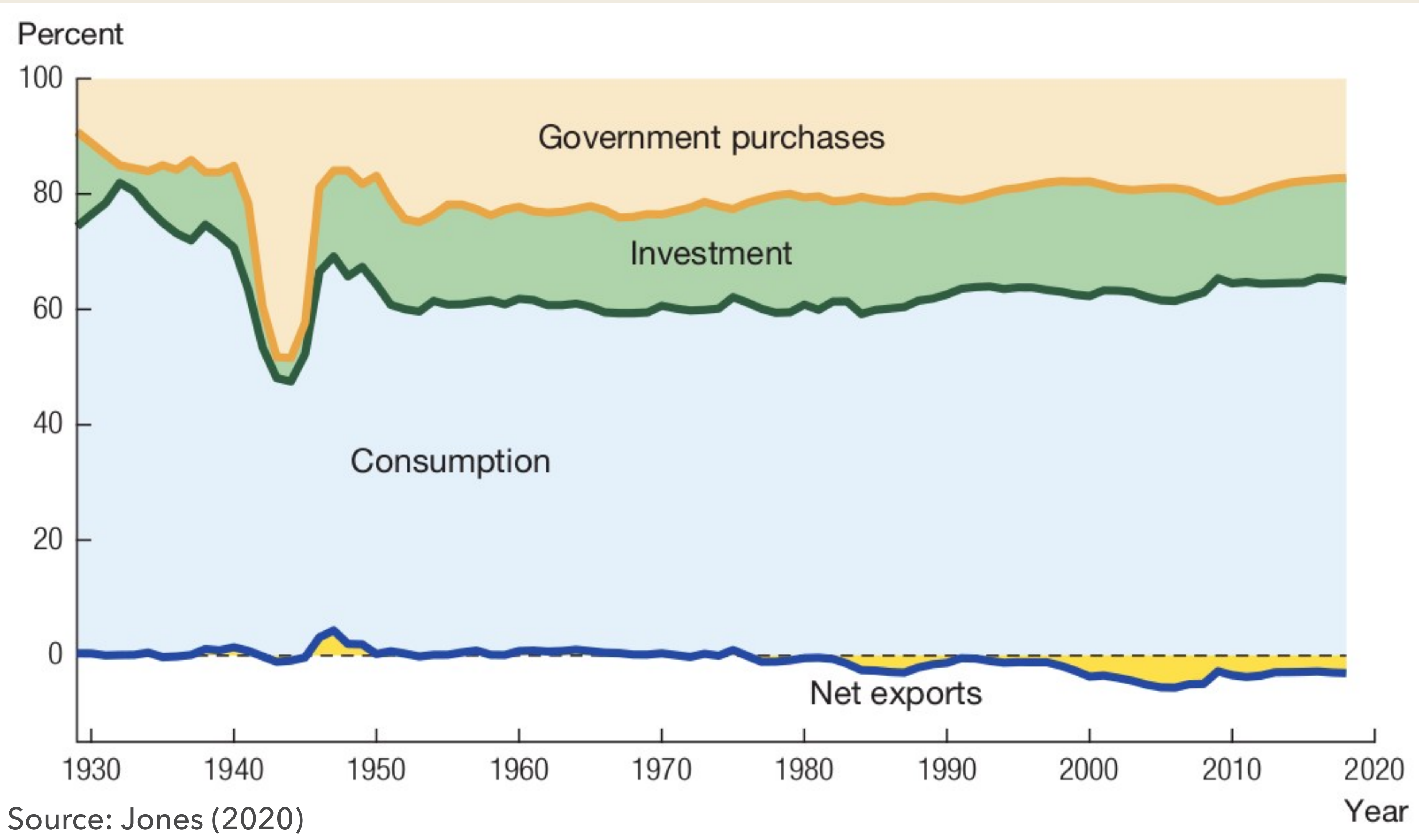

Investment

EC502 Macroeconomics Topic 8

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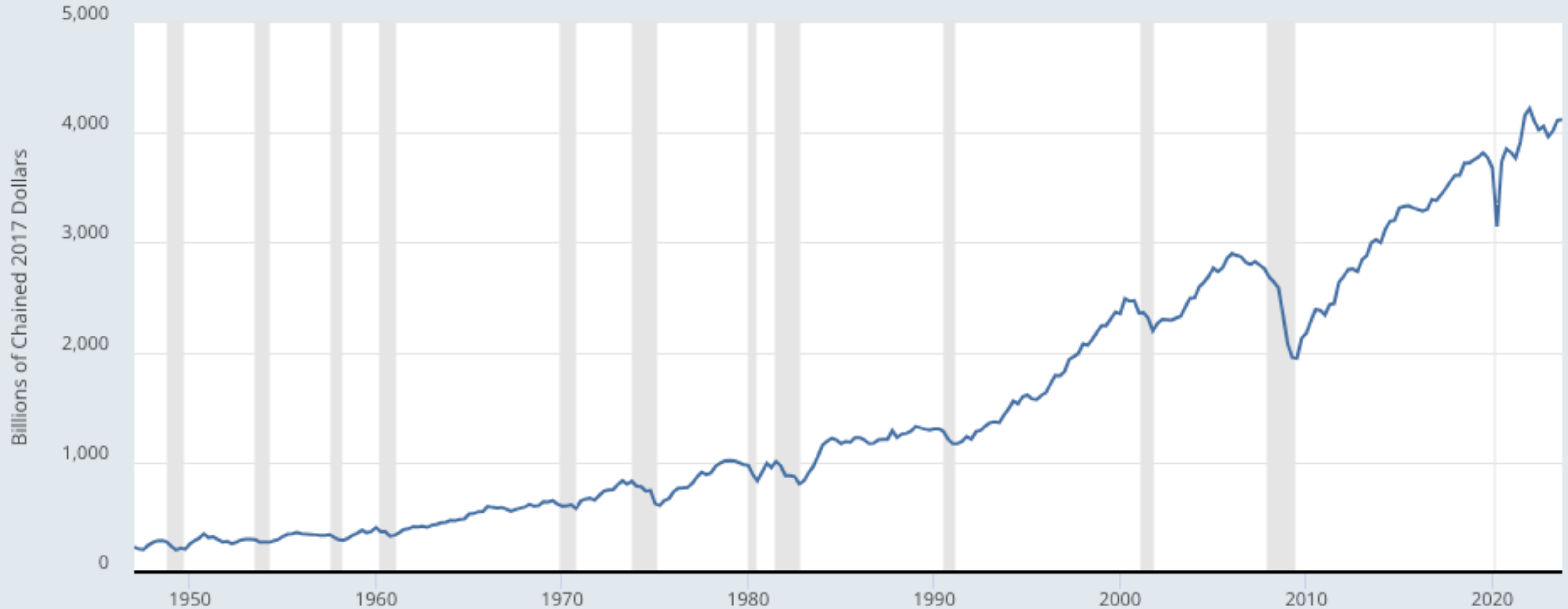
Investment in GDP



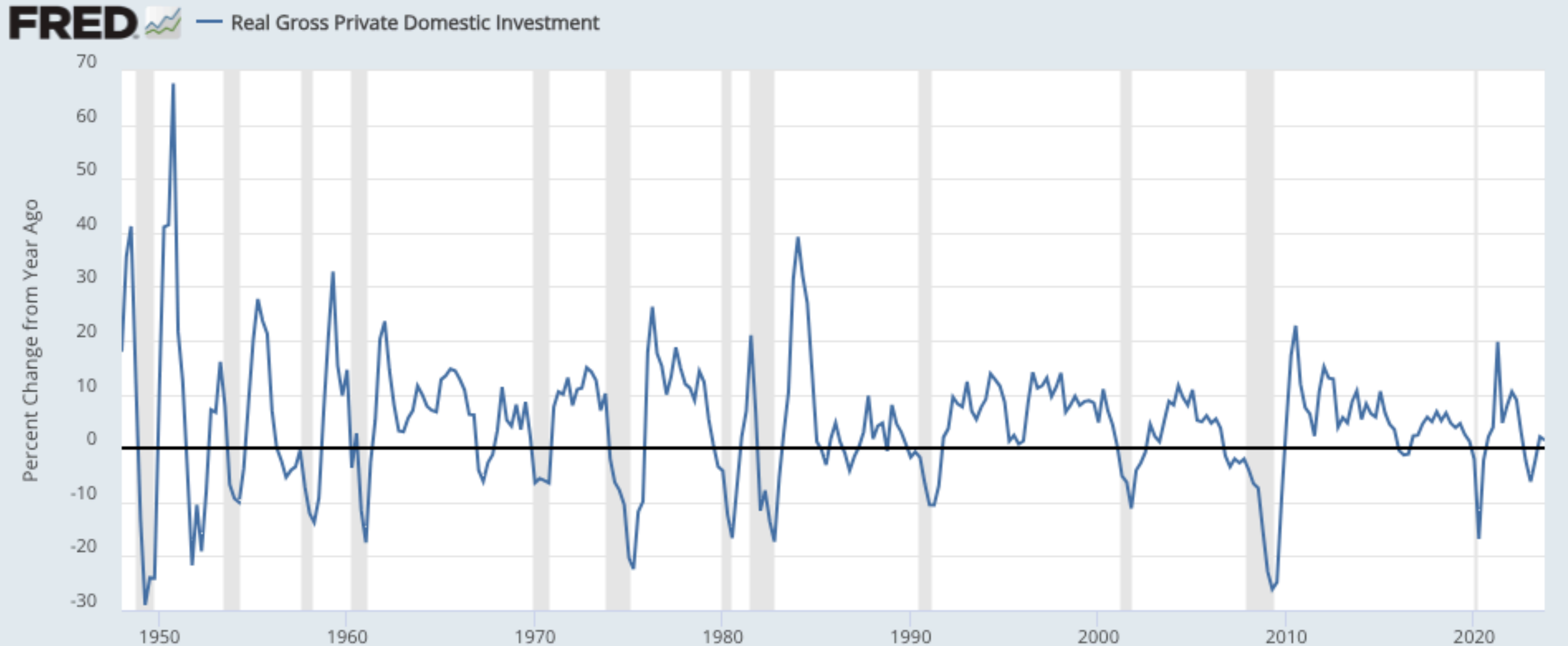
Investment

FRED

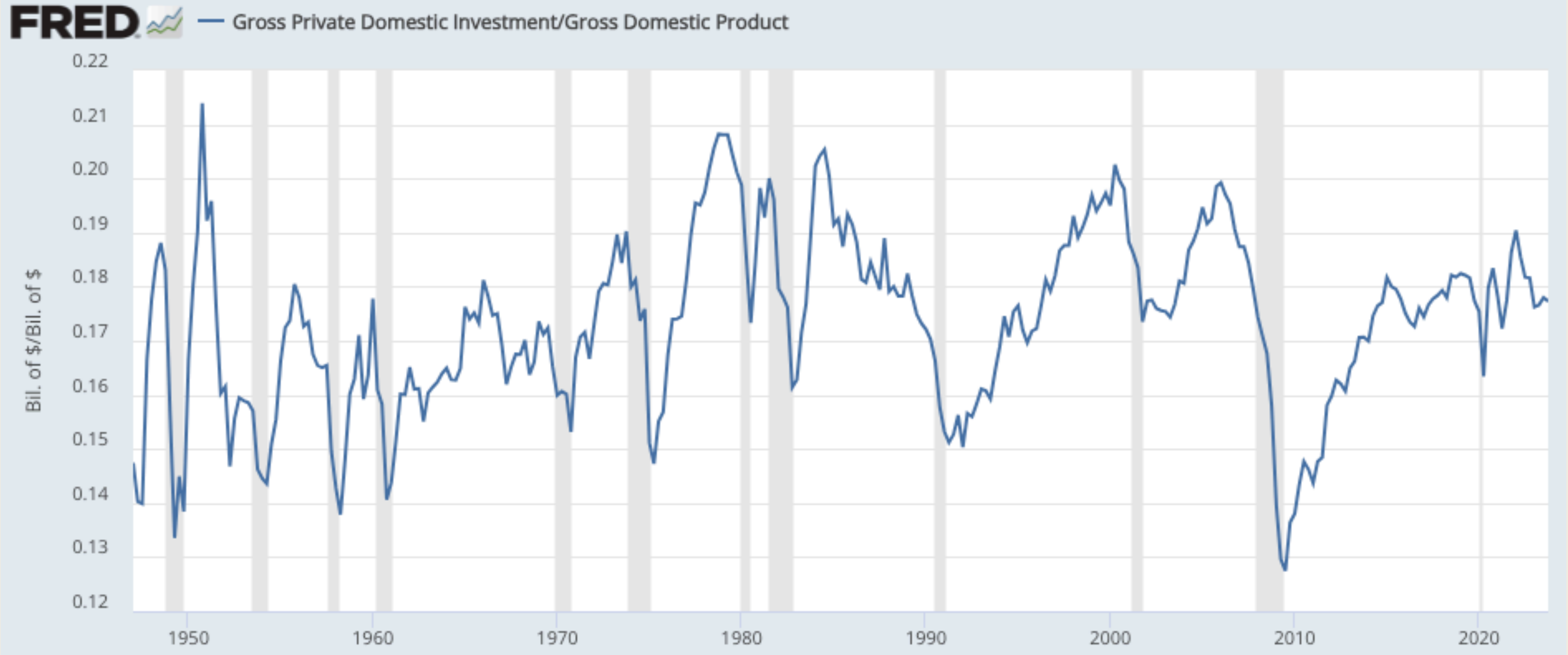
— Real Gross Private Domestic Investment



Investment Growth



Investment over GDP



Questions

- Investment constitutes $\approx 20\%$ of GDP
- Yet, it is the most volatile component of GDP
- What determines investment?
 - Recall in Solow model, this was mechanical, $I_t = sY_t$
- How can a policy stimulate investment in recessions?

Investment with Two Periods

Setup

- Consider a firm operating the following production function

$$F_t(K_t, L_t) = A_t K_t^\alpha L_t^{1-\alpha}$$

- Firms own capital stock K_t and invest with convex adjustment costs $\Phi(I_t, K_t)$

$$K_1 = (1 - \delta)K_0 + I_0, \quad \delta : \text{depreciation rate}$$

- Firms hire labor in the competitive labor market with wage w_t
- The firm maximizes the presented discounted value of dividends

$$D_0 + \frac{1}{1+r} D_1$$

where $D_t = F_t(K_t, L_t) - w_t L_t - I_t - \Phi(I_t, K_t)$ is the profit of a firm in period t

Adjustment Costs

- We assume the following adjustment cost function

$$\Phi(I, K) = \frac{\phi}{2} \left(\frac{I}{K} \right)^2 K$$

- This function is increasing and convex in I
 - The additional investment costs more when you are already investing a lot
- This function is constant returns to scale in (I, K)
 - doubling your investment and capital also doubles the cost of investment

Firm's Problem

- Given K_0 , a firm solves

$$\max_{L_0, I_0, K_1, L_1} \left[F_0(K_0, L_0) - w_0 L_0 - I_0 - \Phi(I_0, K_0) \right] + \frac{1}{1+r} \left[F_1(K_1, L_1) - w_1 L_1 \right]$$

subject to

$$K_1 = K_0(1 - \delta) + I_0$$

- The first-order conditions with respect to L_t :

$$\frac{\partial F_t(K_t, L_t)}{\partial L_t} = w_t \quad (1)$$

- The first-order condition with respect to I_0 is

$$1 + \frac{\partial \Phi(I_0, K_0)}{\partial I_0} = \frac{1}{1+r} \frac{\partial F_1(K_1, L_1)}{\partial K_1} \quad (2)$$

LHS: marginal cost of investment, RHS: marginal benefit of investment

Investment Solution

- With our functional forms, we can solve for labor demand using (1):

$$L_t = (1 - \alpha)^{1/\alpha} A_t^{1/\alpha} w_t^{-1/\alpha} K_t \quad (3)$$

- Equation (2) is

$$1 + \phi \frac{I_0}{K_0} = \frac{1}{1 + r} \alpha A_1 K_1^{\alpha-1} L_1^{1-\alpha} \quad (4)$$

- Combining (3) and (4),

$$\frac{I_0}{K_0} = \frac{1}{\phi} \left[\frac{1}{1 + r} \alpha (1 - \alpha)^{\frac{1-\alpha}{\alpha}} A_1^{\frac{1}{\alpha}} w_1^{-\frac{1-\alpha}{\alpha}} - 1 \right]$$

Comparative Statics

$$\frac{I_0}{K_0} = \frac{1}{\phi} \left[\frac{1}{1+r} \alpha (1-\alpha)^{\frac{1-\alpha}{\alpha}} A_1^{\frac{1}{\alpha}} w_1^{-\frac{1-\alpha}{\alpha}} - 1 \right] \quad (5)$$

Investment is higher when

- interest rate, r , is lower
- future productivity, A_1 , is higher
- future wage, w_1 , is lower

All should be intuitive

Value of Firms

- Let us rewrite firm's investment in a different way
- Define the value of firms (discounted future profits):

$$V_1 = \frac{1}{1+r} D_1$$

- In principle, V_1 should correspond to the stock price of the firm
- Using the definition, $D_1 = F_1(K_1, L_1) - w_1 L_1$, and labor demand (3),

$$V_1 = \frac{1}{1+r} \alpha A_1^{\frac{1}{\alpha}} (1-\alpha)^{\frac{1-\alpha}{\alpha}} w_1^{-\frac{1-\alpha}{\alpha}} K_1$$

- Define $q_1 \equiv V_1/K_1$, which we call "q"

Q-Theory of Investment

- With the definition of “q”, we can rewrite investment equation (5) as

$$\frac{I_0}{K_0} = \frac{1}{\phi} [q_1 - 1]$$

- We often refer to the above expression as “q-theory of investment”
- Investment is positive if and only if $q_1 > 1$
 - The average value of capital is higher than its cost
- Investment is negative if and only if $q_1 < 1$
 - The average value of capital is lower than its cost
- Importantly, q_1 summarizes the impact of r, w_1, A_1 (“sufficient statistics”)

Investment with Many Periods

Investment Problem with Many Periods

- We generalize the previous model to many periods, $t = 0, \dots, T$
- The firm solves

$$\max_{\{I_t, K_{t+1}, D_t, L_t\}} \sum_{t=0}^T \frac{1}{\prod_{s=0}^{t-1} (1 + r_s)} D_t$$

subject to

$$D_t = F_t(K_t, L_t) - w_t L_t - I_t - \Phi(I_t, K_t)$$

$$K_{t+1} = (1 - \delta)K_t + I_t$$

Lagrangian

- The Lagrangian is

$$\mathcal{L} = \sum_{t=0}^T \frac{1}{\prod_{s=0}^{t-1} (1 + r_s)} \left\{ \left[F_t(K_t, L_t) - w_t L_t - I_t - \Phi(I_t, K_t) \right] + q_{t+1} \left[K_{t+1} - (1 - \delta)K_t - I_t \right] \right\}$$

- First-order conditions with respect to L_t, I_t, K_t are

$$\frac{\partial F_t(K_t, L_t)}{\partial L_t} = w_t$$

$$1 + \frac{\partial \Phi(I_t, K_t)}{\partial I_t} = q_{t+1}$$

$$q_t = \frac{1}{1 + r_{t-1}} \left[\frac{\partial F_t(K_t, L_t)}{\partial K_t} - \frac{\partial \Phi(I_t, K_t)}{\partial K_t} + (1 - \delta)q_{t+1} \right]$$

Optimality Conditions

- With our functional form assumptions, the first two conditions can be written as

$$L_t = (1 - \alpha)^{1/\alpha} A_t^{1/\alpha} w_t^{-1/\alpha} K_t \quad (7)$$

$$\frac{I_t}{K_t} = \frac{1}{\phi} [q_t - 1]$$

- Using the above two, the third condition is

$$\begin{aligned} q_t &= \frac{1}{1 + r_t} \left[\alpha A_{t+1} K_{t+1}^{\alpha-1} L_{t+1}^{1-\alpha} + \frac{\phi}{2} \left(\frac{I_{t+1}}{K_{t+1}} \right)^2 + (1 - \delta) q_{t+1} \right] \\ &= \frac{1}{1 + r_t} \left[\alpha (A_{t+1})^{\frac{1}{\alpha}} (1 - \alpha)^{\frac{1-\alpha}{\alpha}} w_{t+1}^{-\frac{1-\alpha}{\alpha}} - \frac{I_{t+1}}{K_{t+1}} - \frac{\phi}{2} \left(\frac{I_{t+1}}{K_{t+1}} \right)^2 + \left(\frac{I_{t+1}}{K_{t+1}} + (1 - \delta) \right) q_{t+1} \right] \quad (8) \end{aligned}$$

Firm's Value

- Define the firm's value as the cumulative discounted sum of future profits

$$\begin{aligned} V_t &= \sum_{k=t+1}^T \frac{1}{\prod_{s=t}^t (1 + r_s)} D_k \\ &= \frac{1}{1 + r_t} \left[D_{t+1} + \sum_{k=t+2}^T \frac{1}{\prod_{s=t+1}^t (1 + r_s)} D_k \right] \\ &= \frac{1}{1 + r_t} \left[F_{t+1}(K_{t+1}, L_{t+1}) - w_{t+1}L_{t+1} - I_{t+1} - \Phi(I_{t+1}, K_{t+1}) + V_{t+1} \right] \end{aligned}$$

Firm's Value per unit Capital = Q

- The firm's value per unit capital is, using (6),

$$\begin{aligned}\frac{V_t}{K_t} &= \frac{1}{1+r_t} \left[\alpha(A_{t+1})^{\frac{1}{\alpha}}(1-\alpha)^{\frac{1-\alpha}{\alpha}} w_{t+1}^{-\frac{1-\alpha}{\alpha}} - \frac{I_{t+1}}{K_{t+1}} - \frac{\phi}{2} \left(\frac{I_{t+1}}{K_{t+1}} \right)^2 + \frac{K_{t+1}}{K_t} \frac{V_{t+1}}{K_{t+1}} \right] \\ &= \frac{1}{1+r_t} \left[\alpha(A_{t+1})^{\frac{1}{\alpha}}(1-\alpha)^{\frac{1-\alpha}{\alpha}} w_{t+1}^{-\frac{1-\alpha}{\alpha}} - \frac{I_{t+1}}{K_{t+1}} - \frac{\phi}{2} \left(\frac{I_{t+1}}{K_{t+1}} \right)^2 + \left(\frac{I_{t+1}}{K_{t+1}} + (1-\delta) \right) \frac{V_{t+1}}{K_{t+1}} \right] \quad (9)\end{aligned}$$

- Comparing (8) and (9), we conclude

$$q_t = \frac{V_t}{K_t}$$

Q-Theory of Investment

- Q-theory of investment:

$$\frac{I_t}{K_t} = \frac{1}{\phi} [q_t - 1]$$

- Investment is positive if and only if $q_t > 1$
 - The average value of capital, "q", is higher than its cost
- "q" is of course a function of parameters:

$$q_t = \frac{1}{1 + r_t} \left[\alpha(A_{t+1})^{\frac{1}{\alpha}} (1 - \alpha)^{\frac{1-\alpha}{\alpha}} w_{t+1}^{-\frac{1-\alpha}{\alpha}} - \frac{I_{t+1}}{K_{t+1}} - \frac{\phi}{2} \left(\frac{I_{t+1}}{K_{t+1}} \right)^2 + \left(\frac{I_{t+1}}{K_{t+1}} + (1 - \delta) \right) q_{t+1} \right]$$

- q_t is higher if A_t is higher, r_t is lower, and w_t is lower

Stimulating Investment through Temporary Tax Incentives

– Zwick and Mahon (2017)

Background

- Background: weak investment during 00-01 and 07-08 recessions
- In response, Congress passed a bill that allows “bonus depreciation”
- The policies were intended as economic stimulus
- What is “bonus depreciation”?
- Was it successful in stimulating investment?

Tax System

- Consider a firm buying \$1 million worth of computers
- The firm owes corporate taxes on income net of business expenses
- Expenses on nondurable items (e.g., wages):
the firm can immediately deduct the full cost of these items on its tax return
- Expenses on investment:
the firm split deduction over multiple years (exact schedule differs by investment)

Example (corporate tax rate = 35%)

| Year: | 0 | 1 | 2 | 3 | 4 | 5 | Total |
|------------------------------------|-----|-----|------|------|------|------|-------|
| <i>Normal depreciation</i> | | | | | | | |
| Deductions (000s) | 200 | 320 | 192 | 115 | 115 | 58 | 1,000 |
| Tax benefit ($\tau = 35$ percent) | 70 | 112 | 67.2 | 40.3 | 40.3 | 20.2 | 350 |

Bonus Depreciation

- Bonus depreciation allows to deduct $b\%$ of remaining expenses immediately
- The total amount deducted over time does not change
- Bonus depreciation only accelerates the deductions. Why stimulate investment?

Example of 50% bonus depreciation

| Year: | 0 | 1 | 2 | 3 | 4 | 5 | Total |
|--|-----|-----|------|------|------|------|-------|
| <i>Normal depreciation</i> | | | | | | | |
| Deductions (000s) | 200 | 320 | 192 | 115 | 115 | 58 | 1,000 |
| Tax benefit ($\tau = 35$ percent) | 70 | 112 | 67.2 | 40.3 | 40.3 | 20.2 | 350 |
| <i>Bonus depreciation (50 percent)</i> | | | | | | | |
| Deductions (000s) | 600 | 160 | 96 | 57.5 | 57.5 | 29 | 1,000 |
| Tax benefit ($\tau = 35$ percent) | 210 | 56 | 33.6 | 20.2 | 20.2 | 10 | 350 |

Modeling Taxes

- Back to the two-period model
- Let τ be corporate tax rate
- Let z_0 be the percentage of $t = 0$ investment that firms can deduct immediately
- The remaining $z_1 \equiv 1 - z_0$ are deducted at $t = 1$

Investment Problem with Taxes

$$\max_{L_0, I_1, K_1, L_1} \left[(1 - \tau) [F_0(K_0, L_0) - w_0 L_0] - I_0 - \Phi(I_0, K_0) + \tau z_0 (I_0 - \Phi(I_0, K_0)) \right] \\ + \frac{1}{1 + r} \left[(1 - \tau) (F_1(K_1, L_1) - w_1 L_1) + \tau z_1 (I_0 + \Phi(I_0, K_0)) \right]$$

subject to

$$K_1 = K_0(1 - \delta) + I_0$$

Optimality Conditions

- The first-order conditions are

$$L_t = (1 - \alpha)^{1/\alpha} A_t^{1/\alpha} w_t^{-1/\alpha} K_t \quad (10)$$

$$\left[1 - \tau z_0 - \frac{1}{1+r} \tau z_1 \right] \left(1 + \phi \frac{I_0}{K_0} \right) = \frac{1 - \tau}{1+r} \alpha A_1 K_1^{\alpha-1} L_1^{1-\alpha} \quad (11)$$

- Define $z_N \equiv \tau z_0 + \frac{1}{1+r} \tau z_1$:
 - The presented discounted value of deductions per unit investment
- Plugging (10) into (11),

$$\frac{I_0}{K_0} = \frac{1}{\phi} \left[\frac{1}{1+r} \frac{1 - \tau}{1 - z_N} \alpha (1 - \alpha)^{\frac{1-\alpha}{\alpha}} A_1^{\frac{1}{\alpha}} w_1^{-\frac{1-\alpha}{\alpha}} - 1 \right]$$

Impact of Bonus Depreciation in the Model

$$\frac{I_0}{K_0} = \frac{1}{\phi} \left[\frac{1}{1+r} \frac{1-\tau}{1-z_N} \alpha (1-\alpha)^{\frac{1-\alpha}{\alpha}} A_1^{\frac{1}{\alpha}} w_1^{-\frac{1-\alpha}{\alpha}} - 1 \right]$$

- Bonus depreciation b allows firms to deduct extra $b\%$ of z_1 at $t = 0$

$$z_N = \tau(z_0 + bz_1) + \frac{1}{1+r} \tau(z_1 - bz_1)$$

- How does bonus depreciation (an increase in b) affect investment?

$$\frac{d(I_0/K_0)}{dz_N} > 0, \quad \frac{dz_N}{db} = \tau z_1 \left(1 - \frac{1}{1+r} \right) > 0$$

- Bonus depreciation increases present discounted value of deductions if $r > 0$
- More deductions lower the effective investment costs and stimulate investment

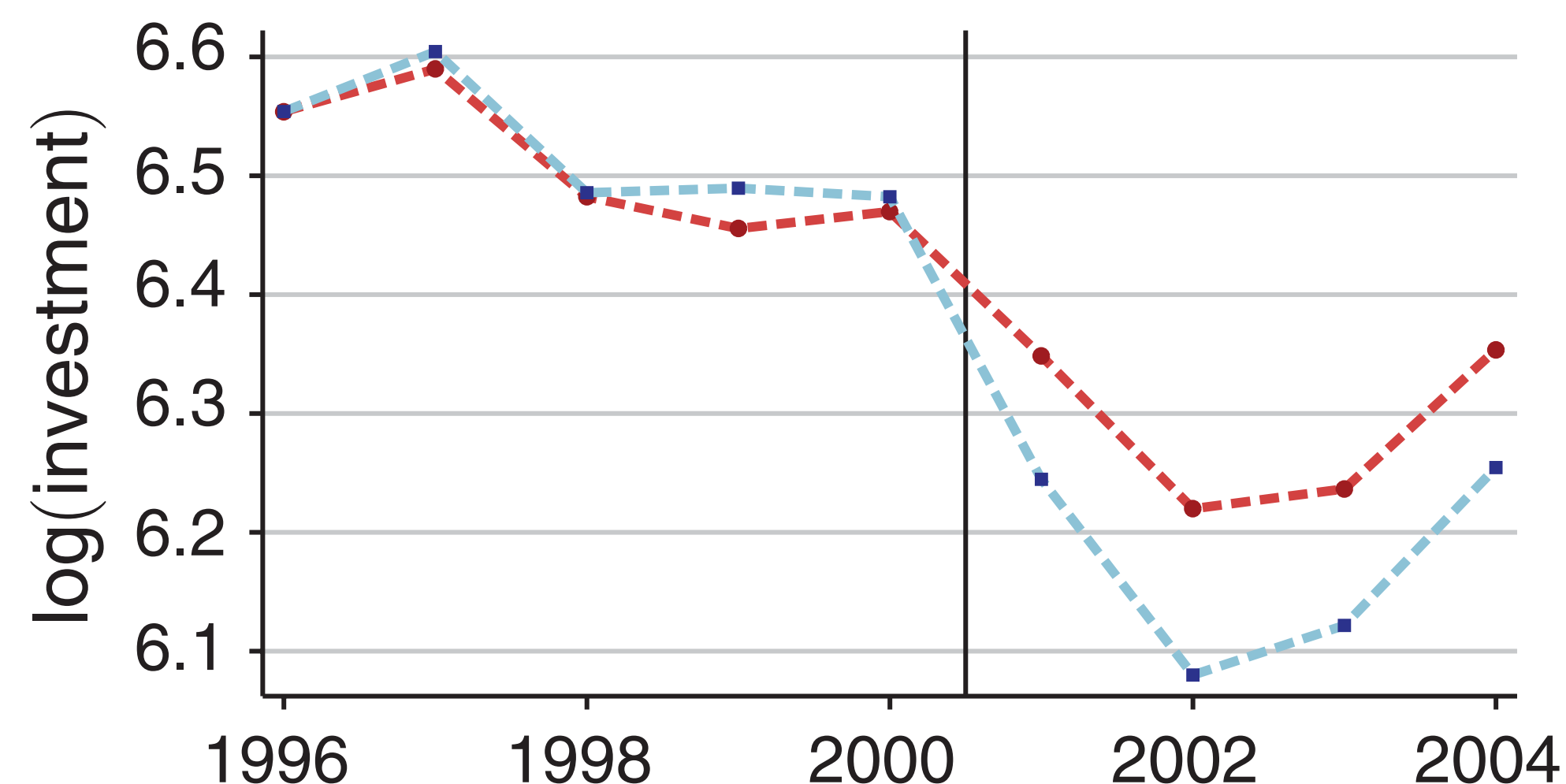
Empirical Setup

- Bonus depreciation implementation (b in our model):
 - 2001-2003: 30%
 - 2003-2004: 50%
 - 2008-2010: 50%
 - 2009-2010: 100%
- Construct present discounted value of deductions (z_N in our model) by industry
- Industries had the same b but differed in the original deductions schedule, z_0 & z_1
- If industries have long-duration schedules (higher z_1), the impact of b is higher

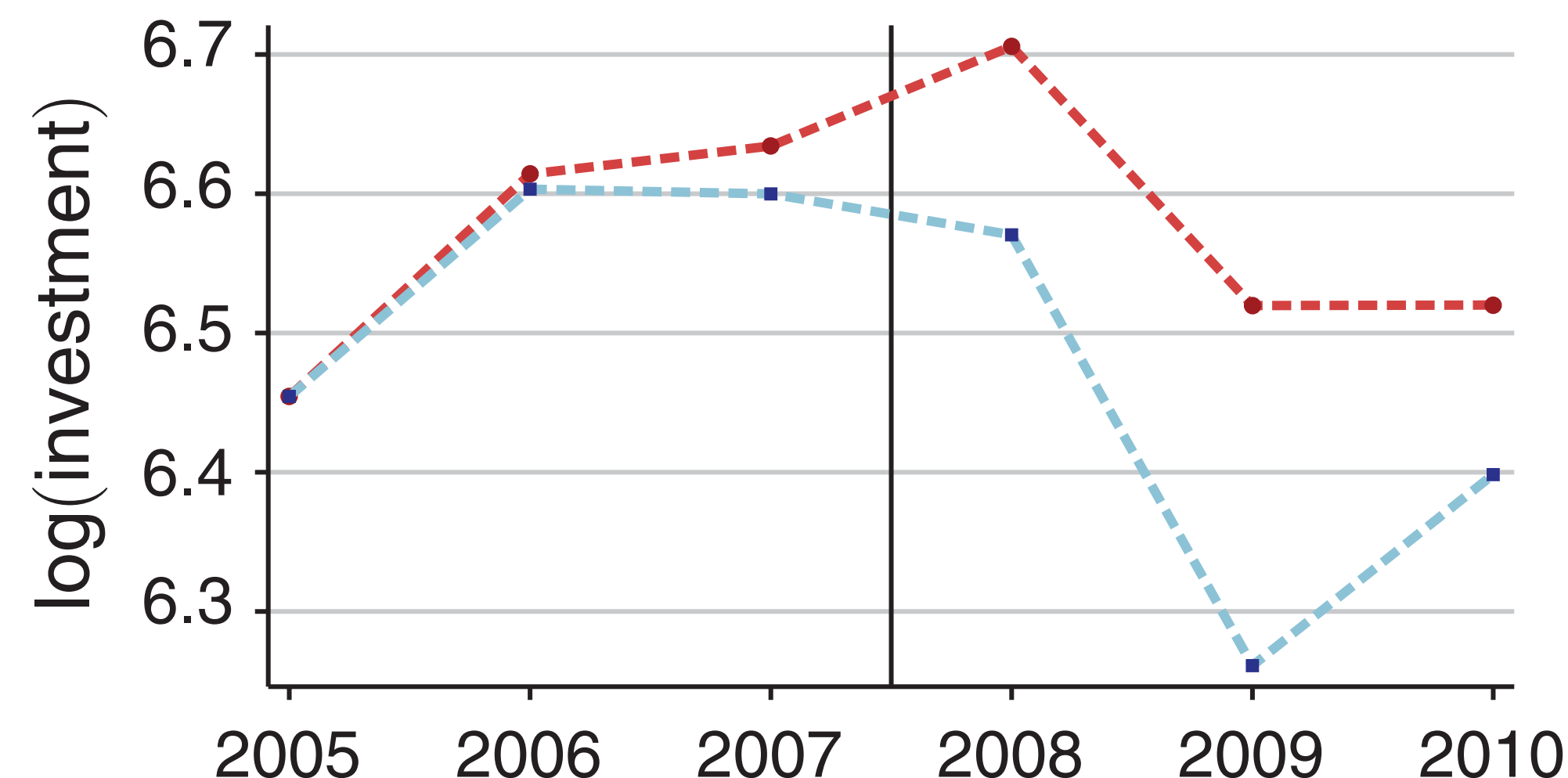
$$\frac{dz_N}{db} = \tau z_1 \left(1 - \frac{1}{1+r} \right)$$

Impact of Bonus Depreciation

Panel A. Intensive margin: bonus I



Panel B. Intensive margin: bonus II



---●--- Treatment group (long duration industries)
---■--- Control group (short duration industries)

- Higher-duration industries increase investment relative to low-duration industries
- Bonus depreciation raised investment of eligible capital by 10-15%

Heterogeneous Responses

TABLE 6—HETEROGENEITY BY EX ANTE CONSTRAINTS

| | Sales | | Div payer? | | Lagged cash | |
|------------------|----------------|----------------|----------------|----------------|----------------|----------------|
| | Small | Big | No | Yes | Low | High |
| $z_{N,t}$ | 6.29 (1.21) | 3.22 (0.76) | 5.98 (0.88) | 3.67 (0.97) | 7.21 (1.38) | 2.76 (0.88) |
| Equality test | $p = 0.030$ | | $p = 0.079$ | | $p = 0.000$ | |
| Observations | 177,620 | 255,266 | 274,809 | 127,523 | 176,893 | 180,933 |
| Clusters (firms) | 29,618 | 29,637 | 39,195 | 12,543 | 45,824 | 48,936 |
| R^2 | 0.44 | 0.76 | 0.69 | 0.80 | 0.81 | 0.76 |

■ Investment response is larger for

- smaller firms
- firms paying no dividends
- firms with low cash holdings

Why Do Size, Dividend, and Cash Matter?

- Do size, dividend, and cash matter in our model?
- No. Recall:

$$\frac{I_0}{K_0} = \frac{1}{\phi} \left[\frac{1}{1+r} \frac{1-\tau}{1-z_N} \alpha (1-\alpha)^{\frac{1-\alpha}{\alpha}} A_1^{\frac{1}{\alpha}} w_1^{-\frac{1-\alpha}{\alpha}} - 1 \right]$$

- Whether firms are large, pay dividends, or hold cash is irrelevant (on their own)
- Then why do we find such heterogeneity in the data?
- One explanation is financial friction
- The prevalence of financial friction is correlated with size, dividend, and cash
- Constrained firms could react more to bonus depreciation