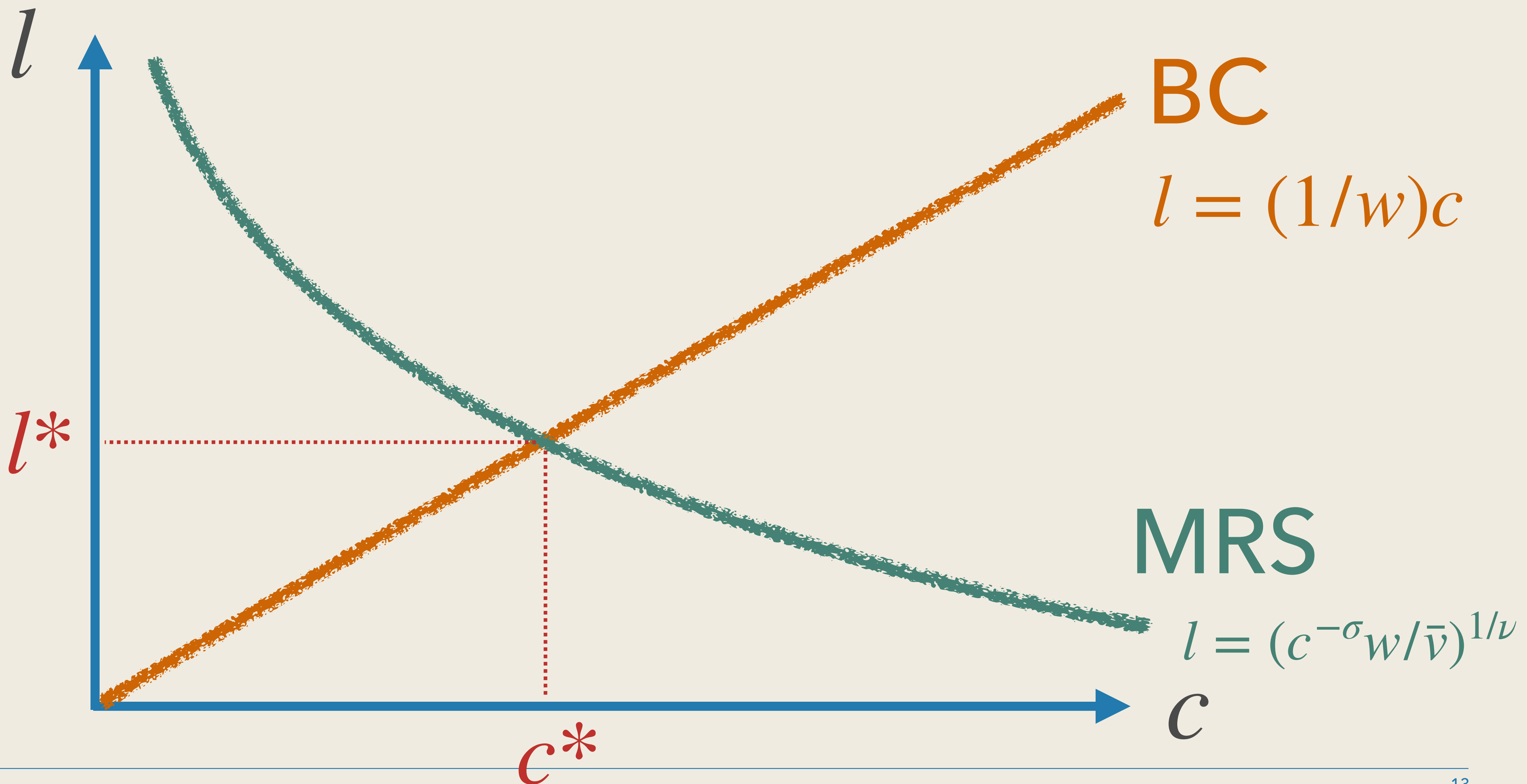
- Consumption and hours worked,  $\{c, l\}$ , jointly solve

$$c^{-\sigma} w = \bar{v} l^{\nu} \quad \text{(MRS)}$$

$$c = wl \quad \text{(BC)}$$

1. **(MRS)** defines a decreasing relationship between  $c$  and  $l$
2. **(BC)** defines an increasing relationship between  $c$  and  $l$

# Graphical Representation



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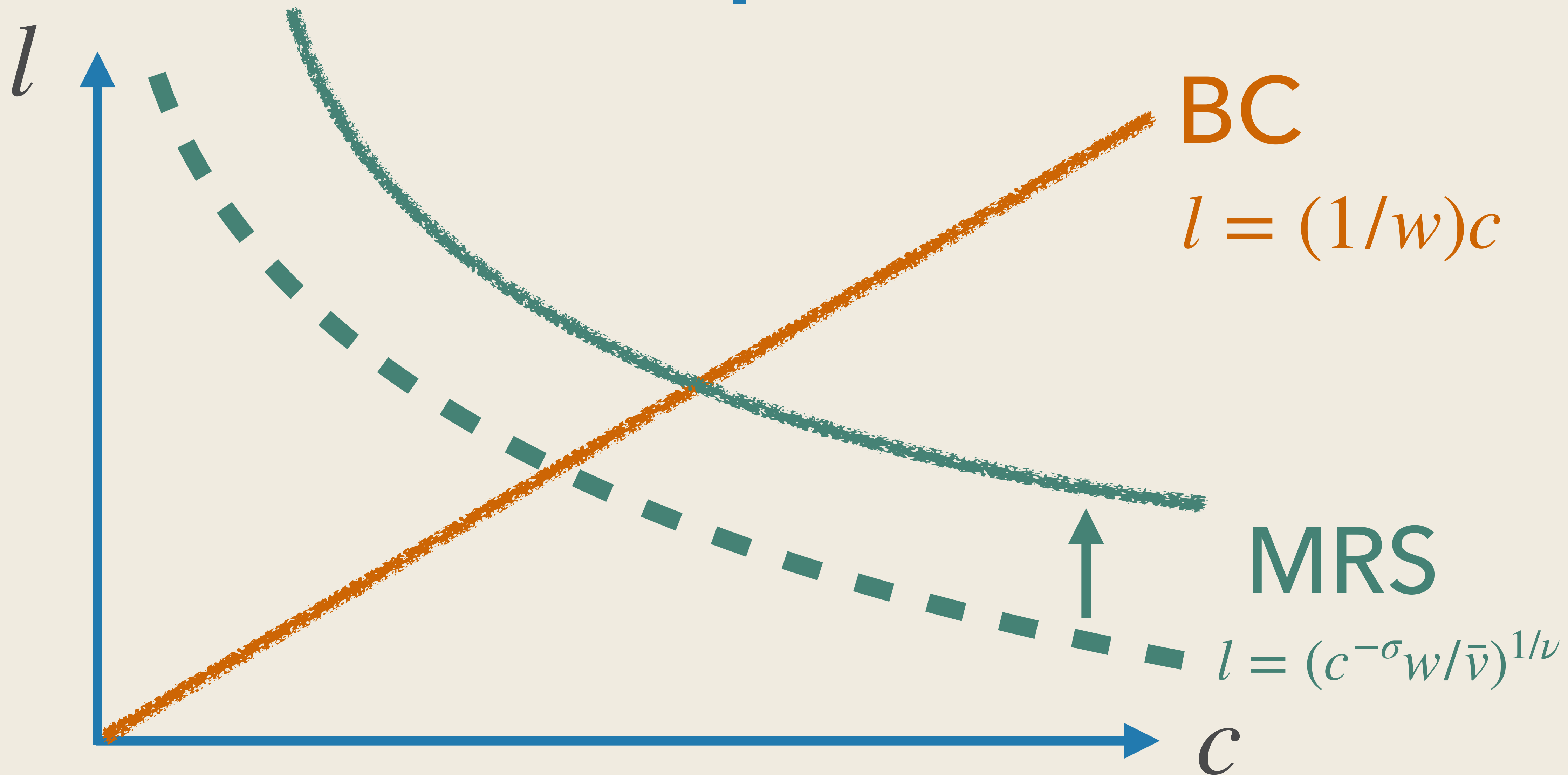
# Question

$$c^{-\sigma} w = \bar{v} l^{\nu} \quad (\text{MRS})$$

$$c = w l \quad (\text{BC})$$

- As a country gets richer, what happens to the labor supply?
- We will consider an increase in wage,  $w$
- Note that wage,  $w$ , appears in two places (orange and green)

# First Effect: Shift Up in MRS Curve



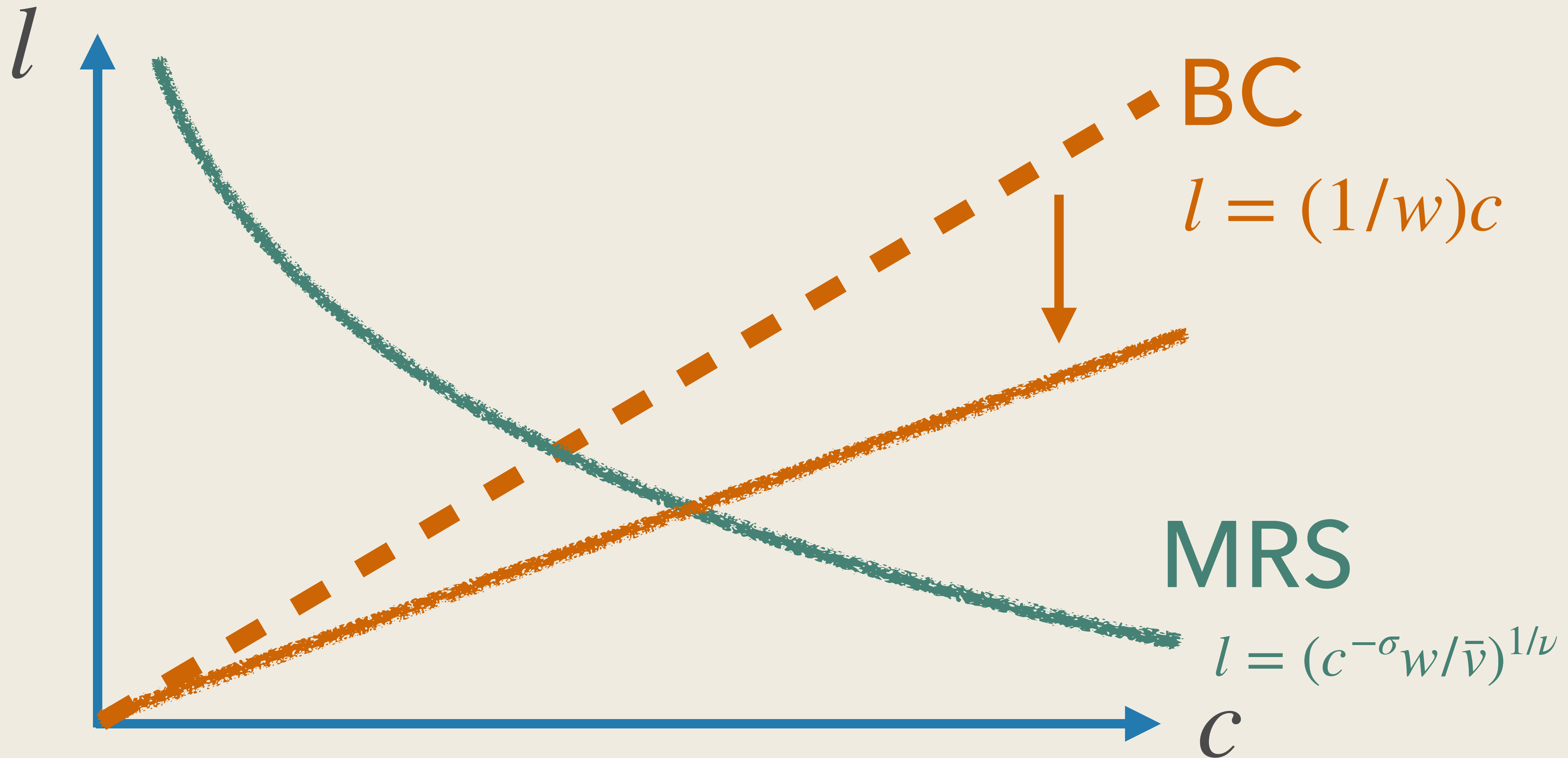
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# Substitution Effect

- MRS curve shifts up when  $w$  goes up
  - If wages are higher, the marginal benefit of working is higher for any given  $c$
- Holding the BC curve fixed, this means the labor supply,  $l$ , goes **up**!
- We call this a **substitution effect**



# Second Effect: Shift Down in BC Curve

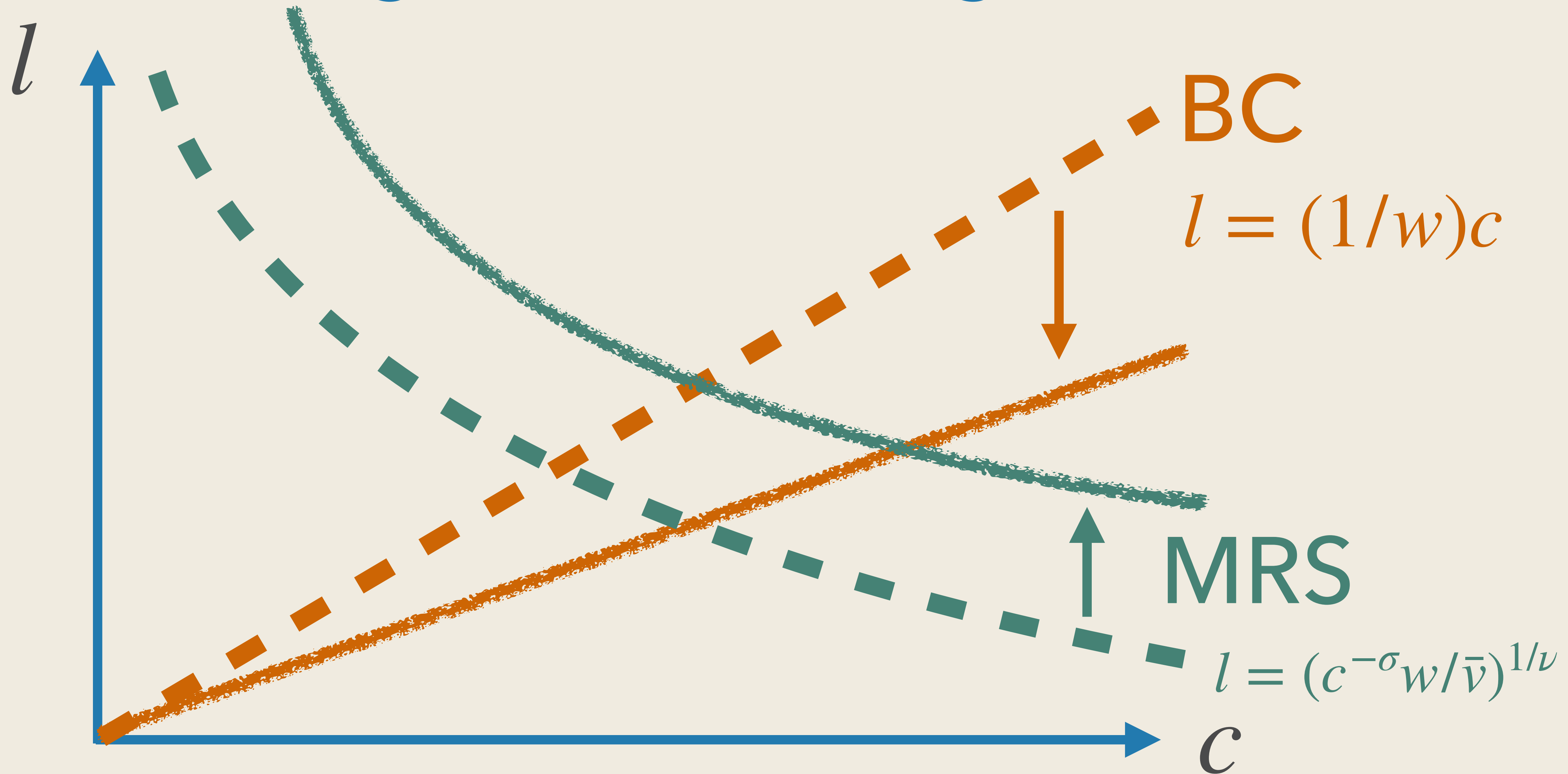


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# Income Effect

- BC curve shifts down when  $w$  goes up
  - If wages are higher, the budget constraint implies  $c$  is higher for any given  $l$
- Holding the MRS curve fixed, this means the labor supply,  $l$ , goes **down**!
- If I am richer, I don't need to work hard
- We call this as **income effect**

# Putting Two Effects Together



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# Higher or Lower Labor Supply?

- So, does the labor supply go up or down when  $w$  goes up?
- Not clear
- In fact, it can go either way

## Solving for $l$

$$c^{-\sigma} w = \bar{v} l^{\nu} \quad (\text{MRS})$$

$$c = w l \quad (\text{BC})$$

$$\Rightarrow l = \bar{v}^{\frac{-1}{\nu + \sigma}} w^{\frac{1}{\sigma + \nu}} w^{\frac{-\sigma}{\sigma + \nu}}$$

# $\sigma$ Determines the Relative Importance

$$\begin{aligned} l &= \bar{v}^{\frac{-1}{\nu + \sigma}} \mathcal{W}^{\frac{1}{\sigma + \nu}} \mathcal{W}^{\frac{-\sigma}{\sigma + \nu}} \\ &= \bar{v}^{\frac{-1}{\nu + \sigma}} \mathcal{W}^{\frac{1 - \sigma}{\sigma + \nu}} \end{aligned}$$

1.  $\sigma < 1$ :  $l$  is increasing in  $w$ . Substitution effect dominates income effect
2.  $\sigma > 1$ :  $l$  is decreasing in  $w$ . Income effect dominates substitution effect
3.  $\sigma = 1$ :  $l$  is invariant to  $w$ . Income effect and substitution effect cancel

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# Endogeneizing $w$

- Now we endogenize wages,  $w$
- Suppose the firm operates the following production function

$$y = Al$$

- Firms solve

$$\max_l Al - wl$$

- In equilibrium,

$$w = A$$

- Plugging it back,

$$l = \bar{v}^{\frac{1}{\nu + \sigma}} A^{\frac{1 - \sigma}{\sigma + \nu}}$$

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# Can We Qualitatively Explain Two Facts?

- Taking log,

$$\log l = \frac{1 - \sigma}{\sigma + \nu} \log A + \text{const}.$$

- Suppose that  $\sigma > 1$

1. Rich (high  $A$ ) countries work less than poor countries
2. As countries grow (higher  $A$ ), they work less

- If income effect dominates substitution effect, we can explain aggregate data



# Can We Quantitatively Explain Two Facts?

## ■ Over time-series

$$g_l = \frac{1 - \sigma}{\sigma + \nu} g_A + \epsilon_t$$

- Calculations from the US data suggest  $g_l \approx -0.4\%$  and  $g_A \approx 2\%$
- This suggests  $\frac{1 - \sigma}{\sigma + \nu} \approx -0.2$

## ■ In the cross-section,

$$\log l_i = \frac{1 - \sigma}{\sigma + \nu} \log A_i + \epsilon_i$$

- Regression estimates by Bick, Fuchs-Schündeln, Lgakos (2015),  $\frac{1 - \sigma}{\sigma + \nu} \approx -0.15$

## ■ Time-series and cross-sectional relationships line up well

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# **Income Effect from Labor Supply: Direct Evidence**

- Golosov, Graber, Mogstad & Novgorodsky (2023)**

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# Direct Evidence?

- Households work less as the economy grows and gets richer
- The model suggests that a strong income effect,  $\sigma > 1$ , is the key reason
- Do we have direct evidence of income effect?
- What is the ideal experiment?
- What is the concern with using the aggregate data?

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# Isolating Income Effect

- Maybe wage is not the only thing that changes over time and across countries
- The ideal experiment that isolates the income effect:
  - Give money to people and see how they change the labor supply
- If income effect is big, we will see a big reduction in labor supply
- If income effect is small, we will not see a major change in labor supply

# Conceptual Framework

- Add non-labor income  $T$  to the previous model,

$$\max_{c,l} u(c) - v(l)$$

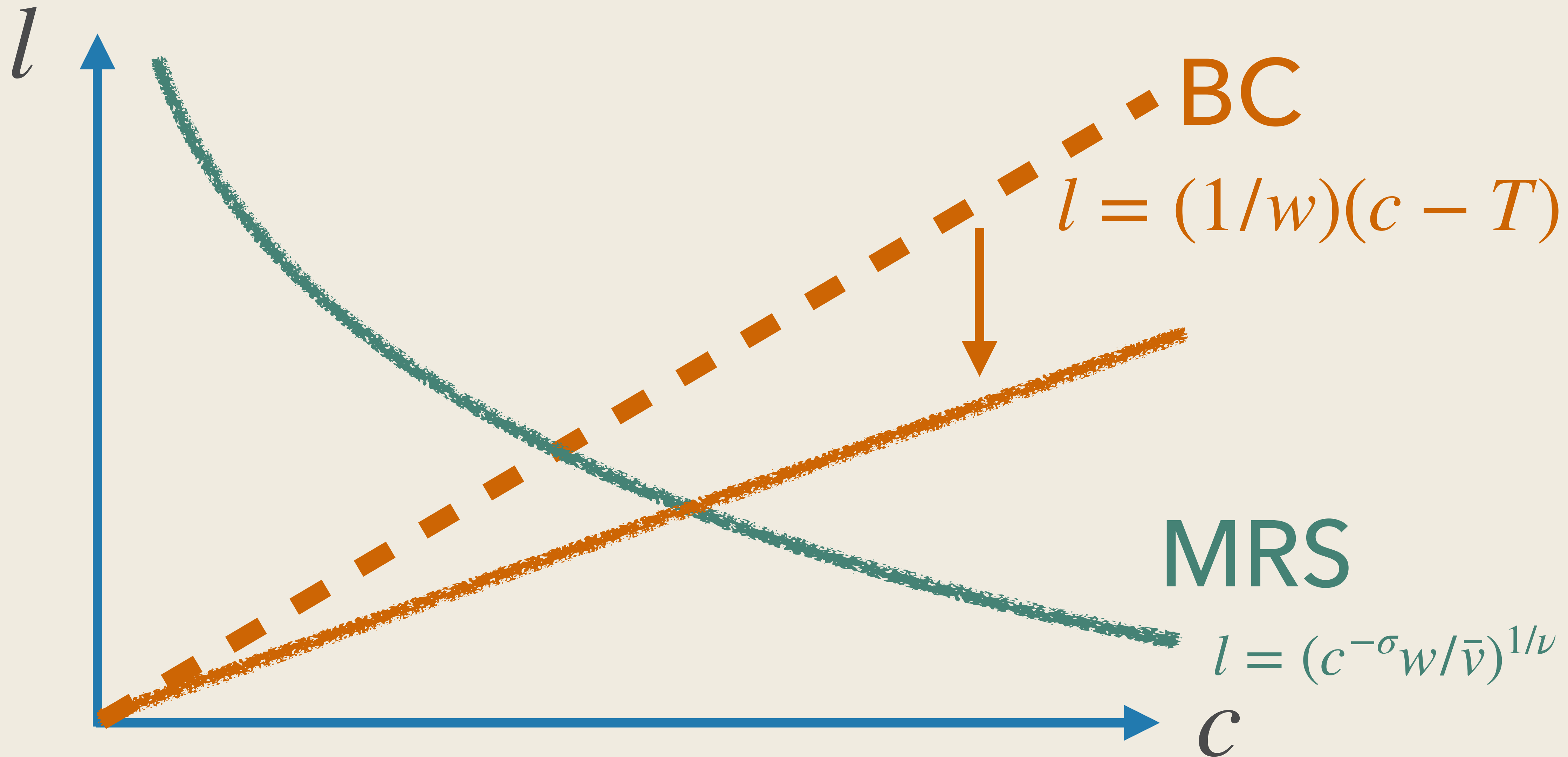
$$\text{s.t. } c = wl + T$$

- Assuming  $u(c) = \frac{c^{1-\sigma}}{1-\sigma}$  and  $v(l) = \bar{v} \frac{l^{1+\nu}}{1+\nu}$ ,  $\{c, l\}$ , jointly solve

$$c^{-\sigma} w = \bar{v} l^{\nu} \quad (\text{MRS})$$

$$c = wl + T \quad (\text{BC})$$

# Impact of Lottery Winning $T \uparrow$



# Marginal Propensity to Earn

- Empirically, it is convenient to look at MPE out of  $T \equiv \frac{d(wl)}{dT}$ 
  - MPE = marginal propensity to earn
- One can show:

$$MPE = \frac{d(wl)}{dT} = \frac{-\sigma s_w}{[\sigma s_w + \nu]}$$

where  $s_w = \frac{wl}{wl + T}$  is the share of wage income in total income

- MPE speaks to the importance of  $\sigma$ :  
If  $\sigma = 0$ ,  $MPE = 0$ . If  $\sigma$  is very big,  $MPE$  is very negative.

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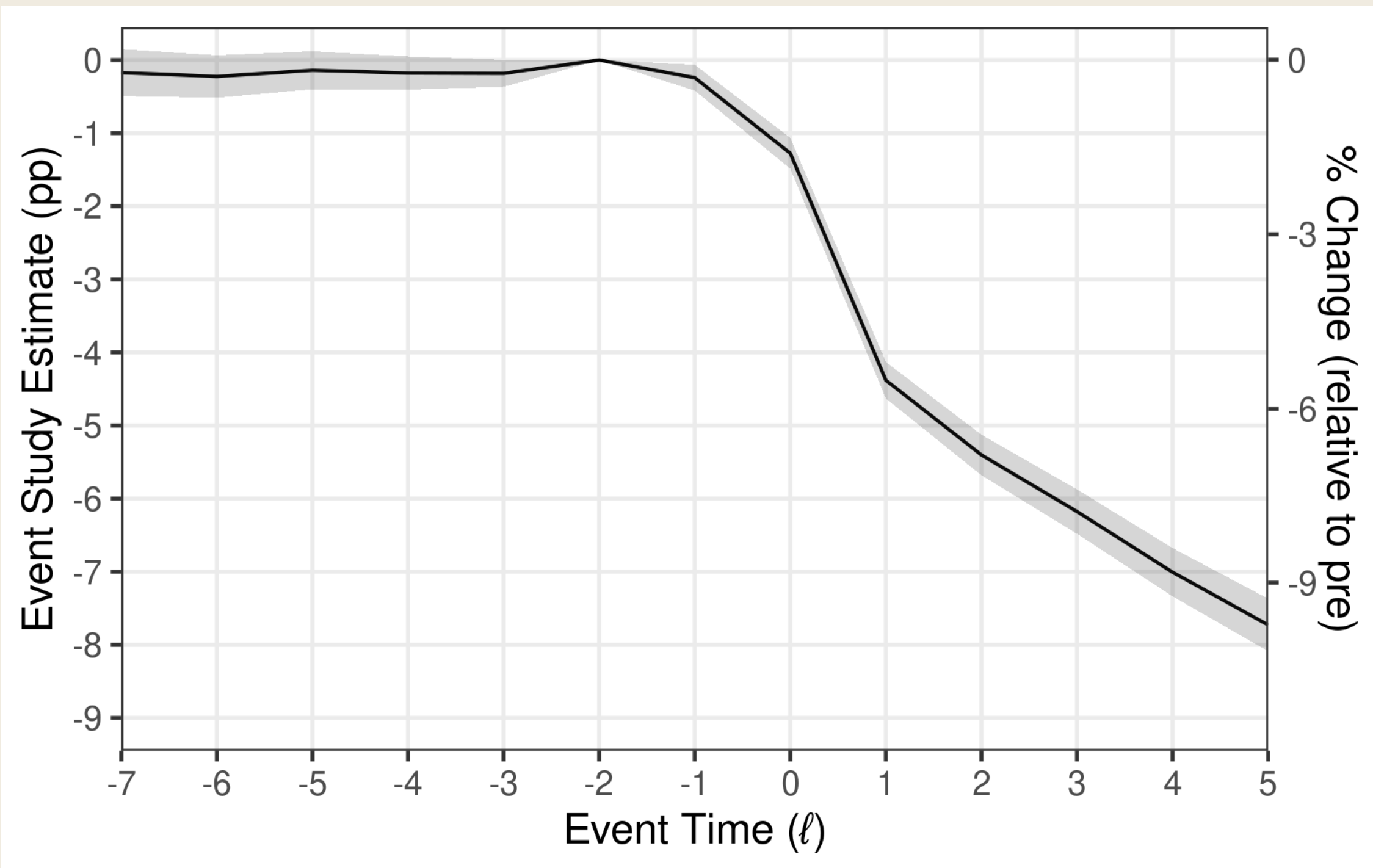
# Labor Supply Response of Lottery Winners

- Golosov, Graber, Mogstad & Novgorodsky (2023):  
Study the labor supply responses of US lottery winners
- US tax data for 1999-2016
  - Lottery winnings are taxable income
- Median size of post-tax winning: \$43,600
- 90,731 lottery winners in the sample
- Compare the response to lottery winnings relative to later winners

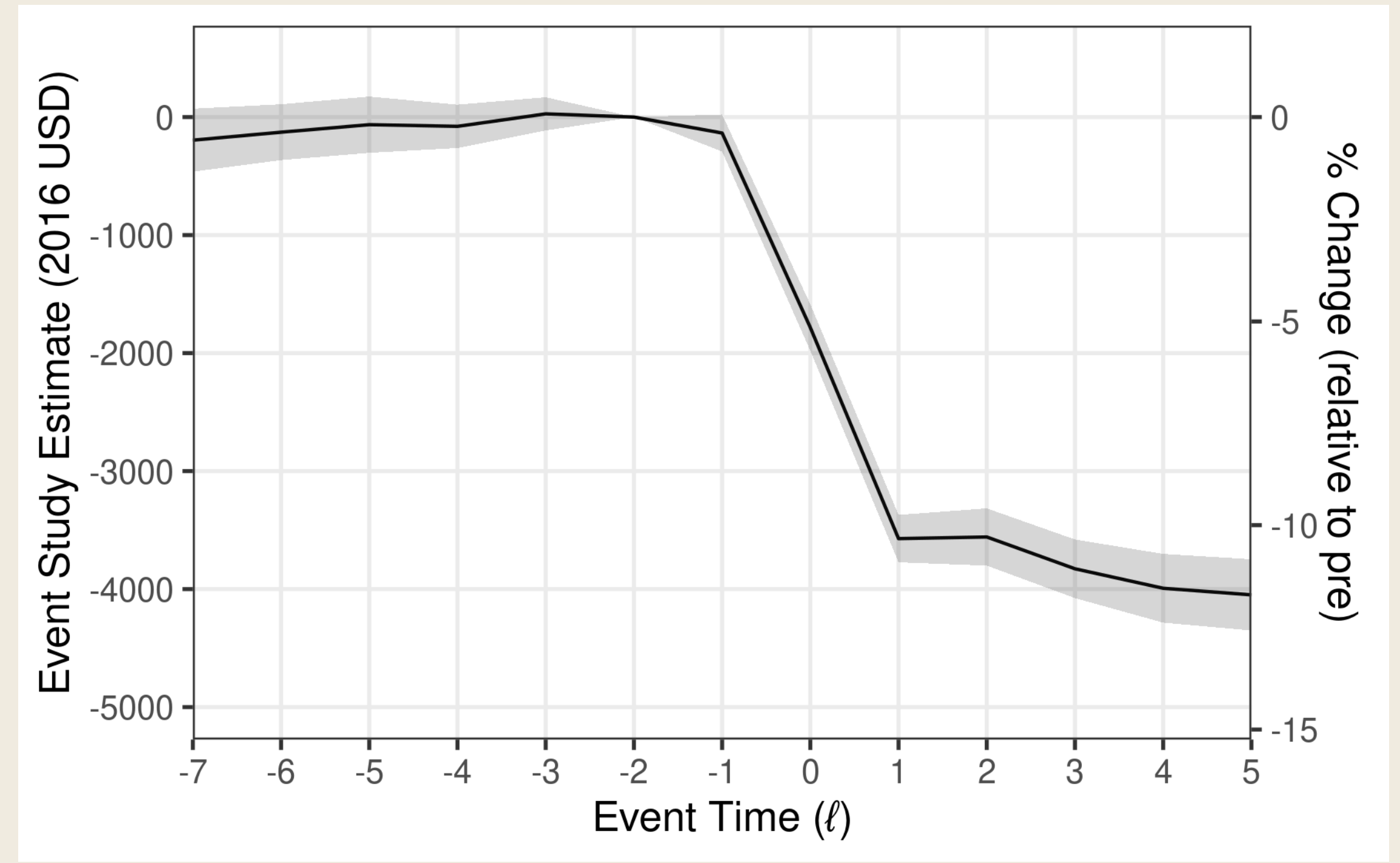


# Labor Supply Response to Lottery Winning

## Employment



## Wage Earnings



Source: Golosov, Graber, Mogstad & Novgorodsky (2023)

# MPE Estimates

Table 4.1: IV estimates of the effect of exogenous change in unearned income

Outcome	Sample				
	Full Sample	Quartile 1	Quartile 2	Quartile 3	Quartile 4
	Pre-Win Income	Pre-Win Income	Pre-Win Income	Pre-Win Income	Pre-Win Income
	(1)	(2)	(3)	(4)	(5)
Per-Adult Total Labor Earnings	-0.5227 (0.0146)	-0.3080 (0.0240)	-0.5204 (0.0197)	-0.5893 (0.0221)	-0.6735 (0.0389)

Source: Golosov, Graber, Mogstad & Novgorodsky (2023)

## ■ Therefore

$$\frac{d(wl)}{dT} = - \frac{\sigma s_w}{[\sigma s_w + \nu]} \approx -0.52$$

## ■ Assuming $s_w \approx 2/3$ (labor share) and $\nu \in [0.1, 0.5]$ (micro estimates) implies

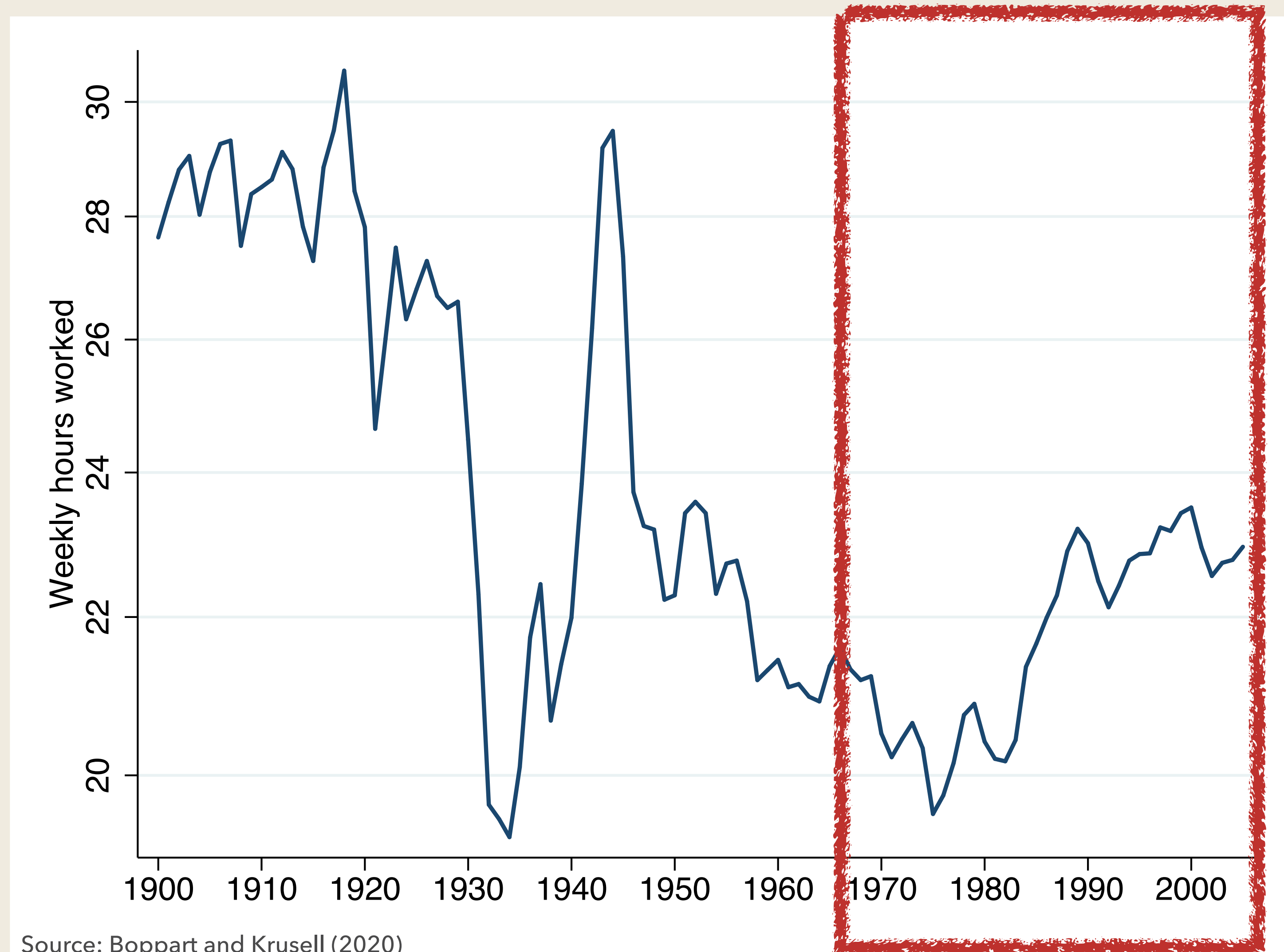
$$\sigma \in [0.2, 0.8]$$

## ■ Can be large but not as large as $\sigma > 1$

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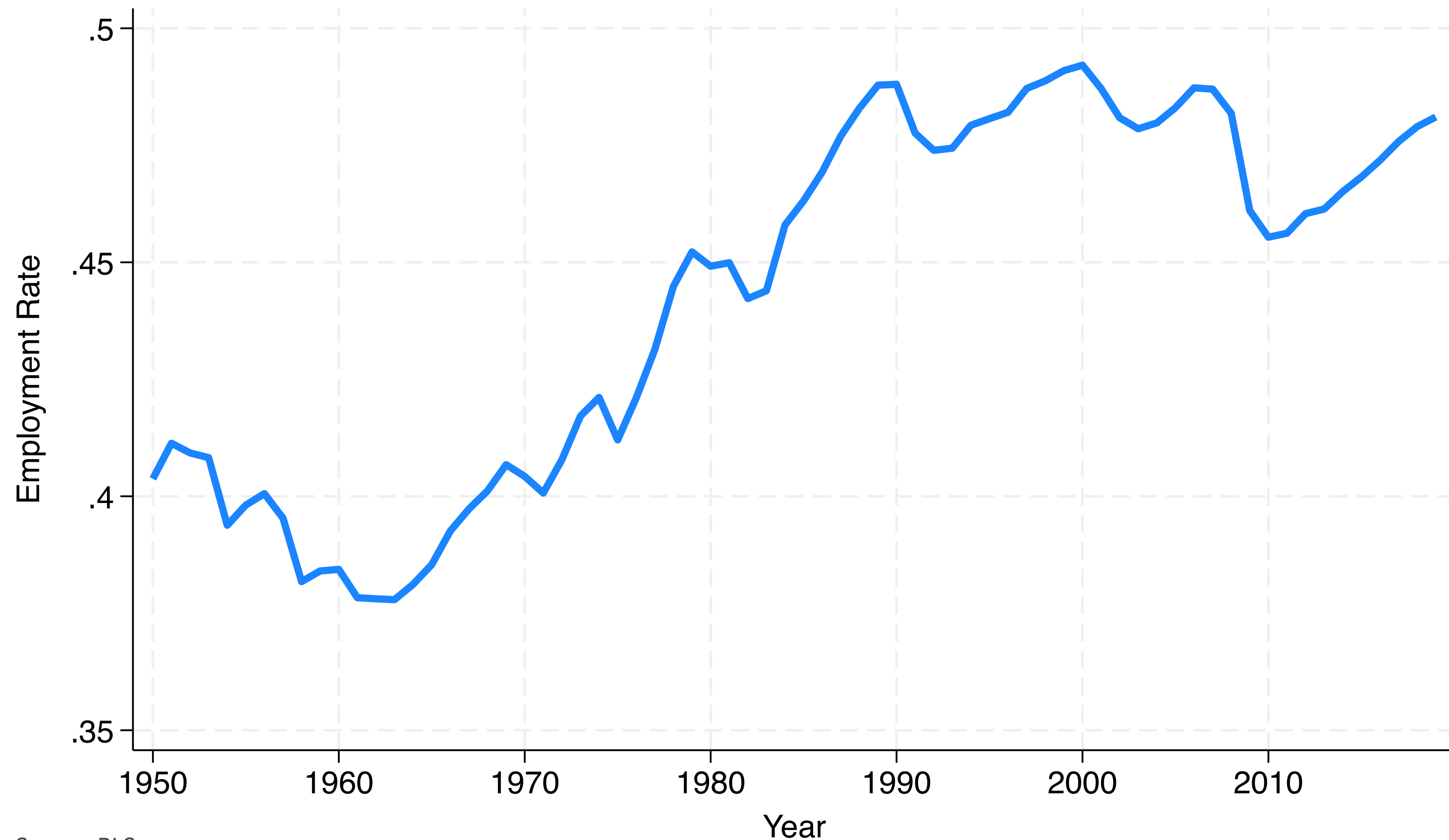
# **Who is Reducing the Labor Supply? (Recently)**

# Average Hours Worked in the US



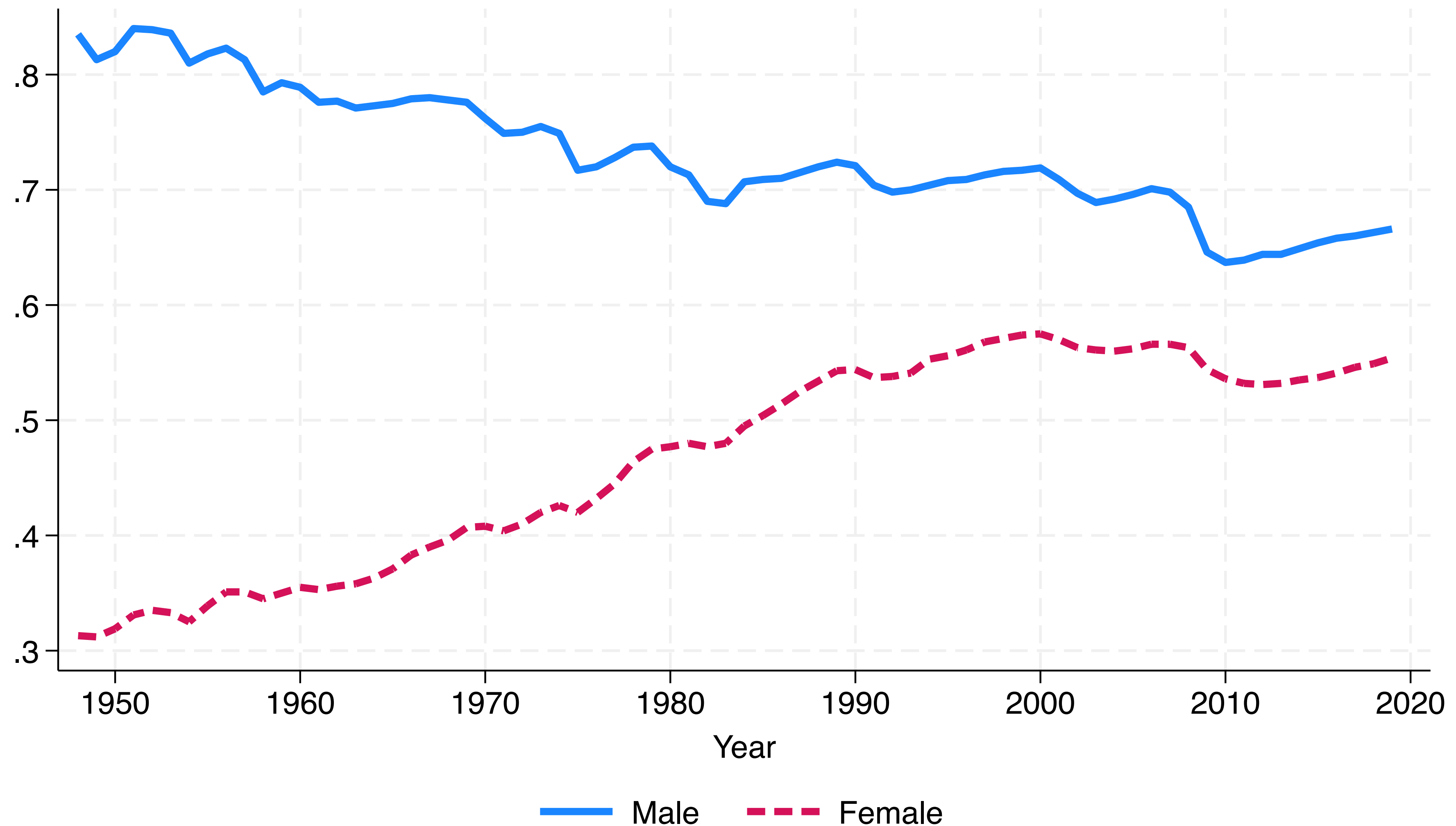
Source: Boppart and Krusell (2020)

# Rise in Employment Rate since 1950



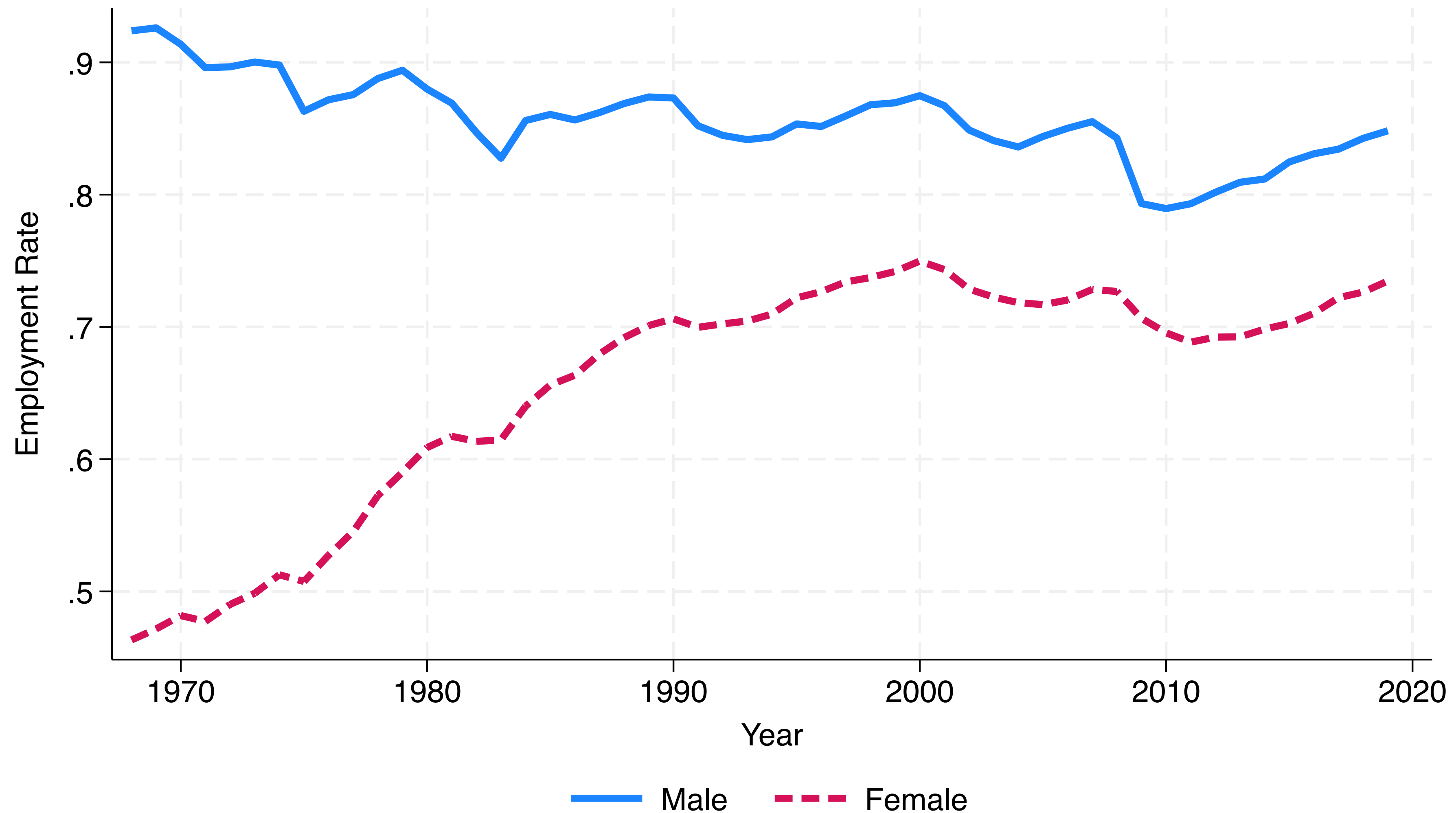
Source: BLS

# Employment Rate by Gender



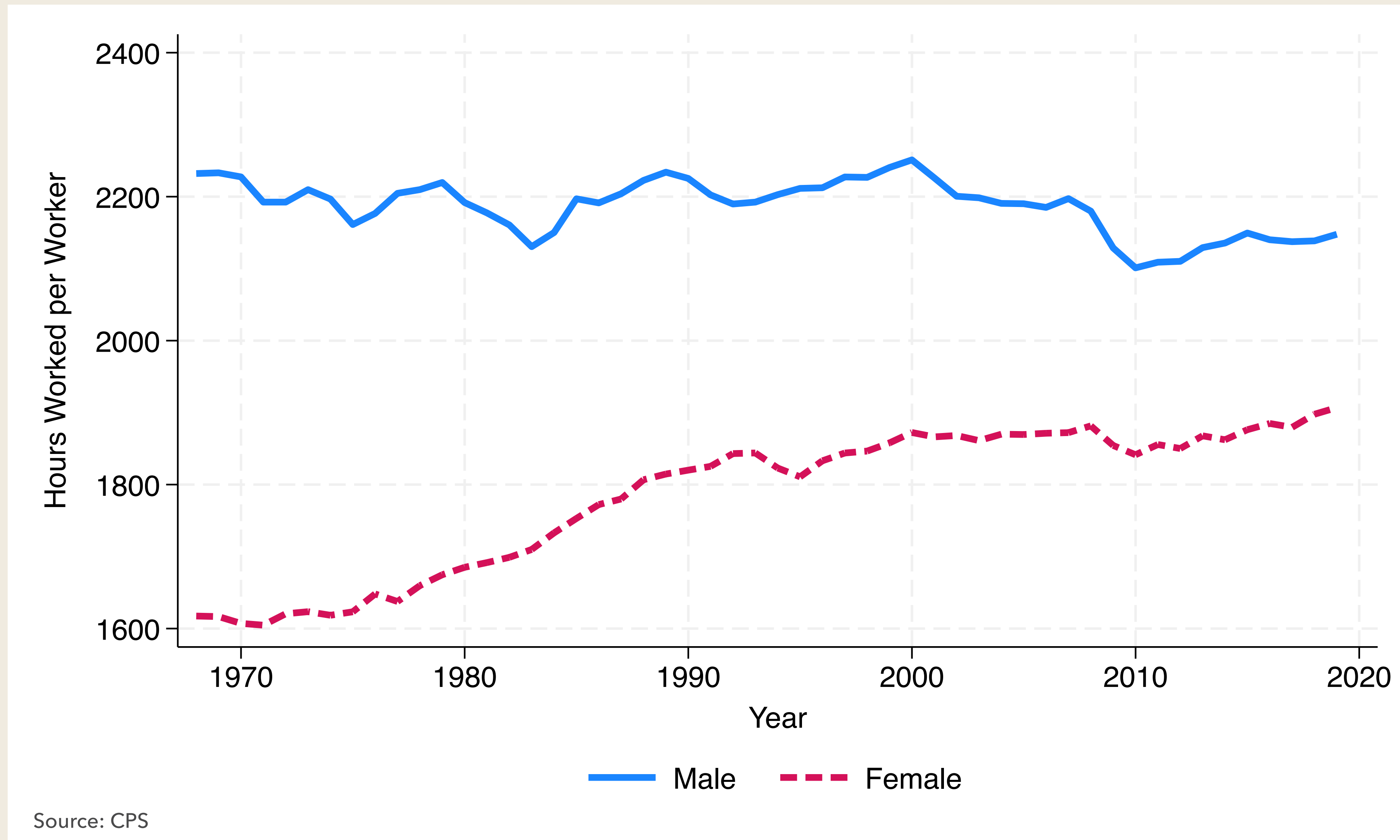
Source: BLS

# Prime-Age 25-54



Source: CPS

# Hours Worked Conditional on Being Employed



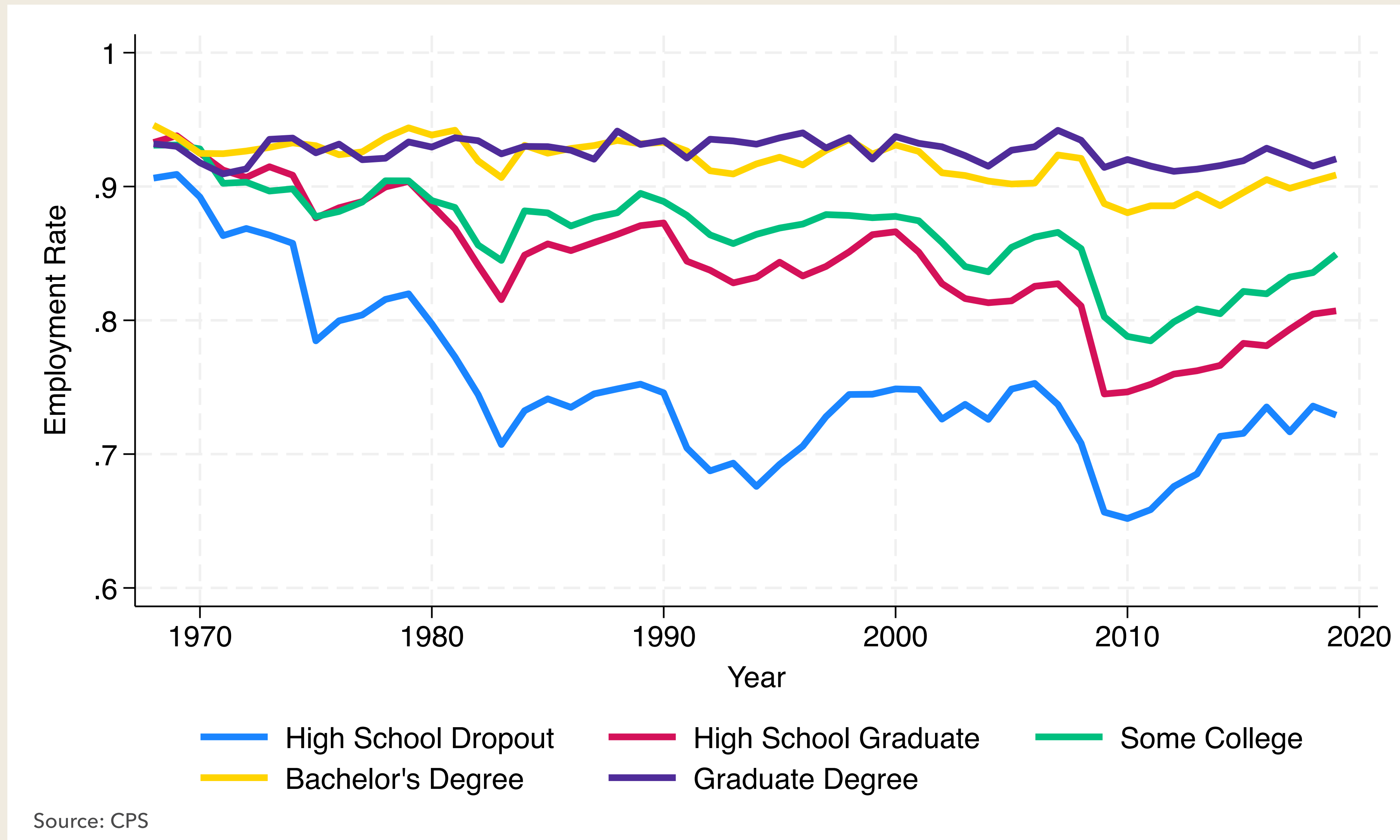


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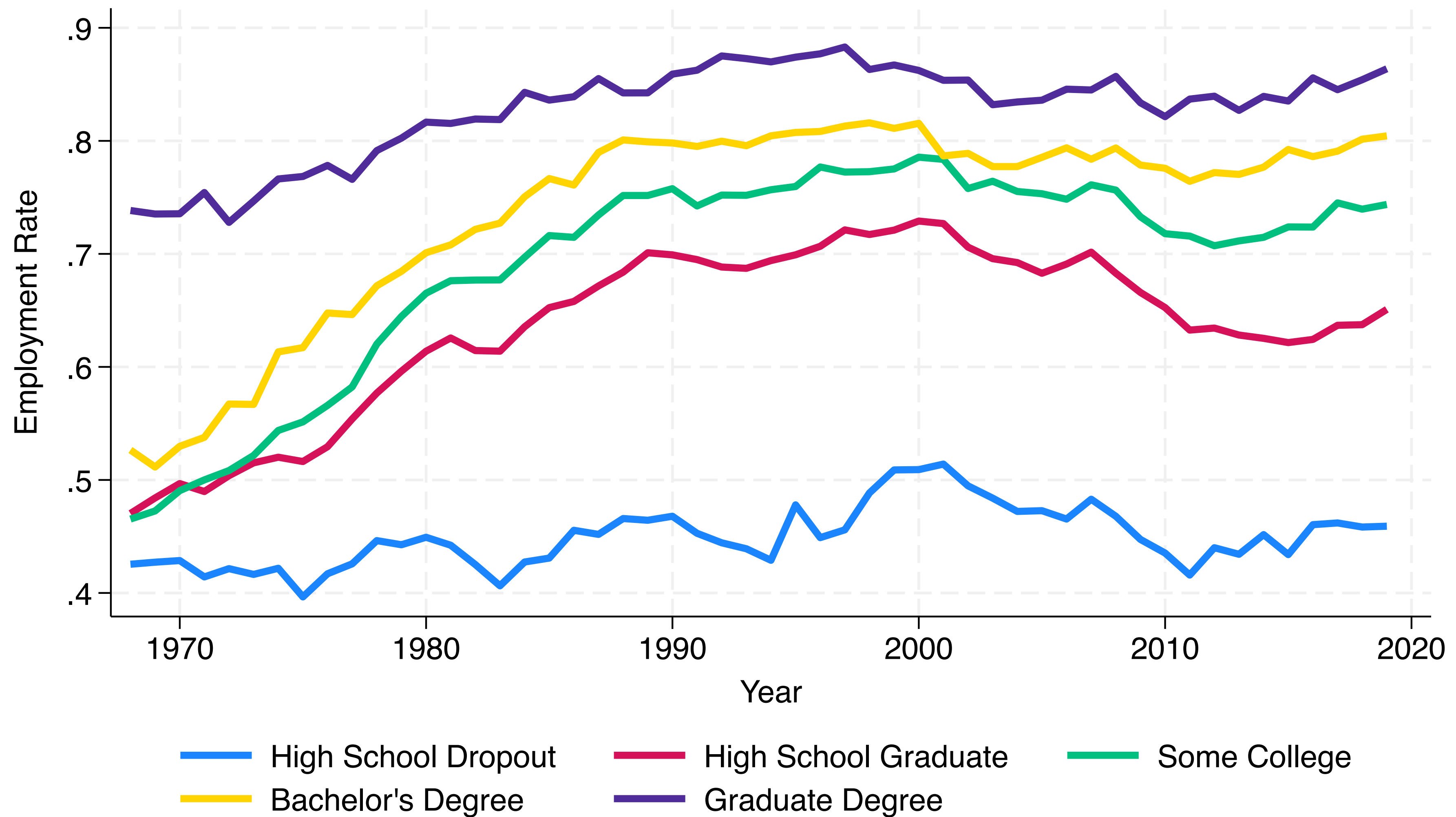
# What Drives Increase in Female Employment?

1. Changes in social norm (less discrimination)
2. Improvement in home production technology
3. The rise of the service sector
4. Medical technology  
(the birth control pill, maternal health after birth, infant formula)

# Male Employment Rate by Education

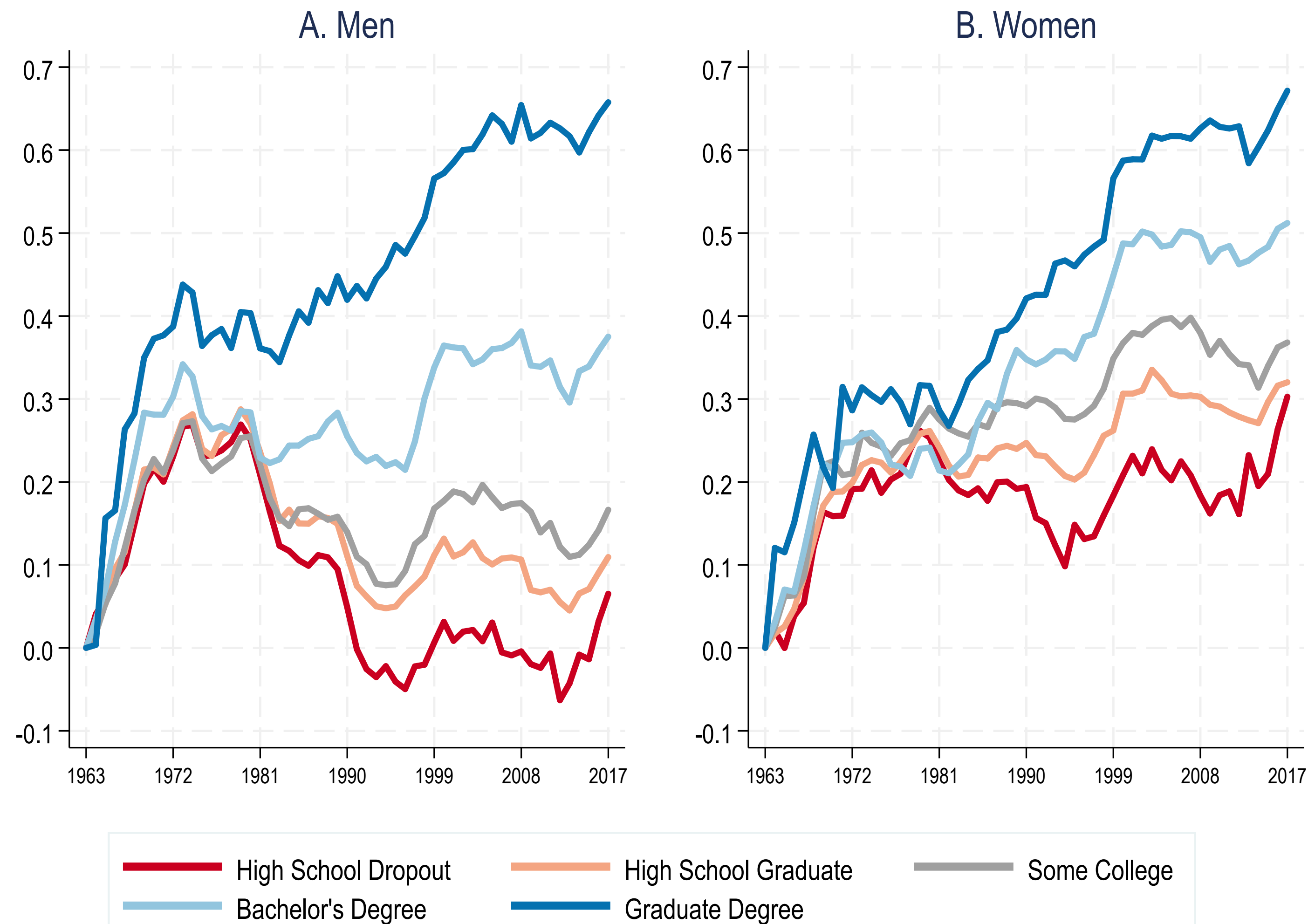


# Female Employment Rate by Education



Source: CPS

# Who's Wages are Getting Higher?



Source: Autor (2019)

# What Do Non-Employed Young Men Do?

Table 4: Leisure Activities for Men 21-30 (Hours per Week): By Employment Status

Activity	Employed			Non-Employed		
	2004-2007	2012-2015	Change	2004-2007	2012-2015	Change
Total Leisure	57.6	59.6	2.0	86.9	82.1	-4.8
Recreational Computer	3.0	4.3	1.3	5.4	9.6	4.3
Video Game	1.8	2.9	1.0	3.4	5.9	2.5
ESP	23.6	23.9	0.3	30.1	29.9	-0.2
TV/Movies/Netflix	15.9	15.5	-0.5	27.8	25.0	-2.7
Socializing	7.4	7.8	0.3	10.6	8.9	-1.7
Other Leisure	7.7	8.1	0.5	13.0	8.6	-4.4
Job Search and Education	2.0	1.9	-0.1	9.4	14.1	4.7

Source: Aguiar, Bils, Kofi Charles, Hurst (2021)

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## My Take

$$l = \bar{v} \frac{-1}{\nu + \sigma} w \frac{1}{\sigma + \nu} w \frac{-\sigma}{\sigma + \nu}$$

- Micro evidence points toward  $\sigma < 1$
- Income effect, on its own, is not strong enough to explain declining hours with income
- Then why do hours work decline with income?
- As a country develops, it develops leisure enhancing activity that raises  $\bar{v}$ 
  - TV, smart phones, tablets, video games, SNS, Youtube, netflix, etc...



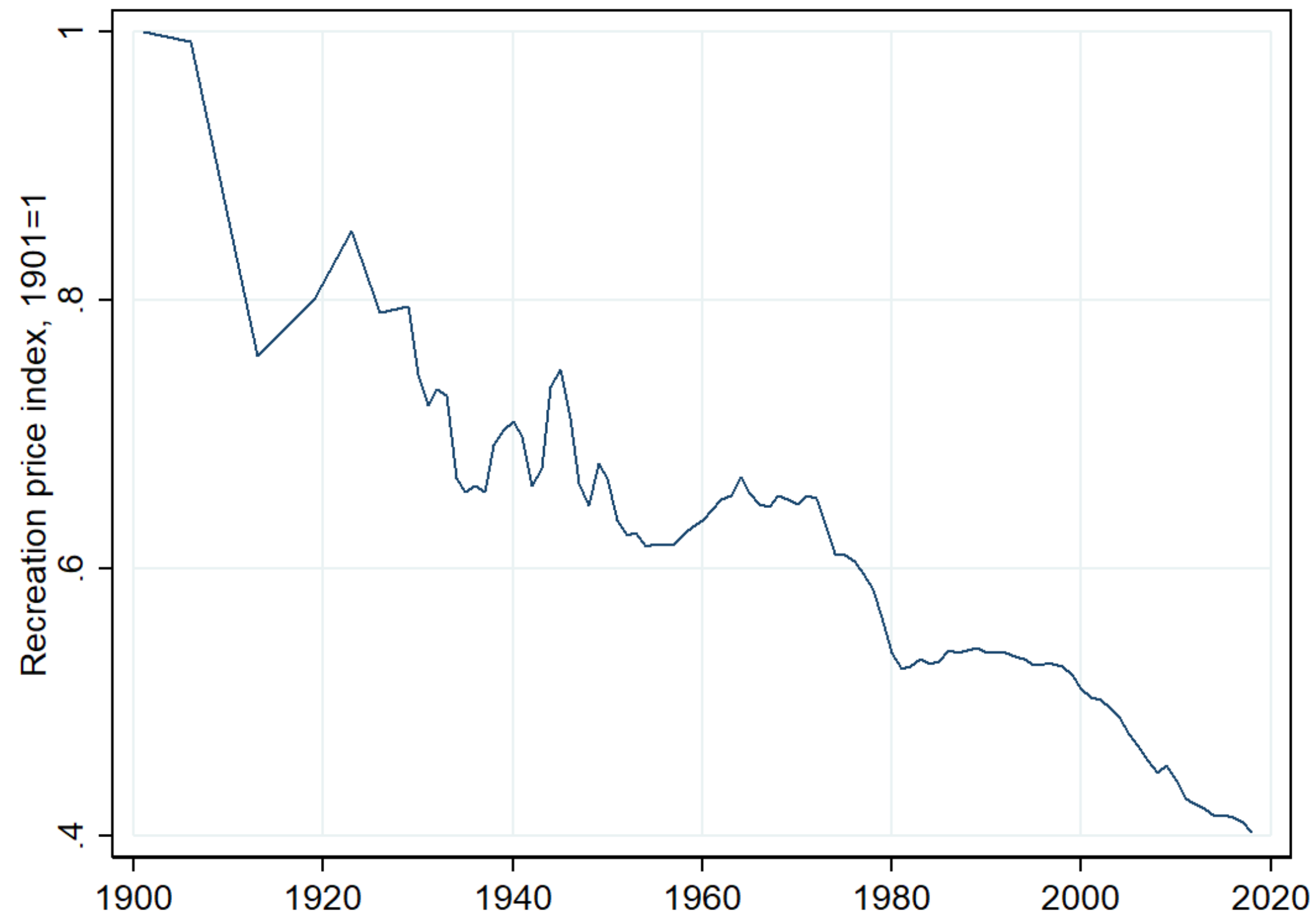
# Leisure Technology



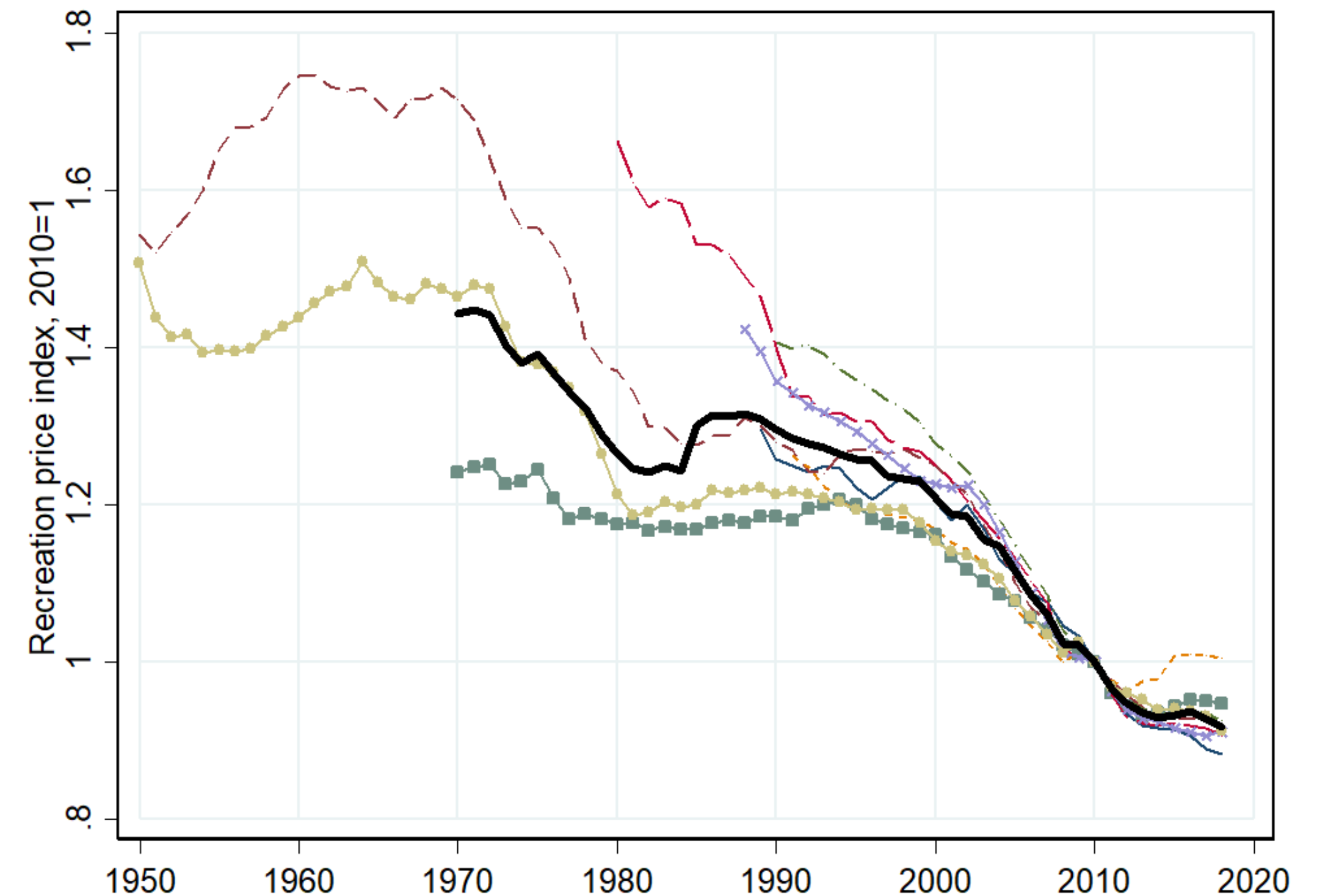
- Real TV price has fallen by a factor of 1000 since 1950
- Netflix, Spotify:  $\approx \$10$  for unlimited use
- Apple iOS Store: 900,000 games, 2/3 are free



# Real Price of Recreation Goods and Services



(a) U.S.



(b) All countries

Source: Kopytov, Roussanov, Taschereau-Dumouchel (2023)