Business Cycles

EC502 Macroeconomics Topic 9

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Business cycles: joint movements of economic activity at medium frequency

How do we extract medium frequency from the data?









One option is to detrend using a linear trend

One may argue this is not very sensible

Linear Trend



- There are various ways to isolate medium-frequency movements
- We will focus on Baxter-King bandpass filter (because we are at BU!)
- Statistical procedure to distinguish medium- and low-frequency movements
- Popular alternative: Hodrick-Prescott filter

Baxter-King Bandpass Filter





Use Baxter-King filter to extract low-frequency (more than 8 years) components





Baxter-King filter to extract medium-frequency movements (1.5-8 years)



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Consumption



Investment





Unemployment Rate



Log Government Expenditure



In Search of Unified Explanation

One is led by the facts to conclude that, with respect to the qualitative behavior of comovements among series, business cycles are all alike.

To theoretically inclined economists, this conclusion should be attractive and challenging, for it suggests the possibility of a unified explanation of business cycles.

Robert Lucas (1977)







Search for a theory that explains

- 1. Positive comovements between *Y*, *C*, *I*, *L*
- 2. $\operatorname{std}(\log I) \gg \operatorname{std}(\log Y) \approx \operatorname{std}(\log C) \approx \operatorname{std}(\log L)$



We will build a model of the macroeconomy that endogeneizes

- 1. labor supply
- 2. consumption
- 3. investment
- At this point, we already have all the tools. We learned theories of
 - 1. labor supply
 - 2. consumption
 - 3. investment

Now we put everything together in one model!





Real Business Cycle Theory with Two-Periods













- Two-periods, t = 0, 1
- The economy is populated by
 - 1. a continuum of identical households: consume, save, & supply labor
 - 2. a continuum of identical firms: hire labor & invest

Setup



Households

- Households earn labor income and profits income (households own firms)
- We assume labor supply at t = 1 is exogenous (simplification)
- Households have the following preferences $u(C_0) - v(l_0) + \beta u(C_1)$
- The budget constraints are

- $C_0 + C_1 = (1$
- Given (r, w_0, w_1) , households choose $\{C_0, C_1, l_0, a_0\}$ to maximize (1) s.t. (2)-(3)

$$-a_0 = w_0 l_0 + D_0$$

$$(+r)a_0 + w_1l_1 + D_1$$









The firms solve the same problem as in the previous lecture note

subject to

Firms

 $\max_{L_0, I_1, K_1, L_1} D_0 + \frac{1}{1+r} D_1$

 $D_0 = F_0(K_0, L_0) - w_0 L_0 - I_0 - \Phi(I_0, K_0)$ $D_1 = F_1(K_1, L_1) - w_1 L_1$ $K_1 = (1 - \delta)K_0 + I_0$



Market Clearing Conditions

- Unlike before, now all prices, (r, w_0, w_1) are endogenous
- How are they pinned down? demand = supply
- Market clearing conditions:
 - $C_0 + I_0 + \Phi(I_0, K_0) = F_0(K_0, L_0)$ $C_1 = F_1(K_1, L_1)$ $l_0 = L_0$ $l_1 = L_1$
- When all prices are endogenous, we call it as "general equilibrium model"



Equilibrium Definition

- **1.** Given $\{r, w_0, w_1\}$, households optimally choose $\{C_0, C_1, l_0, l_1, a_0\}$
- **2.** Given $\{r, w_0, w_1\}$, firms optimally choose $\{I_0, L_0, L_1, K_1\}$
- 3. Markets clear

lly choose { C_0, C_1, l_0, l_1, a_0 } ose { I_0, L_0, L_1, K_1 }



Functional Form Assumptions

We will impose the following familiar functional form assumptions

u(C)

v(l)

 $F_t(K,L)$

 $\Phi(I,K)$

$$C^{1-\sigma}$$

$$= \frac{C^{1-\sigma}}{1-\sigma}$$

$$l^{1+\nu}$$

$$= \bar{\nu} \frac{l^{1+\nu}}{1+\nu}$$

$$= A_t K_t^{\alpha} L_t^{1-\alpha}$$

$$=\frac{\phi}{2}\left(\frac{I}{K}\right)^2 K$$



Characterizing Equilibrium



Optimality Conditions

- The housheolds' optimal choice of labor supply imlies $\bar{v}l_0^{\nu} = w_0 C_0^{-\sigma}$
- The households' optimal consumption-saving decision implies
- The firm's optimal labor demand:
- The firm's optimal investment:

$$1 + \phi \frac{I_0}{K_0} =$$

 $W_{t} = (1)$

 $C_0^{-\sigma} = \beta(1+r)C_1^{-\sigma}$

$$(-\alpha)A_tK_t^{\alpha}L_t^{-\alpha}$$

$$\frac{1}{1+r} \alpha K_1^{\alpha-1} L_1^{1-\alpha}$$



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Simplifying

Impose $l_0 = L_0$ and substitute (6) into (4) to obtain

Solve (7) for 1 + r and substitute it into (5) to obtain

Recall the goods market clearing conditions and the evolution of capital stock are $C_0 + I_0 + \frac{\phi}{2} \frac{I_0^2}{K_0} = Y_0$ $Y_0 = A_0 K_0^{\alpha} L_0^{1-\alpha}$ $C_1 = A_1 K_1^{\alpha} L_1^{1-\alpha}$ $K_1 = (1 - \delta)K_0 + I_0$ $\blacksquare \{C_0, C_1, I_0, L_0, K_1, Y_0\} \text{ solve (8)-(13)}$

- $\bar{v}L_{0}^{\alpha+\nu} = (1-\alpha)A_{0}K_{0}^{\alpha}C_{0}^{-\sigma}$
- $C_0^{-\sigma} = \beta \frac{\alpha A_1 K_1^{\alpha 1} L_1^{1 \alpha}}{1 + \frac{\phi I_0}{K_0}} C_1^{-\sigma}$



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$$C_0^{-\sigma} = \beta \frac{\alpha A_1 \left[K_0 (1-\delta) + I_0 \right]^{\alpha - 1} L_1^{1-\alpha}}{1 + \phi I_0 / K_0} \left(A_1 \left[K_0 (1-\delta) + I_0 \right]^{\alpha} L_1^{1-\alpha} \right)^{-\sigma}$$

Eq. (8) is giving a relationship between c_0 and L_0 :

$$\bar{\nu}L_0^{\alpha+\nu} =$$

• Use (13) to rewrite (10), which gives a relationship between Y_0 and I_0 : $\hat{C}_0(I_0) + I_0 + \frac{\phi}{2} \frac{I_0^2}{K_0} = Y_0$ **Eq. (11)** gives a relationship between Y_0 and L_0 : $Y_0 = A_0 K_0^{\alpha} L_0^{1-\alpha}$

Four Equations with Four Unknowns (C_0, I_0, Y_0, L_0) Into (9) gives a relationship btwn $C_0 \& I_0$, which we write as $\hat{C}_0(I_0)$:

$$(1 - \alpha)A_0K_0^{\alpha}C_0^{-\sigma}$$





Eq. (13) defines an increasing relationship between C_0 and I_0

- If firms invest more, future consumption will be higher
- Consumption smoothing implies today's consumption will be also higher

Equation

$$\frac{A_1 \left[K_0(1-\delta) + I_0 \right]^{\alpha-1} L_1^{1-\alpha}}{1 + \phi I_0 / K_0} \int_{-1/\sigma}^{-1/\sigma} \left(A_1 \left[K_0(1-\delta) + I_0 \right]^{\alpha} + \frac{\hat{C}_0(I_0)}{1 + \phi I_0 / K_0} \right) = \hat{C}_0(I_0)$$







Eq. (8) defines a decreasing relationship between C_0 and L_0

- As households consume more, marginal utility of consumption declines • This discourages households to work (income effect)







Eq. (14) defines an increasing relationship between Y_0 and I_0

- More investment leads to higher consumption today

$\hat{C}_0(I_0) + I_0 + \frac{\phi}{2} \frac{I_0^2}{K_0} = Y_0$

Investment, I_0

• In order to sustain higher consumption and investment, output needs to be higher







- **Eq. (11)** defines an increasing relationship between Y_0 and L_0
 - More employment leads to more production

 $Y_0 = A_0 K_0^{\alpha} L_0^{1-\alpha}$

Employment, L₀

hip between Y_0 and L_0 roduction














































































What Drives Business Cycles?





- We consider various "shocks" to our economy
 Shocks: exogenous changes in some aspects of the economy
 We then study how (C₀, I₀, L₀, Y₀) endogenously respond to the shocks
- Ask: do the endogenous responses look like business cycles?



- What shocks should we study?
- Let us explore various possibilities
 - 1. Changes in productivity today, A_0
 - 2. Changes in productivity in the future, A_1
 - 3. Changes in households' discount factor, β
 - 4. Changes in firms' desire to invest, ϕ
 - 5. Changes in households' incentive to work, \bar{v}

























Changes in Today's Productivity

- An increase in A₀ increases GDP, consumption, and investment
- The impact of hours worked is generally ambiguous
- What is the mechanism?

 - 1. An increase in $A_0 \Rightarrow$ increases w_0 and $D_0 \Rightarrow$ Households are wealthier 2. Households increase consumption both at time 0 and 1 (consumption smoothing)
 - 3. In order to increase consumption at t = 1, households need to save: Y C go up
 - 4. In order Y C to go up, I must go up because $Y C = I + \Phi$
 - 5. Because w_0 increases, substitution effect increases labor supply
 - 6. Because (w_0, D_0) increase, income effect decreases labor supply
- As long as σ is not too large, we can show that 5 dominates 6 \Rightarrow hours worked increases









- Therefore, an increase in A₀ produces something that looks like business cycles!



	Y	C	I	L
$A_0 \uparrow$				
$A_1\uparrow$				
β				
ϕ \uparrow				
$\overline{\mathcal{V}}$				





Optimism and Pessimism

- What about other shocks?
- What if we shock future productivity A_1 ?
 - Booms are the time when people expect future to be bright (optimistic)
 - Recessions are the time when people are pessimistic



























Can Optimism Generate Business Cycles?

- An increase in A_1 increases D_1 and investment and thereby r
 - If r is higher, households would like to save more through substitution effect
 - If (r, D_1) are higher, would like to consume more today through income effect
- When $\sigma > 1$, the latter dominates and C_0 increases
 - Then L_0 decreases through income effect, and Y_0 goes down
 - As a result, $I_0 = Y_0 C_0$ decreases as well.
- When $\sigma < 1$, the former dominates and C_0 decreases • Then L_0 increases through income effect, and Y_0 goes up
- - As a result, $I_0 = Y_0 C_0$ increases
- When $\sigma = 1$, two effects cancel and nothing happens
- Does this look like a business cycle? No.



	Y	
$A_0 \uparrow$		
$A_1 \uparrow (\sigma > 1)$		
$A_1 \uparrow (\sigma < 1)$	$\mathbf{\uparrow}$	
β		
$\phi \uparrow$		
\overline{v}		









• How about changes in β ?

Changes in Discount Factor

 Booms are the times when households would like to consume more today Recessions are the times when households would like to consume less today





















Postponing Consumption

• An increase in β decreases C_0

- This increases L_0 through income effect
- Investment increases because $I_0 = Y_0 C_0$
- Does this look like a business cycle? No.



	Y	
$A_0 \uparrow$		
$A_1 \uparrow (\sigma > 1)$		
$A_1 \uparrow (\sigma < 1)$	$\mathbf{\uparrow}$	
β	$\mathbf{\uparrow}$	
$\phi \uparrow$		
\overline{v}		






Investment Shock

Let now us consider the shock to investment cost, ϕ

- Booms are the time when firms find it easy to invest
- Recessions are the time when firms find it difficult to invest



















Investment Shock

An increase in ϕ decreases I_0

- Since $C_0 = Y_0 I_0 \Phi$, consumption increases
- L_0 decreases through income effect, and Y_0 also decreases
- Does this look like a business cycle? No.



	Y	
$A_0 \uparrow$		
$A_1 \uparrow (\sigma > 1)$		
$A_1 \uparrow (\sigma < 1)$	$\mathbf{\uparrow}$	
β \uparrow	$\mathbf{\uparrow}$	
$\phi \uparrow$		
$\overline{v}\uparrow$		







Labor Disutility Shock

• What about changes in \bar{v} ? Literal interpretation:

- Booms are the times when households want to or can work more
- Recessions are the times when households do not or cannot work enough





















Labor Disutility Shock

- An increase in \bar{v} decreases L_0 , which decreases the output, Y_0
- Does this look like a business cycle? Yes.

• Then *r* needs to rise to lower $C_0 \& I_0$ and clear the market



	Y	
$A_0 \uparrow$		
$A_1 \uparrow (\sigma > 1)$		
$A_1 \uparrow (\sigma < 1)$	\uparrow	
β	$\mathbf{\uparrow}$	
$\phi \uparrow$		
\overline{v}		





Real Business Cycle with Infinite Horizon

- We now extend the previous model to conduct quantitative analaysis
- We will assume the time horizon is infinite, $t = 0, ..., \infty$
- Can our model replicate business cycles quantitatively?

Quantiative Model

Households and Firms

Households solve

subject to

Firms solve

subject to

 $D_t = \mathbf{A} K_t^{\alpha} L_t^{1-}$

$$\begin{bmatrix} C_t^{1-\sigma} & l_t^{1+\nu} \\ \hline 1-\sigma & 1+\nu \end{bmatrix}$$

$$C_t + a_t = (1 + r_{t-1})a_{t-1} + w_t l_t + D_t$$

$$\sum_{t=0}^{n} \frac{1}{\prod_{s=0}^{t-1} (1+r_s)} D_t$$

$$-\alpha - w_t L_t - I_t - \frac{\phi}{2} \left(\frac{I_t}{K_t}\right)^2 K_t$$

$$(1-\delta)K_t + I_t$$

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Equilibrium Definition

- Equilibrium consists of $\{C_t, l_t, I_t, L_t, K_{t+1}\}$ and $\{w_t, r_t\}$ such that
- **1.** Given $\{w_t, r_t\}$, households optimally choose $\{C_t, l_t, a_t\}$
- **2.** Given $\{w_t, r_t\}$, firms optimally choose $\{I_t, K_{t+1}, L_t\}$
- 3. Markets clear

 $C_t + I_t + \Phi(I_t, K_t) = F_t(K_t, L_t)$ $l_t = L_t$

Equilibrium Cond

- 1. Euler equation: \mathcal{U}^{\prime}
- 2. Labor supply:
- 3. Labor demand:
- 4. Investment:

ditions: {
$$C_t$$
, L_t , I_t , K_{t+1} , q_t , w_t , r_t }
 $f(C_t) = \beta(1 + r_t)u'(C_{t+1})$

$$w_t u'(C_t) = v'(L_t)$$

 $\partial F_t(k)$

$$\frac{I_t}{K_t} = \frac{1}{\phi} \left[q_t - 1 \right]$$

$$q_t = \frac{1}{1+r_t} \left[\frac{\partial F_{t+1}(L_{t+1}, K_{t+1})}{\partial K_{t+1}} - \frac{I_{t+1}}{K_{t+1}} - \frac{\phi}{2} \left(\frac{I_{t+1}}{K_{t+1}} \right)^2 + \left(\frac{I_{t+1}}{K_{t+1}} + (1-\delta) \right) q_{t+1} \right]$$

- 5. Capital stock evolution:
- 6. Goods market clearing:

$$\frac{K_t, L_t}{L_t} = w_t$$

$$(1-\delta)K_t + I_t$$

 $C_t + I_t + \Phi(I_t, K_t) = F_t(K_t, L_t)$

Procedure

- We set the parameter values to reasonable values ("calibration")
- We then compute the steady state, where all the variables are constant over time
- Next, we simulate the model in response to a sudden shock
- The shock process is assumed to be AR(1). For example, in the case of productivity,
 - $(\log A_t \log A) = \rho(\log A_{t-1} \log A) + \epsilon_t^A$
 - with $\rho \in [0,1)$ and $\epsilon_t^A \sim N(0,\sigma_A^2)$

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- One period is a quarter
- Set $\alpha = 1/3$ to match labor share
- Set $\sigma = 1$ to be consistent with (roughly) constant hours worked in the long-run • This implies a *permanent* change in A does not change L
- Set $\beta = 0.96^{1/4}$ to match 4% interest rate
- Set $\nu = 1$ to be (upper-end of) micro-level labor supply elasticity estimates
- Set $\delta = 5\%$ to match $K/Y \approx 3.5$
- Set $\phi = 10$ (match estimates of Zwick-Mahon (2017))
- We assume all shocks have the same persistence of $\rho = 0.9$

Parameterization

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Impulse Response to Productivity Shock

Y

Future Productivity Shock

Patience Shock

Investment Cost Shock

Labor Disutility Shock

	Data	Model			
		A	ß	ϕ	V
$\operatorname{corr}(\Delta Y, \Delta C)$	0.65	0.99	-0.91	-0.80	0.99
$\operatorname{corr}(\Delta Y, \Delta I)$	0.81	0.99	0.97	0.93	0.99
$\operatorname{corr}(\Delta Y, \Delta L)$	0.66	0.99	0.96	0.91	0.99

Correlation

	Data	Model			
		A	ß	ϕ	V
$std(\Delta C)/std(\Delta Y)$	0.80	0.75	2.1	2.1	0.75
$std(\Delta I)/std(\Delta Y)$	4.9	1.53	9.1	13.3	1.5
$std(\Delta h)/std(\Delta Y)$	0.98	0.12	1.52	1.5	1.5

- Two shocks are fairly successful in accounting for business cycles
 - 1. Productivity shock
 - 2. Labor disutility shock
- The former is what Kydland & Prescott (1982) argued for
- Why? because A_t is directly measurable as Solow residual:

$$\log A_t = \log Y_t - ($$

 $(\alpha \log K_t + (1 - \alpha) \log L_t)$

TFP in the Data

Log TFP

TFP in the Data and in the Model

	Data	Model			
		A	ß	ϕ	V
$\operatorname{corr}(\Delta Y, \Delta A)$	0.80	0.99	0	0	0
$std(\Delta A)/std(\Delta Y)$	0.58	0.92	0	0	0

Criticisms of the RBC Model and Where We Are

Cheap Criticisms of the RBC

1. Not plausible (most common and non-scientific criticism)

- Changes in A_t due to technological progress is plausible
- But this should be lower frequency than business cycles
- Technological *regress* does not make sense
- 2. TFP is endogenous (unconstructive criticism)
 - Changes in A_t cannot be treated as exogenous "shock"
 - Changes in A, could be a result of innovation or misallocation

Both of the above criticisms apply to labor disutility shocks as well

Deeper Critisms of the RBC Model

	Data	Model			
		A	ß	ϕ	V
$std(\Delta C)/std(\Delta Y)$	0.80	0.75	2.1	2.1	0.75
$std(\Delta I)/std(\Delta Y)$	4.9	1.53	9.1	13.3	1.5
$\operatorname{std}(\Delta h)/\operatorname{std}(\Delta Y)$	0.98	0.12	1.52	1.5	1.5

3. The model generates too little volatility in *L*

- This is a valid point. RBC mechanism lacks forces to generate volatile L
- This led many researchers to focus on shocks that look like \bar{v} shocks

h lacks forces to generate volatile L on shocks that look like \bar{v} shocks

Deeper Critisms of the RBC Model

4. The model fails to replicate the behavior of prices, (r, w), in the data

 $Corr(\Delta Y, \Delta w) = 0.99$ $Corr(\Delta Y, \Delta r) = -0.99$

 $std(\Delta w)/std(\Delta Y) = 0.87$ $std(\Delta r)/std(\Delta Y) = 0.05$

Prices in the Data $Corr(\Delta Y, \Delta w) = 0.22$ $Corr(\Delta Y, \Delta r) = 0.002$ $std(\Delta w)/std(\Delta Y) = 0.45$ $std(\Delta r)/std(\Delta Y) = 2.5$

Deeper Critisms of the RBC Model

Suppose that

 $F_t(K_t, L_t) =$

 u_t : capital utilization rate.

- Many factories or machines are not utilized in recessions
- Correct measure of TFP is

 $\log A_t = \log Y_t - (\alpha(\log u_t + \log K_t) + (1 - \alpha)\log L_t)$

5. Once we measure TFP accurately, the correlation between TFP and GDP is weak:

$$= A_t (\boldsymbol{u}_t K_t)^{\alpha} L_t^{1-\alpha}$$



Where We Are

- Did macroeconomists find a unified explanation of business cycles? Perhaps not
- Most economists do not accept RBC as the final answer
- But RBC is an extremely useful benchmark model
- Ironically, all the attempts to criticize RBC are still based on RBC
- So in the end, what drives business cycles? Some recently suggested alternatives:
 - Risk/Uncertainty
 - Financial frictions





