Investment

EC502 Macroeconomics
Topic 9

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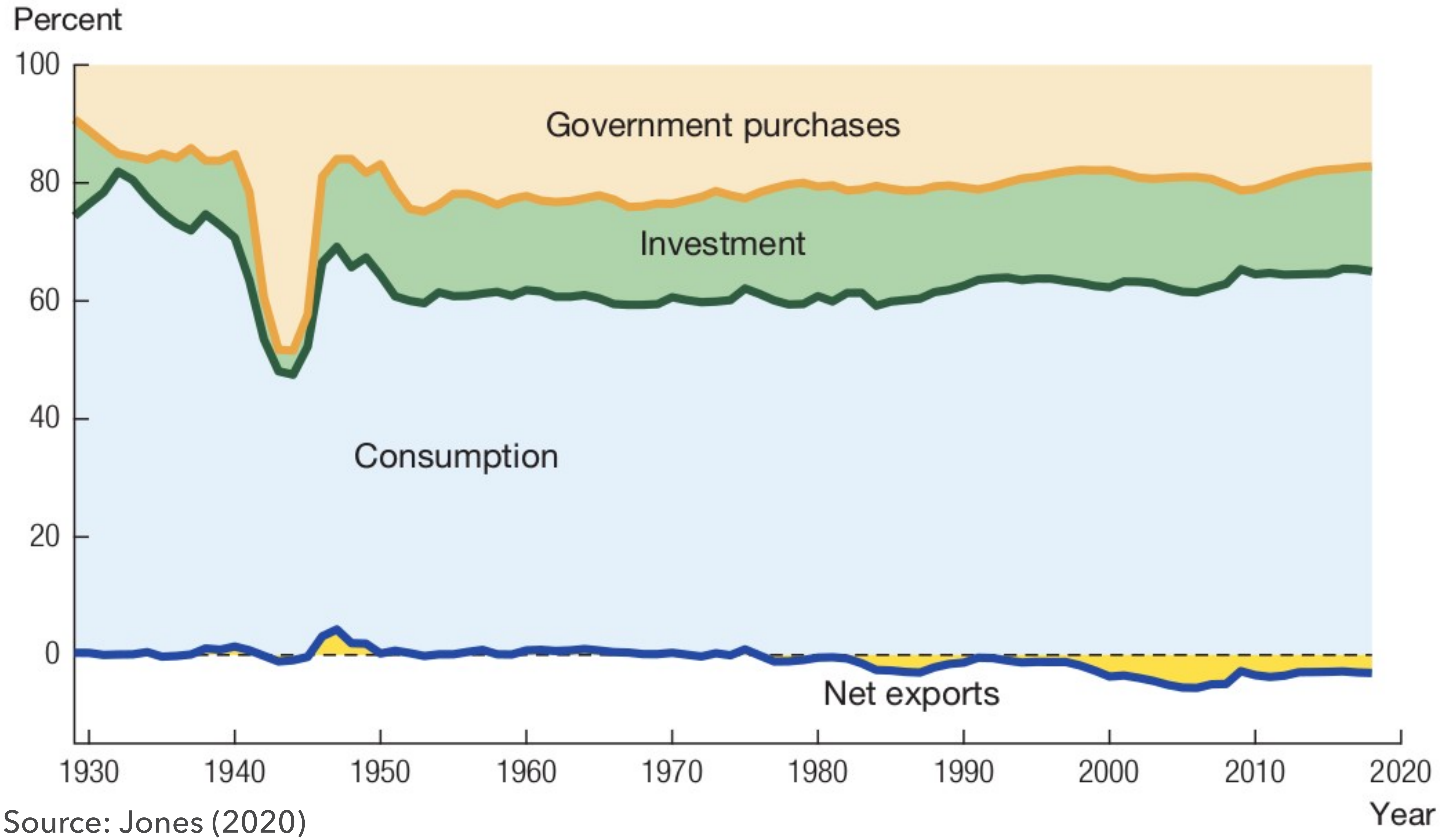
Investment in GDP



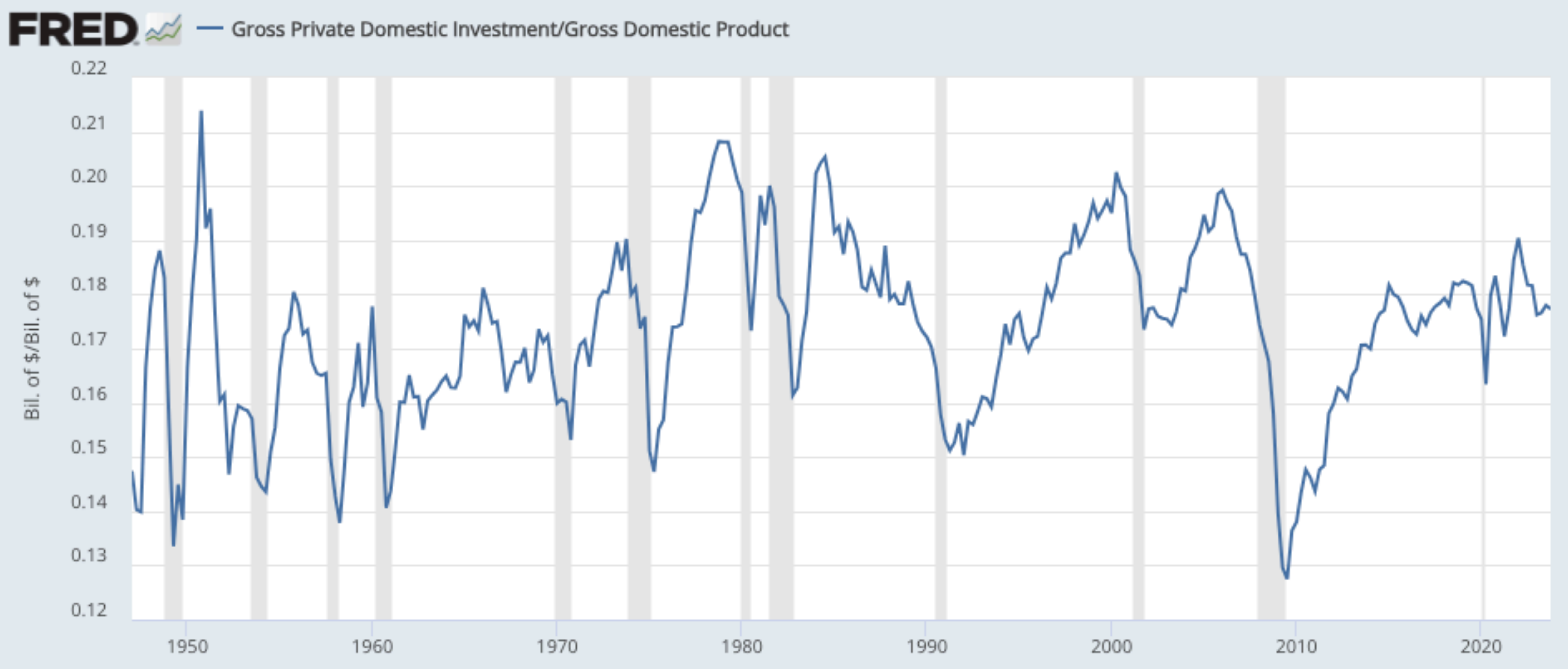
Source: Jones (2020)

Investment


FRED
Real Gross Private Domestic Investment
5,000
4,000
3,000
2,000
1,000
0
Billions of Chained 2017 Dollars
1950
1960
1970
1980
1990
2000
2010
2020

Investment Growth

Investment over GDP



Questions

- Investment constitutes $\approx 20\%$ of GDP
- Yet, it is the most volatile component of GDP
- What determines investment?
  - Recall in Solow model, this was mechanical, $I_t = sY_t$
- How can a policy stimulate investment in recessions?

Investment with Two Periods

Setup

- Consider a firm operating the following production function

$$F_t(K_t, L_t) = A_t K_t^{\alpha} L_t^{1-\alpha}$$

- Firms own capital stock K_t and invest with convex adjustment costs $\Phi(I_t, K_t)$

$$K_1 = (1-\delta)K_0 + I_0, \quad \delta : \text{depreciation rate}$$

- Firms hire labor in the competitive labor market with wage w_t
- The firm maximizes the presented discounted value of dividends

$$D_0 + \frac{1}{1+r} D_1$$

where $D_t = F_t(K_t, L_t) - w_t L_t - I_t - \Phi(I_t, K_t)$ is the profit of a firm in period t

Adjustment Costs

- We assume the following adjustment cost function

$$\Phi(I,K) = \frac{\phi}{2}\left(\frac{I}{K}\right)^2 K$$

- This function is increasing and convex in I
    - The additional investment costs more when you are already investing a lot
- This function is constant returns to scale in (I, K)
    - doubling your investment and capital also doubles the cost of investment

Firm's Problem

- Given K_0, a firm solves

$$\max_{L_0, I_1, K_1, L_1} \left[F_0(K_0, L_0) - w_0 L_0 - I_0 - \Phi(I_0, K_0)\right] + \frac{1}{1+r}\left[F_1(K_1, L_1) - w_1 L_1\right]$$

subject to

$$K_1 = K_0(1-\delta) + I_0$$

- The first-order conditions with respect to L_t:

$$\frac{\partial F_t(K_t, L_t)}{\partial L_t} = w_t \tag{1}$$

- The first-order condition with respect to I_0 is

$$1 + \frac{\partial \Phi(I_0, K_0)}{\partial I_0} = \frac{1}{1+r}\frac{\partial F_1(K_1, L_1)}{\partial K_1} \tag{2}$$

LHS: marginal cost of investment, RHS: marginal benefit of investment

Investment Solution

- With our functional forms, we can solve for labor demand using (1):

$$L_t = (1-\alpha)^{1/\alpha} A_t^{1/\alpha} w_t^{-1/\alpha} K_t \tag{3}$$

- Equation (2) is

$$1 + \phi \frac{I_0}{K_0} = \frac{1}{1+r} \alpha A_1 K_1^{\alpha - 1} L_1^{1-\alpha} \tag{4}$$

- Combining (3) and (4),

$$\frac{I_0}{K_0} = \frac{1}{\phi} \left[\frac{1}{1+r} \alpha (1-\alpha)^{\frac{1-\alpha}{\alpha}} A_1^{\frac{1}{\alpha}} w_1^{-\frac{1-\alpha}{\alpha}} - 1 \right]$$

Comparative Statics

$$\frac{I_0}{K_0} = \frac{1}{\phi}\left[\frac{1}{1+r}\alpha(1-\alpha)^{\frac{1-\alpha}{\alpha}}A_1^{\frac{1}{\alpha}}w_1^{-\frac{1-\alpha}{\alpha}} - 1\right] \tag{5}$$

Investment is higher when

- interest rate, r, is lower
- future productivity, A_1, is higher
- future wage, w_1, is lower

All should be intuitive

Value of Firms

- Let us rewrite firm's investment in a different way
- Define the value of firms (discounted future profits):

$$V_1 = \frac{1}{1+r} D_1$$

  - In principle, V_1 should correspond to the stock price of the firm

- Using the definition, $D_1 = F_1(K_1, L_1) - w_1 L_1$, and labor demand (3),

$$V_1 = \frac{1}{1+r} \alpha A_1^{\frac{1}{\alpha}} (1-\alpha)^{\frac{1-\alpha}{\alpha}} w_1^{-\frac{1-\alpha}{\alpha}} K_1$$

- Define $q_1 \equiv V_1 / K_1$, which we call "q"

Q-Theory of Investment

- With the definition of "q", we can rewrite investment equation (5) as

$$\frac{I_0}{K_0} = \frac{1}{\phi}\left[q_1 - 1\right]$$

- We often refer to the above expression as "q-theory of investment"
- Investment is positive if $q_1 > 1$, and negative if $q_1 < 1$
  - Invest if the average value of capital is higher than its cost for the first investment
  - Disinvest if the average value of capital is lower than its cost
- Importantly, q_1…
  - is easily measurable: just divide the stock value by the capital stock!
  - summarizes the impact of r, w_1, A_1 ("sufficient statistics")

Investment with Many Periods

Investment Problem with Many Periods

- We generalize the previous model to many periods, $t = 0, \dots, T$
- The firm solves

$$\max_{\{I_t, K_{t+1}, D_t, L_t\}} \sum_{t=0}^{T} \frac{1}{\prod_{s=0}^{t-1}(1 + r_s)} D_t$$

subject to

$$D_t = F_t(K_t, L_t) - w_t L_t - I_t - \Phi(I_t, K_t)$$

$$K_{t+1} = (1 - \delta)K_t + I_t$$

Lagrangian

- The Lagrangian is

$$\mathscr{L} = \sum_{t=0}^{T} \frac{1}{\prod_{s=0}^{t-1}(1+r_s)} \left\{ \left[F_t(K_t, L_t) - w_t L_t - I_t - \Phi(I_t, K_t)\right] + q_{t+1}\left[(1-\delta)K_t + I_t - K_{t+1}\right] \right\}$$

- First-order conditions with respect to L_t, I_t, K_t are

$$\frac{\partial F_t(K_t, L_t)}{\partial L_t} = w_t$$

$$1 + \frac{\partial \Phi(I_t, K_t)}{\partial I_t} = q_{t+1}$$

$$q_t = \frac{1}{1+r_{t-1}} \left[\frac{\partial F_t(K_t, L_t)}{\partial K_t} - \frac{\partial \Phi(I_t, K_t)}{\partial K_t} + (1-\delta) q_{t+1} \right]$$

Optimality Conditions

- With our functional form assumptions, the first two conditions can be written as

$$L_t = (1-\alpha)^{1/\alpha} A_t^{1/\alpha} w_t^{-1/\alpha} K_t \qquad (7)$$

$$\frac{I_t}{K_t} = \frac{1}{\phi}\left[q_t - 1\right]$$

- Using the above two, the thrid condition is

$$q_t = \frac{1}{1+r_t}\left[\alpha A_{t+1} K_{t+1}^{\alpha-1} L_{t+1}^{1-\alpha} + \frac{\phi}{2}\left(\frac{I_{t+1}}{K_{t+1}}\right)^2 + (1-\delta) q_{t+1}\right]$$

$$= \frac{1}{1+r_t}\left[\alpha (A_{t+1})^{\frac{1}{\alpha}} (1-\alpha)^{\frac{1-\alpha}{\alpha}} w_{t+1}^{-\frac{1-\alpha}{\alpha}} - \frac{I_{t+1}}{K_{t+1}} - \frac{\phi}{2}\left(\frac{I_{t+1}}{K_{t+1}}\right)^2 + \left(\frac{I_{t+1}}{K_{t+1}} + (1-\delta)\right) q_{t+1}\right] \qquad (8)$$

Firm's Value

- Define the firm's value as the cumulative discounted sum of future profits

$$
\begin{aligned}
V_t &= \sum_{k=t+1}^{T} \frac{1}{\prod_{s=t}^{t}(1+r_s)} D_k \\
&= \frac{1}{1+r_t}\left[D_{t+1} + \sum_{k=t+2}^{T} \frac{1}{\prod_{s=t+1}^{t}(1+r_s)} D_k\right] \\
&= \frac{1}{1+r_t}\left[F_{t+1}(K_{t+1}, L_{t+1}) - w_{t+1}L_{t+1} - I_{t+1} - \Phi(I_{t+1}, K_{t+1}) + V_{t+1}\right]
\end{aligned}
$$

Firm's Value per unit Capital = Q

- The firm's value per unit capital is, using (6),

$$\frac{V_t}{K_t} = \frac{1}{1+r_t}\left[\alpha(A_{t+1})^{\frac{1}{\alpha}}(1-\alpha)^{\frac{1-\alpha}{\alpha}}w_{t+1}^{-\frac{1-\alpha}{\alpha}} - \frac{I_{t+1}}{K_{t+1}} - \frac{\phi}{2}\left(\frac{I_{t+1}}{K_{t+1}}\right)^2 + \frac{K_{t+1}}{K_t}\frac{V_{t+1}}{K_{t+1}}\right]$$

$$= \frac{1}{1+r_t}\left[\alpha(A_{t+1})^{\frac{1}{\alpha}}(1-\alpha)^{\frac{1-\alpha}{\alpha}}w_{t+1}^{-\frac{1-\alpha}{\alpha}} - \frac{I_{t+1}}{K_{t+1}} - \frac{\phi}{2}\left(\frac{I_{t+1}}{K_{t+1}}\right)^2 + \left(\frac{I_{t+1}}{K_{t+1}} + (1-\delta)\right)\frac{V_{t+1}}{K_{t+1}}\right] \quad (9)$$

- Comparing (8) and (9), we conclude

$$q_t = \frac{V_t}{K_t}$$

Q-Theory of Investment

- Q-theory of investment:

$$\frac{I_t}{K_t} = \frac{1}{\phi}\left[q_t - 1\right]$$

- Investment is positive if and only if $q_t > 1$
  - The average value of capital, "q", is higher than its cost
- "q" is of course a function of parameters:

$$q_t = \frac{1}{1+r_t}\left[\alpha(A_{t+1})^{\frac{1}{\alpha}}(1-\alpha)^{\frac{1-\alpha}{\alpha}}w_{t+1}^{-\frac{1-\alpha}{\alpha}} - \frac{I_{t+1}}{K_{t+1}} - \frac{\phi}{2}\left(\frac{I_{t+1}}{K_{t+1}}\right)^2 + \left(\frac{I_{t+1}}{K_{t+1}} + (1-\delta)\right)q_{t+1}\right]$$

  - q_t is higher if A_t is higher, r_t is lower, and w_t is lower

Corporate Tax and Investment

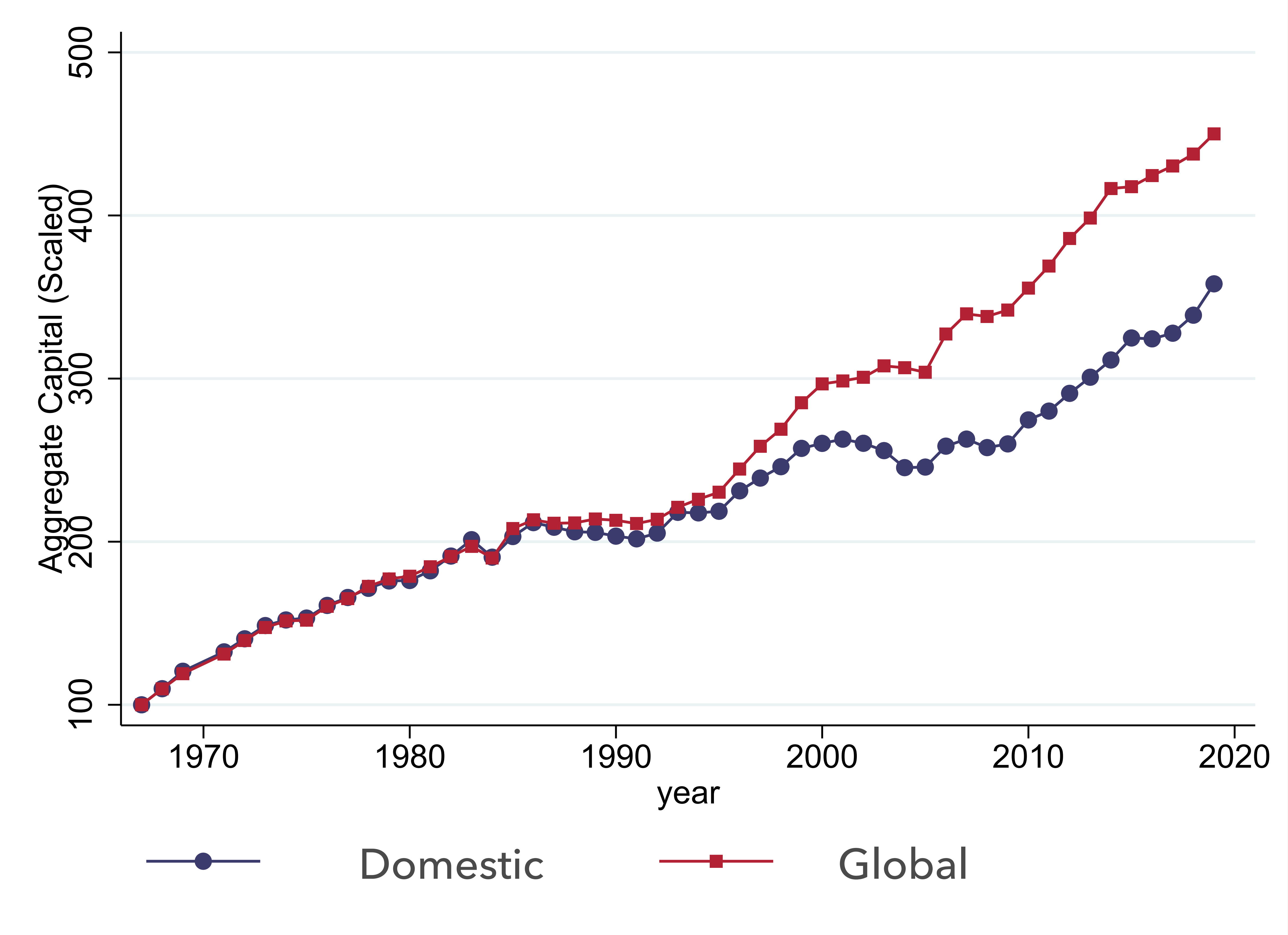
– Chodorow-Reich, Smith, Zidar, and Mahon (2025)

TCJA of 2017

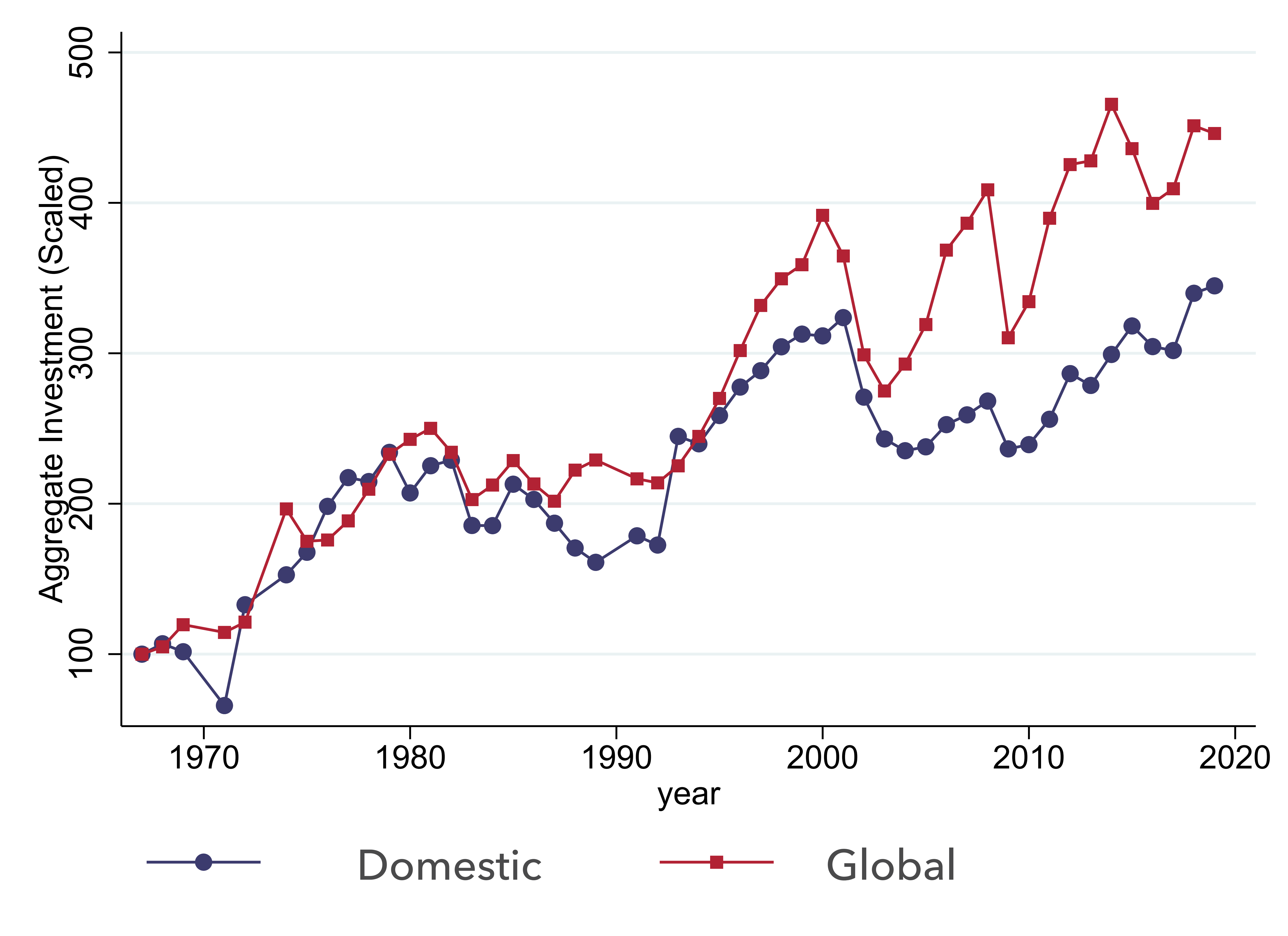
- In 2017, President Trump signed the Tax Cuts and Jobs Act (TCJA)
- It was the largest corporate tax cut in the history of the US
  - lowered the top statutory corporate tax rate from 35% to 21%
- The main goal was to increase investment by aligning with international levels
  - The US corporate tax rates had remained stable for 30 years
  - In contrast, corporate tax rates fell in many other countries
- How did TCJA affect U.S. firms' investment?

Foreign Investment Outpaces Domestic Investment

Global vs. U.S. Capital, 1967–2019



Global vs. U.S. Investment, 1967–2019



TCJA Package

Table 1: Main Provisions of the TCJA Affecting Investment

Provision	Pre-TCJA	Post-TCJA	Cost ($)
Domestic Provisions			
1. Top corporate rate	35%	21%	−1.35T
2. Accelerated depreciation	50% bonus	Full expensing for 5 years, then phase-out	−86B
3. Domestic Production Activities Deduction (DPAD)	9% of qualified production activity income	None	+98B
4. Alternative Minimum Tax	Applicable if mean revenues >$7.5M	None	−40B
5. Foreign-Derived Intangible Income (FDII)	None	37.5% deduction on export share of deemed intangible income	−64B
6. Net operating losses	2 year carryback + carryforward	No carryback and limited to 80% of income	+201B
Foreign Provisions			
1. Foreign subsidiary income	Taxable when repatriated	Not taxed	−224B
2. Global Intangible Low Tax Income (GILTI)	None	Minimum tax of 10.5% on foreign deemed intangible income	+112B
Total			−1.35T

Corporate Tax System

- Consider a firm buying $1 million worth of computers
- The firm owes corporate taxes on income net of business expenses
- Expenses on nondurable items (e.g., wages):
  the firm can immediately deduct the full cost of these items on its tax return
- Expenses on investment:
  the firm split deduction over multiple years (exact schedule differs by investment)

Example (corporate tax rate = 35%)

Year:	0	1	2	3	4	5	Total
Normal depreciation							
Deductions (000s)	200	320	192	115	115	58	1,000
Tax benefit ($\tau = 35$ percent)	70	112	67.2	40.3	40.3	20.2	350

Source: Zwick and Mohn (2017)

Modeling Taxes

- Back to the two-period model
- Let τ be the corporate tax rate
- Let Γ be the presented discounted value of deductions per unit investment
- For example, the firm's profit at period 0 is

$$(1-\tau)\left[F_0(K_0, L_0) - w_0 L_0\right] - (1-\Gamma)\left[I_0 + \Phi(I_0, K_0)\right]$$

Investment Problem with Taxes

$$\max_{L_0, I_1, K_1, L_1} \left[(1-\tau)\Big(F_0(K_0, L_0) - w_0 L_0\Big) - (1-\Gamma)\Big(I_0 + \Phi(I_0, K_0)\Big)\right]$$

$$+\frac{1}{1+r}\left[(1-\tau)\Big(F_1(K_1, L_1) - w_1 L_1\Big)\right]$$

subject to

$$K_1 = K_0(1-\delta) + I_0$$

Optimality Conditions

- The first-order conditions are

$$L_t = (1-\alpha)^{1/\alpha} A_t^{1/\alpha} w_t^{-1/\alpha} K_t \tag{10}$$

$$(1-\Gamma)\left(1+\phi\frac{I_0}{K_0}\right) = \frac{1-\tau}{1+r}\alpha A_1 K_1^{\alpha-1} L_1^{1-\alpha} \tag{11}$$

- Plugging (10) into (11),

$$\frac{I_0}{K_0} = \frac{1}{\phi}\left[\frac{1}{1+r}\frac{1-\tau}{1-\Gamma}\alpha(1-\alpha)^{\frac{1-\alpha}{\alpha}} A_1^{\frac{1}{\alpha}} w_1^{-\frac{1-\alpha}{\alpha}} - 1\right]$$

Impact of Taxes in the Model

$$I_0 = \frac{1}{\phi}\left[\frac{1}{1+r}\frac{1-\tau}{1-\Gamma}\alpha(1-\alpha)^{\frac{1-\alpha}{\alpha}}A_1^{\frac{1}{\alpha}}w_1^{-\frac{1-\alpha}{\alpha}} - 1\right]K_0$$

- How does a change in corporate tax affect investment?

$$d\log I_0 = -\left[1 + \frac{1}{I_0/K_0}\frac{1}{\phi}\right]\times\hat{\tau}, \quad \hat{\tau} \equiv \frac{d\tau}{1-\tau}$$

- How does a change in deduction affect investment?

$$d\log I_0 = \left[1 + \frac{1}{I_0/K_0}\frac{1}{\phi}\right]\times\hat{\Gamma}, \quad \hat{\Gamma} \equiv \frac{d\Gamma}{1-\Gamma}$$

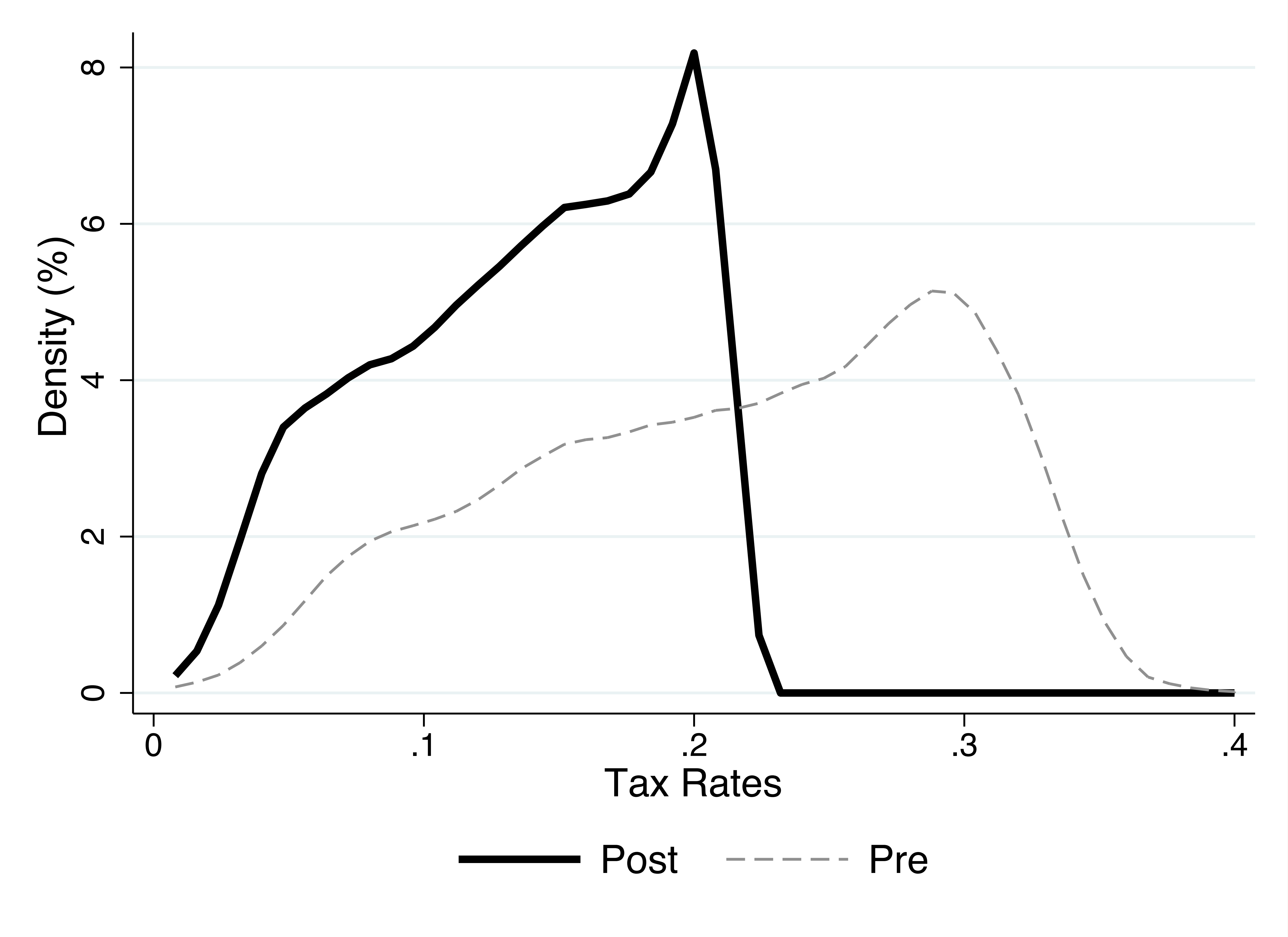
- The model predicts that the investment should respond symmetrically to τ & Γ

Impact of Taxes in the Data?

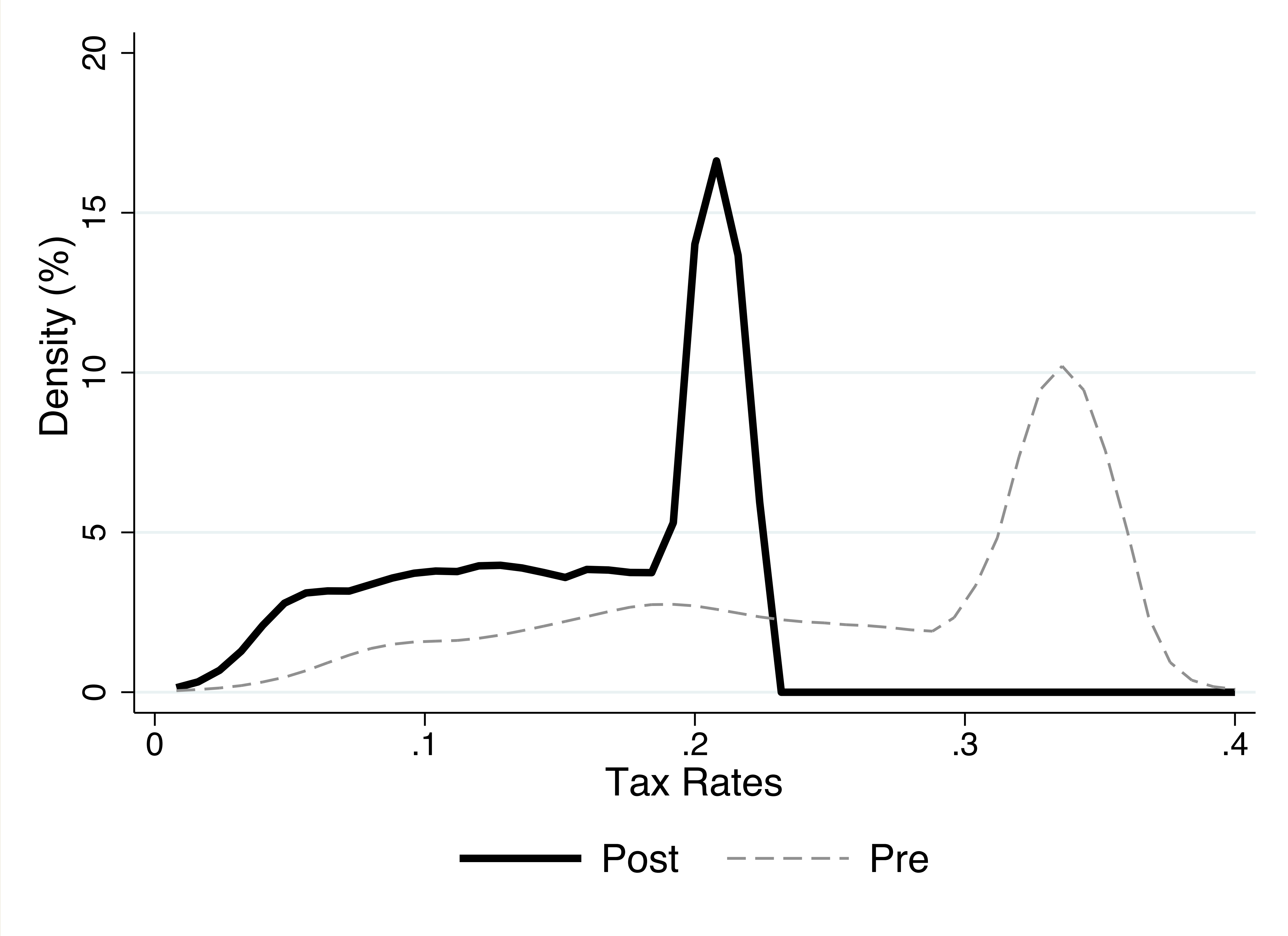
- Data:
  - Corporate tax returns data 2011-2019
  - Focus on mid- and large-size firms $\approx$ 10,000 firms
- Empirical Strategy:
  - Compare firms experiencing different changes in (τ, Γ)
  - Why do firms differ in changes in Γ?
    - The investment types differ in depreciation allowance
    - Firms differed in investment types $\Rightarrow$ different changes in Γ
  - Why do firms differ in changes in τ?
    - Corporate tax differs depending on income, credits, and deductions
    - Firm-level simulation to compute implied changes in τ

Density of Tax Changes

A: Pre- and Post-TCJA Γ



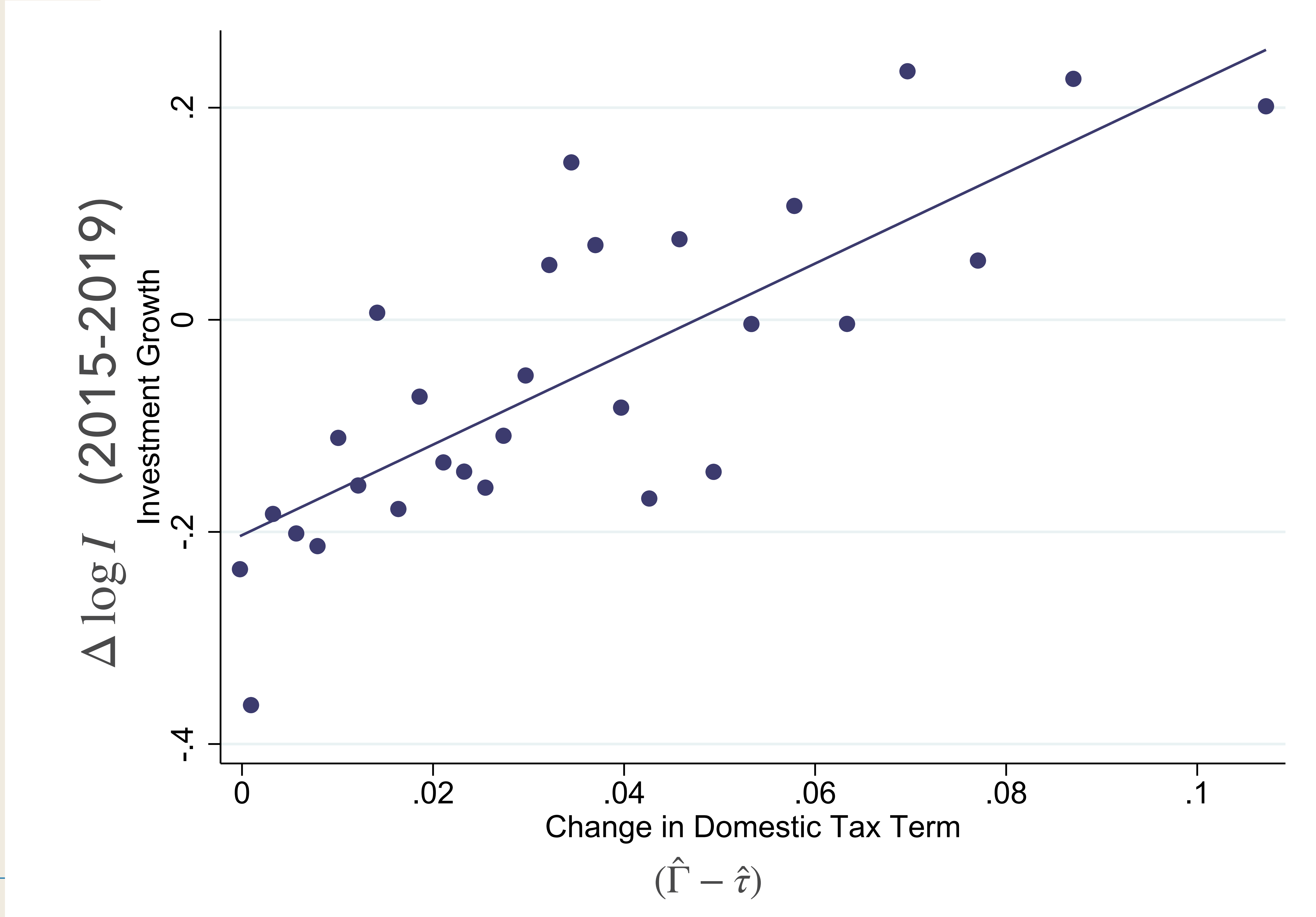
B: Pre- and Post-TCJA τ



Lower Taxes, Higher Investment

- The model predicts

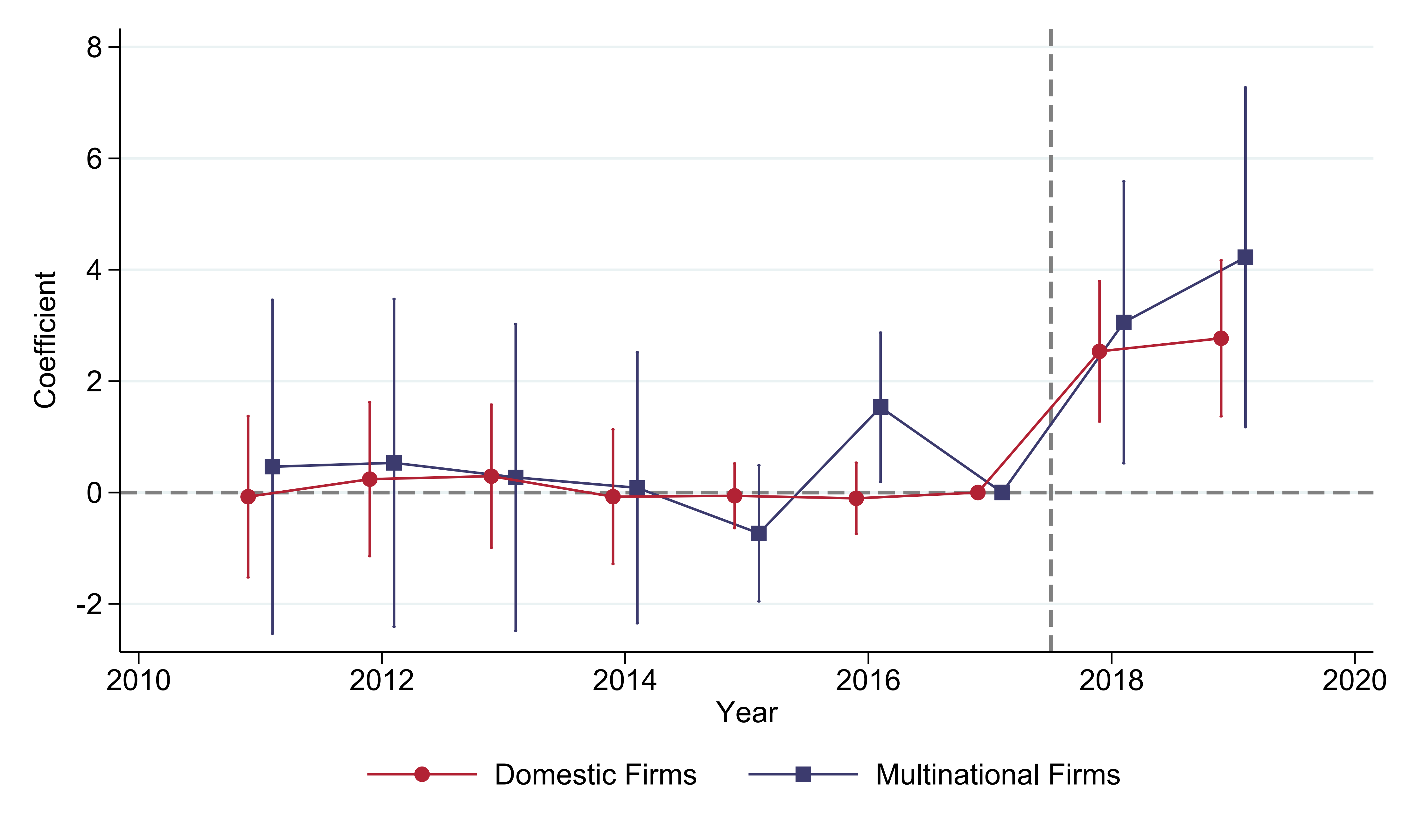
$$\Delta \log I_i = \alpha - \beta \hat{\tau}_i + \beta \hat{\Gamma}_i + \epsilon_i$$



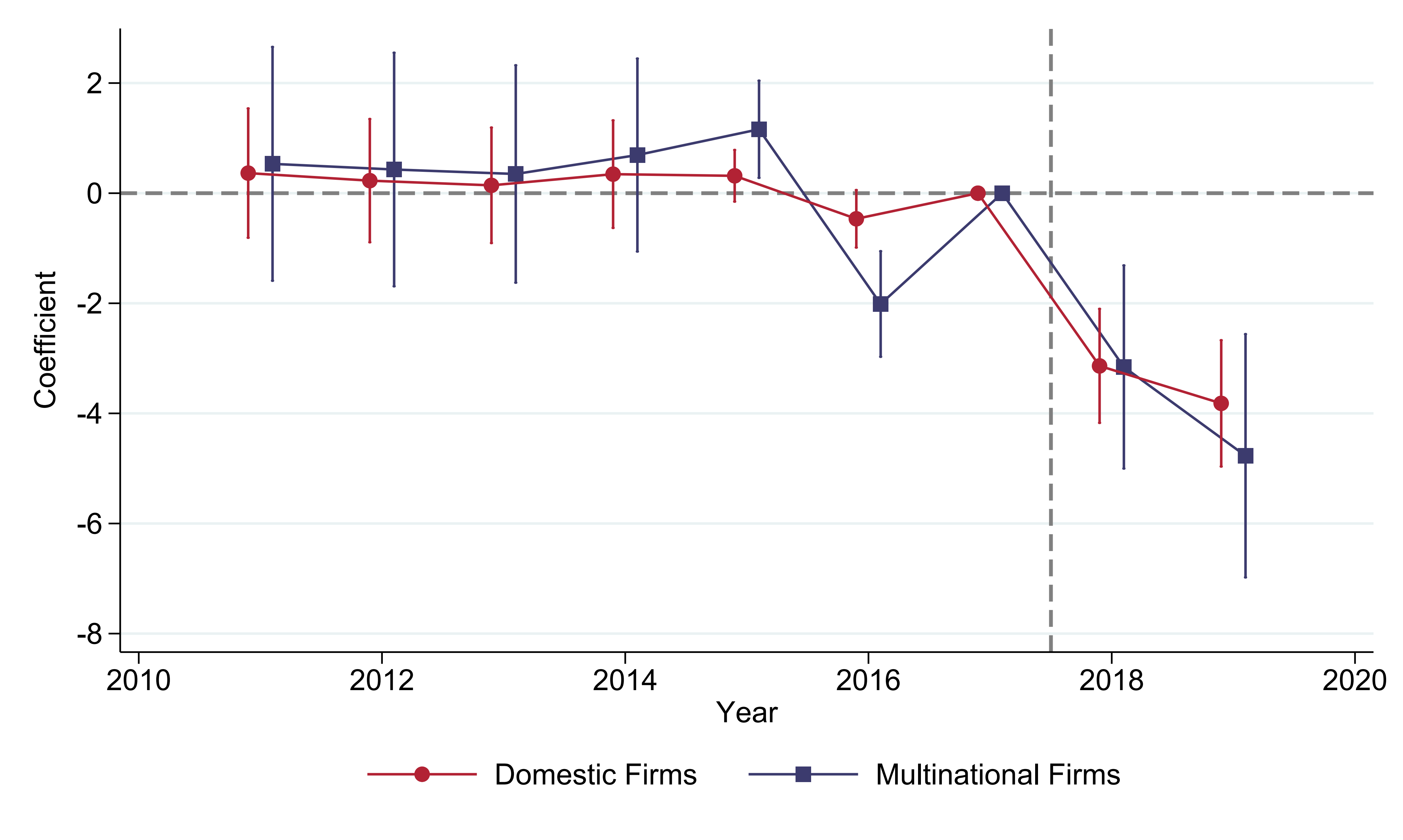
Dynamic Response

$$\log I_{i,t} - \log I_{i,2017} = \alpha_t + \beta_t^{\Gamma}\hat{\Gamma}_i + \beta_t^{\tau}\hat{\tau}_i + \epsilon_{i,t}$$

A. Domestic Cost of Capital $\hat{\Gamma}$



B. Domestic Tax Rate $\hat{\tau}$



- In 2019, 1% higher $\Gamma \Rightarrow$ 4% higher investment
- In 2019, 1% lower $\tau \Rightarrow$ 4% higher investment

Long-Run Macro Impact

Use the structural model to infer the long-run and macro impact of TCJA

- TCJA increased investment by 6.4% in the long-run ($\approx$ 15 years)
- Holding investment fixed, TCJA reduces 42p.p. of the corporate tax revenue
- The increase in investment partially offsets the decline in revenue by 5p.p.