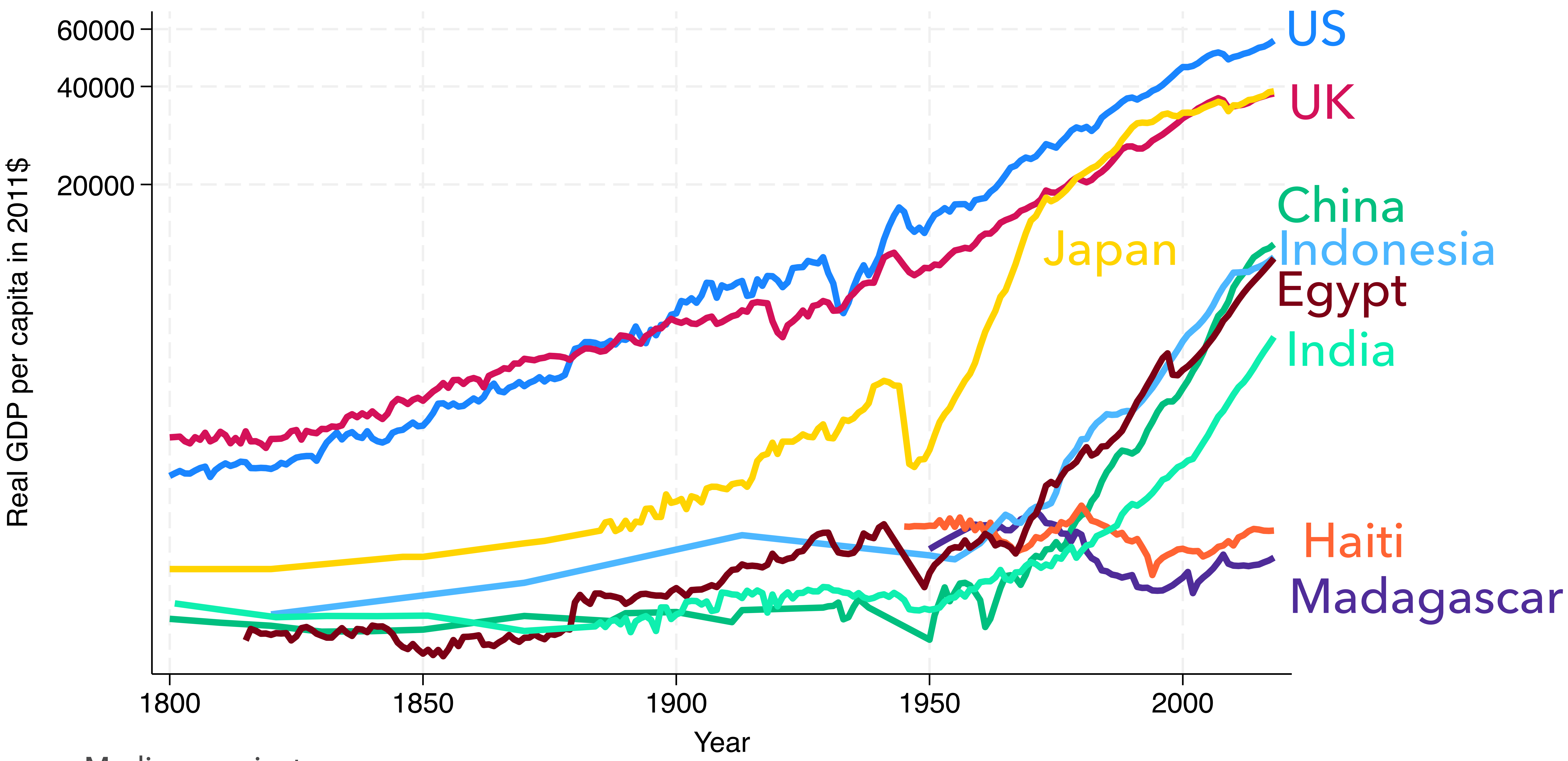
Development and Growth Accounting

EC502 Macroeconomics
Topic 1

Masao Fukui

2026 Spring

Why are Some Countries Richer than Others?



Source: Madison project

Cross-Country Income Differences

- United States today are
  1. 5 times richer than people in China
  2. 10 times richer than people in India
  3. more than 40 times richer than people in Haiti

- What drives these enormous differences in standards of living across countries?

Role of Models

All theory depends on assumptions which are not quite true. That is what makes it theory. The art of successful theorizing is to make the inevitable simplifying assumptions in such a way that the final results are not very sensitive.

—Robert Solow

Production Function

- Suppose the output of a country is produced using
  1. Labor, L
  2. Physical capital (machines, building, etc), K
- A production function tells us how much we can produce output given L and K:

$$Y = F(K, L)$$

- We say $F(K, L)$ features
  - constant returns to scale if $F(\lambda L, \lambda K) = \lambda F(L, K)$
  - decreasing returns to scale if $F(\lambda L, \lambda K) < \lambda F(L, K)$
  - increasing returns to scale if $F(\lambda L, \lambda K) > \lambda F(L, K)$

Cobb-Douglas Production Function

- A popular functional form is Cobb-Douglas production function

$$Y = F(K, L) = AK^{\alpha}L^{\beta}$$

  - A: the level of technology
  - $\alpha, \beta \in [0,1]$: importance of each factor
- Using the previous definition,
  - $\alpha + \beta = 1 \Rightarrow$ constant returns to scale
  - $\alpha + \beta < 1 \Rightarrow$ decreasing returns to scale
  - $\alpha + \beta > 1 \Rightarrow$ increasing returns to scale
- We will assume constant returns to scale. Why?
  Replication argument: If all the inputs double, output should double

Important Distinction

$$F(K, L) = AK^{\alpha}L^{1-\alpha}$$

- Here, $F(K, L)$ is constant returns to scale to all inputs
- But, $F(K, L)$ features diminishing returns to a particular input
  - If we only double K, output less than doubles:

$$F(2K, L) = 2^{\alpha}F(K, L) < 2F(K, L)$$

  - Equivalently, $F(K, L)$ is concave in both arguments:

$$F_{KK}(K, L) < 0, \quad F_{LL}(K, L) < 0$$

Development Accounting

Decomposing GDP per Capita

$$Y_i = A_i K_i^{\alpha} L_i^{1-\alpha}$$

- i: country
- Divide both sides by population size, N_i, and taking log:

$$\log(Y_i/N_i) \quad = \quad \log A_i \quad + \quad \alpha \log(K_i/N_i) \quad + \quad (1-\alpha)\log(L_i/N_i)$$

Decomposing GDP per Capita

$$Y_i = A_i K_i^{\alpha} L_i^{1-\alpha}$$

- i: country
- Divide both sides by population size, N_i, and taking log:

$$\log(Y_i/N_i) = \log A_i + \alpha \log(K_i/N_i) + (1-\alpha)\log(L_i/N_i)$$

GDP per capita | Technology | Capital per capita | Employment per capita

Decomposing GDP per Capita

$$Y_i = A_i K_i^{\alpha} L_i^{1-\alpha}$$

- i: country

- Divide both sides by population size, N_i, and taking log:

$$\log(Y_i/N_i) = \log A_i + \alpha \log(K_i/N_i) + (1-\alpha)\log(L_i/N_i)$$

GDP per capita — Technology — Capital per capita — Employment per capita

- How much of differences in GDP per capita due to

1. capital
2. labor
3. technology (which we don't directly observe)

Development Accounting

$$\log(Y_i/N_i) \quad = \quad \log A_i \quad + \quad \alpha \log(K_i/N_i) \quad + \quad (1-\alpha)\log(L_i/N_i)$$

- This exercise called **development accounting**
  - It is accounting because we do not theorize how each component is determined
- Nevertheless, it helps us to guide what theoretical model we should write down
- In order to implement development accounting, we need to take a stand on α
- What value should we use for α?

Factor Shares

- Factor shares: what fraction of GDP is paid to each factor?
- Suppose firms need pay w to hire workers and r to rent machines
- Firms take (w, r) as given (competitive market) and choose (L, K):

  $$\max_{K,L} AK^{\alpha}L^{1-\alpha} - wL - rK$$

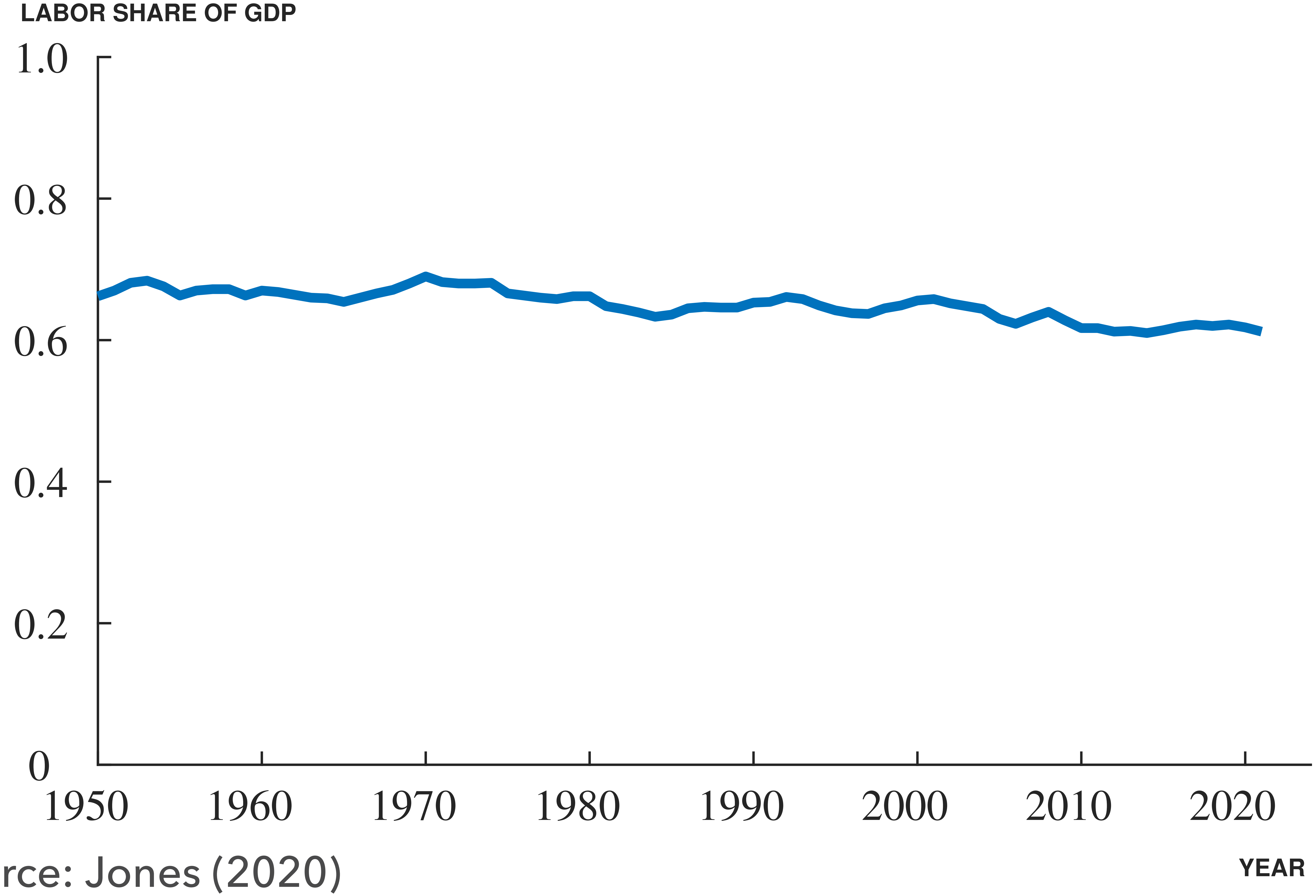
  Taking the first-order condition with respect to L

  $$(1-\alpha)AK^{\alpha}L^{-\alpha} = w \qquad (1)$$

  The firm equalizes the marginal product of labor to wages
- Multiplying both sides of (1) by L,

  $$\frac{wL}{Y} = (1-\alpha) \quad \Rightarrow \text{Labor share of GDP is } 1-\alpha$$

Stable Labor Share



Source: Jones (2020)

Technology as Residual

- Labor share $\approx 2/3$ and stable over time, so we assume $\alpha = 1/3$
- With the assumed value of α, we can construct a measure of "technology"

$$\log A_i = \log(Y_i/N_i) - \alpha \log(K_i/N_i) - (1-\alpha)\log(L_i/N_i)$$

  - Also referred to as "total factor productivity (TFP)" or "Solow residual"
  - $\log A_i$ captures differences in GDP not captured by K/N or L/N
  - Measure of our ignorance

First Look at the Data 2019

	Y/N	K/N	L/N	A
U.S.	100	100	100	100
China	22	33	116	30
India	10	12	76	26
Haiti	2.5	7	84	7

Data: Penn World Table 2019

- Large differences in K/N and A
- Little difference in L/N (employment per person)

Variance Decomposition

- We can explore more systematically

$$\text{Var}\left(\log Y_i/N_i\right) = \text{Cov}\left(\log(Y_i/N_i), \alpha \log K_i/N_i\right)$$ Variance in GDP due to K/N

$$+\text{Cov}\left(\log Y_i/N_i, (1-\alpha)\log L_i/N_i\right)$$ Variance in GDP due to L/N

$$+\text{Cov}\left(\log Y_i/N_i, \log A_i\right)$$ Variance in GDP due to A

- Therefore, $\dfrac{\text{Cov}(\log Y_i/N_i, \log X_i)}{\text{Var}(\log Y_i/N_i)}$ corresponds to the share explained by a factor X

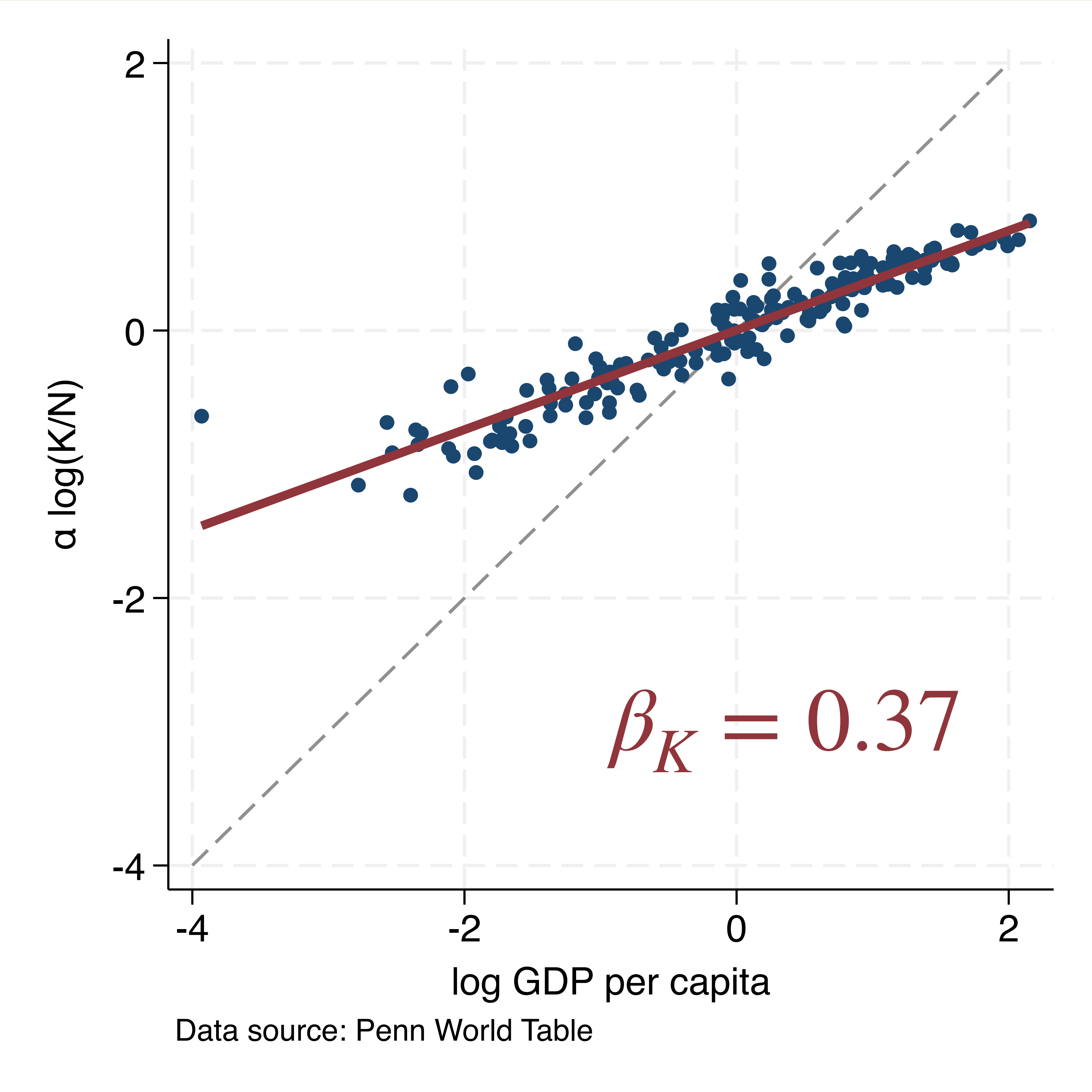
- This can be obtained as a regression coefficient β_X of

$$\log X_i = \beta_X \log(Y_i/N_i) + \gamma + \epsilon_i$$

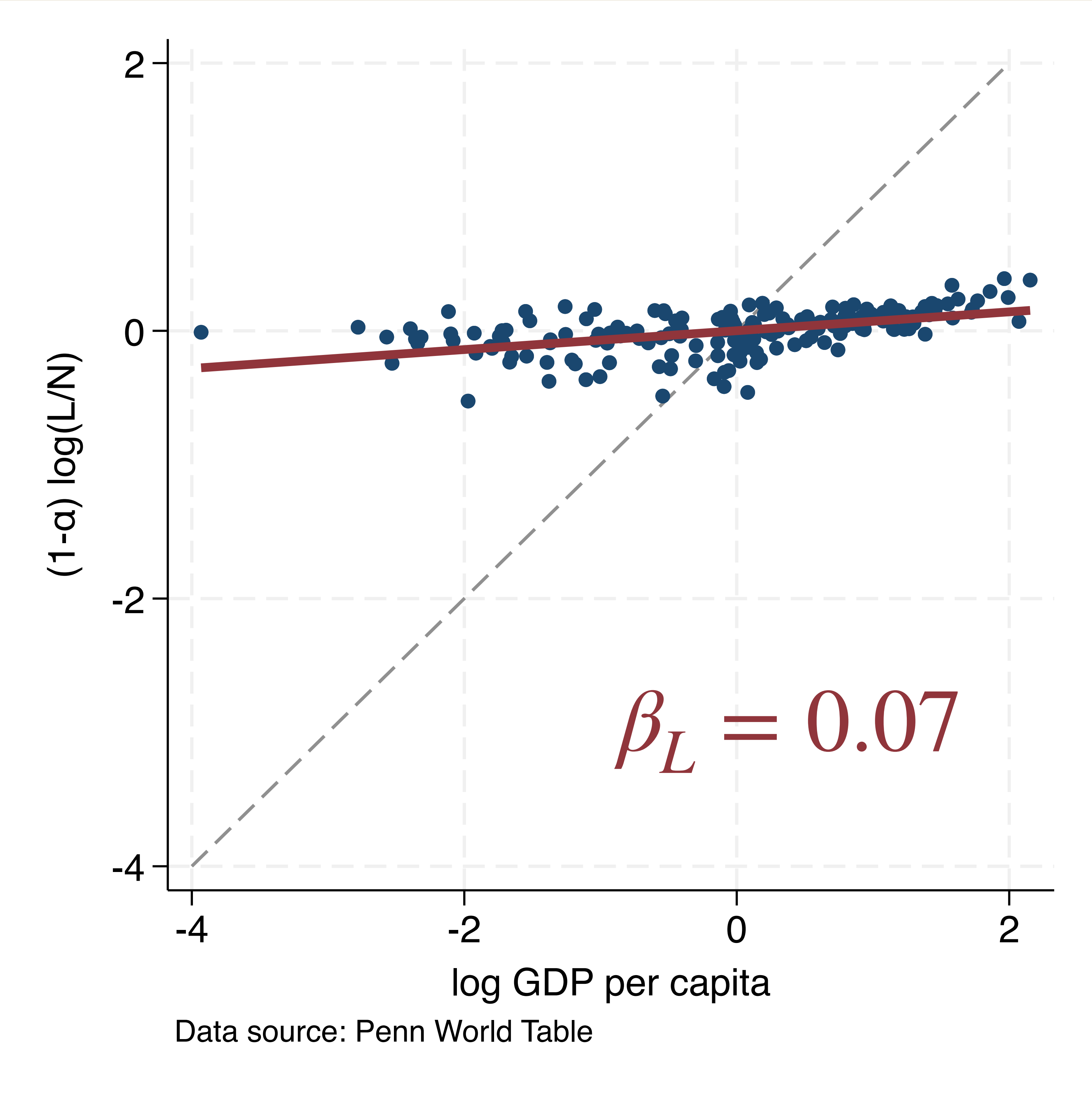
If $\beta_X = 1$, differences in GDP per capita entirely due to X

Development Accounting 2019

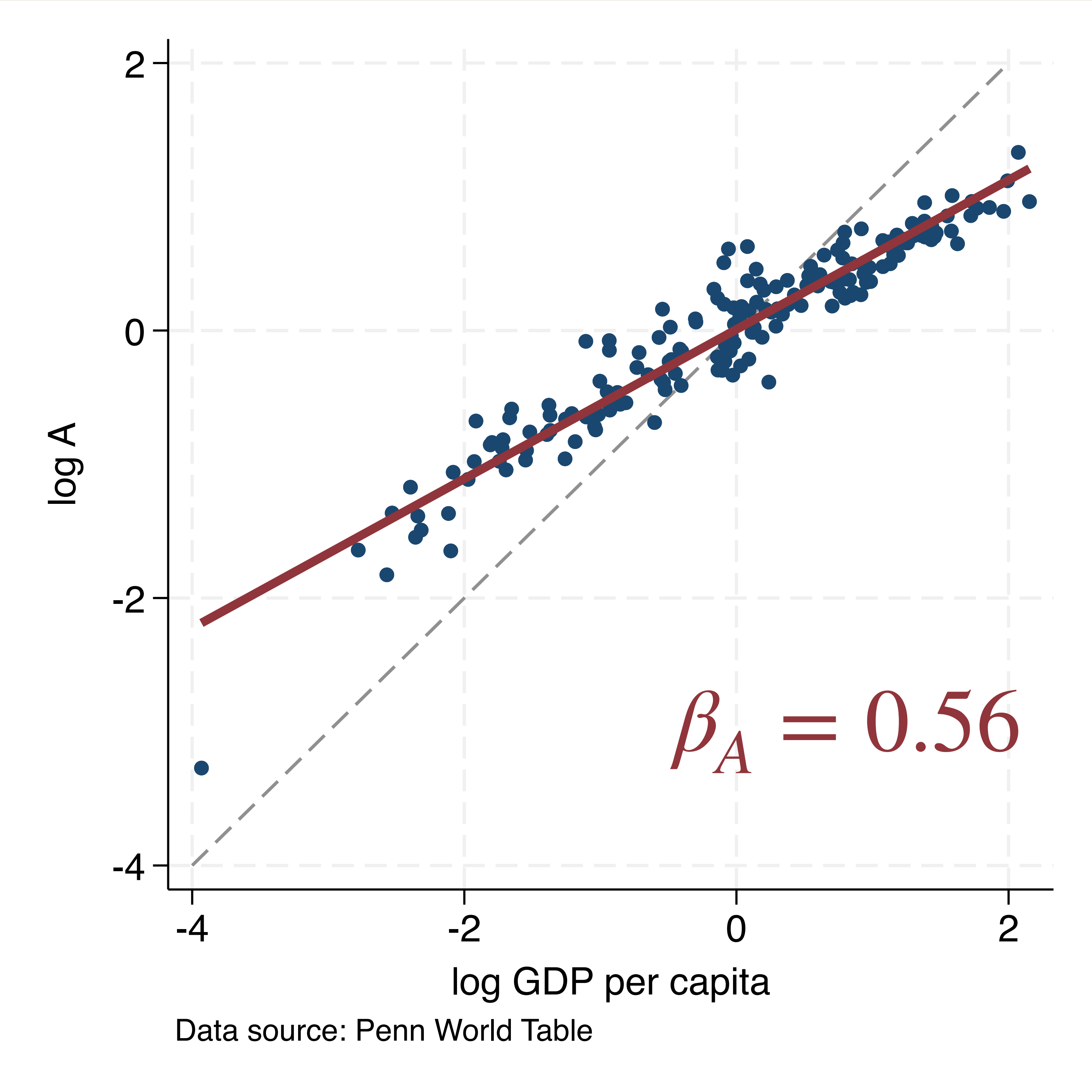
$\alpha \log(K/N)$



$(1-\alpha)\log(L/N)$



$\log A$



- Cross-country income differences due to K/N : 37%, L/N: 7%, A: 56%

What Did We Miss?

What Did We Miss?

- Non-trivial fraction of income differences due to differences in capital
  - This motivates us to build a theory that determines capital
- However, more than half of the differences due to TFP
- Disappointing because more than half attributed to something we don’t observe
  - Observable country characteristics explain less than half of income differences
- Are you convinced? What did we potentially miss?

The Plan of Attacks

Maybe we did not measure “labor inputs” correctly

1. Hours worked
2. Human capital
   - Constructive approach
   - Deductive approach using immigrants

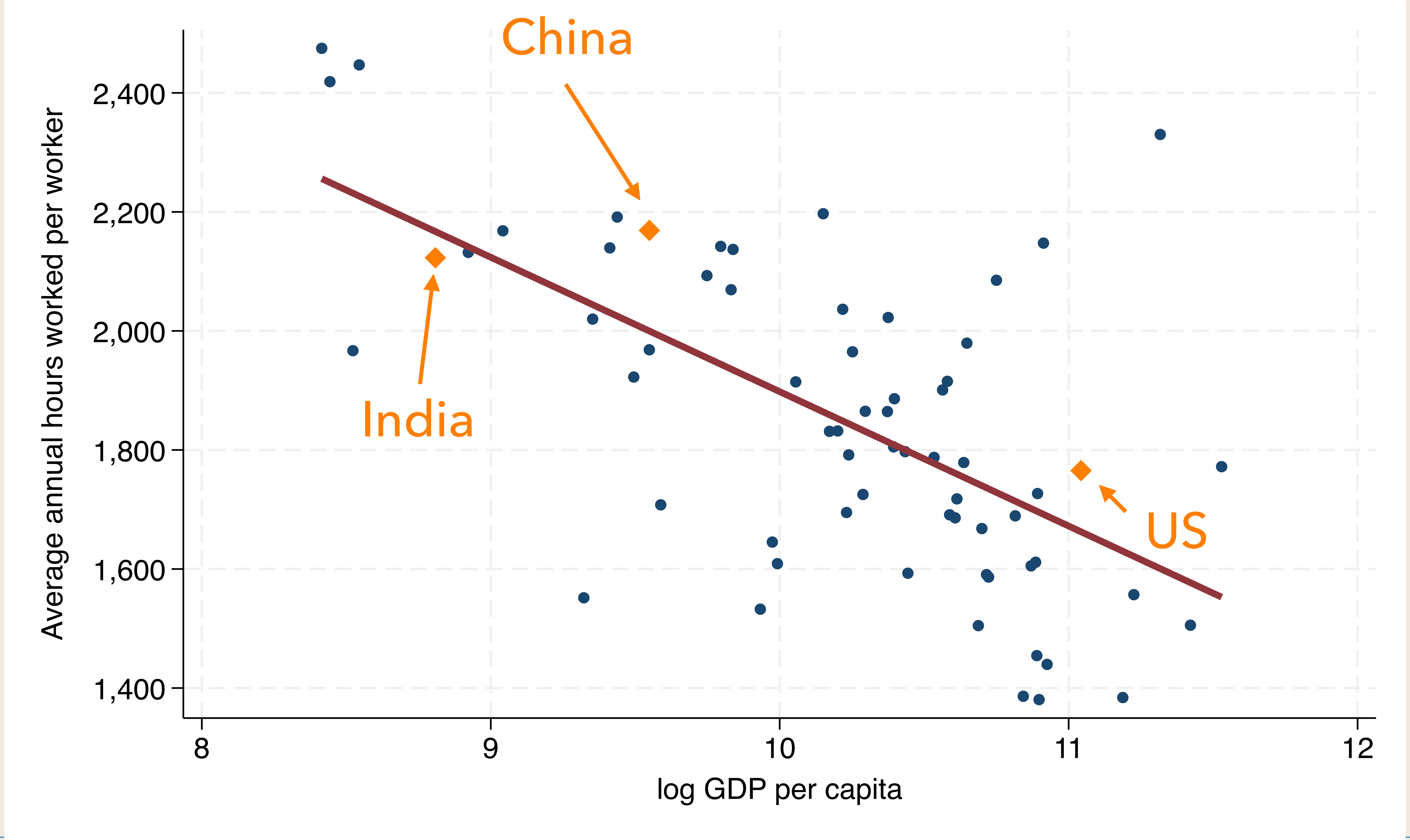
1. Hours Worked

$$Y_i = A_i K_i^{\alpha} (h_i L_i)^{1-\alpha}$$

h_i: hours worked per worker

- Before, we assumed all workers worked for the same hours in all countries
- If h_i is higher for richer countries, this may help explain income differences

Hours Worked Declines with GDP



Development Accounting with Hours Worked

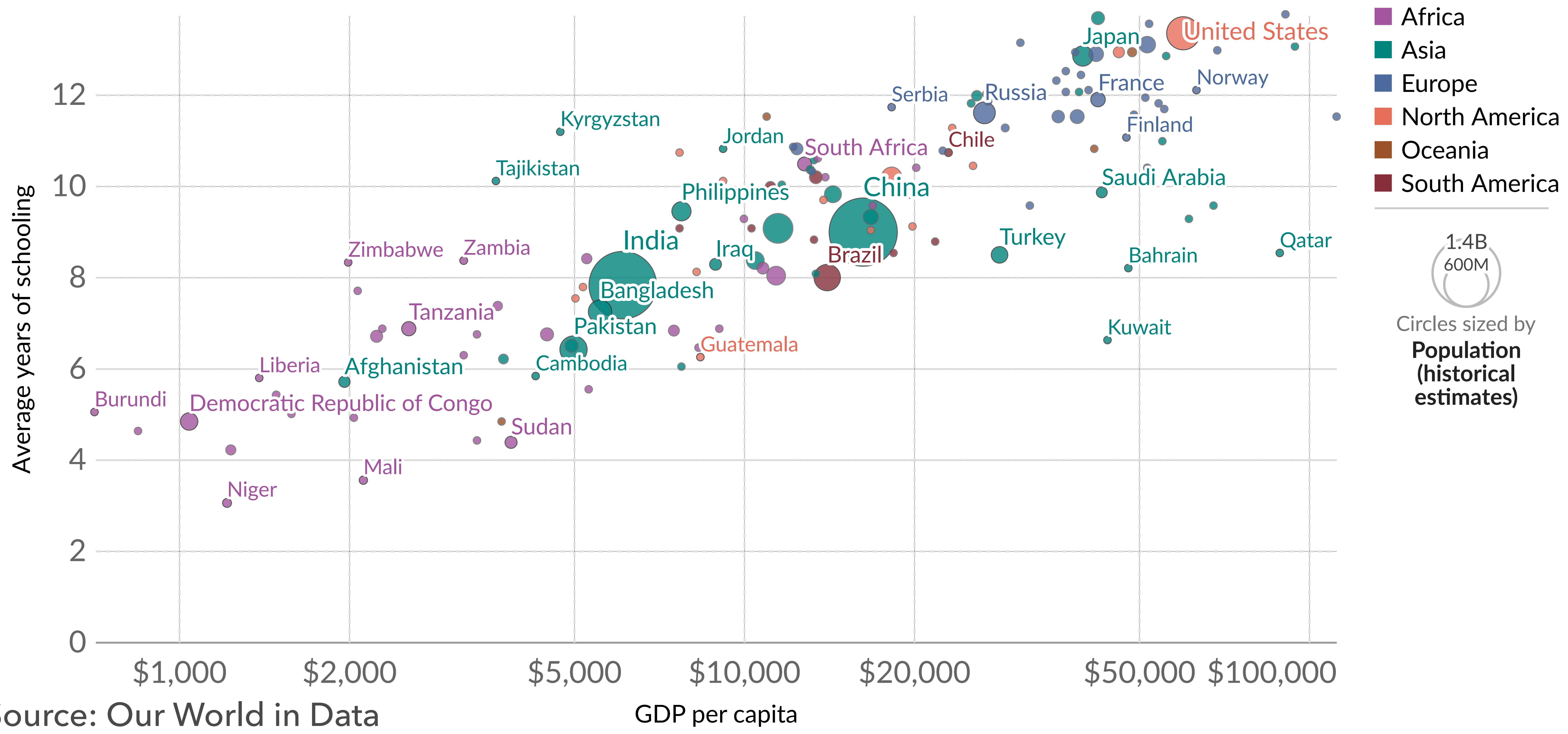
- Even more important role of A once we allow hours worked to vary

2. The Importance of Human Capital:

Constructive Approach

2. Human Capital

- We have assumed that workers in rich countries and poor countries are the same
- Is this plausible? – Perhaps not



Source: Our World in Data

How Do We Measure Human Capital?

- Now we construct the human capita index:

$$L = \sum_{s=0}^{S} \phi^s L^s$$

  - L^s: number of workers with schooling year s
  - ϕ^s: relative efficiency of workers with schooling year s

- We normalize $\phi^0 = 1$

- How do we obtain ϕ^s?

Inferring Human Capital from Wages

- Suppose workers with different schooling years are paid different wages
- The profit maximization is now

$$\max_{K,L^s} AK^\alpha \left(\sum_s \phi^s L^s \right)^{1-\alpha} - \sum_s w_i^s L^s - rK$$

- Taking the first-order condition with respect to L_i^s,

$$(1-\alpha)\phi^s AK^\alpha \left(\sum_s \phi^s L^s \right)^{-\alpha} = w^s$$

- Taking ratio,

$$\frac{\phi^s}{\phi^0} = \frac{w^s}{w^0}$$

$\Rightarrow$ relative wages informative about ϕ^s

Human Capital Index

- Many estimates of $\{w_i^s\}$ in the labor economics literature
  - How wages vary depending on education
  - Let's talk more about this in a few slides
- We assume

$$\log \phi^s = 0.1 \times s$$

- Now we plug estimates of ϕ^s and construct our human capital index:

$$L_i = \sum_{s=0}^{S} \phi^s L_i^s$$

- With new L_i, let us re-do development accounting

Differences in Human Capital

	Y/N	K/N	L/N employment	L/N human capital	A human capital
U.S.	100	100	100	100	100
China	22	33	116	83	37
India	10	12	76	44	38
Haiti	2.5	7	84	38	12

Data: Penn World Table 2019

- More differences in L/N, but not quite as much as A or K/N

Development Accounting with Human Capital

$\alpha \log(K/N)$ | $(1-\alpha)\log(L/N)$ | $\log A$

- Cross-country income differences due to K/N : 37%, L/N: 18%, A: 45%

Ongoing Debate

- Human capital explains 18% of cross-country income differences
- This reduces the contribution of our measure of ignorance to less than half
- Lots of debate on the role of human capital:
  1. Functional form: $L_i = G(\{L_i^s\}_{s=0}^S)$ rather than $L_i = \sum_{s=0}^S \phi^s L_i^s$
  2. ϕ^s could be different across countries (e.g., quality of education)
  3. Schooling is not the only source of human capital (e.g., experience)
- Some argue human capital can explain almost all cross-country differences

Detour:
How Do We Estimate Returns to Education?

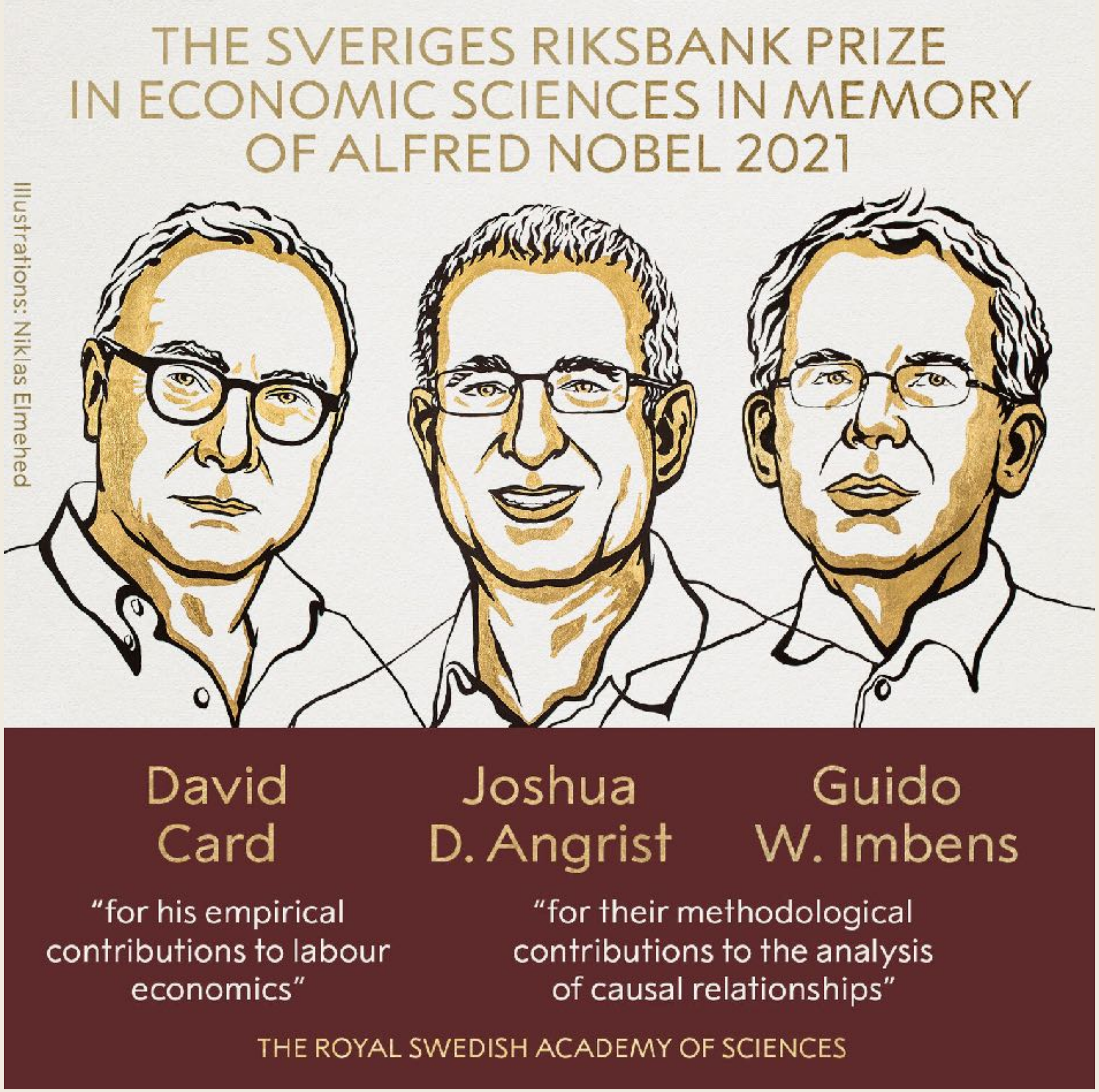
How Do Wages Change with Schooling?

- How should we obtain estimates of returns to education, ϕ^s?
- How wages vary depending on education:

Determinants of wage other than schooling

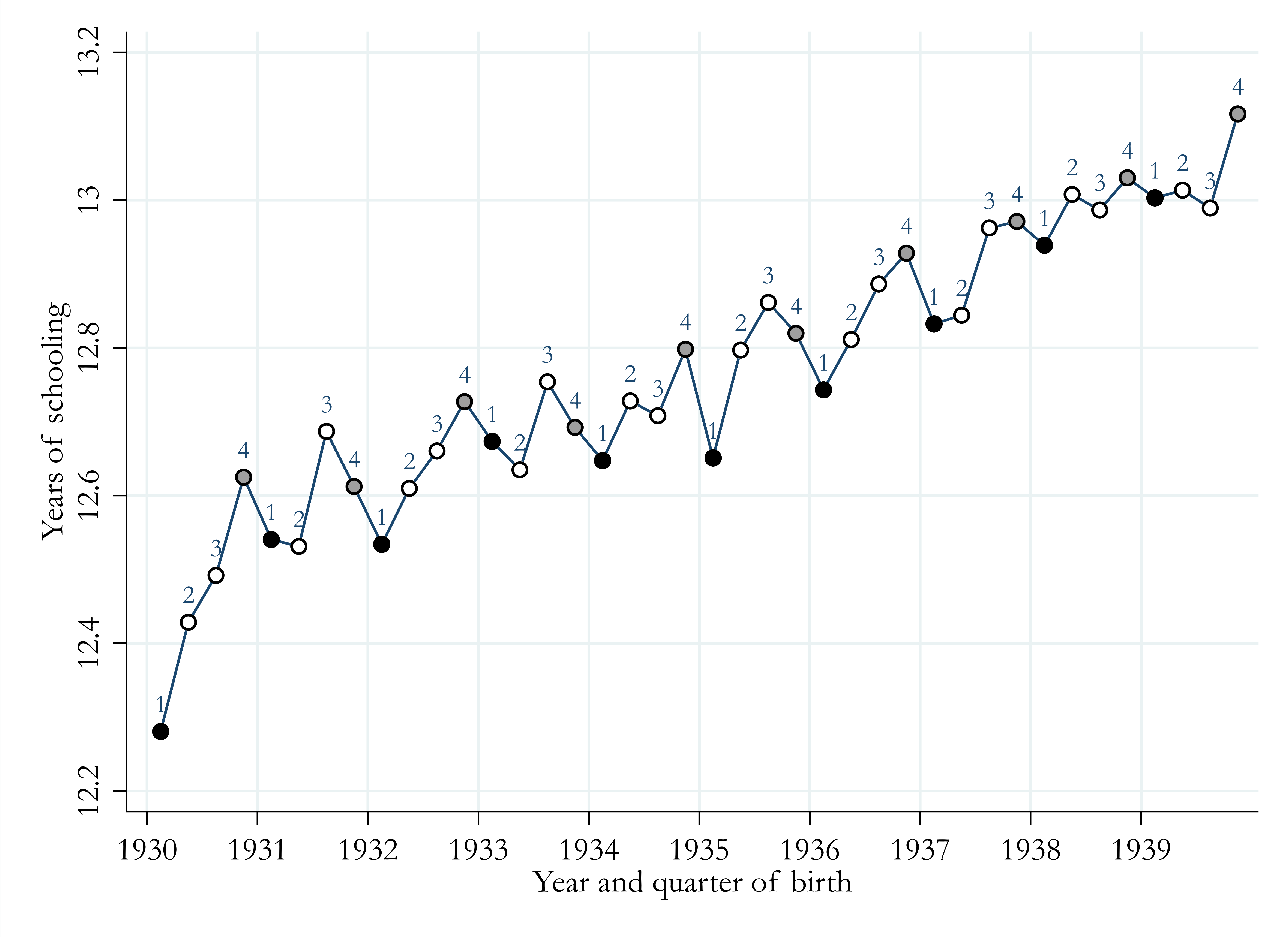
$$\log \text{wage}_i = \beta \times (\text{Years of Schooling})_i + \epsilon_i$$

- Natural to expect that $\text{Cov}(\text{Years of schooling}_i, \epsilon_i) \neq 0$
  - Maybe more motivated individuals are more likely to go to schools
- Cannot run OLS to estimate β
  - More educated people are highly paid, not necessarily because of education!

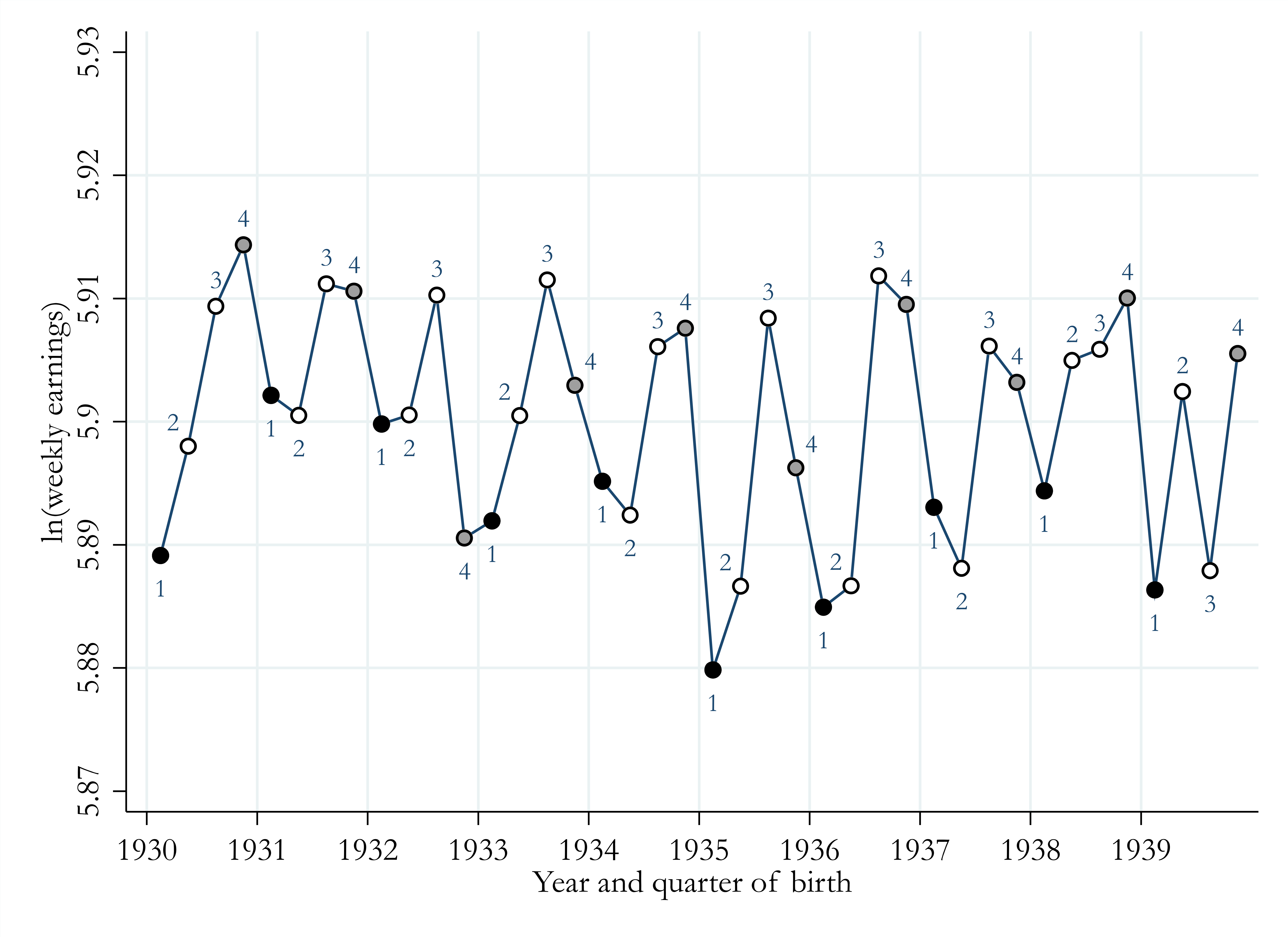

THE SVERIGES RIKSBANK PRIZE
IN ECONOMIC SCIENCES IN MEMORY
OF ALFRED NOBEL 2021
Illustrations: Niklas Elmehed
David
Card
Joshua
D. Angrist
Guido
W. Imbens
"for his empirical
contributions to labour
economics"
"for their methodological
contributions to the analysis
of causal relationships"
THE ROYAL SWEDISH ACADEMY OF SCIENCES

Schooling and Timing of Birth

- Did we corre



Earnings and Timing of Birth



What are Returns to Education?

- One additional year of schooling $\Rightarrow \approx 10\%$ increase in earnings
- Why do earnings increase?
  - Maybe classes are useless, but degrees are

Contents lists available at ScienceDirect

Journal of Public Economics

journal homepage: www.elsevier.com/locate/jpube

ELSEVIER

The effect of human capital on earnings: Evidence from a reform at Colombia's top university☆

Carolina Arteaga

Department of Economics, UCLA, United States

ARTICLE INFO

JEL classifications:
I23
I25
I26
J24
J31

Keywords:
Education
Human capital
Signaling

ABSTRACT

In this paper I test whether the return to college education is the result of human capital accumulation or instead reflects the fact that attending college signals higher ability to employers. I exploit a reform at Universidad de Los Andes, which in 2006 reduced the amount of coursework required to earn degrees in economics and business by 20% and 14%, respectively, but did not change the quality of incoming or graduating students. The size of the entering class, their average high school exit exam scores, and graduation rates were not affected by the reform, indicating that selection of students into the degrees remained the same. Using administrative data on wages and college attendance, I estimate that wages fell by approximately 16% in economics and 13% in business. These results suggest that human capital plays an important role in the determination of wages and reject a pure signaling model. Surveying employers, I find that the reduction in wages may have resulted from a decline in performance during the recruitment process, which led students to be placed in lower-quality firms. Using data from the recruitment process for economists at the Central Bank of Colombia, I find that the reform reduced the probability of Los Andes graduates' being hired by 17 percentage points.

2. The Importance of Human Capital:

Deductive Approach using Immigrants

What Do Immigrants Tell Us?

- Let us tackle the problem from a different angle (Hendricks and Schoellman, 2018)
- Focus on immigrants to the US
- How much wage gains do immigrants experience upon arrival to the US?
- Immigrants bring their human capital (L) but do not bring A or K of home country
  - Instead, they can now use technology or physical capital in the US
- If A or K very important, their wages rise one-for-one with GDP gap
- If A or K not important, their wages should not change

Formal Argument

- The wage of worker s with human capital ϕ^s working in country i:

$$w_i^s = (1-\alpha)\,\phi^s \times \underbrace{A_i K_i^\alpha \left(\bar{\phi}_i^s L_i\right)^{-\alpha}}_{\text{country specific component} \equiv Z_i}$$

- (Log-)wage change after migrating to country j:

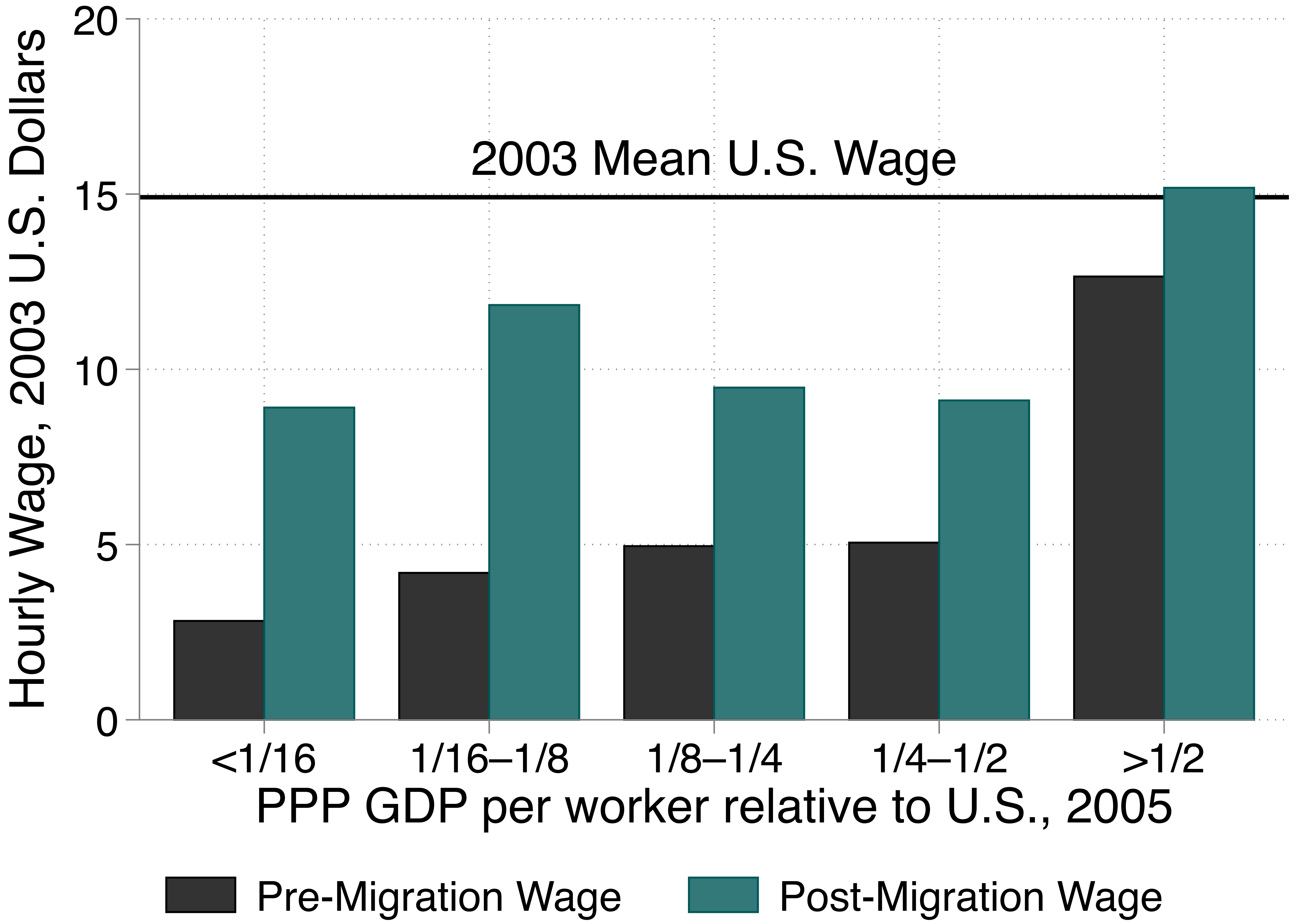
$$\log w_j^s - \log w_i^s = \log Z_j - \log Z_i$$

- (Log-)GDP gap:

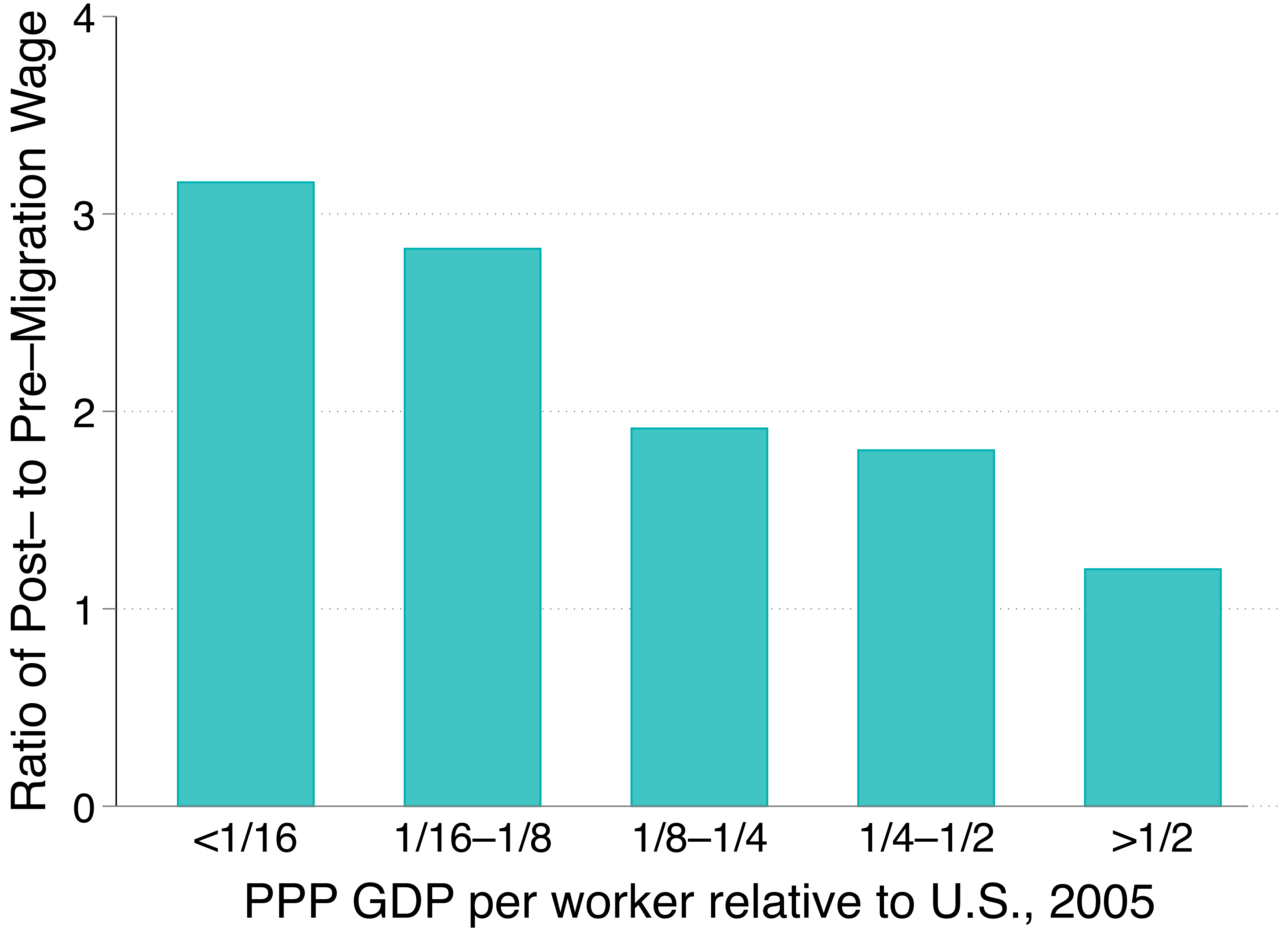
$$\log Y_j - \log Y_i = \log Z_j - \log Z_i + \log(\bar{\phi}_j^s L_j) - \log(\bar{\phi}_i^s L_i)$$

- If human capital is unimportant, $\log w_j^s - \log w_i^s \approx \log Y_j - \log Y_i$

Wage Gains from Immigration



Pre- and Post-Migration Wages



Wage Gains at Migration

Source: Hendricks and Schoellman (2018)

How Do Wage Gains Compare to GDP Gap?

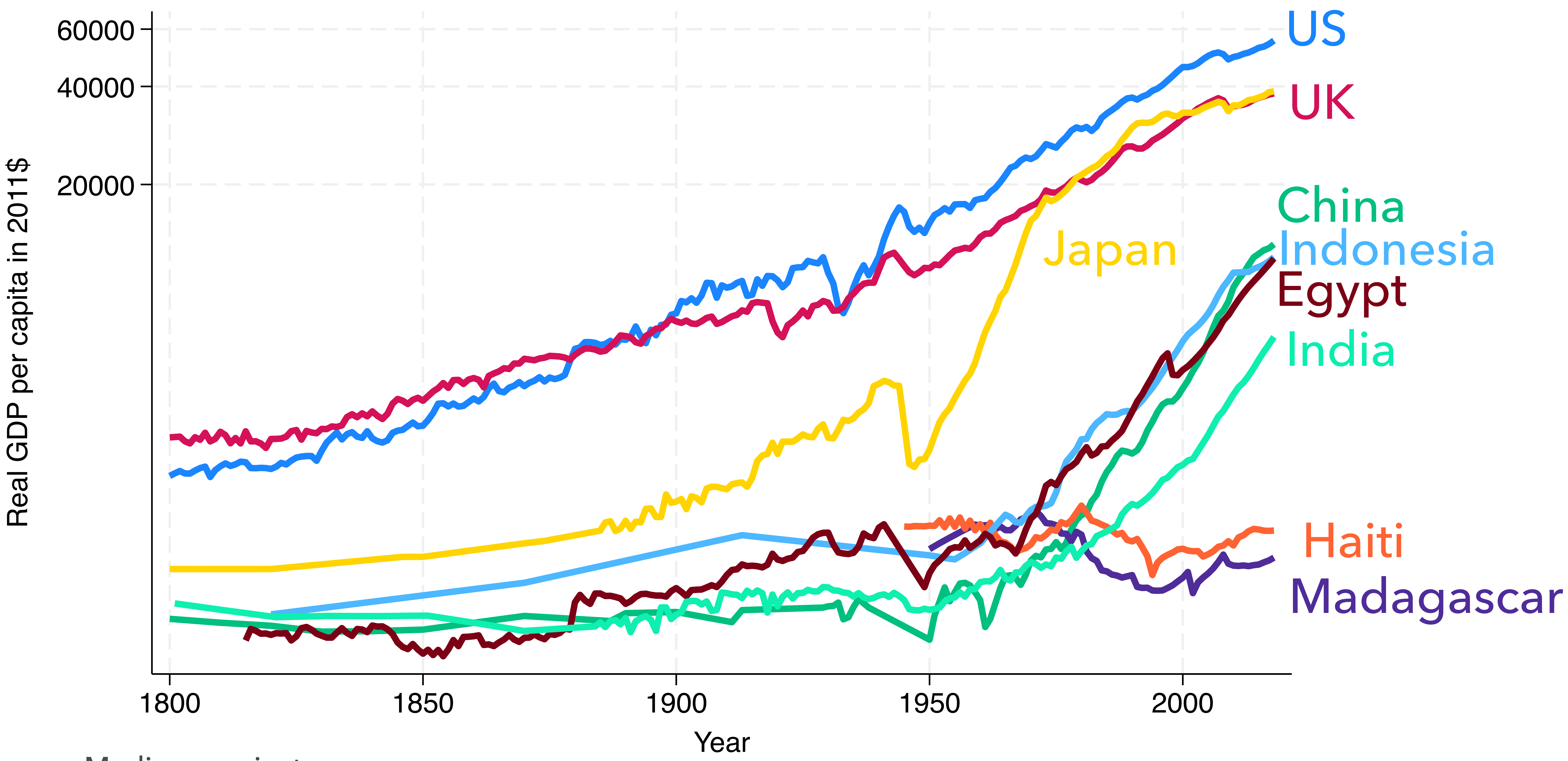
Group	Hourly Wage		Development Accounting			
	Pre-Mig.	Post-Mig.	Wage Gain	GDP Gap	h share	95% C.I.
Panel A: NIS Sample by GDP per worker category						
$< 1/16$	$2.82	$8.91	3.2	31.8	0.66	(0.60, 0.73)
$1/16 - 1/8$	$4.19	$11.83	2.8	11.9	0.58	(0.54, 0.62)
$1/8 - 1/4$	$4.95	$9.48	1.9	5.6	0.63	(0.55, 0.71)
$1/4 - 1/2$	$5.05	$9.11	1.8	3.0	0.48	(0.34, 0.62)
$1/2 - 1$	$12.64	$15.18	1.2	1.3	0.48	(-0.23, 1.19)

Source: Hendricks and Schoellman (2018)

- Wage gains are typically much smaller than GDP gap
- This implies that human capital is an important component of income differences
- Differences in TFP or physical cannot be the whole story

Growth Accounting

Why Do Countries Grow?



Source: Madison project

Growth Accounting

- Why do countries grow?
- The growth rate of the economy between t and $t+T$:

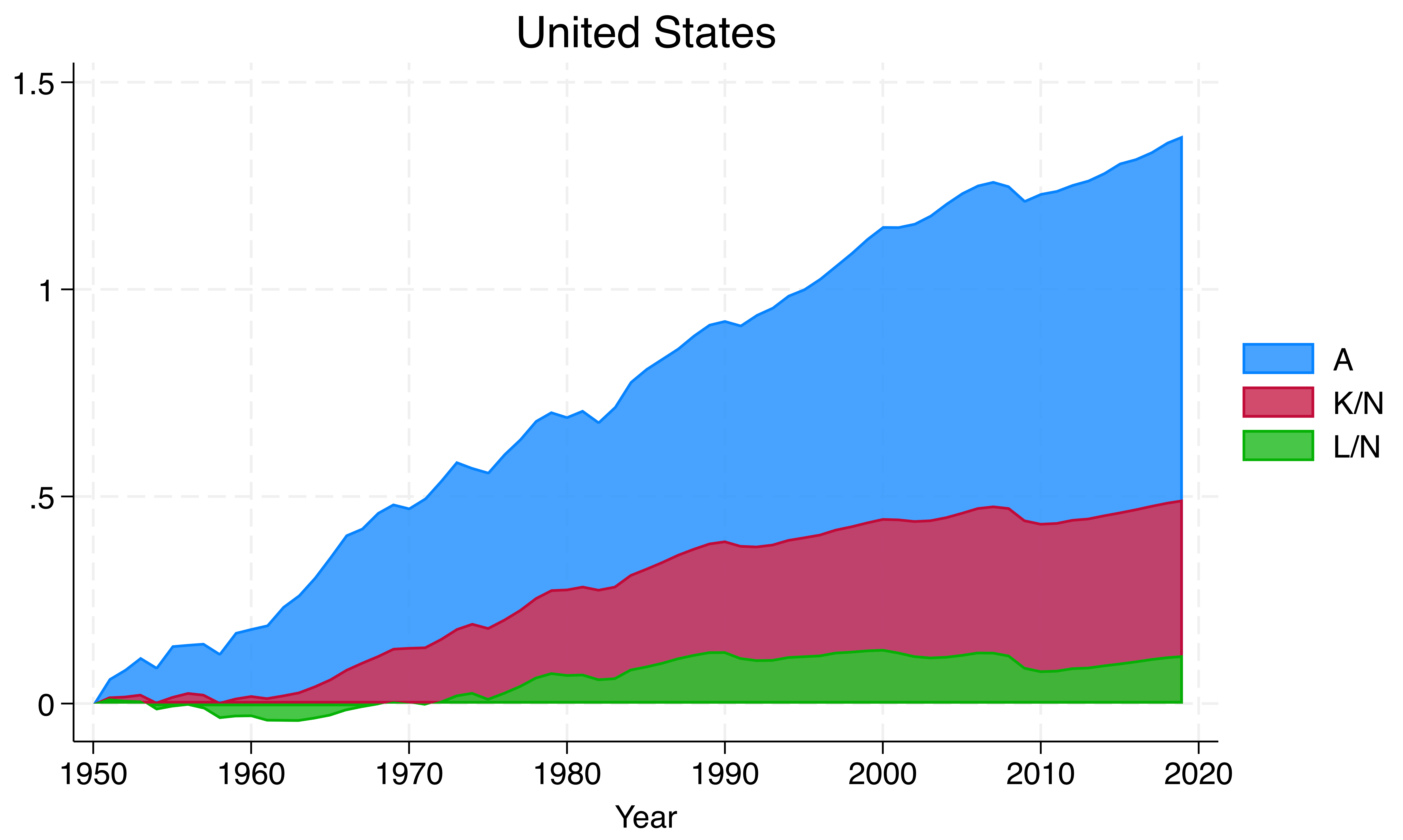
$$\Delta_T \log(Y_t/N_t) \equiv \log(Y_{t+T}/N_{t+T}) - \log(Y_t/N_t)$$

- With $Y_t = A_t K_t^{\alpha} L_t^{1-\alpha}$, we can decompose growth into:

$$\begin{aligned}\Delta_T \log(Y_t/N_t) = {} & \alpha \Delta_T \log(K_t/N_t) && \text{Growth due to } K \\ & +(1-\alpha)\Delta_T \log(L_t/N_t) && \text{Growth due to } L \\ & +\Delta_T \log(A_t) && \text{Growth due to } A\end{aligned}$$

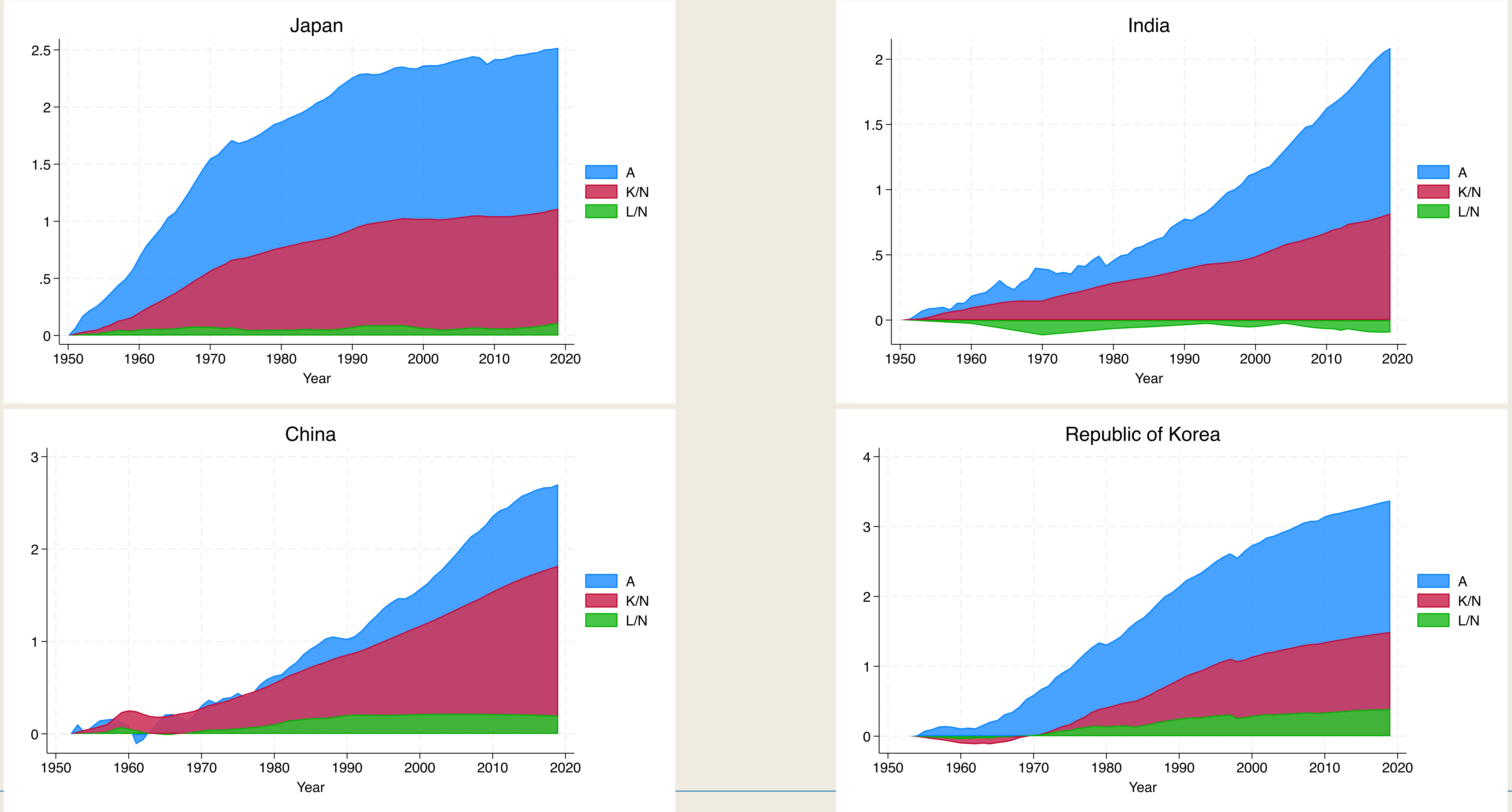
  - **Growth accounting:** decomposition over time-series
  - **Development accounting**: decomposition over cross-section

Growth Accounting: US

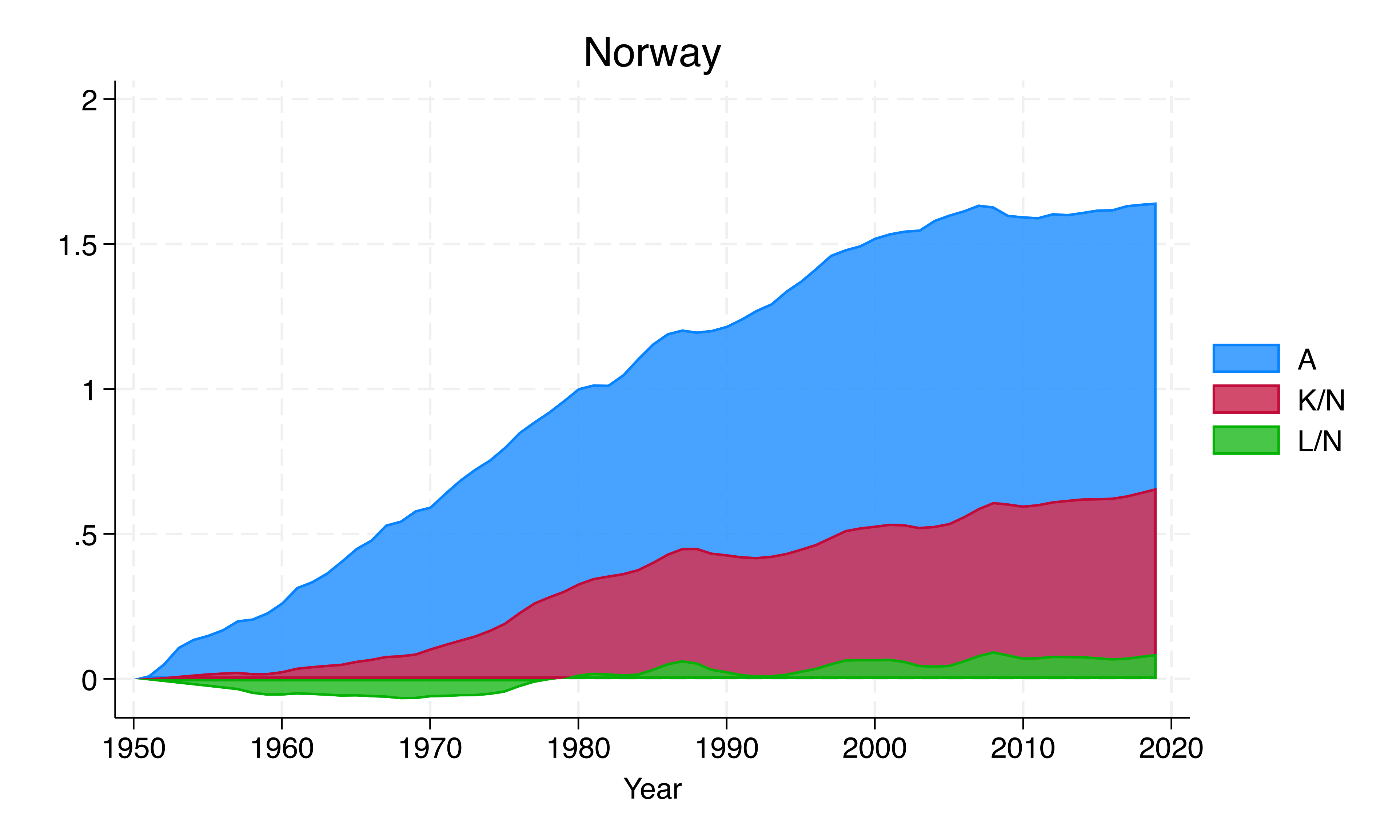


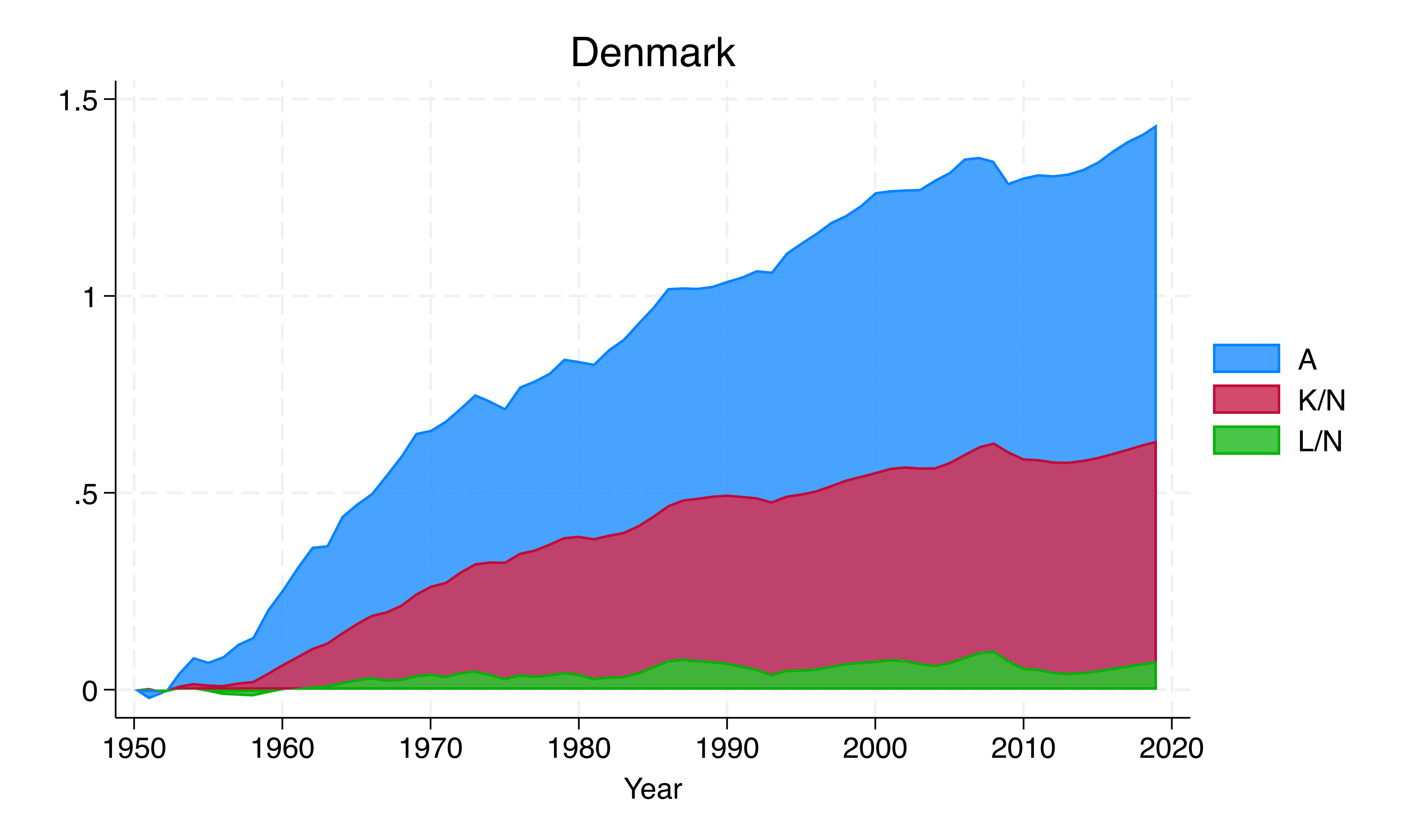
Source: PWT 10.01

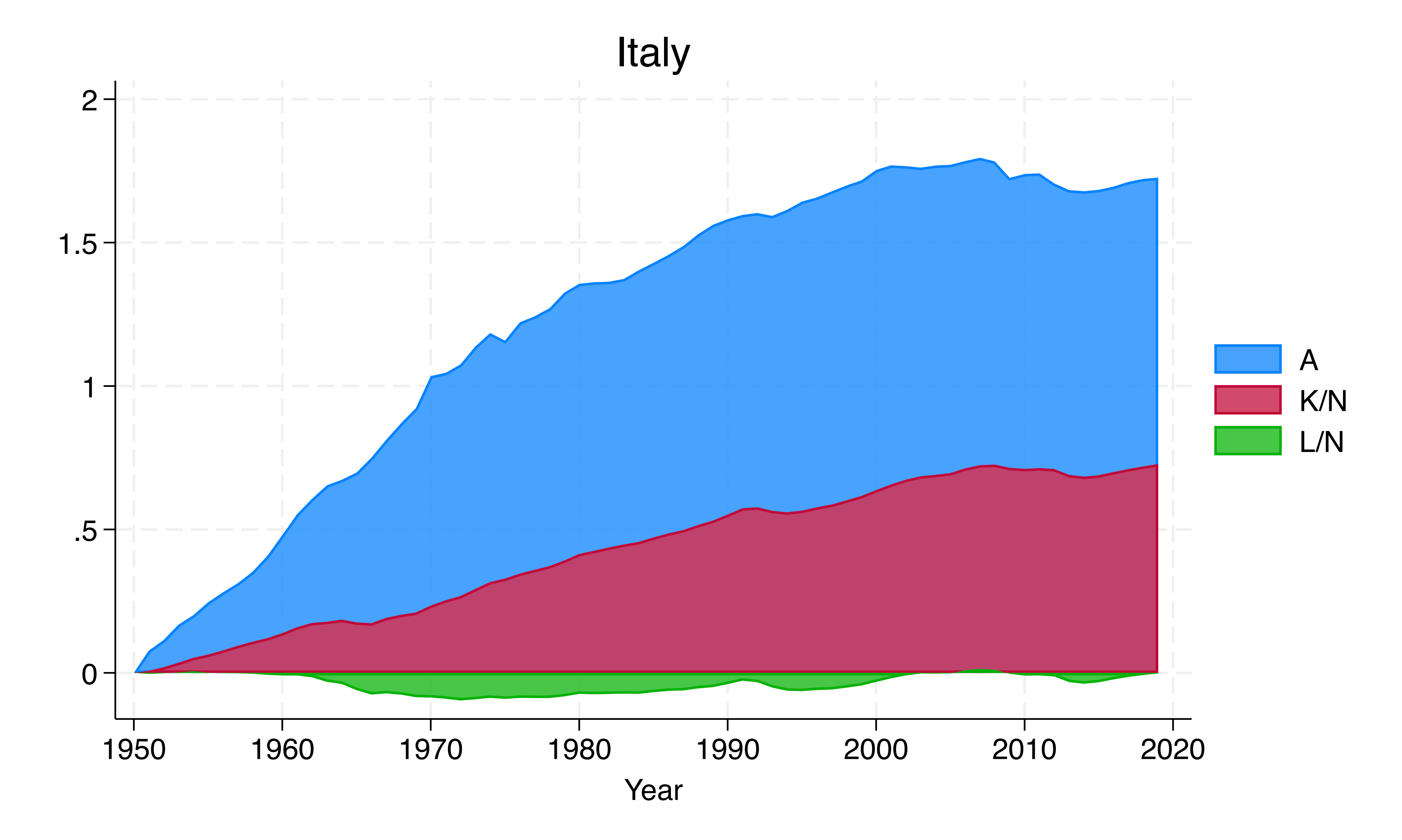
Growth Accounting: Asia

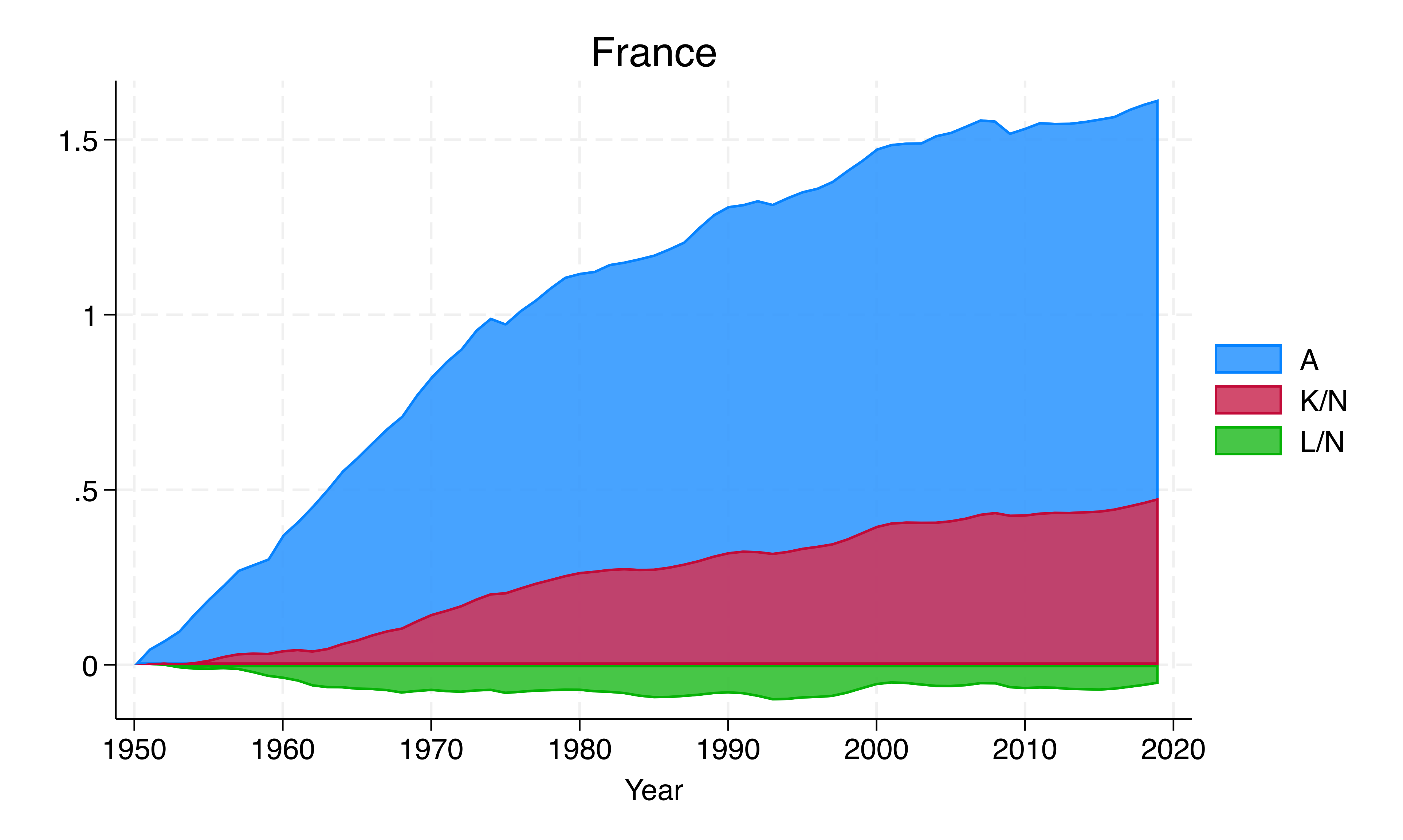


Growth Accounting: Europe

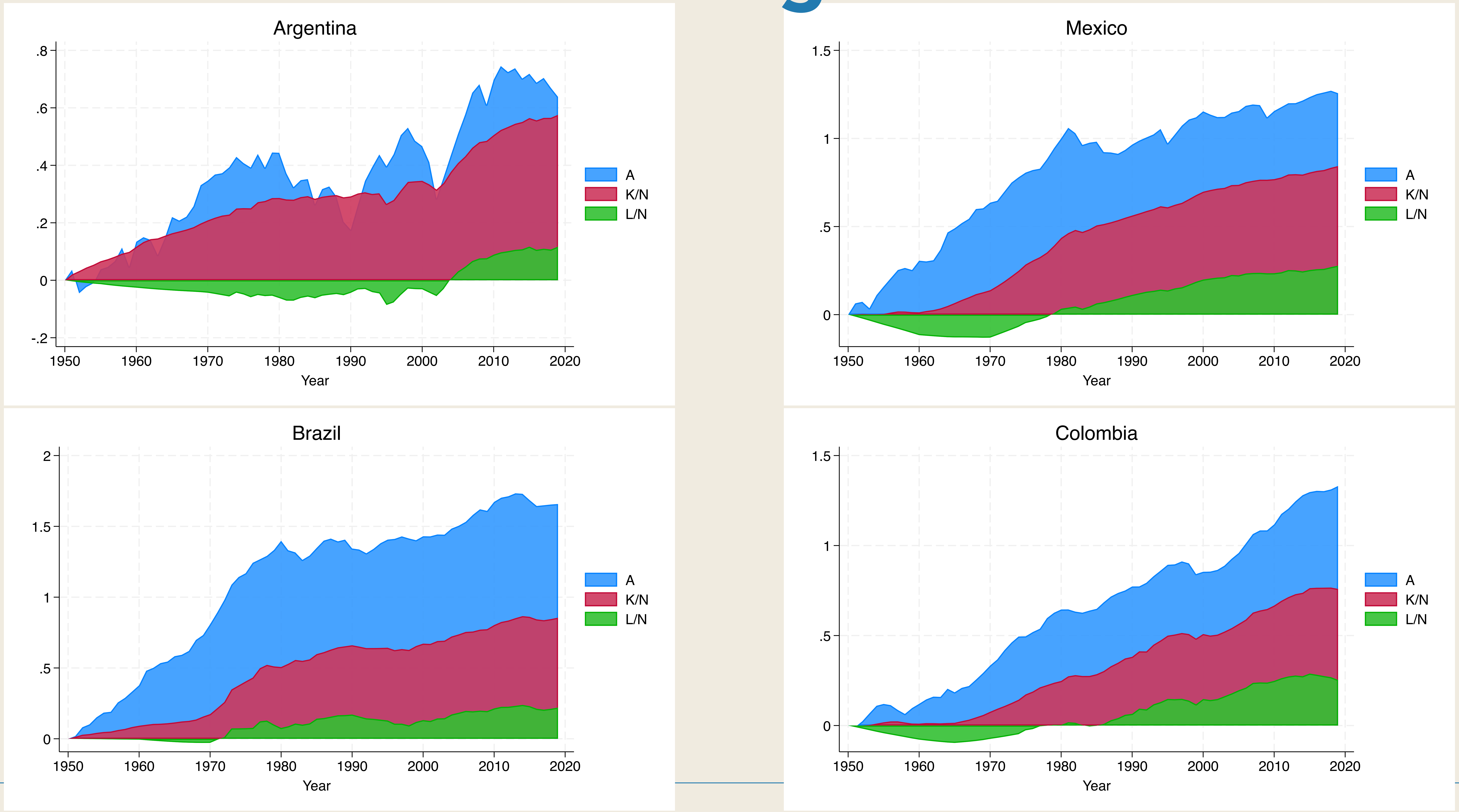








Growth Accounting: Latin America



Takeaway from Growth Accounting

- In almost all countries, the predominant driver of growth is TFP
- Capital is also important
- Labor seems to matter less

Looking Ahead

- We have learned two accounting tools
- **Development accounting**:
  Cross-sectional decomposition of difference in GDP per capita
- **Growth accounting**:
  Time-series decomposition of growth in GDP per capita
- Both exercises suggest that
  1. important role of K
  2. even more important role of A
- Next lectures develop theories that determine K and A