Online Supplement for "Currency Pegs, Trade Deficits and Unemployment: A Reevaluation of the China Shock"

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January 23, 2026

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A Stylized Two-Period Model

To clarify the mechanisms underlying our quantitative results, we present a tractable two-country, two-period model. This framework isolates the interaction between an exchange rate peg and nominal wage rigidity. We show that Foreign productivity growth generates concurrent unemployment and trade deficits because the exchange rate cannot absorb the shock; consequently, the required relative price adjustment must occur through sticky nominal wages, creating real distortions. We use this framework to derive theoretical implications for welfare and optimal policy.

A.1 Model setup

Our environment has two countries, Home (H) and Foreign (F). In our application, Home will be the United States and Foreign will be China. There are two periods: $t = 0$ (short-run) and $t = 1$ (long-run). A representative household in each country consumes goods from both countries and supplies labor to firms that produce goods. Each country has its own nominal account; the price of country j's currency in units of country i's currency at time t is e_{jit}, with $e_{HHt} = e_{FFt} = 1$ and $e_{FHt} = \frac{1}{e_{HFt}}$. We denote $e_t = e_{FHt}$. Hence an increase in e_t is a depreciation of the Home currency.

Household preferences. In each country j, there is a representative agent who consumes goods C_{ijt} across origins i aggregated into a final good C_{jt}, supplies labor ℓ_{jt}. The household has preferences represented by

$$\mathcal{U}_j = [u(C_{j0}) - v(\ell_{j0})] + \beta[u(C_{j1}) - v(\ell_{j1})], \tag{A.1}$$

$$\text{where } u(C) = \frac{C^{1-\gamma^{-1}} - 1}{1-\gamma^{-1}}, \text{ and } C_{jt} = (C_{Hjt}^{\frac{\sigma-1}{\sigma}} + C_{Fjt}^{\frac{\sigma-1}{\sigma}})^{\frac{\sigma}{\sigma-1}}.$$

Here σ is the elasticity of substitution between domestic and foreign goods (the Armington elasticity), and γ is the elasticity of intertemporal substitution. We assume that the Armington elasticity is larger than unity, and the intertemporal elasticity is smaller: formally, $\sigma > 1$ and $\sigma > \gamma$.[1] $v(\cdot)$ is the disutility of supplying labor, which we assume is increasing and convex with $v(0) = 0$.

Technology. A representative firm in country i uses labor as input and has a constant returns

[1] Empirical estimates of σ range from 3-10 (Anderson and van Wincoop, 2003; Imbs and Mejean, 2017) to 1.5-3 (Boehm et al., 2023), but is consistently greater than 1. Estimates of γ are less than 1 and sometimes indistinguishable from 0. Section 2.4 of the main text draws on the literature to discuss this assumption. If we instead had $\sigma = \gamma = 1$, we are in the Cole and Obstfeld (1991) case, where the equilibrium always features trade balance. Thus this assumption is key to predicting the direction of trade imbalance.

to scale production function that requires $\frac{1}{A_{ij}}$ labor to supply a unit of good to market j. Thus for a firm in country i selling Y_{ij} goods to country j at time t using ℓ_{ijt} labor, we have

$$Y_{ijt} = A_{ij}\ell_{ijt}.$$

A_{ij} implicitly incorporates trade frictions. Throughout we assume $A_{HF} \leq A_{HH}$ and $A_{FH} \leq A_{FF}$, implicitly assuming home bias in consumption.

Savings. Each country issues a domestic bond with zero net supply. In period 0, households in each country j have access to a claim of a unit of currency i in period 1, with the price of a claim being $\frac{1}{1+i_{i1}}$ in country i currency. We let B_{ij1} denote the amount of claims for i currency that households in country j own. We assume there is no risk, and bonds from Home and Foreign are perfect substitutes.

Labor Market and Nominal Rigidity. We consider the simplest form of short-run nominal wage rigidity. We assume that nominal wages in both countries are completely fixed in period $t = 0$ to an exogenous level $\{W_{j0}\}$, while wages $\{W_{j1}\}$ are flexible for $t = 1$. Since wages are rigid in period 0, we assume that the labor market is demand-determined in both countries, and workers supply whatever labor is demanded. In period 1, we assume that wages equalize labor supply and labor demand.[2]

Monetary policy and exchange rates. The monetary authority at Home sets the nominal interest rate according to a CPI-based Taylor rule with a coefficient of 1 on inflation:

$$\log(1 + i_{H1}) = -\log(\beta) + \log(\frac{P_{H1}}{P_{H0}}) + \epsilon_{H0}, \tag{A.2}$$

where ϵ_{H0} is the discretionary monetary policy.[3] This rule implicitly sets the real rate $R_{H1} = (1 + i_{H1})\frac{P_{H0}}{P_{H1}}$ at

$$R_{H1} = \frac{1}{\beta}\exp(\epsilon_{H0}).$$

We say a monetary policy *does not respond to shocks* if it sets $\epsilon_{H0} = 0$, or equivalently $R_{H1} = \frac{1}{\beta}$. In Sections 2 onwards, we consider a more standard Taylor rule, which delivers similar results.

Turning to Foreign monetary policy, we are interested in the equilibrium dynamics when Foreign pegs the nominal exchange rate to Home. We assume that Foreign monetary policy

[2]The assumption that wages are completely fixed is to highlight the intuition; any short-run friction in wage adjustment will yield qualitatively identical results.

[3]This follows McKay et al. (2016), Auclert et al. (2021), and allows our analysis to be orthogonal to the effects of monetary policy *rules*.

directly chooses the exchange rate

$$e_0 = e_1 = \bar{e}, \tag{A.3}$$

at an exogenous level $\bar{e}$.[4]

Trade taxes and subsidies. The government can also levy taxes on imports and subsidize exports. We assume that the Home government unilaterally chooses the short-run import tariff t_{FHt} and export subsidy s_{HFt}. If we denote the pre-tariff price of i goods to j at time t by P_{ijt}, Home government revenue is

$$T_{Ht} = t_{FHt} P_{FHt} C_{FHt} - s_{HFt} e_{FHt} P_{HFt} C_{HFt}. \tag{A.4}$$

We assume that the revenue T_{Ht} is rebated lump-sum to the representative household.

A.2 Competitive Equilibrium

In a competitive equilibrium, households maximize their utility, firms maximize their profit, and markets clear. We briefly derive each condition and relegate the details to the .

Utility maximization. The household at country j chooses consumption $\{C_{ijt}\}, \{\ell_{it}\}_{t=1}, \{B_{ijt}\}$ to maximize utility $\mathcal{U}_H$ as described in Equation A.1 subject to the sequential budget constraints,

$$\sum_i (1 + t_{ij0}) P_{ij0} C_{ij0} + \sum_i \frac{B_{ij1}}{1 + i_{ijt}} e_{ij0} \leq W_{j0} \ell_{j0} + \Pi_{j0} + T_{j0}, \tag{A.5}$$

$$\sum_i (1 + t_{ij1}) P_{ij1} C_{ij1} \leq W_{j1} \ell_{jt} + \sum_i B_{ij1} e_{ij1} + \Pi_{j1} + T_{j1}, \tag{A.6}$$

where P_{ijt} is the (pre-tariff) prices for goods from country i to j in units of j currency, B_{j1} is a tradable claim to one nominal unit of account in period 1 with price $\frac{1}{1+i_{jt}}$, W_{jt} is the nominal wage, Π_{jt} is the profit of country j firms and T_{jt} is the government revenue rebated lump-sum.

[4] An explicit monetary rule setting i_{Ft} that leads to the exchange rate peg can be found in Benigno et al. (2007).

The first-order conditions to this utility maximization problem are standard and given by:

$$P_{jt} = (\sum_i ((1+t_{ijt})P_{ijt})^{1-\sigma})^{1/(1-\sigma)}, \tag{A.7}$$

$$\lambda_{ijt} = \frac{((1+t_{ijt})P_{ijt})^{1-\sigma}}{\sum_l P_{ljt}^{1-\sigma}}, \tag{A.8}$$

$$v'(\ell_{j1}) = \frac{u'(C_{j1})W_{j1}}{P_{j1}}, \tag{A.9}$$

$$u'(C_{jt}) = \beta(1+i_{jt})\frac{P_{jt}}{P_{jt+1}}u'(C_{jt+1}) = \beta R_{jt} u'(C_{jt+1}), \tag{A.10}$$

$$\frac{1+i_{F1}}{1+i_{H1}} = \frac{e_1}{e_0}, \tag{A.11}$$

where P_{jt} denotes the consumer price index (CPI) in country j and λ_{ijt} the expenditure share. With the peg $e_1 = e_0 = \bar{e}$, the last condition becomes $i_{F1} = i_{H1}$ (trilemma).

Since wages $\{W_{j0}\}$ are rigid at $t = 0$ and the labor market is demand determined, we may have $v'(\ell_{j0}) \neq \frac{u'(C_{j0})W_{j0}}{P_{j0}}$. We define the *labor wedge* in period 0 as

$$\vartheta_{j0} = v'(\ell_{j0}) - \frac{u'(C_{j0})W_{j0}}{P_{j0}}, \tag{A.12}$$

how much the marginal value of working for households is away from the marginal return from working in utility terms. If $\vartheta_{j0} < 0$, households would like to supply more labor but cannot, so there is *involuntary unemployment*. If $\vartheta_{j0} > 0$, households are supplying more labor than they would want to, so the economy is *overheated*.

Firm optimization. The profits of a representative firm from j selling Y_{ijt} goods to market i is given by

$$\Pi_{it} = \sum_j \left[(1+s_{ijt})\frac{1}{e_{ijt}}P_{ijt} - \frac{W_{it}}{A_{ij}}\right] Y_{ijt}$$

where s_{ijt} is an ad-valorem sales subsidy to i. Since firms are competitive, profits Π_{jt} are equal to 0, and the unit price is equal to marginal cost:

$$P_{ijt} = \frac{1}{1+s_{ijt}} e_{ijt} \frac{W_{it}}{A_{ij}}. \tag{A.13}$$

Market clearing. For each (i, t), the goods market clearing conditions are given by

$$\ell_{it} = \sum_j \frac{C_{ijt}}{A_{ij}}, \tag{A.14}$$

and the bonds market clearing condition is given by

$$B_{H1} + e_1 B_{F1} = 0. \tag{A.15}$$

Equilibrium. We are ready to define an equilibrium in the model as follows:

Definition 1. *Given fundamentals $\{A_{ij}\}$, rigid short-run wage W_{H0}, W_{F0}, policy $\{R_{H1}, t_{ijt}, s_{ijt}\}$ and pegged exchange rate $\bar{e} = e_0 = e_1$, a pegged equilibrium consists of prices $\{W_{it}, P_{it}, P_{ijt}\}$, household's choice variables $\{C_{ijt}\}$,$\{B_{it}\}$,$\{\ell_{it}\}_{t\geq 1}$ and demand-determined short-run labor $\{\ell_{i0}\}$ such that Equations A.5 to A.15 hold.*

A.3 Consequences of a trade shock

In this subsection, we highlight the equilibrium response to trade shocks in this model. As a benchmark, we consider the laissez-faire equilibrium where $t_{FHt} = s_{HFt} = 0$.

The timing of the model and the shock is as follows. Before the start of our setup ($t = -1$), productivities were at a level $\{A_{ij,-1}\}$, and nominal wages $W_{i,-1}$ and exchange rate e_{-1} were such that trade is balanced and labor wedge is zero. Right before $t = 0$, a shock permanently increases Foreign export productivity A_{FH}; we call this the *trade shock*. We assume that wages $\{W_{i0}\}$ are rigid at the pre-shock level $\{W_{i,-1}\}$, and the Foreign policymaker pegs the exchange rate $e_0 = e_1$ at the pre-shock level e_{-1}.

Equilibrium responses. To investigate the effects of the trade shock on trade balance and employment levels, we first observe how the terms-of-trade responds to a trade shock under a peg. We denote by $S_{HFt} = \frac{P_{HFt}\bar{e}}{P_{FHt}}$ the Home terms-of-trade at time t, where a higher terms-of-trade means a higher price of exports relative to imports. S_{HFt} is given by:

$$S_{HFt} = \frac{(\frac{W_{Ht}}{\bar{e}A_{HF}})\bar{e}}{\frac{W_{Ft}\bar{e}}{A_{FH}}} = \underbrace{(\frac{W_{Ht}}{W_{Ft}\bar{e}})}_{\text{relative wage}} \underbrace{(\frac{A_{FH}}{A_{HF}})}_{\text{productivity}} \tag{A.16}$$

As discussed in Section 2.3 of the main text, when wages are rigid and the exchange rate is pegged, we have $\omega_0 > \omega_1$: Home's relative wage is higher in the short-run than the long-run. This results in the following comparative static:

Proposition A.1. *In the pegged equilibrium, in response to a trade shock ($A_{FH} \uparrow$), Home runs a trade deficit ($B_{H1} < 0$). Moreover, if Home monetary policy does not respond ($R_{H1} = \frac{1}{\beta}$), then there is involuntary unemployment at Home ($\vartheta_{H0} < 0$).*

Proof. See Subsection A.5. □

The intuition behind Proposition A.1 is discussed in Section 2.3 of the main text.

Welfare effects. Next, we turn to the welfare implications of the trade shock. We first highlight that trade balances affect the future terms-of-trade: specifically, a deterioration in balances B_{H1} leads to a decrease in future relative wage ω_1. The intuition is closely related to the transfer problem: debt accumulated today becomes a future *transfer* for Foreign, which, combined with a home bias for demand, increases global demand for Foreign goods, improving their terms-of-trade and worsening Home's.

Using this fact, the next proposition highlights the possibility that Home's aggregate welfare may decrease as a result of Foreign growth:

Proposition A.2. *In the pegged equilibrium where monetary policy does not respond ($R_{H1} = \frac{1}{\beta}$), a small increase in A_{FH} reduces Home welfare when σ is sufficiently high and improves Home welfare when σ is small (i.e. close to 1).*

Proof. See Subsection A.5. □

An intuitive explanation is as follows. There are three channels through which productivity growth A_{FH} affects Home welfare:

$$\frac{d\mathcal{U}_H}{dA_{FH}} = \underbrace{-\frac{u'(C_{H0})}{P_{H0}} C_{FH0} \frac{dP_{FH0}}{dA_{FH}}}_{\text{terms-of-trade at } t=0} - \underbrace{\vartheta_0 \frac{d\ell_0}{dA_{FH}}}_{\text{labor wedge}} + \underbrace{\frac{\beta u'(C_{H1})}{P_{H1}} \left[C_{HF1} \frac{dP_{HF1}}{dA_{FH}} - C_{FH1} \frac{dP_{FH1}}{dA_{FH}} \right]}_{\text{terms of trade at } t=1} \quad \text{(A.17)}$$

The terms correspond to (1) the short-run effect of cheaper import goods, (2) labor market friction caused by wage rigidity, and (3) change in long-run terms-of-trade, including direct productivity effects and general equilibrium effects. If $\sigma \to 1$, preference becomes Cobb-Douglas, the pegged equilibrium coincides with the flexible-wage equilibrium, and trade is balanced as in Cole and Obstfeld (1991). Then the effects (2) and the general equilibrium component of (3) go to zero, leaving cheaper goods as the primary welfare benefit. In the opposite case, when $\sigma \to \infty$, short-run demand for Home goods becomes 0. Then, a small change in A_{FH} can cause a discrete loss of utility from the labor wedge and the trade deficit worsening future terms-of-trade, dwarfing welfare gains from cheaper goods.

The possibility of Foreign productivity growth harming Home welfare echoes immiserizing growth where Home's productivity growth worsens its terms-of-trade, negating gains from the expansion of the production frontier (Bhagwati, 1958). In our case, Foreign productivity growth improves Home terms-of-trade, and the peg magnifies this gain today, but unemployment moves Home production into the interior of the PPF and harms future terms-of-trade through trade deficit, offsetting the gains.

Proposition A.2 cautions against using trade balance as a welfare indicator. Public discourse often views trade deficits as inherently undesirable. However, whenever σ exceeds 1 and surpasses γ, a trade deficit is the predicted outcome for Home under a trade shock under a peg. The shock may benefit Home welfare if σ is not excessively high. Conversely, a large γ with $\sigma \to 1$ results in Home's trade surplus and welfare gains, whereas with $\gamma > \sigma$ both large, Home faces welfare losses despite a trade surplus. In the next sections, we undertake a quantitative analysis of the substitution, rigidity, and productivity growth to assess whether the China shock improved or harmed aggregate US welfare.[5]

A.4 Policy response

In this subsection, we consider the unilateral problem of the Home government facing a growth in A_{FH} and an exchange rate peg. We assume the Home government can choose its short-run tariff level t_{FH0}, domestic subsidy s_{HF0} and monetary policy R_{H1}.[6] We assume the government cannot choose long-run tariff t_{FH1}, as the motivation for long-run tariffs as terms-of-trade manipulation is well understood since Graaff (1949).

Formally, the policy problem that the Home government faces is:

$$\max_{t_{FH0}, s_{HF0}, R_{H1}} \mathcal{U}_{\mathcal{H}} = \max_{t_{FH0}, s_{HF0}, R_{H1}} \sum_{t=0}^{1} \beta^t [u(C_{Ht}) - v(\ell_{Ht})] \tag{A.18}$$

subject to the same equilibrium conditions.

We first note that the planner can replicate the flexible price outcome. Indeed, if $\omega_{peg} = \frac{W_{H0}}{W_{F0}\bar{e}}$ is the short-run relative wage under peg, and $\omega_f = \frac{W^f_{H0}}{W^f_{F0}e^f}$ is the relative wage under flexible price (after the trade shock), the planner can set $R_{H1} = \frac{1}{\beta}$ and $t_{FH0} = s_{FH0} = \frac{\omega_f}{\omega_{peg}} - 1$. This tax and subsidy level sets the relative prices equal to the flexible price level, and the tax revenue and cost of subsidy cancel out exactly. Thus, we know the planner can undo the wedges and the potential welfare losses in Proposition A.2.[7]

However, this policy may not be optimal for the Home government. As an extreme example, if Foreign is offering goods for free, Home would be much better off taking those goods than setting high tariffs that distort consumption.

To solve for the optimal policy, we proceed in two steps. First, we solve for the optimal trade policy (t_{FH0}, s_{HF0}) given monetary policy R_{H1}, then we proceed to solve for the optimal R_{H1}. This approach makes the problem more tractable, and the inner problem may be a more reasonable benchmark of reality, where monetary policy is unable to fully respond to a sector-

[5] Whether trade deficits are symptoms of welfare gains or losses is a different question to whether capital controls are beneficial. The next subsection shows that capital controls unambiguously hurt Home welfare.

[6] Since wages are rigid, we do not have Lerner symmetry, and subsidies and tariffs are independent.

[7] This connects with Farhi et al. (2014) that fiscal instruments can replicate currency devaluations.

origin specific trade shock.[8]

Optimal trade policy

Given monetary policy R_{H1}, an indirect formula for the optimal trade policy can be obtained via a first-order variation argument. Starting from the optimal policy, the marginal effect of policy change in welfare must be zero, yielding the following formula:[9]

Lemma A.1. *The optimal short-run tariff rate on imports t_{FH0} satisfies*

$$t_{FH0} = \frac{1}{P_{FH0}} \left[\underbrace{\frac{\vartheta_0}{\tilde{\lambda}} \frac{\partial \ell_{H0}}{\partial C_{FH0}}}_{\text{labor wedge}} - \frac{1}{(1+i_{H1})} \underbrace{(\ell_{HF1} \frac{\partial W_{H1}}{\partial C_{FH0}} - \ell_{FH1} \frac{\partial W_{F1}}{\partial C_{FH0}})}_{\text{future terms-of-trade}} + \underbrace{s_{HF0} P_{HF0} \frac{\partial C_{HF0}}{\partial C_{FH0}}}_{\text{subsidy externality}} \right] \tag{A.19}$$

The optimal short-run subsidy rate on exports s_{HF0} satisfies

$$s_{HF0} = \frac{1}{P_{HF0}} \left[- \underbrace{\frac{\vartheta_0}{\tilde{\lambda}} \frac{\partial \ell_{H0}}{\partial C_{HF0}}}_{\text{labor wedge}} + \frac{1}{(1+i_{H1})} \underbrace{(\ell_{HF1} \frac{\partial W_{H1}}{\partial C_{HF0}} - \ell_{FH1} \frac{\partial W_{F1}}{\partial C_{HF0}})}_{\text{future terms-of-trade}} - \underbrace{P_{HF0} C_{HF0} \frac{\partial s_{HF0}}{\partial C_{HF0}}}_{\text{terms-of-trade today}} \right] \tag{A.20}$$

where $\tilde{\lambda}$ is the Lagrange multiplier on the lifetime budget constraint.

Proof. See Subsection A.6. □

The first-order formula for tariffs succinctly captures the three *externalities* of imports that the Home government seeks to address via a tariff. First, tariffs and subsidies both reduce the labor wedge by stimulating demand for domestic labor. Second, tariffs and subsidies, by affecting relative prices of goods, improve current trade balance, which improves the terms-of-trade in the future. Third, the fiscal externality (deadweight loss) of tariffs and subsidies interact in general equilibrium. In a competitive equilibrium, home households do not internalize any of these effects of an extra unit of import. Thus the tax level $t_{FH0}P_{FH0}$ and the subsidy level $s_{HF0}P_{HF0}$ can be considered a Pigouvian tax that corrects for the three externalities of consuming an extra unit of import or exporting an extra unit.

Using the formula, we can sign the optimal tariff and show that its magnitude *increases* with the Foreign shock A_{FH0}:

[8] In the early 2000s, the government was tightening monetary policy in response to concerns over inflation and tightening of unused resources; loosening in response to the China shock was not the Federal Reserve Bank's goal (Federal Reserve Board, 2005). Following the Great Recession, the Federal Reserve Bank was subject to the Zero Lower Bound.

[9] A similar argument can be found in Costinot et al. (2022).

Proposition A.3. *If there is unemployment at the zero-tariff economy ($\vartheta_{H0} < 0$ when $t_{FH0} = 0$), the optimal tariff t_{FH0} is positive and is increasing in the size of the trade shock A_{FH0}.*

Proof. See Subsection A.6. □

The intuition that we can and should use tariffs as second-best instruments to fix distortions is well-known. The prediction obtained in Proposition A.3 is sharper. We show that in an environment where trade shocks cause unemployment and trade deficits, the tariff should be positive and increase in the magnitude of the trade shock. In this context, the short-run tariff t_{FH0} is akin to *safeguard* tariffs allowed under the WTO Agreement on Safeguards.

But this is not the only role of tariffs in our model, as highlighted in the future terms-of-trade term in Equation A.19. While tariffs do not affect today's terms-of-trade (due to wage rigidity and peg), a unilateral short-run tariff reduces Home's trade deficit, improving Home's future terms-of-trade. Hence, Home would want to set tariffs beyond the globally optimal "distortion-fixing" level, at the expense of Foreign welfare. As such, short-run tariffs are *safeguard* and *beggar-thy-neighbor* at the same time, even when the short-run terms-of-trade is rigid.[10]

Our model underscores that under an exchange rate peg, the optimal short-run tariff is increasing in the magnitude of the trade shock. This contrasts with the flexible exchange rate case, where the optimal tariff is pinned down primarily by the trade elasticity (Gros, 1987) and does not depend on the shock magnitude. Our framework focuses on tariffs that correct a distortion caused by the peg and the trade shock, so the magnitude of the optimal tariff scales with the size of the distortion. We discuss this in more detail in the .

Proposition A.3 assumes monetary policy does not clear unemployment. As aforementioned, the central bank may be unable to clear the output gap caused by sector-specific trade shocks because of multisector considerations, financial concerns, and liquidity constraints such as the Zero Lower Bound. Tariffs will be a useful tool in this second-best world.

Optimal monetary policy

What is the optimal monetary policy R_{H1}? An analogous first-order condition on monetary policy highlights the channels in which monetary policy affects welfare. We highlight a special case when the intertemporal elasticity is equal to 1 (consumption is log):

Proposition A.4. *When $\gamma = 1$, optimal monetary policy R_{H1} satisfies the following equation:*

$$0 = \underbrace{-\vartheta_0 \frac{d\ell_0}{dR_{H1}}}_{wedge} + \tilde{\lambda}_r [\underbrace{R_{H1} t_{FH0} \frac{P_{FH0}}{P_{H0}} \frac{dC_{FH0}}{dR_{H1}}}_{tariff\ fiscal\ externality} + \underbrace{(NX_0)}_{intertemporal\ TOT}], \tag{A.21}$$

[10] By nature of being beggar-thy-neighbor, Foreign can retaliate with its own tariffs to undo the imbalance-adjusting channel of Home tariffs.

where $\tilde{\lambda}_r$ is the Lagrange multiplier on the Home lifetime real budget constraint normalized by P_{H0}.

As a special case, when $t_{FH0} = 0$, the optimal monetary policy R_{H1} is such that $\vartheta_0 > 0$: it is optimal to loosen monetary policy beyond clearing the output gap.

Proof. See Subsection A.6. □

Proposition A.4 highlights that when Foreign pegs, the optimal monetary policy for a borrowing Home will *overshoot* the output gap. This leverages Home's control of *global* monetary policy and manipulate intertemporal terms-of-trade to its favor. Particularly for the US, which influences global rates as the dominant currency (Gopinath et al., 2020) and runs current account deficits, the central bank may want to set a lower interest rate, with minimal risk of bond liquidation from pegging countries.

The proposition also clarifies that tariffs are second-best instruments when monetary policy cannot respond – whether due to the ZLB or multisectoral considerations. In fact, under a positive tariff, the additional losses from tariff fiscal externality compels Home to set a higher interest rate, reducing overall welfare.[11]

The assumption $\gamma = 1$ allows us to circumvent the effect of today's monetary policy on the magnitude of the trade deficit. When $\gamma = 1$, the effect of interest rate on consumption and output is proportionate in both countries: thus the real value of the deficit does not change, and monetary policy R_{H1} does not affect the intratemporal terms-of-trade in the future. On the other hand, when $\gamma \neq 1$, the optimal monetary policy equation (Equation A.21) comes with an additional "future terms of trade" term: monetary policy may affect the magnitude of the deficit in real terms (but not the sign, as we discussed in Section A.3), affecting the optimal policy.

Capital Controls

Lastly, we study the welfare effects of the endogenous deficits we highlighted in Proposition A.1 by considering *capital controls* in addition to the tariffs and subsidies. We have established that deficits and unemployment can come from the same cause – trade shock and exchange rate peg – but are deficits inherently bad for Home welfare? While this is where some policy narratives go, the next proposition shows that this is not the case.

Proposition A.5. *In the pegged equilibrium, removing international financial flows (forcing $B_{H1} = 0$) worsens Home unemployment (ϑ_{H0} decreases), and reduces Home welfare $\mathcal{U}_0$.*

Proof. See Subsection A.6. □

[11] In the , we numerically solve for the joint optimal trade and monetary policy for various levels of the trade shock A_{FH0}. We find that the joint optimal policy involves no tariffs and a very loose monetary policy, highlighting the distortionary nature of tariffs. In a first-best one-sector world, Home would take advantage of the cheap goods and solve the labor wedge solely through monetary policy.

Removing financial flows worsens Home unemployment because of home bias in consumption. Indeed, with trade costs, under the same price levels, Home borrowing to consume will increase demand for Home goods, while Foreign saving will decrease demand for Foreign goods. Since unemployment is determined by aggregate demand, Home's trade deficit in the short-run actually ameliorates unemployment, and capital controls will only worsen unemployment. As such, while deficits may be symptoms of a friction that may harm the economy, deficits themselves are not a friction to solve, and capital controls may harm Home welfare. The fact that financial transfers are welfare-improving under an exchange rate peg is closely related to the idea that fiscal unions are desirable under currency unions (Farhi and Werning, 2017); we highlight that the possibility of a dynamic budget-balanced (net current value zero) transfer is welfare-improving.

A.5 Proofs for Subsection A.3

In this section we prove the Propositions in Section A.3. In the equilibrium under the exchange rate peg, we assume without loss of generality that $\bar{e} = 1$. We first highlight a number of properties of the laissez-faire equilibrium that we extensively use in the proof.

Lemma A.2. *Denote by $\omega_t = \frac{W_{Ht}}{W_{Ft}}$ the relative wage of Home at period $t \in \{0, 1\}$. The following properties hold:*

(a) The real wage $\frac{W_{jt}}{P_{jt}}$ and expenditure share λ_{ijt} depend on $\{W_{Ht}, W_{Ft}\}$ only through ω_t.

(b) Home real wage $\frac{W_{Ht}}{P_{Ht}}$ increases in ω_t, while Foreign real wage decreases in ω_t.

(c) Expenditure share for Home goods λ_{Hjt} is a decreasing function of ω_t; $\lambda_{Fjt} = 1 - \lambda_{Hjt}$ is an increasing function of ω_t

(d) Home relative wage is higher in period 0: $\omega_0 > \omega_1$.

(e) The real wage of Home is higher in period 0: $\frac{W_{H0}}{P_{H0}} > \frac{W_{H1}}{P_{H1}}$.

(f) Relative inflation is higher at Foreign. If we define $\pi_j = \frac{P_{j1}}{P_{j0}}$, we have $\pi_F > \pi_H$.

Proof. (a) We have

$$\frac{W_{Ht}}{P_{Ht}} = \frac{W_{Ht}}{(P_{HHt}^{1-\sigma} + P_{FHt}^{1-\sigma})^{1/(1-\sigma)}} = \frac{W_{Ht}}{((W_{Ht}/A_{HH})^{1-\sigma} + (W_{Ft}/A_{FH})^{1-\sigma})^{1/(1-\sigma)}}$$
$$= \frac{1}{((1/A_{HH})^{1-\sigma} + (\omega_t/A_{FH})^{1-\sigma})^{1/(1-\sigma)}}$$

and analogously for W_{Ft}/P_{Ft}. Likewise, we have

$$\lambda_{Hjt} = \frac{P_{Hjt}^{1-\sigma}}{P_{Hjt}^{1-\sigma} + P_{Fjt}^{1-\sigma}} = \frac{1}{1 + (\frac{W_{Ft}/A_{Fj}}{W_{Ht}/A_{Hj}})^{1-\sigma}} = \frac{1}{1 + (\omega_t)^{\sigma-1}(\frac{A_{Hj}}{A_{Fj}})^{1-\sigma}}$$

and $\lambda_{Fjt} = 1 - \lambda_{Hjt}$. In general, the real wage and expenditure share are functions of ω_t for *any* homothetic aggregator of Home and Foreign goods $\mathcal{C}_j = \mathcal{C}_j(C_{Hjt}, C_{Fjt})$.

(b) By inspection of the previous formula, we see that when $\sigma > 1$, $\frac{W_{Ht}}{W_{Ft}}$ is increasing in ω_t.

(c) Likewise, when $\sigma > 1$, λ_{Hjt} is decreasing in ω_t.

(d) Denote by $\omega^*(\{A_{ij}\})$ the Home relative wage under a *static, flexible-price* economy under productivity $\{A_{ij}\}_{i,j\in\{H,F\}}$, which can be solved by the trade balance equation:

$$\lambda_{FH} W_H \ell_H = \lambda_{HF} W_F \ell_F \;\;\Rightarrow\;\; \omega^* \frac{\ell_H}{\ell_F} = \frac{\lambda_{HF}(\omega^*)}{\lambda_{FH}(\omega^*)}$$

Now since ℓ_j is increasing in $\frac{W_j}{P_j}$, the left-hand side is increasing in ω^* while the right-hand side is decreasing in ω^*. Thus there is a unique ω^*.

Consider the trade shock that increases A_F. Since λ_{FH} is increasing in A_F, λ_{FH} is decreasing in A_F, we have that a higher A_F decreases the right-hand side. Thus to satisfy equality, an increase in A_F must be accompanied by a *decrease* in ω^*.

We assumed that Home relative wage ω_0 is rigid at $\omega_0 = \omega^*(\{A_{ij,-1}\})$. Given an increase in A_F, $\omega_0 = \omega^*(\{A_{ij,-1}\}) > \omega^*(\{A_{ij0}\})$. Now, if we assumed for sake of contradiction that $\omega_1 \geq \omega_0 > \omega^*(\{A_{ij0}\}) = \omega^f$, we would have

$$\omega_t \frac{\ell_H(\omega_t)}{\ell_F(\omega_t)} > \frac{\lambda_{HF}(\omega_t)}{\lambda_{FH}(\omega_t)} \text{ for } t = 0, 1$$

but this would break the lifetime trade balance condition – Home's relative wage is too high in both periods, so Home cannot balance the lifetime budget. Thus we have $\omega_0 > \omega_1$.

(e) This follows from 2 and 5.

(f) We have

$$\left(\frac{P_{Ht}}{P_{Ft}}\right)^{1-\sigma} = \frac{P_{HHt}^{1-\sigma} + P_{FHt}^{1-\sigma}}{P_{HFt}^{1-\sigma} + P_{FFt}^{1-\sigma}} = \frac{(\omega_t \frac{A_{FF}}{A_{HH}})^{1-\sigma} + (\frac{A_{FF}}{A_{FH}})^{1-\sigma}}{(\omega_t \frac{A_{FF}}{A_{HF}})^{1-\sigma} + 1}$$
$$= (\frac{A_{HF}}{A_{HH}})^{1-\sigma}(1 + \frac{(\frac{A_{HH}A_{FF}}{A_{HF}A_{FH}})^{1-\sigma} - 1}{(\omega_t \frac{A_{FF}}{A_{HF}})^{1-\sigma} + 1})$$

Since $\sigma > 1$ and $\frac{A_{HH}A_{FF}}{A_{HF}A_{FH}} > 1$ (Home bias, equivalently $\tau_{FH}\tau_{HF} \geq 1$), the last expression is decreasing in ω_t. Then since $\omega_0 > \omega_1$ and again $\sigma > 1$, we have $\frac{P_{H0}}{P_{F0}} > \frac{P_{H1}}{P_{F1}}$. Rearranging, we get $\pi_F > \pi_H$.

□

Using these properties, we prove the propositions.

Proposition A.1. *In the pegged equilibrium, in response to a trade shock ($A_{FH} \uparrow$), Home runs a trade deficit ($B_{H1} < 0$). Moreover, if Home monetary policy does not respond ($R_{H1} = \frac{1}{\beta}$), then there is involuntary unemployment at Home ($\vartheta_{H0} < 0$).*

Proof. For the first part ($B_{H1} < 0$), note that Home borrows in the short-run if the following inequalities hold:

$$\underbrace{\lambda_{HF0}P_{F0}C_{F0}}_{t=0 \text{ Home exports}} < \underbrace{\lambda_{FH0}P_{H0}C_{H0}}_{t=0 \text{ Home imports}} \text{ and } \underbrace{\lambda_{HF1}P_{F1}C_{F1}}_{t=1 \text{ Home exports}} > \underbrace{\lambda_{FH1}P_{H1}C_{H1}}_{t=1 \text{ Home imports}} \tag{A.22}$$

Invert the second inequality and multiply with the first to have

$$\frac{\lambda_{HF0}}{\lambda_{HF1}}\frac{P_{F0}C_{F0}}{P_{F1}C_{F1}} < \frac{\lambda_{FH0}}{\lambda_{FH1}}\frac{P_{H0}C_{H0}}{P_{H1}C_{H1}}$$

Rearrange to have:

$$\frac{\lambda_{HF0}/\lambda_{HF1}}{\lambda_{FH0}/\lambda_{FH1}} < \frac{\pi_F}{\pi_H}\frac{C_{H0}/C_{H1}}{C_{F0}/C_{F1}} \tag{A.23}$$

where $\pi_j = \frac{P_{j1}}{P_{j0}}$ denote inflation in country j. Note that if $B_1 > 0$, both inequalities are flipped in Inequality A.22, so we have the exact opposite inequality, so Inequality A.23 is a necessary and sufficient condition for Home borrowing. Since both countries face the same nominal interest rate under a peg, we have

$$C_{j0}^{-1/\gamma} = \beta(1+i)\frac{1}{\pi_j}C_{j1}^{-1/\gamma} \quad \Rightarrow \quad \frac{C_{j0}}{C_{j1}} = [\beta(1+i)\pi_j^{-1}]^{-\gamma}$$

Use this to rewrite Inequality A.23 as

$$\frac{\lambda_{HF0}/\lambda_{HF1}}{\lambda_{FH0}/\lambda_{FH1}} < [\frac{\pi_F}{\pi_H}]^{1-\gamma} \Leftrightarrow B_{H1} < 0$$

(Note that the left-hand-side is the first 'variation in terms-of-trade across time' governed by σ, while the right-hand-side is the second 'home bias and relative prices' governed by γ, as described in the main text.)

With the CES parametric assumption, we may rewrite the expenditure shares λ_{ij} as

$$\frac{\lambda_{HF0}}{\lambda_{HF1}} = \frac{(P_{HF0}^{1-\sigma}/P_{F0}^{1-\sigma})}{(P_{HF1}^{1-\sigma}/P_{F1}^{1-\sigma})} = \pi_F^{1-\sigma}(\frac{W_{H0}}{W_{H1}})^{1-\sigma}$$
$$\frac{\lambda_{FH0}}{\lambda_{FH1}} = \frac{(P_{FH0}^{1-\sigma}/P_{H0}^{1-\sigma})}{(P_{FH1}^{1-\sigma}/P_{H1}^{1-\sigma})} = \pi_H^{1-\sigma}(\frac{W_{F0}}{W_{F1}})^{1-\sigma}$$

Hence,

$$\frac{\lambda_{HF0}/\lambda_{HF1}}{\lambda_{FH0}/\lambda_{FH1}} = (\frac{\pi_F}{\pi_H})^{1-\sigma}(\frac{W_{H0}/W_{H1}}{W_{F0}/W_{F1}})^{1-\sigma}$$

This is smaller than $[\frac{\pi_F}{\pi_H}]^{1-\gamma}$ if and only if

$$(\frac{\pi_F}{\pi_H})^{1-\sigma}(\frac{W_{H0}/W_{H1}}{W_{F0}/W_{F1}})^{1-\sigma} < (\frac{\pi_F}{\pi_H})^{1-\gamma}$$
$$\Leftrightarrow (\frac{W_{H0}/W_{H1}}{W_{F0}/W_{F1}})^{1-\sigma} < (\frac{\pi_F}{\pi_H})^{\sigma-\gamma}$$

We have that the left-hand side is less than 1 by $\sigma > 1$ and part (d) of Lemma A.2. We have that the right-hand side is greater than 1 by $\sigma > \gamma$ and part (f) of Lemma A.2. Thus we have $RHS > 1 > LHS$.

For the second part ($\vartheta_{H0} < 0$ when $R_{H0} = 1/\beta$), we first have

$$v'(\ell_{H1}) = u'(C_{H1})\frac{W_{H1}}{P_{H1}}$$

From part (e) of Lemma A.2, we have $\frac{W_{H0}}{P_{H0}} > \frac{W_{H1}}{P_{H1}}$. At the same time, we have $u'(C_{H1}) = u'(C_{H0})$ with $R_H = \frac{1}{\beta}$. Thus, if we can show $\ell_{H1} > \ell_{H0}$, we have

$$\vartheta_{H0} = v'(\ell_{H0}) - u'(C_{H0})\frac{W_{H0}}{P_{H0}} < v'(\ell_{H1}) - u'(C_{H1})\frac{W_{H1}}{P_{H1}} = 0$$

We proceed to show $\ell_{H1} > \ell_{H0}$. Goods market clearing condition is $\ell_{Ht} = \tau_{HH}C_{HHt} + \tau_{HF}C_{HFt}$, and since $C_{H1} = C_{H0}$ and $\lambda_{HH0} < \lambda_{HH1}$ by $\frac{W_{H0}}{W_{F0}} > \frac{W_{H1}}{W_{F1}}$, we have $C_{HH0} < C_{HH1}$. Moreover, with $\sigma > 1$ and $\sigma > \gamma$, we have

$$\frac{C_{HF0}}{C_{HF1}} = \frac{(\frac{P_{HF0}}{P_{F0}})^{-\sigma} C_{F0}}{(\frac{P_{HF1}}{P_{F1}})^{-\sigma} C_{F1}} = \frac{(\frac{P_{HF0}}{P_{F0}})^{-\sigma}}{(\frac{P_{HF1}}{P_{F1}})^{-\sigma}} \cdot (\beta(1+i)\frac{P_{F0}}{P_{F1}})^{-\gamma}$$

$$< \frac{(\frac{P_{HF0}}{P_{F0}})^{-\gamma}}{(\frac{P_{HF1}}{P_{F1}})^{-\gamma}} \cdot (\frac{P_{H1}}{P_{H0}} \frac{P_{F0}}{P_{F1}})^{-\gamma}$$

$$= (\frac{P_{HF0}}{P_{HF1}} \frac{P_{H1}}{P_{H0}})^{-\gamma} = (\frac{W_{H0}}{W_{H1}} \frac{P_{H1}}{P_{H0}})^{-\gamma} < 1$$

where we have the intermediate inequality because $(\frac{P_{HF0}}{P_{F0}} / \frac{P_{HF1}}{P_{F1}}) > 1$ (which follow from $\omega_0 > \omega_1$) and $\sigma \geq \gamma$, and the last inequality from part (e) of Lemma A.2. Thus we have $C_{HH0} < C_{HH1}$ and $C_{HF0} < C_{HF1}$, so $\ell_{H0} < \ell_{H1}$, and we obtain $\vartheta_{H0} < 0$. □

For the next proposition, we first prove that deficits hurt future terms-of-trade.

Lemma A.3. *Suppose Home borrows more in real terms, so that $\frac{B_{H1}}{W_{H1}}$ decreases. Then $\frac{W_{H1}\bar{e}}{W_{F1}}$ falls: Home future relative wage worsens as a result of Home borrowing.*

Proof. The goods market clearing condition for Home goods at $t = 1$ can be rewritten as

$$W_{H1}\ell_{H1} = \lambda_{HH1}(W_{H1}\ell_{H1} + B_{H1}) + \lambda_{HF1}(W_{F1}\ell_{F1} - B_{H1})$$

Rearranging this equation and writing everything in terms of $S_{H1} = \frac{W_{H1}}{W_{F1}}$ and $b = \frac{B_{H1}}{W_{H1}}$, we may write

$$1 = \lambda_{HH1}(1 + \frac{b}{\ell_{H1}}) + \lambda_{HF}(\frac{1}{S}\frac{\ell_{F1}}{\ell_{H1}} - \frac{b}{\ell_{H1}})$$

$$b[\frac{\lambda_{HH} - \lambda_{HF}}{\ell_H}] = 1 - \lambda_{HH} - \lambda_{HF}(\frac{1}{S}\frac{\ell_F}{\ell_H})$$

We have $\frac{\partial \lambda_{HH1}}{\partial S}, \frac{\partial \lambda_{HF1}}{\partial S} < 0$ (Home better terms-of-trade $\iff$ Home goods more expensive), $\frac{\partial \ell_H}{\partial S} > 0, \frac{\partial \ell_F}{\partial S} < 0$ (Home better TOT $\iff$ Home workers have better real wage, want to work more). Then the *RHS* is increasing in S. Moreover, from home bias we have $\lambda_{HH} + \lambda_{FF} > 1 \rightarrow \lambda_{HH} > \lambda_{HF}$, so the coefficient on b is positive. Thus $\frac{\partial b}{\partial S} > 0$; then $\frac{\partial S}{\partial b} = \frac{1}{\frac{\partial b}{\partial S}} > 0$ so running more debt ($b \downarrow$) will lead to worsening terms of trade $S \downarrow$. □

Proposition A.2. *In the equilibrium where policy does not respond ($R_{H1} = \frac{1}{\beta}$), the effect of a small increase of A_{FH} on Home welfare $\mathcal{U}_H$ is ambiguous, and depends on σ. For small changes in $\epsilon_A = A_{FH0} - A_{FH-1}$, we have that:*

- *When $\sigma \rightarrow 1$, we have Home welfare increases as a result of the Foreign shock: $\frac{d\mathcal{U}_H}{dA_{FH}} > 0$.*

- *When $\sigma \to \infty$, we have Home welfare decreases as a result of the Foreign shock: $\frac{d\mathcal{U}_H}{dA_{FH}} < 0$*

Proof. We first derive the first-order welfare equation A.17:

$$\frac{d\mathcal{U}_H}{dA_{FH}} = \underbrace{-\frac{u'(C_{H0})}{P_{H0}} C_{FH0} \frac{dP_{FH0}}{dA_{FH}}}_{\text{cheap goods}} + \underbrace{\vartheta_0 \frac{d\ell_0}{dA_{FH}}}_{\text{labor wedge}} + \underbrace{\frac{\beta u'(C_{H1})}{P_{H1}} [C_{HF1} \frac{dP_{HF1}}{dA_{FH}} - C_{FH1} \frac{dP_{FH1}}{dA_{FH}}]}_{\text{terms of trade at } t=1}$$

Home agent's lifetime utility is

$$\mathcal{U}_H = U(C_{HH0}, C_{FH0}, C_{HH1}, C_{FH1}, \ell_{H0}, \ell_{H1})$$

and is subject to the lifetime budget constraint

$$P_{HH0}C_{HH0} + P_{FH0}C_{FH0} + \frac{1}{1+i_{Ht}}(P_{HH1}C_{HH1} + P_{FH1}C_{FH1}) = W_{H0}\ell_{H0} + \frac{1}{1+i_{H1}} W_{H1}\ell_{H1}$$

Invoking the Envelope theorem, the first-order effect of A_F on $\mathcal{U}_H$ can be written as

$$\frac{d\mathcal{U}_H}{dA_{FH}} = \sum_{t=0}^{1} \sum_{i\in\{H,F\}} \frac{dU}{dC_{iHt}} \frac{dC_{iHt}}{dA_{FH}} + \sum_{t=0}^{1} \frac{dU}{d\ell_{Ht}} \frac{d\ell_{Ht}}{dA_{FH}} \tag{A.24}$$

If we denote by $\tilde{\lambda}$ the Lagrange multiplier on the lifetime budget constraint, we have:

$$\frac{dU}{dC_{iH0}} = \tilde{\lambda} P_{iH0}, \quad \frac{dU}{dC_{iH1}} = \frac{\tilde{\lambda}}{1+i_{H1}} P_{iH1}, \quad \frac{dU}{d\ell_{H1}} = -\frac{\tilde{\lambda}}{1+i_{H1}} W_{H1}$$

while we may have $\frac{dU}{d\ell_{H0}} \neq -\tilde{\lambda} W_{H0}$ because households do not choose ℓ_{H0}: in fact, we have

$$\frac{dU}{d\ell_{H0}} + \tilde{\lambda} W_{H0} = -v'(\ell_{H0}) + \frac{u'(C_{H0})}{P_{H0}} W_{H0} = -\vartheta_0.$$

Plugging these into Equation A.24, we get

$$\frac{d\mathcal{U}_H}{dA_{FH}} = \tilde{\lambda} \left[\sum_{i\in\{H,F\}} (P_{iH0} \frac{dC_{iH1}}{dA_F} + \frac{P_{iH1}}{1+i_{H1}} \frac{dC_{iH0}}{dA_F}) - W_{H0} \frac{d\ell_{H0}}{dA_{FH}} - \frac{W_{H1}}{1+i_{H1}} \frac{d\ell_{H1}}{dA_{FH}} \right] - \vartheta_0 \frac{d\ell_0}{dA_{FH}} \tag{A.25}$$

Now, if we take the derivative of the budget constraint, we have

$$
\begin{aligned}
&\sum_{i\in\{H,F\}}\left(P_{iH0}\frac{dC_{iH0}}{dA_F}+\frac{P_{iH1}}{1+i_{H1}}\frac{dC_{iH1}}{dA_F}\right)-W_{H0}\frac{d\ell_{H0}}{dA_{FH}}-\frac{1}{1+i_{H1}}W_{H1}\frac{d\ell_{H1}}{dA_{FH}}\\
=&-\sum_{i\in\{H,F\}}\left(C_{iH0}\frac{dP_{iH0}}{dA_F}+\frac{C_{iH1}}{1+i_{H1}}\frac{dP_{iH1}}{dA_F}\right)+\ell_{H0}\frac{dW_{H0}}{dA_{FH}}+\frac{\ell_{H1}}{1+i_{H1}}\frac{dW_{H1}}{dA_{FH}}\\
=&-C_{FH0}\frac{dP_{FH0}}{dA_{FH}}-\sum_{i\in\{H,F\}}\frac{C_{iH1}}{1+i_{H1}}\frac{dP_{iH1}}{dA_F}+\frac{\ell_{H1}}{1+i_{H1}}\frac{dW_{H1}}{dA_{FH}}
\end{aligned}
$$

where the last expression follows from the fact that W_{H0} is fixed, so we have $\frac{dW_{H0}}{dA_{FH}}=\frac{dP_{HH0}}{dA_{FH}}=0$. Now to further simplify the last term $-\sum_{i\in\{H,F\}}\frac{C_{iH1}}{1+i_{H1}}\frac{dP_{iH1}}{dA_F}+\frac{\ell_{H1}}{1+i_{H1}}\frac{dW_{H1}}{dA_{FH}}$, we note that the Home goods market clearing condition in period 1 is

$$
\ell_{H1}=\frac{1}{A_H}C_{HH1}+\frac{\tau_{HF1}}{A_H}C_{HF1}
$$

and $P_{HH1}=W_{H1}/A_H$ so $dP_{HH1}=\frac{1}{A_H}dW_{H1}$. From this, we can rewrite

$$
\begin{aligned}
-\sum_{i\in\{H,F\}}C_{iH1}\frac{dP_{iH1}}{dA_F}+\ell_{H1}\frac{dW_{H1}}{dA_{FH}}&=-C_{HH1}\frac{dP_{HH1}}{dA_F}+C_{FH1}\frac{dP_{FH1}}{dA_{FH}}+(\frac{1}{A_H}C_{HH1}+\frac{\tau_{HF1}}{A_H}C_{HF1})\frac{dW_{H1}}{dA_{FH}}\\
&=-C_{FH1}\frac{dP_{FH1}}{dA_{FH}}+\frac{\tau_{HF1}}{A_H}C_{HF1}\frac{dW_{H1}}{dA_{FH}}\\
&=-C_{FH1}\frac{dP_{FH1}}{dA_{FH}}+C_{HF1}\frac{dP_{HF1}}{dA_{FH}}
\end{aligned}
$$

Substitute everything into Equation A.25 to obtain

$$
\frac{d\mathcal{U}_H}{dA_{FH}}=-\tilde{\lambda}C_{FH0}\frac{dP_{FH0}}{dA_{FH}}-\vartheta_0\frac{d\ell_0}{dA_{FH}}+\frac{\tilde{\lambda}}{1+i_{H1}}(C_{HF1}\frac{dP_{HF1}}{dA_{FH}}-C_{FH1}\frac{dP_{FH1}}{dA_{FH}}) \tag{A.26}
$$

and we substitute in $\tilde{\lambda}=\frac{u'(C_{H0})}{P_{H0}}=\frac{\beta(1+i_{H1})u'(C_{H1})}{P_{H1}}$ to obatin Equation A.17.

The terms have natural interpretations:

- The first term, $-\tilde{\lambda}C_{FH0}\frac{dP_{FH0}}{dA_{FH}}$ correspond to utility gains from cheaper consumption at $t=0$. As A_F increases, $\frac{dP_{FH0}}{dA_{FH}}$ takes on a negative value, so the utility increases.
- The second term $-\vartheta_0\frac{d\ell_0}{dA_{FH}}$ is the *labor wedge* at $t=0$. Labor is away from where the consumer wants to supply it. As a result of a higher A_F we have $\vartheta_0<0$ (from Proposition A.1) and $d\ell_0<0$, so there is a loss in welfare.
- The third term $C_{HF1}\frac{dP_{HF1}}{dA_{FH}}-C_{FH1}\frac{dP_{FH1}}{dA_{FH}}$ can be interpreted as the terms-of-trade in $t=1$; it pins down how much total revenue changes from an additional import versus an

additional export, multiplied by the marginal utility of a dollar at $t = 1$. This is affected by both the permanent increase in A_F and the trade imbalance that is incurred that affects future terms-of-trade (Lemma A.3).

Now we can prove the proposition. Consider a small shock that increases $A_F \to A_F + \epsilon$.

When $\sigma \to 1$, we know that $\vartheta_0 \to 0$, and $B_{H1} \to 0$. (This is known from Cole and Obstfeld (1991), but we can directly inspect the proof of Proposition A.1 and see that all the inequalities become equalities at $\sigma = 1$). So the *first-order relevant* welfare changes are the decrease in prices resulting from the productivity gains (term (1) and the productivity component of term (3)). Thus there is a welfare gain when $\sigma \to 1$.

On the other hand, as $\sigma \to \infty$, the welfare losses from term (2) are discrete. Specifically, consider the following formulation:

$$d\mathcal{U}_H = -\tilde{\lambda} C_{FH0} dP_{FH0} - \vartheta_0 d\ell_0 + \frac{\tilde{\lambda}}{1 + i_{H1}} (C_{HF1} dP_{HF1} - C_{FH1} dP_{FH1})$$

When $0 < dA_{FH} < \epsilon$, the first and third terms are bounded by the price changes, which are also at most epsilon: so we have

$$\| - \tilde{\lambda} C_{FH0} dP_{FH0} + \frac{\tilde{\lambda}}{1 + i_{H1}} (C_{HF1} dP_{HF1} - C_{FH1} dP_{FH1}) \| < \epsilon_M$$

On the other hand, as $\sigma \to \infty$, we have $\ell_0 \to 0$, and $\vartheta_0 \to \vartheta < 0$; there is a *discrete* loss of welfare associated with an *infinitesimal* change in A_F. As such, we have that for small ϵ and large σ, $\frac{d\mathcal{U}_H}{dA_{FH}} < 0$: there is a welfare loss associated with trade.

Remark. We conjecture that $\frac{d\mathcal{U}_H}{dA_{FH}}$ is *monotonic* in σ, so that there exists a σ^* such that there are welfare gains when $\sigma < \sigma^*$ and losses when $\sigma > \sigma^*$. This seems intuitive, as all three effects (gains from cheaper goods, labor wedge, and future terms-of-trade) should naturally be monotonic in σ. However, we are unable to prove this, and leave this as a possibility. □

A.6 Proofs for Subsection A.4

Here we prove the propositions for the optimal policy subsection. For this, we prove the following Lemma.

Lemma A.4. *The first-order effect of a tariff and subsidy on Home welfare can be written as:*

$$d\mathcal{U}_H = - \underbrace{\vartheta_0 d\ell_0}_{\text{labor wedge}} + \frac{u'(C_{H0})}{P_{H0}} [\underbrace{t_{FH0} P_{FH0} dC_{FH0}}_{C_{H0} \text{ distortion}} - \underbrace{d(s_{HF0} P_{HF0} C_{HF0})}_{\text{cost of subsidy}}]$$
$$+ \frac{\beta u'(C_{H1})}{P_{H1}} (\underbrace{C_{HF1} dP_{HF1} - C_{FH1} dP_{FH1}}_{\text{future terms-of-trade}})$$

Proof. Re-normalize the tariffs $t_{FH0} \to t_{FH0}/P_{FH0}$, and subsidies $s_{HF0} \to s_{HF0}/P_{HF0}$ so that they have the interpretation of a 'flat addition in price', and we can renormalize them back later.

The rest of the argument is similar to the proof of Proposition A.2 above. Home agent's lifetime utility is

$$\mathcal{U}_H = U(C_{HH0}, C_{FH0}, C_{HH1}, C_{FH1}, \ell_{H0}, \ell_{H1})$$

and is subject to the lifetime budget constraint

$$\begin{aligned} &P_{HH0}C_{HH0} + (P_{FH0} + t_{FH0})C_{FH0} + \frac{1}{1+i_{Ht}}(P_{HH1}C_{HH1} + P_{FH1}C_{FH1}) \\ &= W_{H0}\ell_{H0} + \frac{1}{1+i_{H1}}W_{H1}\ell_{H1} + T_{H0} \end{aligned}$$

with $T_{H0} = t_{FH0}C_{FH0} - s_{HF0}C_{HF0}$.

Analogously to the proof of Proposition A.2, the first-order effect of any policy on welfare can be written as

$$d\mathcal{U}_H = \sum_{t=0}^{1}\sum_{i\in\{H,F\}} \frac{dU}{dC_{iHt}}dC_{iHt} + \sum_{t=0}^{1}\frac{dU}{d\ell_{Ht}}d\ell_{Ht} \tag{A.27}$$

If we denote by $\tilde{\lambda}$ the Lagrange multiplier on the lifetime budget constraint, we have:

$$\begin{aligned} \frac{dU}{dC_{HH0}} &= \tilde{\lambda}P_{HH0}, & \frac{dU}{dC_{FH0}} &= \tilde{\lambda}(P_{FH0} + t_{FH0}) \\ \frac{dU}{dC_{HH1}} &= \frac{\tilde{\lambda}}{1+i_{H1}}P_{HH1}, & \frac{dU}{dC_{FH1}} &= \frac{\tilde{\lambda}}{1+i_{H1}}P_{FH1} \\ \frac{dU}{d\ell_{H0}} &= -\vartheta_0 - \tilde{\lambda}W_{H0}, & \frac{dU}{d\ell_{H1}} &= -\frac{\tilde{\lambda}}{1+i_{H1}}W_{H1} \end{aligned}$$

Plugging these into Equation A.27, we get

$$\begin{aligned} d\mathcal{U}_H =& \tilde{\lambda}\left[\sum_{i\in\{H,F\}}\left(P_{iH0}dC_{iH0} + \frac{P_{iH1}}{1+i_{H1}}dC_{iH1}\right) - W_{H0}d\ell_{H0} - \frac{W_{H1}}{1+i_{H1}}d\ell_{H1}\right] \\ &+ \tilde{\lambda}t_{FH0}dC_{FH0} - \vartheta_0 d\ell_0 \end{aligned}$$

Now the household lifetime budget constraint, with the tax revenue plugged in, is

$$\begin{aligned} &P_{HH0}C_{HH0} + P_{FH0}C_{FH0} + \frac{1}{1+i_{Ht}}(P_{HH1}C_{HH1} + P_{FH1}C_{FH1}) \\ &= W_{H0}\ell_{H0} + \frac{1}{1+i_{H1}}W_{H1}\ell_{H1} - s_{HF0}C_{HF0} \end{aligned}$$

Take the derivative of this, and rearrange to obtain

$$\sum_{i\in\{H,F\}}\left(P_{iH0}dC_{iH0}+\frac{P_{iH1}}{1+i_{H1}}dC_{iH1}\right)-W_{H0}d\ell_{H0}-\frac{1}{1+i_{H1}}W_{H1}d\ell_{H1}$$
$$=\frac{1}{1+i_{H1}}(C_{HF1}dP_{HF1}-C_{FH1}dP_{FH1})-d(s_{HF0}C_{HF0})$$

where we use the fact that $dP_{HH0}=dP_{FH0}=dW_{H0}=0$ by rigidity, and then further simplify using the Home labor market clearing condition. Then the first-order welfare effects are given by

$$\begin{aligned}d\mathcal{U}_H&=-\vartheta_0 d\ell_0+\tilde{\lambda}t_{FH0}dC_{FH0}-\tilde{\lambda}d(s_{HF0}C_{HF0})+\frac{\tilde{\lambda}}{1+i_{H1}}(C_{HF1}dP_{HF1}-C_{FH1}dP_{FH1})\\&=-\vartheta_0 d\ell_0+\frac{u'(C_{H0})}{P_{H0}}[t_{FH0}dC_{FH0}-d(s_{HF0}C_{HF0})]+\frac{\beta u'(C_{H1})}{P_{H1}}(C_{HF1}dP_{HF1}-C_{FH1}dP_{FH1})\end{aligned}$$

□

Lemma A.1. *The optimal short-run tariff rate on imports t_{FH0} satisfies*

$$t_{FH0}=\frac{1}{P_{FH0}}\left[\underbrace{\frac{\vartheta_0}{\tilde{\lambda}}\frac{\partial\ell_{H0}}{\partial C_{FH0}}}_{\text{labor wedge}}-\frac{1}{(1+i_{H1})}\underbrace{(\ell_{HF1}\frac{\partial W_{H1}}{\partial C_{FH0}}-\ell_{FH1}\frac{\partial W_{F1}}{\partial C_{FH0}})}_{\text{future terms-of-trade}}+\underbrace{s_{HF0}P_{HF0}\frac{\partial C_{HF0}}{\partial C_{FH0}}}_{\text{subsidy externality}}\right] \quad (A.28)$$

The optimal short-run subsidy rate on exports s_{HF0} satisfies

$$s_{HF0}=\frac{1}{P_{HF0}}\left[-\underbrace{\frac{\vartheta_0}{\tilde{\lambda}}\frac{\partial\ell_{H0}}{\partial C_{HF0}}}_{\text{labor wedge}}+\frac{1}{(1+i_{H1})}\underbrace{(\ell_{HF1}\frac{\partial W_{H1}}{\partial C_{HF0}}-\ell_{FH1}\frac{\partial W_{F1}}{\partial C_{HF0}})}_{\text{future terms-of-trade}}-\underbrace{P_{HF0}C_{HF0}\frac{\partial s_{HF0}}{\partial C_{HF0}}}_{\text{terms-of-trade today}}\right] \quad (A.29)$$

where $\tilde{\lambda}$ is the Lagrange multiplier on the lifetime budget constraint.

Proof. Under variation in tariffs, the optimal tariff rate with $d\mathcal{U}_H=0$ will satisfy

$$t_{FH0}=\frac{1}{P_{FH0}\frac{dC_{FH0}}{dt_{FH0}}}\left[\frac{\vartheta_0}{\tilde{\lambda}}\frac{d\ell_{H0}}{dt_{FH0}}+\frac{d(s_{HF0}P_{HF0}C_{HF0})}{dt_{HF0}}-\frac{1}{(1+i_{H1})}(\ell_{HF1}\frac{dW_{H1}}{dt_{FH0}}-\ell_{FH1}\frac{dW_{F1}}{dt_{FH0}})\right]$$

The multiplier $\frac{1}{P_{FH0}\frac{dC_{FH0}}{dt_{FH0}}}<0$ corresponds to the inverse elasticity of domestic demand with respect to tariffs; a lower elasticity implies a higher tariff rate. The first term is the effect of tariff on the labor wedge. Since $\frac{d\ell_{H0}}{dt_{FH0}}>0$, when there is unemployment ($\vartheta_0<0$), we want a higher tariff. The second term is the effect of tariffs on subsidy revenue; a higher tariff will decrease real wage in Foreign, leading them to work/consume less, decreasing subsidy revenue. The third term is how much future terms-of-trade moves, in terms of how much marginal revenue from exports vs expenditure from imports move. A higher tariff will lead to less borrowing, leading to improving terms-of-trade, increasing the term.

In summary, when there is unemployment ($\vartheta_0 < 0$), the three terms inside the bracket are all negative; thus the optimal tariff t_{FH0} is *positive*.

A special case is when the Home economy is small; here today's tariffs cannot affect (1) tomorrow's terms-of-trade and (2) the subsidy revenue, so the optimal tariff is simply

$$t_{FH0} = \frac{1}{P_{FH0}\frac{dC_{FH0}}{dt_{FH0}}} \frac{\vartheta_0}{\tilde{\lambda}} \frac{d\ell_{H0}}{dt_{FH0}}$$

and this immediately shows that (1) the tariff is positive and (2) the tariff leaves some unemployment ($\vartheta_0 < 0$; otherwise, we have a contradiction.)

Now, considering variation in subsidies, we have

$$\begin{aligned} s_{HF0} =& \frac{1}{P_{HF0}\frac{dC_{HF0}}{ds_{HF0}}}[-P_{HF0}C_{HF0} + t_{FH0}P_{FH0}\frac{dC_{FH0}}{ds_{HF0}} \\ & - \frac{\vartheta_0}{\tilde{\lambda}}\frac{d\ell_{H0}}{ds_{HF0}} + \frac{1}{(1+i_{H1})}(\ell_{HF1}\frac{dW_{H1}}{ds_{HF0}} - \ell_{FH1}\frac{dW_{F1}}{ds_{HF0}})] \end{aligned}$$

The multiplier $\frac{1}{P_{HF0}\frac{dC_{HF0}}{s_{HF0}}} > 0$ corresponds to the inverse elasticity of foreign demand with respect to exports, and is positive. The first term is the resource cost of the subsidy; it costs to sell cheap goods. The second term is how much consumption distortion by tariffs is affected by subsidies; with a positive tariff, domestic subsidies will be a resource cost that reduces spending overall. The last two terms deliver similar intuition to the tariff case, with both forces implying a *positive* subsidy. □

Proposition A.3. *If there is unemployment at the zero-tariff economy ($\vartheta_{H0} < 0$ when $t_{FH0} = 0$), the optimal tariff t_{FH0} is positive and is increasing in the size of the trade shock A_{FH0}.*

Proof. When $\vartheta_{H0} < 0$, all three terms in the optimal tariff formula (Equation A.19) are positive:

- The first term is positive since an increase in imports C_{FH0} reduce demand for Home labor.
- the second is positive since an increase in C_{FH0} decrease W_{H1} relative to W_{F1} tomorrow (transfer affecting future terms-of-trade effect).
- The third term is positive since an increase in C_{FH0} is associated with an increase in exports C_{HF0}.

Likewise, all three forces increase when the magnitude of A_{FH0} increases. $\square$

Proposition A.4. *When $\gamma = 1$, optimal monetary policy R_{H1} satisfies the following equation:*

$$0 = \underbrace{-\vartheta_0 \frac{d\ell_0}{dR_{H1}}}_{\text{wedge}} + \tilde{\lambda}_r [\underbrace{R_{H1} t_{FH0} \frac{P_{FH0}}{P_{H0}} \frac{dC_{FH0}}{dR_{H1}}}_{\text{tariff fiscal externality}} + \underbrace{(NX_0)}_{\text{intertemporal TOT}}], \tag{A.30}$$

where $\tilde{\lambda}_r$ is the Lagrange multiplier on the Home lifetime real budget constraint normalized by P_{H0}.

As a special case, when $t_{FH0} = 0$, the optimal monetary policy R_{H1} is such that $\vartheta_0 > 0$: it is optimal to loosen monetary policy beyond clearing the output gap.

Proof. Since the central bank is choosing the real rate R_{H1}, we rewrite the budget constraint to incorporate R_{H1}:

$$R_{H1} \frac{1}{P_{H0}} (P_{HH0} C_{HH0} + (P_{FH0} + t_{FH0}) C_{FH0}) + \frac{1}{P_{H1}} (P_{HH1} C_{HH1} + P_{FH1} C_{FH1})$$
$$= R_{H1} \frac{1}{P_{H0}} (W_{H0} \ell_{H0} + T_{H0}) + \frac{W_{H1}}{P_{H1}} \ell_{H1}$$

Then the Lagrange multiplier on this *real* budget constraint is $\tilde{\lambda}_r = \frac{u'(C_{H0})}{R_{H1}} = \beta u'(C_{H1})$

Recall that the central bank's monetary policy rule sets interest rate according to Equation A.2:

$$\log(1 + i_{H1}) = -\log(\beta) + \log(\frac{P_{H1}}{P_{H0}}) + \epsilon_{H0} \ \Leftrightarrow \ R_{H1} = \frac{1}{\beta} \exp(\epsilon_{H0})$$

We consider variations in $\exp(\epsilon_{H0})$ that leave inflation constant; notably, P_{H1} does not move in this variation.

Transform the marginal change in utility in a way analogous to Lemma A.4 to write

$$d\mathcal{U}_H = \tilde{\lambda}_r \left[\sum_{i \in \{H,F\}} \left(R_{H1} \frac{P_{iH0}}{P_{H0}} dC_{iH0} + \frac{P_{iH1}}{P_{H1}} dC_{iH1} \right) - R_{H1} \frac{W_{H0}}{P_{H0}} d\ell_{H0} - \frac{W_{H1}}{P_{H1}} d\ell_{H1} \right]$$
$$+ \tilde{\lambda}_r R_{H1} \frac{t_{FH0}}{P_{H0}} dC_{FH0} - \vartheta_0 d\ell_0$$

Taking the derivative of the budget constraint, we get:

$$\sum_{i \in \{H,F\}} \left(R_{H1} \frac{P_{iH0}}{P_{H0}} dC_{iH0} + \frac{P_{iH1}}{P_{H1}} dC_{iH1} \right) - R_{H1} \frac{W_{H0}}{P_{H0}} d\ell_{H0} - \frac{W_{H1}}{P_{H1}} d\ell_{H1}$$
$$= \frac{1}{P_{H1}} (C_{HF1} dP_{HF1} - C_{FH1} dP_{FH1}) + dR_{H1} (\frac{1}{P_{H0}} NX_{H0})$$

where $NX_{H0} = (W_{H0} \ell_{H0} + T_{H0}) - P_{HH0} C_{HH0} - (P_{FH0} + t_{FH0}) C_{FH0} = \frac{B_{H1}}{R_{H1}}$ is the net export in period 0. Plugging this in and replacing $t_{FH0} \to t_{FH0}P_{FH0}$, we get

$$d\mathcal{U}_H = -\vartheta_0 d\ell_0 + \tilde{\lambda}_r[R_{H1}\frac{t_{FH0}P_{FH0}}{P_{H0}}dC_{FH0} \\ + \frac{1}{P_{H1}}(C_{HF1}dP_{HF1} - C_{FH1}dP_{FH1}) + dR_{H1}(\frac{1}{P_{H0}}NX_{H0})]$$

Now we note that when $\gamma = 1$, the equilibrium level of real balances $\frac{B_{H1}}{P_{H1}}$ do not depend on R_{H1}. This is because after any change in $R_{H1} \to \zeta R_{H1}$ for some constant ζ, the equilibrium conditions exactly hold if we replace $C_{ij1}, C_{i1}, \ell_{i1}$ with $\zeta C_{ij1}, \zeta C_{i1}, \zeta \ell_{i1}$; monetary policy affects period 0 without affecting any real variables in period 1. (We can verify by inspecting the equilibrium conditions)

Thus, the period 1 variables do not depend on R_{H1}, and under the optimal monetary policy, the above equation becomes

$$0 - \vartheta_0 d\ell_0 + \tilde{\lambda}_r[R_{H1}\frac{t_{FH0}P_{FH0}}{P_{H0}}dC_{FH0} + dR_{H1}(\frac{1}{P_{H0}}NX_{H0})] \tag{A.31}$$

which is exactly the equation in the proposition. □

B Model of Quantity Frictions

This section studies a variant of the stylized model in Section A in which we replace *nominal* wage rigidity with a *quantity* rigidity in labor supply. The key message is that the sign of the short-run trade balance hinges on the impact path of the relative wage. Under nominal rigidity and a peg, the Home (US) relative wage is "too high" on impact, generating a short-run trade deficit and involuntary unemployment. Under quantity rigidity, market clearing instead requires the relative wage to *overshoot*, generating a short-run trade surplus and a (temporarily) overheated labor market.

The restriction that labor quantities cannot adjust on impact is a reduced-form way to capture real-world frictions that slow the reallocation of employment across firms, sectors, or regions. Examples include: (i) *search and matching frictions*, where hiring requires time-consuming vacancy posting, screening, and match formation (e.g. Mortensen and Pissarides, 1994), so employment responds gradually even when wages are flexible; (ii) *sectoral or occupational mobility costs*, as in models where workers face switching frictions across sectors or regions (e.g. Artuç et al., 2010; Dix-Carneiro et al., 2023), so short-run employment shares are effectively predetermined

Environment. There are two countries $i \in \{H, F\}$ (Home and Foreign), one sector, and two dates $t \in \{0, 1\}$ followed by a terminal condition. There is no nominal rigidity; nominal variables only pin down units, so we normalize the exchange rate to $e_t \equiv 1$ and take Home's nominal wage as numéraire (equivalently, all allocations are real). Preferences are

$$U_i = u(C_{i0}) - v(L_{i0}) + \beta\big(u(C_{i1}) - v(L_{i1})\big), \qquad u'(C) = C^{-1/\gamma},$$

with v increasing and convex. Final consumption in each country is CES over origins with elasticity $\sigma > 1$, yielding the standard CPI and expenditure shares:

$$P_{it} = \left(P_{Hit}^{1-\sigma} + P_{Fit}^{1-\sigma}\right)^{\frac{1}{1-\sigma}}, \qquad \lambda_{jit} = \frac{P_{jit}^{1-\sigma}}{P_{Hit}^{1-\sigma} + P_{Fit}^{1-\sigma}}.$$

Technology and delivered prices. Delivered unit costs are destination-specific:

$$P_{jit} = \frac{w_{jt}}{A_{ji}}, \tag{B.1}$$

where $A_{ji} > 0$ is productivity of origin j delivering to destination i. "Home bias" corresponds to $A_{HH} > A_{FH}$ and $A_{FF} > A_{HF}$. A trade shock at $t = 0$ is a permanent increase in Foreign export productivity to Home:

$$A_{FH} \uparrow \quad \text{(holding all other } A_{ji} \text{ fixed).}$$

Goods demand and labor-market clearing. Demand satisfies

$$P_{jit}C_{jit} = \lambda_{jit}P_{it}C_{it}.$$

Labor is the only factor. Clearing in origin-j labor market is

$$L_{jt} = \sum_{i\in\{H,F\}} \frac{C_{jit}}{A_{ji}}. \tag{B.2}$$

Intertemporal trade and external assets. Countries trade a one-period bond at gross interest rate $1+i$. The date-t budget constraint is

$$P_{it}C_{it} + B_{i,t+1} = w_{it}L_{it} + (1+i)B_{it}, \qquad B_{i,2} = 0,$$

and world bond market clearing implies $B_{Ht} + B_{Ft} = 0$. Define Home exports and imports in value terms:

$$X_t \equiv \lambda_{HFt}P_{Ft}C_{Ft}, \qquad M_t \equiv \lambda_{FHt}P_{Ht}C_{Ht},$$

so the Home net foreign asset position evolves as

$$B_{H,t+1} = (1+i)B_{Ht} + X_t - M_t, \qquad B_{H0} = 0.$$

Quantity rigidity. Labor is predetermined at $t=0$:

$$L_{H0} = \bar{L}_H, \qquad L_{F0} = \bar{L}_F, \tag{B.3}$$

where $(\bar{L}_H, \bar{L}_F)$ are the pre-shock steady-state labor quantities. At $t=1$ labor is flexible and chosen efficiently:

$$v'(L_{i1}) = \frac{w_{i1}}{P_{i1}}u'(C_{i1}), \qquad i \in \{H,F\}. \tag{B.4}$$

At $t=0$, (B.4) need not hold because (B.3) fixes labor.

Flexible-quantity benchmark. Let $L_i^*(A)$ denote the (static) flexible-quantity labor choice under fundamentals $A \equiv \{A_{ji}\}$. We assume the trade shock reduces desired Home labor and raises desired Foreign labor:

Assumption 1 (Direction of flexible-quantity labor response)**.** *Let A^- denote pre-shock productivities and A^+ denote post-shock productivities with A_{FH} higher. Then*[12]

$$L_H^*(A^+) < L_H^*(A^-), \qquad L_F^*(A^+) > L_F^*(A^-).$$

B.1 Main result

Define the relative wage $\omega_t \equiv w_{Ht}/w_{Ft}$ and the Home labor wedge

$$\vartheta_{Ht} \equiv v'(L_{Ht}) - \frac{w_{Ht}}{P_{Ht}} u'(C_{Ht}).$$

A positive wedge $\vartheta_{Ht} > 0$ corresponds to an overheated labor market (labor is high relative to the MRS at the prevailing real wage).

Lemma B.1 (Relative wage: uniqueness and monotonicity)**.** *Fix productivities $A \equiv \{A_{ji}\}_{j,i \in \{H,F\}}$ and labor supplies (L_H, L_F) at a given date t. Let $\omega \equiv w_H/w_F$ denote the Home relative wage, and normalize $w_F = 1$ so that all delivered prices satisfy $P_{Hit} = \omega/A_{Hi}$ and $P_{Fit} = 1/A_{Fi}$. Then there exists a unique $\omega = \omega(A; L_H, L_F)$ such that labor markets clear:*

$$L_j = \sum_{i \in \{H,F\}} \frac{C_{ji}(\omega; A, L_H, L_F)}{A_{ji}}, \qquad j \in \{H, F\}.$$

Moreover, this equilibrium relative wage is monotone in labor quantities:

$$\frac{\partial \omega(A; L_H, L_F)}{\partial L_H} < 0, \qquad \frac{\partial \omega(A; L_H, L_F)}{\partial L_F} > 0.$$

Proof sketch. Given (A, L_H, L_F), the CES demand system implies that for each destination i, expenditure shares $\lambda_{Hi}(\omega)$ are strictly decreasing in ω and $\lambda_{Fi}(\omega) = 1 - \lambda_{Hi}(\omega)$ are strictly increasing. Using labor-market clearing $L_j = \sum_i C_{ji}/A_{ji}$ and $C_{ji} = \lambda_{ji} P_i C_i / P_{ji}$, one can write equilibrium as a single fixed point in ω:

$$\Psi(\omega; A, L_H, L_F) \equiv \frac{L_H}{L_F} - \frac{\mathcal{D}_H(\omega; A, L_H, L_F)}{\mathcal{D}_F(\omega; A, L_H, L_F)} = 0,$$

where $\mathcal{D}_j(\omega; \cdot)$ denotes effective labor demand for origin j (the right-hand side implied by goods demand). Gross substitutability ($\sigma > 1$) implies $\mathcal{D}_H/\mathcal{D}_F$ is strictly decreasing in ω, so Ψ is strictly increasing and admits a unique zero. Finally, an increase in L_H shifts the left-hand side L_H/L_F up; to restore $\Psi(\omega) = 0$ with Ψ increasing, ω must fall, implying $\partial\omega/\partial L_H < 0$. Similarly, increasing L_F lowers L_H/L_F, so ω must rise, implying $\partial\omega/\partial L_F > 0$. □

[12] This assumption, along with the assumption of completely rigid labor supply, can be replaced by any assumption that says sticky labor supply is less (more) than the flexible labor supply at Home (Foreign).

Proposition B.1 (Quantity rigidity reverses the short-run adjustment). *Maintain the above environment and Assumption 1. Then following an increase in A_{FH}:*

(a) Relative wage overshoots in the short-run: $\omega_0 < \omega_1$.

(b) Home runs a short-run trade surplus: $B_{H1} > 0$ (equivalently, $X_0 - M_0 > 0$).

(c) There is overheating at Home on impact: $\vartheta_{H0} > 0$.

Proof. We prove each statement in turn.

1. $\omega_0 < \omega_1$ **(relative wage overshooting).** By Lemma B.1, for fixed productivities A the equilibrium relative wage $\omega(A; L_H, L_F)$ is uniquely pinned down by labor-market clearing and is strictly decreasing in L_H and strictly increasing in L_F.

For each date t, equilibrium prices and expenditure shares depend on wages only through the relative wage ω_t. Given (A, L_t), the labor-market clearing system (B.2) pins down a unique ω_t.

Moreover, the market-clearing relative wage is monotone in labor quantities: holding A fixed, increasing Home labor L_H (raising Home supply) requires a lower Home relative price to clear goods markets and hence a *lower* Home wage relative to Foreign, i.e. ω weakly decreases; increasing Foreign labor L_F (raising Foreign supply) weakly increases ω.

Under quantity rigidity, $(L_{H0}, L_{F0}) = (\bar{L}_H, \bar{L}_F)$ are fixed at pre-shock levels. Under flexibility at $t = 1$, Assumption 1 implies $L_{H1} < \bar{L}_H$ and $L_{F1} > \bar{L}_F$ in the post-shock allocation. By the monotonicity just stated, moving from (L_{H0}, L_{F0}) to (L_{H1}, L_{F1}) raises ω, hence $\omega_0 < \omega_1$.

2. $B_{H1} > 0$ **(short-run surplus).** The proof follows the same expenditure-switching vs. relative-inflation decomposition as in Proposition 2, with the key difference that the relative wage now satisfies $\omega_0 < \omega_1$ rather than $\omega_0 > \omega_1$. In particular, one can show that the Home export-to-import ratio satisfies

$$\frac{X_0}{M_0} > \frac{X_1}{M_1} \iff \underbrace{\left(\frac{\omega_1}{\omega_0}\right)^{\sigma-1}}_{\text{expenditure switching}} > \underbrace{\left(\frac{P_{F1}/P_{F0}}{P_{H1}/P_{H0}}\right)^{\sigma-\gamma}}_{\text{relative inflation}},$$

where the left term is governed by σ and the right term by γ. Under home bias, the CPI ratio P_{Ht}/P_{Ft} is monotone in ω_t, so $\omega_0 < \omega_1$ implies $P_{H1}/P_{H0} > P_{F1}/P_{F0}$ (Home experiences relatively higher inflation between 0 and 1). When $\sigma > \gamma$, expenditure switching dominates, yielding $X_0/M_0 > X_1/M_1$, hence $X_0 > M_0$ and therefore $B_{H1} > 0$.

3. $\vartheta_{H0} > 0$ **(overheating).** At $t = 1$, labor is flexible and satisfies (B.4), so $\vartheta_{H1} = 0$. At $t = 0$, labor is fixed at the pre-shock level $L_{H0} = \bar{L}_H$, while the flexible-quantity benchmark would choose $L_H^*(A^+) < \bar{L}_H$ by Assumption 1. Thus, relative to the flexible-quantity allocation, Home labor is *too high* on impact. Since v' is increasing, this pushes up the marginal disutility term $v'(L_{H0})$. At the same time, Step 1 implies $\omega_0 < \omega_1$, i.e. Home's relative wage is low on impact; in particular, Home's impact real wage w_{H0}/P_{H0} is below its flexible-quantity counterpart. Together these imply

$$v'(L_{H0}) \;>\; \frac{w_{H0}}{P_{H0}} u'(C_{H0}),$$

i.e. $\vartheta_{H0} > 0$, which corresponds to an overheated labor market at Home on impact.

This completes the proof. □

Proposition B.1 delivers the opposite short-run predictions from the nominal-rigidity mechanism in Proposition 2. With quantity rigidity, market clearing requires the impact relative wage to be *too low*, so Home runs a short-run *trade surplus* and the labor market *overheats* ($\vartheta_{H0} > 0$) rather than exhibiting involuntary unemployment. This comparison clarifies why the *nature* of labor market frictions – prices versus quantities – is central for the joint dynamics of trade balances and labor market slack. The stylized facts of the 2000s (Figure 1 in the main text) align with the nominal-wage-rigidity mechanism, not the quantity-rigidity alternative, suggesting that nominal adjustment frictions and the resulting involuntary unemployment are an important part of the China-shock transmission.

C Data and Calibration

This Appendix builds on Section 3.1 and describes the construction of our data and our calibration strategy.

C.1 WIOD data

Our main source of trade data is the World Input-Output Database (WIOD) 2016 release Timmer et al. (2015). The World Input-Output Tables in the WIOD cover 44 countries and a rest-of-world aggregate, and the data span from 2000 to 2014.

List of country aggregates and sectors. We follow Dix-Carneiro et al. (2023) and divide the world into six country aggregates and six sectors, focusing on the US (country 1) and China (country 2). Table C.1 shows our country aggregates, and Table C.2 shows how the 56 sectors in the WIOD are mapped to the six broad sectors considered in our model.

	Group	Countries in group
1	USA	USA
2	China	China
3	Europe	Austria (AUT), Belgium (BEL), Bulgaria (BGR), Switzerland (CHE), Cyprus (CYP), Czech Republic (CZE), Germany (DEU), Denmark (DNK), Spain (ESP), Estonia (EST), Finland (FIN), France (FRA), United Kingdom (GBR), Greece (GRC), Croatia (HRV), Hungary (HUN), Ireland (IRL), Italy (ITA), Lithuania (LTU), Luxembourg (LUX), Latvia (LVA), Malta (MLT), Netherlands (NLD), Norway (NOR), Poland (POL), Portugal (PRT), Romania (ROU), Slovakia (SVK), Slovenia (SVN), Sweden (SWE)
4	Asia/Oceania	Australia (AUS), Japan (JPN), Korea (KOR), Taiwan (TWN)
5	Americas	Brazil (BRA), Canada (CAN), Mexico (MEX)
6	Rest of World	Indonesia (IDN), India (IND), Russia (RUS), Turkey (TUR), ROW

Table C.1: Country definitions

	Sector aggregate	WIOD sector
1	Agriculture and Mining	Agriculture (1-3), Mining (4)
2	LT Manufacturing	Wood (7), Paper and Printing (8-9), Coke and Petroleum (10), Basic and Fabricated Metals (15-16), other mfg (22)
3	MT Manufacturing	Food (5), Textiles (6), Rubber (13), Mineral (14)
4	HT Manufacturing	Chemical and Pharmaceutical (11-12), Machinery, Computers and Motor Vehicles (17-23)
5	LT Services	Utilities (24-26), Construction (27), Wholesale and Retail (28-30), Transportation (31-35), Accommodation (36), Other Service (54), Household (55), Miscellaneous (56)
6	HT Services	Media and Telecommunications (37-39), IT (40), Finance (41-43), Real Estate (44), Legal (45), Architecture (46), Science (47), Advertising (48), Other Professional (49), Government and Education (50-52), Health (53)

Table C.2: Sector definitions

Note: The numbers inside parentheses denote the WIOD sectors, which follow the International Standard Industrial Classification revision 4 (ISIC Rev. 4). The classification of the six broad sectors follow Dix-Carneiro et al. (2023). In the sector aggregate classifications, (L,M,H) stand for Low-, Medium-, High- and T stands for Technology.

C.1.1 Constructed variables

The World Input-Output Table of WIOD contains the following raw data:

- M_{ijt}^{sn}, goods produced in sector s at country i that is used as inputs for goods in sector n at country j.
- F_{ijt}^{s}, goods produced in sector s at country i that is used as final expenditure in country j. (There are five expenditure categories; three consumption and two investment. We aggregate them.)
- GO_{it}^{s}, VA_{it}^{s}, ITM_{it}^{s} denote gross output, value added and international transport margins in country, sector (i, s) respectively.

Since the data comprise 44 countries and 56 sectors, we map this into our 6-sector, 6-country model by a direct sum.

From M_{ijt}^{sn} and F_{ijt}^{s}, we obtain the following:

- X_{ijt}^{s}, the total exports from i to j in sector s, given by

$$X_{ijt}^{s} = F_{ijt}^{s} + \sum_{n} M_{ijt}^{sn}$$

- λ_{ijt}^{s}, the share of sector s expenditure in j that originates from i, given by

$$\lambda_{ijt}^{s} = \frac{X_{ijt}^{s}}{\sum_{i'} X_{i'jt}^{s}}$$

- IO_{it}^{sn}, the input-output table of country i, given by

$$IO_{it}^{sn} = \sum_{i'} M_{i'it}^{sn}$$

- E_{it}^{s}, expenditure of country i in sector s, by

$$E_{it}^{s} = \sum_{i'} F_{i'it}^{s}$$

We also obtain the net exports of country i by

$$NX_{it} = \sum_{s} VA_{it}^{s} + \sum_{s} ITM_{it}^{s} - \sum_{s} C_{it}^{s}$$

To ensure that net exports sum to zero, we assign any error to the rest-of-world.

From the WIOD Socio-Economic Accounts (SEA), we obtain the following:

- Industry-level employment $L_{i,2000}^{s}$ at period $t = 0$: we use the 2000 values as the initial condition for our model.

- Sectoral prices. We obtain $P_{it}^{s,dom}$, the domestic output price (price deflator) of country i in WIOD sector j expressed in millions of dollars. We closely follow the procedure in Dix-Carneiro et al. (2023) to construct $P_{it}^{s,dom}$ for our 6 country aggregates i and 6 sectors j.

We use the constructed $\{X_{ijt}^{s}, \lambda_{ijt}^{s}, IO_{it}^{sn}, E_{it}^{s}, NX_{it}, VA_{it}, GO_{it}, L_{i,2000}^{s}, P_{it}^{s,dom}\}$ in our calibration.

C.2 CPS data

To construct labor transition across sectors, we use the Current Population Survey (CPS). We rely on the annual retrospective questions from the Annual Social and Economics Supplement (ASEC) of the CPS. We map the 1990 Census industry codes in the CPS to the WIOD sector codes (based on ISIC Rev. 4) then into our 6 sectors, and obtain the transition ratio of employment from sector s to sector n at time t:

$$\mu_{t}^{sn} = \frac{1_{s,t-1} 1_{nt} wt_{it}}{\sum_{s'} 1_{s,t-1} 1_{s't} wt_{it}}$$

C.3 Calibration of parameters outside of the model

The parameters in Panel A of Table 1 are calibrated outside the model. We make note of two parameters important in our model, which are σ (Armington elasticity) and κ (slope of the New Keynesian Phillips Curve with respect to the output gap).

Calibration of σ. We use $\sigma = 5$ as the elasticity of within-sector goods substitution across different origins. This is identical to the elasticity used in Rodríguez-Clare et al. (2022), and generates the same gravity trade equation as in Dix-Carneiro et al. (2023)[13]. In Appendix F, we assess the sensitivity of our results to different levels of the elasticity. As long as the elasticity is greater than 1, our results are qualitatively identical, as we show in Section 2.3.

Calibration of κ. Hazell et al. (2022) estimate the slope of the following equation for unemployment:

$$\pi_t = -\kappa' \hat{u}_t + \beta E_t \pi_{t+1} + \nu_t$$

where $\hat{u}_t = \bar{u}_t - u_t$ is the gap from full employment. Using inter-state panel data at a quarterly frequency, they find $\kappa' = 0.0062$. In our context, our time is annual, so the equivalent form is

$$\pi_t = -\kappa'(1 + \beta^{1/4} + \beta^{2/4} + \beta^{3/4})\hat{u}_t + \beta E_t \pi_{t+1}.$$

Moreover, their measure of unemployment is $u_t = 1 - N_{Ht}$. In our context, our wage NKPC is given by

$$\log(1 + \pi_t^w) = \kappa(v'(\ell_t) - \frac{w_t}{P_t} u'(C_t)) + \beta \log(1 + \pi_{t+1}^w)$$

The output gap can be rewritten as $v'(\ell_t) - \frac{w_t}{P_t}u'(C_t) = v'(\ell_t) - v'(\ell_t^D)$ where ℓ_t^D is the desired labor supply at this level. Linearizing v near the full-employment level $\ell_t = 1$, we have

$$\pi_t^w = \kappa \frac{\theta}{\varphi}(\ell_t - 1) + \beta \pi_{t+1}^w$$

Lastly, if wages increase by $X\%$ everywhere, the price index would also increase proportionately because production technology has constant returns to scale. Thus, the κ value consistent with Hazell et al. (2022) is given by

$$\kappa = \varphi \kappa' \frac{1}{\theta}(1 + \beta^{1/4} + \beta^{2/4} + \beta^{3/4}) = 0.05$$

[13] The formulation is different, because Dix-Carneiro et al. (2023) use a Eaton-Kortum model of perfect competition with a continuum of goods. In our model, the gravity equation is governed by a scale of $(1 - \sigma)$, whereas in their model it is governed by $-\lambda$ where λ is the Frechet scale parameter. Dix-Carneiro et al. (2023) use $\lambda = 4$, generating the same gravity equation.

using our values of $\varphi = 2, \beta = 0.95$, and the population average of θ given by 0.966.

C.4 Calibration of parameters in our model

The next paragraphs detail the calibration of parameters in Panel B of Table 1, using the WIOD and CPS data above. In this section, a variable with a bar above ($\overline{X}$) denotes variables directly observable in the data, and all other variables denote equilibrium objects.

We first note that the preference shares and production function parameters are directly measurable from the data:

$$\alpha_{it}^{s} = \frac{\overline{E}_{it}^{s}}{\sum_n \overline{E}_{it}^{n}} \tag{C.1}$$

$$\phi_{it}^{sn} = \frac{\overline{IO}_{it}^{sn}}{\sum_{s'} \overline{IO}_{it}^{s'n}} \tag{C.2}$$

$$\phi_{it}^{s} = \frac{\overline{VA}_{it}^{s}}{\overline{GO}_{it}^{s}} \tag{C.3}$$

The calibration of the remaining parameters $A_{ijt}^{s}, \delta_{it}^{s}, \eta_{it}^{s}, \chi_{it}^{sn}$ requires use of our model. We first calibrate the 2000 values, and then calibrate the 'shocks' to these variables.

C.4.1 Calibration of the initial period

Since δ_{it} governs intertemporal preference shocks, we need not calibrate it for the year 2000. We assume that the model is in steady-state in the year 2000.**Importantly, we do not assume balanced trade in this initial steady state; instead, we match the observed trade imbalances (**$NX_{i,2000} \neq 0$**) from the data.** This implies two conditions: first, the labor market is in full employment (no output gap); second, the observed labor distribution L_i^s in 2000 represents the stationary distribution given the initial parameters $\{A_{ijt}^{s}, \chi_{it}^{sn}, \theta_{it}^{s}\}$.

Suppressing the time subscripts t, we calibrate the 2000 values of $\{A_{ij}^{s}, \chi_i^{sn}, \theta_i^{s}\}$ to match the following observed data:

- Productivity A_{ij}^{s} matches the sector-level expenditure shares λ_{ij}^{s} and sectoral value added.
- Intensity of labor disutility θ_i^s is such that labor supply $\ell_i^s = 1$ in the initial period.
- Migration costs χ_i^{sn} match the observed migration flows (for the US) and ensure the observed sectoral employment shares are stationary (for China). We assume no migration for countries outside of US and China.

Productivity A_{ij}^s. We identify the destination-specific productivity terms A_{ij}^s (which capture both fundamental productivity and trade costs) using data on trade shares, prices, and sectoral value added. The firm pricing equation for goods sent from i to j is given by:

$$P_{ij}^s = e_{ij} \frac{1}{A_{ij}^s} (W_i^s)^{\phi_i^s} \prod_n (P_i^n)^{\phi_i^{ns}} = e_{ij} \frac{A_{ii}^s}{A_{ij}^s} P_i^{s,dom}$$

where $P_i^{s,dom} \equiv P_{ii}^s$ is the domestic producer price in country i. The gravity equation implies that expenditure shares λ_{ij}^s satisfy:

$$\lambda_{ij}^s = \frac{(P_{ij}^s)^{1-\sigma_s}}{(P_j^s)^{1-\sigma_s}}$$

Taking the ratio of bilateral to domestic trade shares yields:

$$\frac{P_{ij}^s}{P_{jj}^s} = \left(\frac{\lambda_{ij}^s}{\lambda_{jj}^s} \right)^{\frac{1}{1-\sigma_s}} \tag{C.4}$$

Combining these, we can express the bilateral productivity wedge relative to the domestic productivity as:

$$\frac{A_{ij}^s}{A_{ii}^s} = \frac{e_j P_j^{s,dom}}{e_i P_i^{s,dom}} \frac{A_{jj}^s}{A_{ii}^s} \left(\frac{\lambda_{jj}^s}{\lambda_{ij}^s} \right)^{\frac{1}{1-\sigma_s}} \tag{C.5}$$

Equation (C.5) identifies the *relative* productivities A_{ij}^s / A_{ii}^s given data on prices and trade shares. To pin down the *levels* of domestic productivity A_{ii}^s, we match the observed sectoral value added (VA_i^s) in the data. We solve for the set of domestic productivities $\{A_{ii}^s\}$ and wages $\{W_i^s\}$ that simultaneously satisfy the market clearing conditions and generate the observed value added shares. Specifically, we solve the following system:

$$P_i = \prod_s (P_i^s)^{\alpha_i^s} \tag{pindex}$$

$$(P_j^s)^{1-\sigma} = \sum_i (P_{ij}^s)^{1-\sigma_s} \tag{pindex}$$

$$\sum_s VA_i^{s,data} = P_i C_i + NX_i^{data} \tag{budget}$$

$$R_i^s = \sum_j \lambda_{ij}^s (\alpha_j^s P_j C_j + \sum_n \phi_j^{sn} R_j^n) \tag{goods market}$$

where the auxiliary variables are given by the unit cost function $P_{ij}^s = \frac{e_{ij}}{A_{ij}^s} (W_i^s)^{\phi_i^s} \prod_n (P_i^n)^{\phi_i^{ns}}$, the gravity equation $\lambda_{ij}^s = (P_{ij}^s / P_j^s)^{1-\sigma}$, and the labor market clearing condition $\phi_i^s R_i^s = W_i^s L_i^s$. In this system, we treat labor supply L_i^s as fixed at the data values and iterate on $\{A_{ii}^s\}$ until the model-implied value added $W_i^s L_i^s$ matches the data $VA_i^{s,data}$. The solution provides the calibrated set of destination-specific productivities $\{A^s_{ij}\}$ for the initial period.

Disutility of labor. We calibrate θ^s_i such that $\ell^s_i = 1$ in equilibrium. In our calibration of productivity, we obtain C_i, W^s_i, P_i. Then the labor supply equation is

$$\theta^s_i(\ell^s_i)^{\varphi^{-1}} = v'(\ell^s_i) = \frac{W^s_i}{P_i} u'\left(\frac{C_i}{L_i}\right)$$

The calibrated value of θ^s_i that satisfies $\ell^s_i = 1$ is therefore $\theta^s_i = \frac{W^s_i}{P_i} u'(\frac{C_i}{L_i})$.

Migration costs. We calibrate χ^{sn}_i to be consistent with the initial labor allocation. For the US, where detailed panel data is available from the CPS, we calibrate χ^{sn}_{US} to match the observed gross migration flows between 1999 and 2000.

For China, where such detailed flow data is unavailable, we calibrate χ^{sn}_i to ensure that the economy is in a steady state with the observed labor distribution. Specifically, we set χ^{sn}_i such that the net flows across sectors are zero given the initial wages and prices, meaning the observed employment distribution $L^s_{i,2000}$ is the stationary distribution of the Markov chain implied by the workers' mobility decisions.

C.4.2 Calibration of the shocks

We calibrate the time-varying paths of destination-specific productivities $\{A^s_{ijt}\}$, intertemporal preference shifters $\{\delta_{it}\}$, UIP wedges $\{\psi_{it}\}$, and migration costs $\{\chi^{sn}_{it}\}$ for the period $t = 2000$ to $T_{data} = 2012$. We assume that after 2012, these fundamentals remain constant at their 2012 levels.

Our calibration strategy relies on the fact that the number of structural parameters equals the number of observable data targets. This allows us to "invert" the model's dynamic equilibrium conditions to recover the unique sequence of fundamentals that rationalizes the observed data.

Joint Determination. It is important to emphasize that while we conceptually map specific parameters to specific targets for identification purposes below, we cannot recover these parameters sequentially. In our general equilibrium framework with nominal rigidities and forward-looking dynamics (e.g., consumption smoothing and the wage Phillips curve), changes in any single parameter path affect all equilibrium outcomes simultaneously. Consequently, we employ a joint calibration algorithm: we solve for the entire set of parameter sequences $\{A^s_{ijt}, \delta_{it}, \psi_{it}, \chi^{sn}_{it}\}$ simultaneously by iterating on the full dynamic model until it perfectly reproduces all target time series.

We target the following observables:

(a) **Sectoral Unit Costs and Value Added:** We match the changes in sectoral output prices (USD) and the level of real value added to identify domestic productivity A^s_{iit}.

(b) **Bilateral Trade Shares:** We match the bilateral expenditure shares λ^s_{ijt} to identify the bilateral productivity wedges A^s_{ijt} (which capture trade costs).

(c) **Net Exports:** We match the trajectory of Net Exports to GDP ratios to identify the discount factor shocks δ_{it} (with $\delta_{US,t}$ normalized to 1).

(d) **Exchange Rates and Interest Rates:** We match the bilateral exchange rate paths e_{ijt} and policy rates i_{it} to identify the UIP wedges ψ_{it}.

(e) **Labor Reallocation:** We match the gross migration flows μ^{sn}_{it} to identify the dynamic migration costs χ^{sn}_{it}.

Productivity and Trade Costs (A^s_{ijt}) We recover the destination-specific productivity A^s_{ijt} to match pricing and trade data. Conceptually, this identification involves two components. First, the domestic component A^s_{iit} is pinned down by the domestic firm pricing equation, conditional on the equilibrium path of nominal wages W^s_{it} (which must satisfy the wage Phillips curve) and intermediate prices:

$$A^s_{iit} = \frac{1}{P^{s,dom}_{it}} (W^s_{it})^{\phi^s_{it}} \prod_n (P^n_{it})^{\phi^{ns}_{it}} \tag{C.6}$$

where $P^{s,dom}_{it}$ is the observed domestic price deflator. Second, the bilateral component is identified by the gravity equation, which relates the productivity wedge to relative prices and trade shares:

$$A^s_{ijt} = A^s_{jjt} \left(\frac{e_{it} P^{s,dom}_{it}}{e_{jt} P^{s,dom}_{jt}} \right) \left(\frac{\lambda^s_{jjt}}{\lambda^s_{ijt}} \right)^{\frac{1}{\sigma_s - 1}} \tag{C.7}$$

In the global solution, these conditions must hold simultaneously with the market clearing conditions that determine the wages and price indices on the right-hand side.

Preference Shifters (δ_{it}) The intertemporal preference shifters δ_{it} (where we normalize $\delta_{US,t} = 1$) are identified to match the trajectory of global trade imbalances. Specifically, we calibrate the sequence of δ_{it} for each country i such that the model-implied ratio of Net Exports to Value Added (NX_{it}/VA_{it}) exactly matches the data. In the model, δ_{it} acts as a demand shifter: a higher δ_{it} increases the desire to save (postponing consumption), thereby reducing current import demand and increasing net exports. We iterate on the sequence of $\{\delta_{it}\}$ until the dynamic equilibrium generates the observed path of trade imbalances for all countries.

UIP Wedges (ψ_{it}) We recover the UIP wedges ψ_{it} to match the realized path of the exchange rate between the Chinese Yuan (CNY) and the US Dollar (USD), conditional on observed interest rate differentials. We use the US Federal Funds Rate for $i_{US,t}$ and the overnight interbank rate for China for $i_{CN,t}$. The wedge ψ_{it} is recovered as the residual in the UIP equation. This approach interprets ex-post deviations from standard UIP as arising from unmodeled financial frictions or explicit exchange rate management policies.

Migration Costs (χ_{it}^{sn}) Finally, we recover the dynamic migration costs χ_{it}^{sn} to match the observed gross flows of workers across sectors. Inverting the migration share equation (Eq. 13), we obtain:

$$\chi_{it}^{sn} = \beta\hat{\delta}_{it+1}V_{it+1}^{n} - \nu\log(\mu_{it}^{sn}) + C_{it} \tag{C.8}$$

where C_{it} is a normalization constant common to all destination sectors. Because this equation depends on the future value function V_{it+1}^{n}—which itself depends on future wages, prices, and preference shifters—these costs must be solved for as part of the full dynamic fixed point.

D Foresight of the China Shock

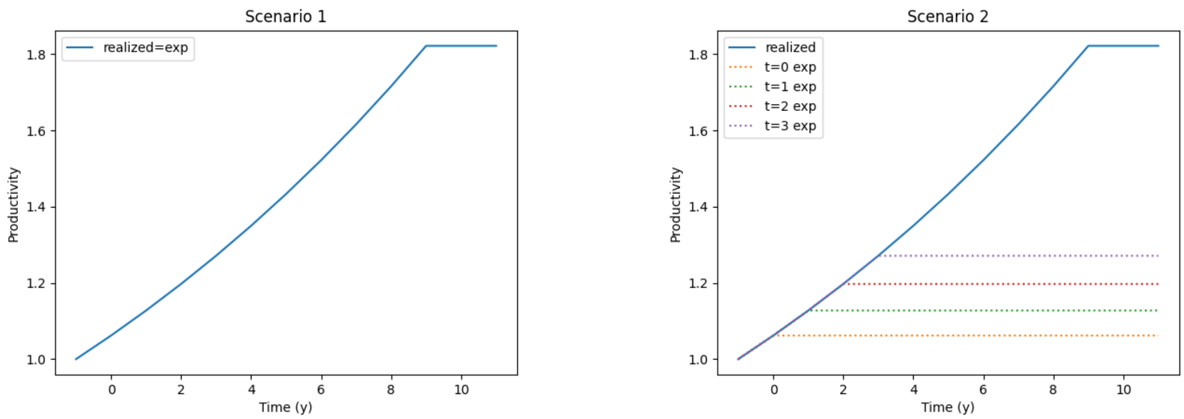
We discuss anticipation of the shock by the households of the model, as agents' foresight of the China shock is important in determining the economy's response to the shock. The literature on structurally estimating the effect of the China shock (Caliendo et al., 2019; Rodríguez-Clare et al., 2022; Dix-Carneiro et al., 2023) all implicitly assume that every agent in the economy at $t = T_0$ have perfect foresight of the full sequence of productivities for $t \in [T_0, T_{data}]$ including the China shock and makes forward looking choices, including sectoral reallocation and consumption-savings, anticipating the development of the full path of the China shock at the start of the model (usually 2000). If the China shock was truly a shock, this is equivalent to assuming that nobody knew of the productivity growth in 1999, but everyone woke up at 2000 and learned the full sequence of the China shock, including that it will plateau at around 2010 (Autor et al., 2021).[14] The problem with this approach is that the model implies a lot of front-loading in transition – wages will adjust incorporating not only the immediate shock but all future shocks, manufacturing workers in 2000 would have a higher desire to leave, and Chinese households will borrow large amounts if they foresaw the full extent of Chinese growth – and the calibrated parameters have to take extreme values to reconcile this with the observed migration and net exports.

We consider an alternative assumption – that agents face a *series* of unanticipated shocks for each t between T_0 and T_{data}. Specifically, in the baseline equilibrium with the realized China shock, at every year t between T_0 and T_{data}, agents learn the new fundamentals at time t $\Theta_t = \{\tilde{\tau}^s_{ijt}, \tilde{\delta}_{it}, \tilde{A}^s_i\}$, and agents (incorrectly) assume that the fundamentals are constant for $t' > t$. In this sense, every year between T_0 and T_{data} is a *China shock*.

To test the validity of this assumption, we estimate the response of our economy to a gradual productivity shock in the low-tech manufacturing sector of China over T_c years, but using two polar opposite assumptions about agents' foresight. In the first exercise, we assume that agents *do not foresee* the shocks in full: for T_c years, the agents face an unanticipated productivity shock every year, and makes decisions assuming that there are no more shocks onwards. In the second exercise, we assume instead, analogously to the literature, that agents in the model have perfect foresight of the full sequence of productivity shocks in $t = T_0 = 2000$. All remaining fundamentals are fixed at calibrated values in $t = T_0$, so the only deviation is the productivity shocks, and to highlight the role productivity shocks play in our model, we assume, for this thought exercise only, that the economy is in steady-state under the initial parameters at $T_0 = 2000$, so any transition dynamics can be fully attributed to the productivity shock.

Exercise 1. Gradual shock, no foresight. First we study the no-foresight assumption, as

[14] One of the reasons why the literature assumes this strong form of perfect foresight is computational tractability. Our modeling framework and solution algorithm (Section 3.2) allows us to bypass these challenges.



(a) Perfect foresight

(b) Compare peg and float

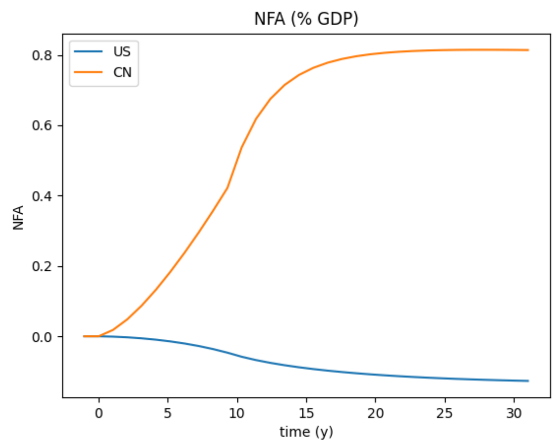
Figure D.1: Productivity growth in low-tech manufacturing. 6% per year for 10 years.

represented by the right panel of Figure D.1. In this case, the economy started at the 2000 levels, then Chinese productivity in low-tech manufacturing grows by 6% for 10 years, but every year, agents are surprised by the new productivity level; in this sense, every year is a China shock for 10 years.

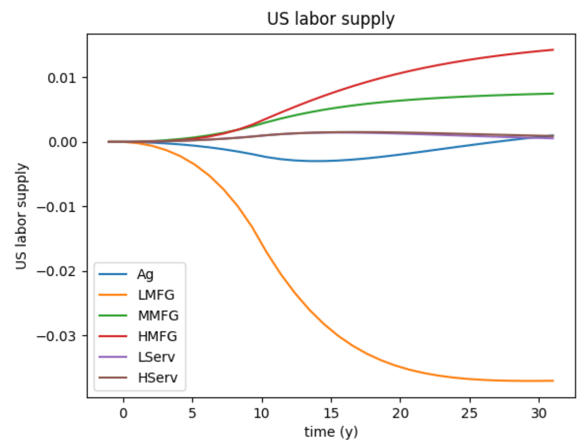
Figure D.2 plots the net foreign asset position, wage, labor reallocation, and unemployment response of the US in response to this shock. From the top left panel, we see that the net foreign asset B_{it} for the US is negative, while the net foreign asset for China is positive; so China saves while the US borrows, in line with the observed data. In this sense, our channel – exchange rate peg interacting with a productivity shock – can *endogenously generate the savings glut*, as seen in Proposition 2 in Section 2.3. The top right plot, which shows labor reallocation, is analogous to the perfect foresight case, where workers slowly move out of the affected sector, and move into and out of other sectors depending on the input-output linkage.

The bottom two figures show the labor market's response in terms of wages and unemployment. Both plots match the empirical facts (Figure 1), theoretical prediction in Section 2.3, and matches evidence found in the literature (Autor et al., 2013, 2021). Wages in the most affected sector fall, but wages in other sectors fall too because of the shock propagating to other sectors through input-output linkage. Lastly, the China shock induces unemployment in the US that grows over time as Chinese productivity grows over time, and reverts to zero as Chinese growth plateaus and the economy slowly adjusts to the new steady-state. Notably, while the directly exposed sector is most harmed, unemployment increases for workers in other sectors as well, because of input-output linkages.

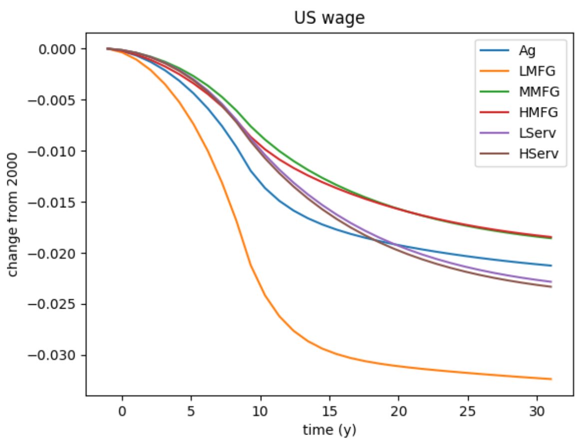
Exercise 2. Gradual shock, perfect foresight. Next we consider the perfect foresight model, as represented in the left panel of Figure D.1. In this case, the economy started at the 2000 levels,



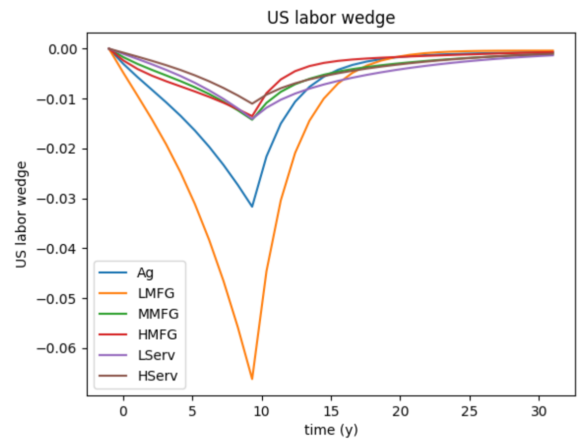
(a) NFA (% GDP)



(b) US labor reallocation



(c) US wage



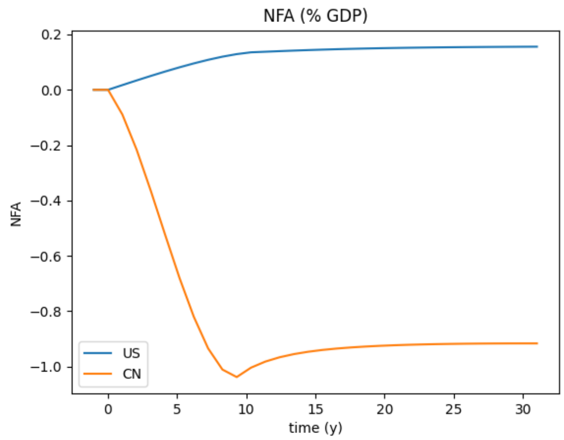
(d) US labor wedge

Figure D.2: Response of the US economy to the gradual shock with no foresight.

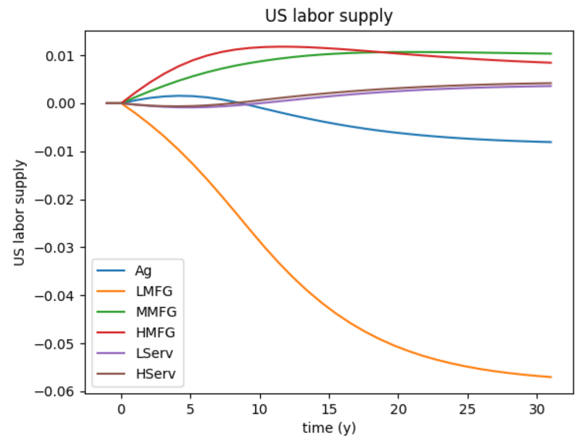
then Chinese productivity in low-tech manufacturing grows by 6% for 10 years, and all agents in the model expect the full path of Chinese productivity growth.

Figure D.3 plots the net foreign asset position, wage, labor reallocation, and unemployment response of the US in response to this shock. As the top left panel shows, if everyone in the model has perfect foresight of the China shock, Chinese agents have an incentive to borrow because they foresee that their productivity in 10 years will be double their productivity today; likewise, US anticipates that Chinese goods will be much cheaper in the future, so it saves. The top right panel shows the labor reallocation response of the China shock, which is in line with what we would expect; since low-tech manufacturing in China grows, workers move out into other sectors. At the same time, some sectors grow more than others because of input-output linkages.

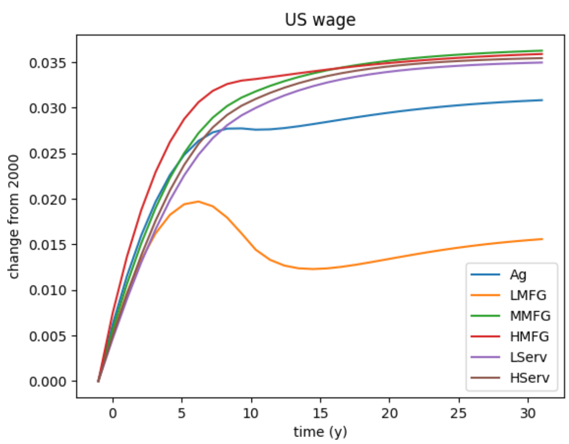
The bottom two panels of Figure D.3 show the wage and unemployment responses of the



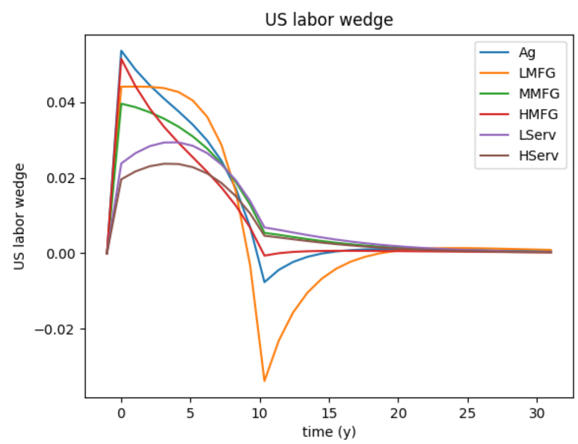
(a) NFA (% GDP)



(b) US labor reallocation



(c) US wage



(d) US labor wedge

Figure D.3: Response of the US economy to the gradual shock with perfect foresight.

China shock. From the left panel, we see that wages *increase* in response to a Chinese productivity growth across all sectors. This is because of the combination of the fact that US borrows to consume more today, and home bias in the model. The most interesting response is the labor wedge, as observed in Figure D.3d. Since the economy faces a sudden surge in US goods demand (due to US saving and home bias), and both wages and labor supply are slow to adjust, there is *excess demand* for domestic goods – the US economy is overheated because of the expectation of future growth in China. As we see, neither the consumption-savings, nor the unemployment responses match those of the China shock.

We note that reality is somewhere in between these polar opposite assumptions (no foresight vs perfect foresight). Because the consumption-savings and labor market responses of the no foresight assumption are more consistent with the empirical evidence (such as Autor et al.

(2021)), in our main text, we calibrate and solve for the baseline and counterfactual economies under the assumption that households did not foresee the China shock.

E Solution Algorithm

This section presents the algorithms we use to estimate the model, calibrate the shocks, and perform counterfactual simulations. We assume convergence to the steady-state in T periods for a large enough T. In our baseline specification, we assume $T = 100$, so the economy converges to the new steady-state in 100 years.

E.1 Variables and equations

As outlined in Section 3.2, we solve the economy in the *sequence-space*. Thus we consider a sequence of variables $\{X_t\}_{t=0}^{T}$, and each period's variables X_t comprise

$$X = (B_i, P_i, C_i, e_i, W_i^s, P_i^s, \ell_i^s, L_i^s, V_i^s).$$

Table E.1 lists the definitions of the variables of interest, and auxiliary variables we use in our solution algorithm.

Panel A. Variables of interest

Variable	Description
B_i	NFA in USD
P_i	Final goods price
C_i	Final goods consumption
e_i	Exchange rate
W_i^s	Sectoral wage
P_i^s	Sectoral goods price
ℓ_i^s	Per-worker labor supply
L_i^s	Distribution of labor
V_i^s	Worker value function

Panel B. Auxiliary variables

Variable	Description
R_i^s	Revenue of i in s
E_i^s	Expenditure of i in s
$\mu_i^{ss'}$	Worker transition matrix
P_{ij}^s	Unit price of good
λ_{ij}^s	Trade shares
i_{it}	Nominal interest rate

Table E.1: Variables to solve for

We denote the *auxiliary* variables as such because they can be directly computed from the variables in X:

$$R_i^s = \frac{W_i^s \ell_i^s L_i^s}{\phi_i^s} \quad \text{(Revenue)}$$

$$E_i^s = \alpha_i^s P_i C_i + \sum_n (1 - \phi_i^n) \phi_i^{ns} R_i^n \quad \text{(Expenditure)}$$

$$\mu_i^{ss'} = \frac{\exp\left(\frac{1}{\nu}(\beta \hat{\delta}_i V_{i,t+1}^{s'} - \chi_i^{ss'})\right)}{\sum_n \exp\left(\frac{1}{\nu}(\beta \hat{\delta}_i V_{i,t+1}^{n} - \chi_i^{sn})\right)} \quad \text{(Worker transition)}$$

$$P_{ij}^s = e_{ij} \tau_{ij}^s \frac{1}{A_i^s} (W_i^s)^{\phi_i^s} \prod_n (P_i^n)^{\phi_i^{ns}} \quad \text{(Unit cost)}$$

$$\lambda_{ij}^s = \frac{(P_{ij}^s)^{1-\sigma}}{\sum_l (P_{lj}^s)^{1-\sigma}} \quad \text{(Trade share)}$$

$$\log(1 + i_{it}) = -\log \beta + \phi_\pi \log(P_{it+1}/P_{it}) + \phi_{lw} \cdot \vartheta_{it} + \epsilon_{it}^{MP} \quad \text{(Taylor rule)}$$

where $\hat{\delta}_{it} \equiv \delta_{it+1}/\delta_{it}$ is the change in the preference shifter, and ϑ_{it} is the aggregate labor wedge (output gap). China's interest rate $i_{CN,t}$ is set equal to the US rate (peg), with a potential wedge ψ for UIP deviations.

We take the logs of the positive variables C, P, W, e, L, ℓ to ensure stability of our algorithm. Given the variables X_t, the equations of the quantitative model (in Section A.1) can be written as:

$$F_1(X_t) = p_i - \sum_s \alpha_i^s p_i^s \qquad \text{(price index)}$$

$$F_2(X_t) = (1-\sigma)p_j^s - \log \sum_i \exp((1-\sigma)p_{ij}^s) \qquad \text{(sector price)}$$

$$F_3(X_t) = R_i^s - \sum_j \frac{e_j}{e_i} \frac{\bar{L}_j}{\bar{L}_i} \lambda_{ij}^s E_j^s \qquad \text{(goods market)}$$

$$F_4(X_t, X_{t+1}) = P_i C_i + \frac{B_{i,t+1}}{(1+i_i)e_{i,t+1}} - \frac{B_{it}}{e_{it}} - \sum_s W_i^s \ell_i^s L_i^s - T_i - \Pi_i \qquad \text{(HH budget)}$$

$$F_5(X_t, X_{t+1}) = \left(-\frac{1}{\gamma}c_i - p_i\right) - \left(-\frac{1}{\gamma}c_{i,t+1} - p_{i,t+1}\right) - \log(\beta(1+i_i)) - \log \hat{\delta}_i \qquad \text{(Euler)}$$

$$F_6(X_t, X_{t+1}) = e_{i,t+1} - e_{it} + \log(1+i_i) - \log(1+i_{US}) - \psi_i \qquad \text{(UIP)}$$

$$F_7(X_t, X_{t+1}) = L_{i,t+1}^s - \sum_n \mu_i^{ns} L_{it}^n \qquad \text{(migration)}$$

$$F_8(X_t, X_{t+1}) = V_{it}^s - \Lambda_{it} W_{it}^s \ell_{it}^s + v(\ell_{it}^s) - \nu \log \sum_n \exp\left(\frac{1}{\nu}(\beta\hat{\delta}_i V_{i,t+1}^n - \chi_i^{sn})\right) \qquad \text{(Value)}$$

$$F_9(\{w_{i,t-1}^s\}, X_t, X_{t+1}) = (w_{it}^s - w_{i,t-1}^s) - \kappa_w \underbrace{\left[v'(\ell_{it}^s) - \frac{W_{it}^s}{P_{it}} u'(C_{it})\right]}_{\vartheta_{it}^s} - \beta\hat{\delta}_i(w_{i,t+1}^s - w_{it}^s) \qquad \text{(NKPC)}$$

where T_i is tariff revenue, Π_i is the UIP wedge profit rebate to China, $\Lambda_{it} = u'(C_{it})/P_{it}$ is the marginal utility of income, and ϑ_{it}^s is the sectoral labor wedge. This set of equations is the main set of equations we use to solve for the equilibrium. Note that the period t equilibrium conditions only depend on $t, t+1$ variables and the previous period wage.

E.2 Solving for the steady-state

We first solve for the long-run steady-state: an equilibrium with persistent net foreign asset positions (in USD) and relative wages. Per our assumptions, countries may have a persistent NFA $B_i \neq 0$. Given any values of the fundamentals and parameters in Table 1, and the terminal real NFA $\{B_i\}_i$, the steady-state comprises $2I + 5IS$ variables $X_T = (P_i, C_i), (W_i^s, P_i^s, L_i^s, \ell_i^s, V_i^s)$

that solve the following system of equations, written using the form in Section E.1:

$$G_{ss}(X_T) = \begin{pmatrix} F_1(X_T) \\ F_2(X_T) \\ F_3(X_T) \\ F_4(X_T, X_T) \\ F_7(X_T, X_T) \\ F_8(X_T, X_T) \\ F_9(X_T, X_T, X_T) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \tag{E.1}$$

taking advantage of the fact that in steady-state, $X_{T-1} = X_T = X_{T+1}$. The algorithm for solving for this steady-state is as follows: it robustly converges for any given parameters.

Step 1. Make an initial guess for the solution $X_T^{(1)}$.

Step 2. Use **gradient descent** to update the guess $X_T^{(1)} \to X_T^{(2)}$ (we use 20 iterations with learning rate 10^{-11}).

Step 3. Use **Newton's method** on $G_{ss}(X_T)$ to update the guess $X_T^{(2)} \to X_T^{(3)}$, until the error tolerance $\|G_{ss}(X_T)\|^2$ is below a certain threshold (we use 10^{-18}).

The resulting set of variables $X_T^{(3)}$ is the set that solves the system G_{ss} given B_T. See Section E.6 for the bolded nonlinear solvers.

E.3 Solution algorithm for pegged economy

Given any set of dynamic parameters and fundamentals in Table 1 and the initial conditions $\{w_{i,-1}^s, L_{i0}^s, B_i\}$, China's pegged exchange rate $e_2 = \bar{e}$, and any policy $\{T_{ijt}^s\}, \{\epsilon_{it}^{MP}\}$, the economy is defined in the sequence-space as the set of variables

$$X = \{X_t\}_{t=0}^T = \{(B_{it}, P_{it}, C_{it}, e_{it}, W_{it}^s, P_{it}^s, \ell_{it}^s, L_{it}^s, V_{it}^s)\}_{t=0}^T$$

that satisfy the equilibrium conditions. The period-t equilibrium conditions are given by

$$G_t(X_t, \{w^s_{it-1}\}, X_{t+1}) = \begin{pmatrix} F_1(X_t) \\ F_2(X_t) \\ F_3(X_t) \\ F_4(X_t, X_{t+1}) \\ F_5(X_t, X_{t+1}) \\ F_6(X_t, X_{t+1}) \\ F_7(X_t, X_{t+1}) \\ F_8(X_t, X_{t+1}) \\ F_9(\{w^s_{it-1}\}, X_t, X_{t+1}) \end{pmatrix} \tag{E.2}$$

The set of equations for the *path* $\{X_t\}_{t=0}^{T-1}$, given a terminal steady-state X_T, is

$$\mathcal{G}(\{X_t\}_{t=0}^{T-1}, X_T) = \begin{pmatrix} G_0(X_0, \{w^s_{i,-1}\}, X_1) \\ G_1(X_1, \{w^s_{i,0}\}, X_2) \\ \cdots \\ G_{T-2}(X_{T-2}, \{w^s_{i,T-3}\}, X_{T-1}) \\ G_{ss-1}(X_{T-1}, \{w^s_{i,T-2}\}, X_T) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \cdots \\ 0 \\ 0 \end{pmatrix} \tag{E.3}$$

where G_{ss-1} is the period $T-1$ condition that links the sequence-space to the terminal steady-state, and is given by:

$$G_{ss-1}(X_{T-1}, \{w^s_{i,T-2}\}, X_T) = \begin{pmatrix} F_1(X_{T-1}) \\ \hat{F}_6(X_{T-1}, X_T) \\ C_{T-1} - C_T \\ F_2(X_{T-1}) \\ F_3(X_{T-1}) \\ \hat{F}_9(\{w^s_{i,T-2}\}, X_{T-1}, X_T) \\ F_8(X_{T-1}, X_T) \end{pmatrix} \tag{E.4}$$

The following are the differences between the last condition G_{ss-1} and a generic G_t:

- We replace the Euler equation F_5 with $C_{T-1} = C_T$, signifying that we have reached a terminal state.
- We replace the first element of the UIP condition F_6 (for China) with the exchange rate peg condition $e_{CN,T-1} = \bar{e}$. The remaining UIP conditions for other countries are unchanged.
- We remove the household budget constraint F_4 and the migration equation F_7. The household budget constraint is implicitly satisfied by the outer loop (Broyden iteration on terminal bonds), and migration is forward-looking.

- In the NKPC F_9, we impose $w^s_{it} = w^s_{i,T-1}$ for all $t \geq T-1$, signifying that we are in steady-state by period T.

Technical note: all of this is necessary because our model is nonstationary and the exchange rate and NFA features a unit root.

Given our construction of $\mathcal{G}$, we implement our solution algorithm in two steps: inner loop and outer loop.

Inner loop. Solve for the *path* $X_{path} = \{X_t\}_{t=0}^{T-1}$ that solves $\mathcal{G}(X_{path}, X_T)$ given a terminal state X_T. In an abuse of notation, we remove the dependency of $\mathcal{G}$ on X_T.

Step 1. Make an initial guess for $X^{(1)}_{path}$. Here it is important that the sequence $\{X_t\}$ *converges* to the terminal state X_T for the algorithm to be stable.

Step 2. Use **gradient descent** on $\|\mathcal{G}(X_{path})\|^2$ to improve the initial guess $X^{(1)}_{path} \to X^{(2)}_{path}$.

Step 3. Use **quasi-Newton's method** on $\mathcal{G}(X_{path})$ to update the guess $X^{(2)}_{path} \to X^{(3)}_{path}$. In practice we repeat until $\|\mathcal{G}(X_{path})\|^2 < 10^{-12}$.

Step 4. Use **Levenberg-Marquardt algorithm** on $\mathcal{G}(X_{path})$ to fine-tune the guess $X^{(3)}_{path} \to X^{(4)}_{path}$. In practice we repeat until $\|\mathcal{G}(X_{path})\|^2 < 10^{-12}$.

Steps 3 and 4 require quick construction and inversion of the Jacobian of $\mathcal{G}(X_{path})$, which is a large matrix (in our main specification, with $I = S = 6$ and $T = 100$, the Jacobian has dimension 20000×20000). We have knowledge of the structure of $\mathcal{G}$: each time t equation G_t depends only on X_t, X_{t+1} and $\{w^s_{i,t-1}\}$. Thus we know the sparsity structure of the Jacobian (i.e. where all the nonzero elements are), so we use automatic differentiation (autodiff) to speed up this process, and construct the Jacobian $J_{\mathcal{G}}$ as a sparse matrix. Then we use Intel's PARDISO package[15] to quickly solve the linear system $J_{\mathcal{G}}\Delta x = -\mathcal{G}(X_{path})$.

Outer loop. Solve for the terminal bond positions B_T that are consistent with the path X_{path}. We use **Broyden's method**, a quasi-Newton algorithm that approximates the inverse Jacobian of the bond market clearing condition.

Step 1. Start from an initial guess of $B^{(0)}_T$. Initialize the inverse Jacobian approximation $H^{(0)} = -\alpha I$ where $\alpha \in (0, 1)$ is a damping parameter.

Step 2. Given $B^{(k)}_T$, solve for $X^{(k)}_T$ using the steady-state solver (Section E.2).

Step 3. Given $X^{(k)}_T$, solve for $X^{(k)}_{path}$ using the inner loop.

[15] See https://www.intel.com/content/www/us/en/docs/onemkl/developer-reference-c

Step 4. Using $X_{T-1}^{(k)}$ and the household budget constraint at $T-1$, compute the implied terminal bonds $B_{T,implied}^{(k)}$ that clear the bond market.

Step 5. Define the residual $F^{(k)} = B_{T,implied}^{(k)} - B_T^{(k)}$. Use the **Broyden update** to compute the next guess $B_T^{(k+1)}$. Repeat until $\|F^{(k)}\| < tol$.

Once the outer loop converges, we have a solution in the sequence-space $\{X_t\}$. When $I = S = 6$ and $T = 100$, with our current code, the solution is usually found within 1-3 minutes.

E.4 Solution algorithm for floating economy

For the floating economy, we make two modifications to the equilibrium conditions:

(a) **Monetary policy**: China follows an independent Taylor rule instead of pegging to the US:

$$\log(1 + i_{CN,t}) = -\log\beta + \phi_\pi \log(P_{CN,t+1}/P_{CN,t}) + \phi_{lw} \cdot \vartheta_{CN,t} + \epsilon_{CN,t}^{MP}$$

(b) **UIP wedge**: The UIP deviation ψ_i is set to zero for all countries, so F_6 becomes the standard UIP condition.

The terminal linking condition G_{ss-1}^{float} has the same structure as the pegged case:

$$G_{ss-1}^{float}(X_{T-1}, \{w_{i,T-2}^s\}, X_T) = \begin{pmatrix} F_1(X_{T-1}) \\ F_6(X_{T-1}, X_T) \\ C_{T-1} - C_T \\ F_2(X_{T-1}) \\ F_3(X_{T-1}) \\ \hat{F}_9(\{w_{i,T-2}^s\}, X_{T-1}, X_T) \\ F_8(X_{T-1}, X_T) \end{pmatrix} \tag{E.5}$$

The key difference from the pegged case is that F_6 is the standard UIP condition for all countries (no exchange rate peg replacement).

The solution algorithm is identical to the pegged case: we use the same inner loop (gradient descent, quasi-Newton, Levenberg-Marquardt) and the same outer loop (Broyden's method) to solve for the terminal bond positions B_T that clear the bond market.

E.5 Solving the economy under no foresight

The algorithms described above assume that agents have perfect foresight over the entire path of shocks. In the *no foresight* case, agents are "surprised" by shocks each period: at each calendar time t, they observe the current period's shocks but expect all future wedges to be zero.

Outer loop over calendar time $t = 0, 1, 2, \ldots, T_{calib}$:

At each period t, we perform the following steps:

Step 1. Generate "naive" parameters. Construct a parameter path where:

- *Period 0* (current): Use actual parameters with period-t shocks.
- *Periods 1 to T* (future): Agents expect all wedges to be zero:

$$\delta = 0, \quad T^{tariff} = 0, \quad \epsilon^{MP} = 0, \quad \psi = 0, \quad \text{ZLB inactive.}$$

The future exchange rate peg is set to the expected value $\bar{e}_{t+1}$.

Step 2. Solve full equilibrium using the algorithm in Section E.3 with the naive parameters:

- Same inner loop (gradient descent → quasi-Newton → Levenberg-Marquardt).
- Same outer loop (Broyden's method for terminal bonds).
- But agents expect shocks to disappear after period 0.

Step 3. Extract period-0 outcome as the actual realization for calendar period t.

Step 4. Update initial conditions for the next calendar period:

- B_{t+1}: bonds from period 1 of the solution.
- L^s_{t+1}: labor distribution from period 1.
- w^s_t: wages from period 0 (for NKPC).

Step 5. Use solution as warm start for the next period's solve.

The sequence of period-0 outcomes across all calendar periods forms the equilibrium path under no foresight. Note that this algorithm is computationally expensive: it solves T_{calib} full equilibrium problems sequentially, each requiring the inner and outer loop iterations.

E.6 Nonlinear solver algorithms

This subsection describes the generic nonlinear solvers we use in our solution algorithms.

Gradient descent. Given a function $f : \mathbb{R}^n \to \mathbb{R}^n$, we approximate the root of f by applying gradient descent on $g = \|f\|_2^2 = \sum_i f_i^2$.

Input: function $g = \|f\|_2^2$; gradient ∇g of g; learning rate λ; number of iterations m; tolerance tol.

Algorithm:

Step 1. Start from an initial guess $x^{(0)}$.

Step 2. Evaluate ∇g, the gradient of g, at $x^{(i)}$.

Step 3. Update the guess $x^{(i+1)} = x^{(i)} - \lambda \cdot \nabla g(x^{(i)})$ for sufficiently small λ.

Step 4. Repeat 2-3 for m iterations, terminate if $g(x^{(i+1)}) < tol$.

In practice, gradient descent is too slow to converge to the root. We use this to *update* the initial guess, to feed in to the next solvers.

Newton's method. Given a function $f : \mathbb{R}^n \to \mathbb{R}^n$, we approximate the root of f by Newton's method.

Input: f, the function; J, the Jacobian J_f of f; $g = \|f\|_2^2$; number of iterations m; tolerance tol.

Algorithm:

Step 1. Start from an initial guess $x^{(0)}$.

Step 2. Use autodiff to compute J_f at $x^{(i)}$.

Step 3. Use PARDISO to evaluate $J_f(x^{(i)})^{-1} f(x^{(i)})$.

Step 4. Update $x^{(i+1)} = x^{(i)} - J_f(x^{(i)})^{-1} f(x^{(i)})$.

Step 5. Repeat 2-4 for m iterations, terminate if $g(x^{(i+1)}) < tol$.

Newton's algorithm requires a good initial guess. In static problems (solving for the terminal state), we use parts of the equation (which are contraction mappings) to construct the initial guess close to the solution. In dynamic problems, our initial guess is close to the terminal steady-state: this 'anchors' the problem and allows for convergence. For efficiency reasons, we use the quasi-Newton method below for the high-dimensional dynamic problem.

Quasi-Newton's method. Given a function $f : \mathbb{R}^n \to \mathbb{R}^n$, we approximate the root of f by quasi-Newton's method.

Input: f, the function; J, the Jacobian J_f of f; $g = \|f\|_2^2$; max step size $\bar{s}$; number of iterations m; tolerance tol.

Algorithm:

Step 0. *(Initial tuning)* Start from an initial guess $x^{(0)}$. For a few iterations, take small Newton steps with a fixed small step size to stabilize the initial guess.

Step 1. Use autodiff to compute J_f at $x^{(i)}$. Here it is essential that our autodiff procedure is sparse-aware, that is, aware of the nonzero elements of J_f.

Step 2. Use PARDISO to solve $J_f(x^{(i)}) \cdot dx = f(x^{(i)})$.

Step 3. Construct *candidate* updates $x(s) = x^{(i)} - s \cdot dx$ for a grid of step sizes $s \in [0, \bar{s}]$.

Step 4. Compute $g(x(s))$ for each s and update $x^{(i+1)}$ to be $x(s)$ with the minimal $g(x)$.

Step 5. If improvement is below 1%, terminate early. Otherwise, repeat Steps 1-4 until $g(x^{(i+1)}) < tol$ or $i = m$.

The advantage of this approach is as follows: the bottleneck in Newton's method is computing the Jacobian and solving the linear system. By searching over a grid of step sizes after each Newton direction computation, we can effectively search for more candidates with minimal time cost. In reality, Newton's method can overshoot in the first few steps so it's better to have small s, whereas closer to the root, the optimal s tends to be larger (2-4).

Levenberg-Marquardt method (modified). Given a function $f : \mathbb{R}^n \to \mathbb{R}^n$, we approximate the root of f using a modified Levenberg-Marquardt algorithm. The standard LM algorithm adjusts the damping parameter λ based on whether the update improves the objective. Our modification additionally incorporates a grid search over step sizes, similar to the quasi-Newton method above.

Input: f, the function; J, the Jacobian J_f of f; $g = \|f\|_2^2$; damping parameter λ; number of iterations m; tolerance tol.

Algorithm:

Step 1. Start from an initial guess $x^{(0)}$ with initial $\lambda = 10^{-4}$.

Step 2. Use autodiff to compute J_f at $x^{(i)}$. Here it is essential that our autodiff procedure is sparse-aware, that is, aware of the nonzero elements of J_f.

Step 3. Compute $A = J^T J + \lambda \cdot \text{diag}(J^T J)$ and the right-hand side $b = J^T f(x^{(i)})$.

Step 4. Use PARDISO to solve $A \cdot dx = b$ for the update direction dx.

Step 5. Construct *candidate* updates $x(s) = x^{(i)} - s \cdot dx$ for a grid of step sizes $s \in [10^{-2}, 40]$ (400 points, geometrically spaced).

Step 6. Compute $g(x(s))$ for each s. Let x_{best} be the candidate with the minimal $g(x)$.

Step 7. If $g(x_{best}) < g(x^{(i)})$, accept $x^{(i+1)} = x_{best}$ and update λ: if step size $s < 0.5$, multiply λ by 5; otherwise divide λ by λ_{down}. If $g(x_{best}) \geq g(x^{(i)})$, multiply λ by λ_{up} and return to Step 3 (up to 20 inner iterations).

Step 8. Terminate if $g(x^{(i+1)}) < tol$, or $i = m$. Otherwise return to Step 2.

In practice we use $\lambda_{up} = 1.5$ and $\lambda_{down} = 5$ with tolerance $tol = 10^{-13}$. The grid search over step sizes allows us to find larger improvements per iteration than the standard LM algorithm.

Broyden's method for terminal bonds. Given a function $f : \mathbb{R}^n \to \mathbb{R}^n$, find the fixed point x^* such that $f(x^*) = x^*$, or equivalently $F(x^*) = 0$ where $F(x) = f(x) - x$. This is used to find terminal bond holdings B_T such that the sequence-space model converges. Each evaluation of f requires solving the full inner loop (the high-dimensional nonlinear system described above).

Input: initial guess $x^{(0)}$; damping parameter $\alpha = 0.5$; tolerance $tol = 10^{-5}$.

Algorithm:

Step 1. Initialize the inverse Jacobian approximation $H_0 = -\alpha I$.

Step 2. Evaluate $f(x^{(0)})$ and compute the residual $F^{(0)} = f(x^{(0)}) - x^{(0)}$.

Step 3. Compute the update $x^{(1)} = x^{(0)} - H_0 F^{(0)} = x^{(0)} + \alpha F^{(0)}$.

Step 4. For $k \geq 1$: evaluate $f(x^{(k)})$ and compute the residual $F^{(k)} = f(x^{(k)}) - x^{(k)}$.

Step 5. Update H via the Sherman-Morrison formula:

$$H_k = H_{k-1} + \frac{(s_{k-1} - H_{k-1} y_{k-1}) s_{k-1}^\top H_{k-1}}{s_{k-1}^\top H_{k-1} y_{k-1}}$$

where $s_{k-1} = x^{(k)} - x^{(k-1)}$ and $y_{k-1} = F^{(k)} - F^{(k-1)}$.

Step 6. Compute the next iterate $x^{(k+1)} = x^{(k)} - H_k F^{(k)}$.

Step 7. Terminate if $\|F^{(k)}\| < tol$. Otherwise return to Step 4.

This method approximates the inverse Jacobian without computing actual derivatives, making it efficient when each function evaluation is expensive. In practice, it converges within 5–7 iterations for the terminal bond problem.

F Robustness Checks

Calibration Strategy and Identification. A key challenge in quantitative policy analysis is ensuring that recovered fundamentals (productivity and preference shifters) and the counterfactual experiments are policy-invariant. To address this, we adopt a full re-calibration approach throughout this robustness section. For each alternative specification (unless otherwise noted), we do not simply swap policy parameters while holding the baseline residuals fixed; instead, we re-recover the sequences of productivity $\{A^s_{ijt}\}$, preference shifters $\{\delta_{it}\}$, and migration cost $\{\chi^{sn}_{it}\}$ to match the exact same observables (trade shares, employment) under the new policy rule. This procedure ensures that our findings are not driven by "baking" the effects of the baseline peg into the residuals. By demonstrating that the counterfactual effects of the peg remain consistent across these re-calibrated specifications, we confirm that our results capture the causal mechanism of the currency regime rather than a measurement artifact of the baseline inversion.

F.1 Alternative monetary policy

In our main text, we assumed that the floating countries (US and the world except China) used a Taylor rule targeting CPI inflation. In this subsection, we consider a generalized rule that also responds to unemployment:

$$\log(1 + i_{1t}) = \log(1 + \bar{i}) + \phi_\pi \log(1 + \pi_{1t}) - \phi_y \log(u_{1t}/\bar{u}), \tag{F.1}$$

where u_{1t} is the average unemployment rate across sectors. Our baseline specification corresponds to $\phi_\pi = 1.5$ and $\phi_y = 0$.

As discussed in Section 2.1, nominal wage rigidity implies that the economy features a sector-specific labor wedge ϑ^s_{jt}, which corresponds to involuntary unemployment. Divine coincidence fails in our model for two reasons:

(a) **Sectoral heterogeneity**: The China shock hits manufacturing disproportionately. A single interest rate cannot simultaneously stabilize all sectors. Lowering rates enough to clear manufacturing labor markets would overheat services.

(b) **Foreign origin of shocks**: Divine coincidence holds for domestic demand shocks but fails for foreign productivity shocks. When China becomes more productive, the optimal relative price of US goods should fall. Under flexible prices, this happens through exchange rate depreciation. Under a peg with sticky wages, the adjustment requires deflation, which CPI-targeting policy resists.

Despite these limitations, we focus on Taylor rules as they are empirically grounded, widely used in policy analysis, and allow for direct comparison with the existing literature. We provide suggestive evidence on the policy trade-offs by exploring alternative coefficients, leaving the formal derivation of the optimal policy rule to future research.

In this subsection, we redo the exercises in Section 4.2 (Reevaluating the China shock) with four alternative monetary policy rules that place different weights on inflation or explicitly target real activity (unemployment):

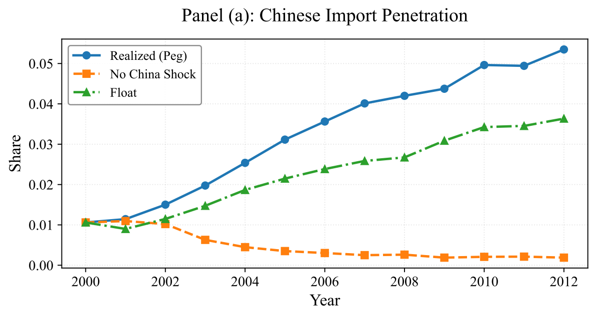
(a) **Dove**: Weaker response to inflation ($\phi_\pi = 1.2, \phi_y = 0$).

(b) **Hawk**: Stronger response to inflation ($\phi_\pi = 2.0, \phi_y = 0$).

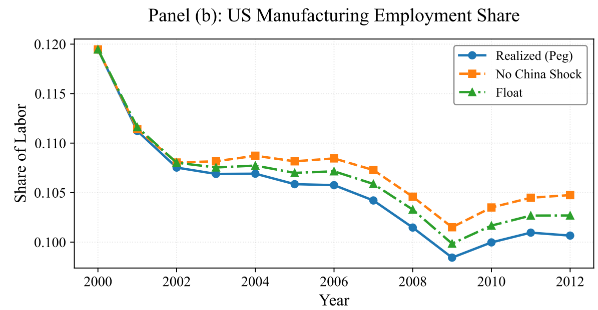
(c) **Taylor with unemployment**: Targets both inflation and unemployment ($\phi_\pi = 1.5, \phi_y = 0.5$).

(d) **NGDP**: Targets both inflation and unemployment with equal weight ($\phi_\pi = 1.5, \phi_y = 1.5$).

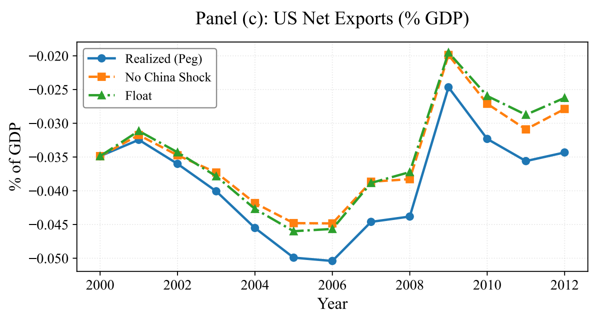
Dove Policy ($\phi_\pi = 1.2$) Figure F.1 shows the effects of the peg under the Dove policy. Results are virtually the same as Figure 4 in the main text.



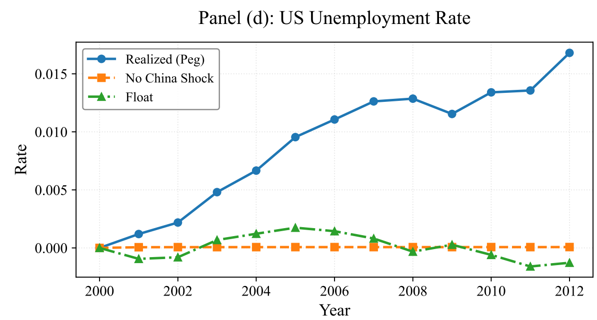
(a) CN import penetration



(b) Manufacturing share



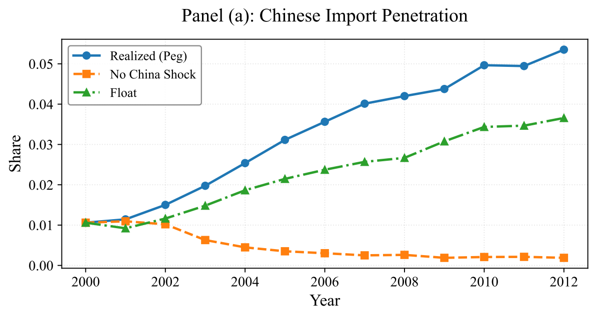
(c) Net exports (% GDP)



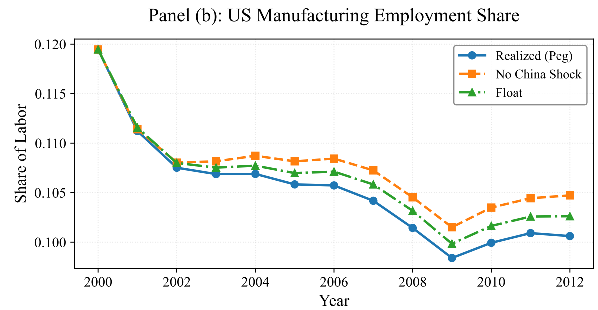
(d) Unemployment rate

Figure F.1: Effect of Peg: Dove Policy.

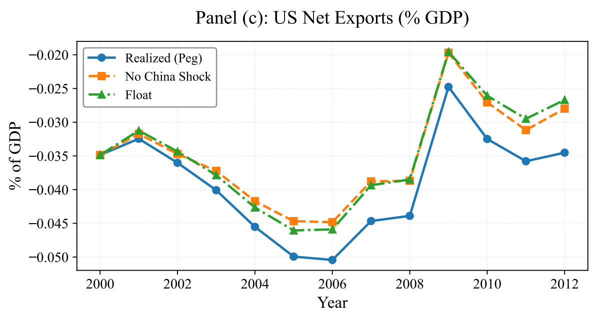
Hawk Policy ($\phi_\pi = 2.0$) Figure F.2 shows the results for the Hawk policy.



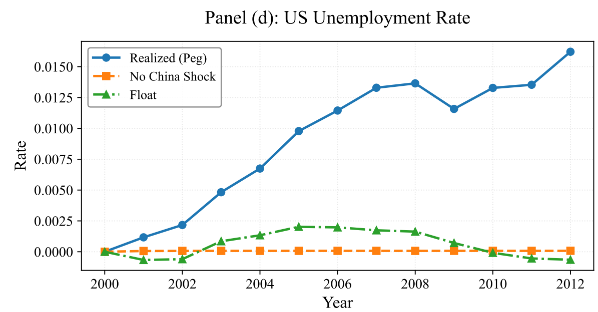
(a) CN import penetration



(b) Manufacturing share



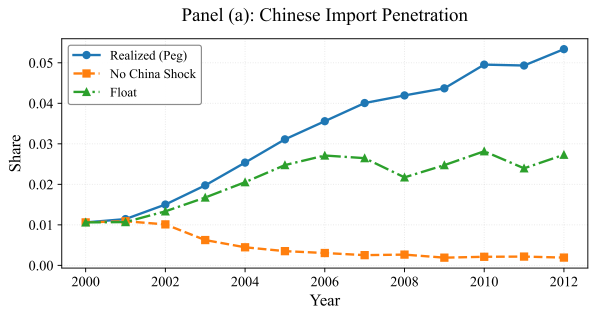
(c) Net exports (% GDP)



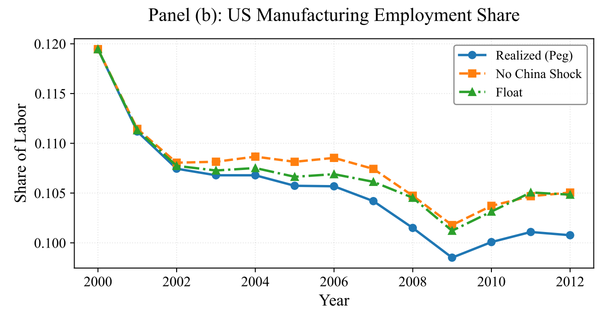
(d) Unemployment rate

Figure F.2: Effect of Peg: Hawk Policy.

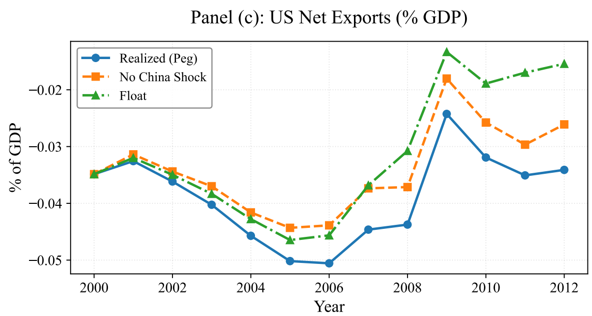
Taylor Rule with unemployment ($\phi_\pi = 1.5, \phi_y = 0.5$) Figure F.3 shows the results for the Taylor rule with an additional unemployment target. In this case, the response to unemployment significantly differs. As China penetrates the US economy and unemployment increases, nominal interest rates decrease and offset part of this unemployment increase.



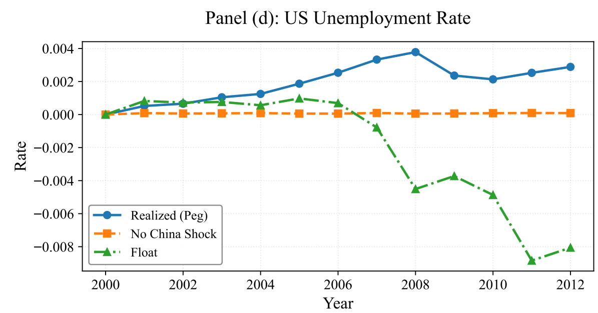
(a) CN import penetration



(b) Manufacturing share



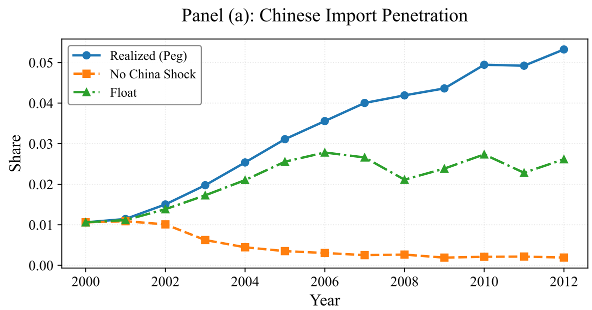
(c) Net exports (% GDP)



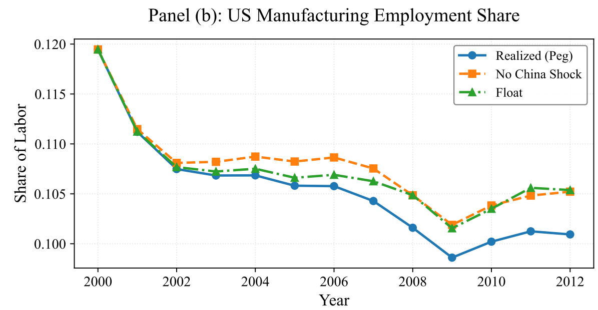
(d) Unemployment rate

Figure F.3: Effect of Peg: Taylor Rule.

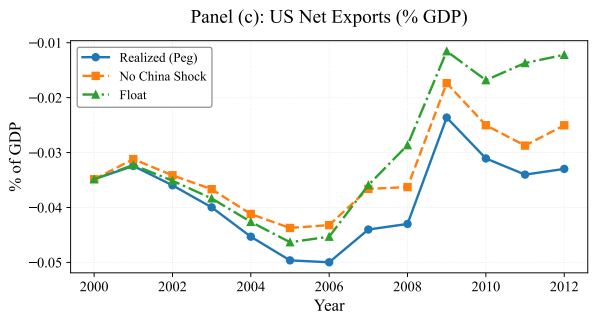
NGDP Targeting ($\phi_\pi = 1.5, \phi_y = 1.5$) Figure F.4 shows the results for NGDP targeting. The response of unemployment is further dampened relative to F.3.



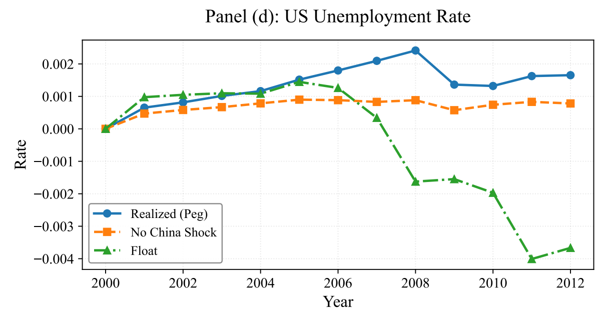
(a) CN import penetration



(b) Manufacturing share



(c) Net exports (% GDP)



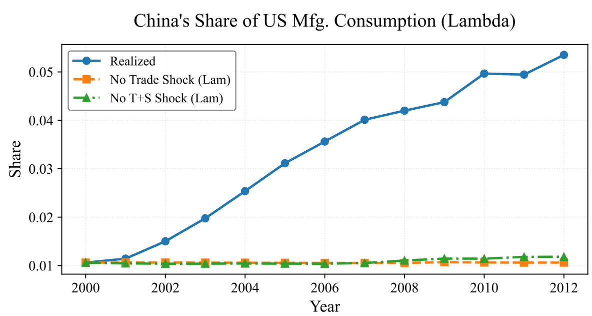
(d) Unemployment rate

Figure F.4: Effect of Peg: NGDP Targeting.

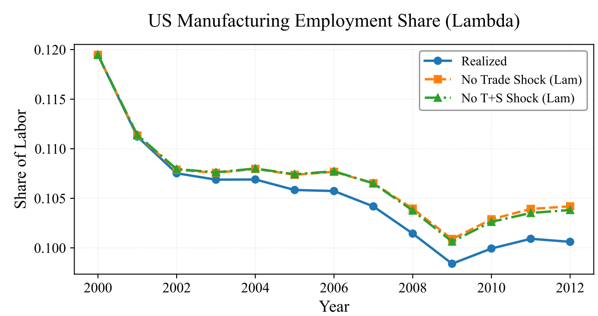
F.2 Alternative China Shocks

In our main text, our baseline assumption on the counterfactual 'no China shock' economy was an economy where productivity A^s_{ijt} for China are fixed at the 2000 level. In this subsection, we consider an alternative definition: the 'no China shock' economy as an economy where productivity A^s_{ijt} are calibrated to values such that λ^s_{ijt} for China is fixed at the 2000 values. This would be closer to specifications that calibrate the China shock to match regression coefficients on observed growth in export shares (Caliendo et al., 2019; Rodríguez-Clare et al., 2022).

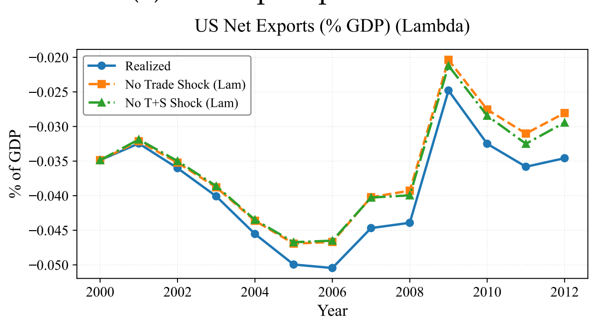
The results are shown in Figures F.5 (China Shock) and F.6 (Peg Effect).



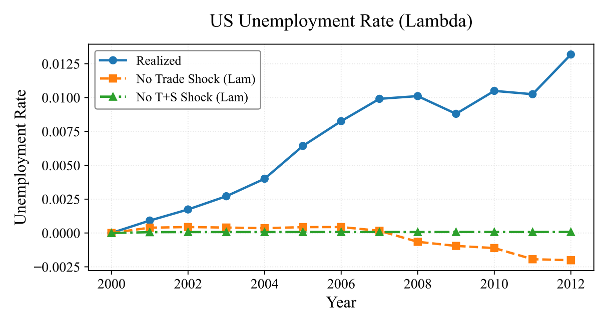
(a) CN import penetration



(b) Manufacturing share

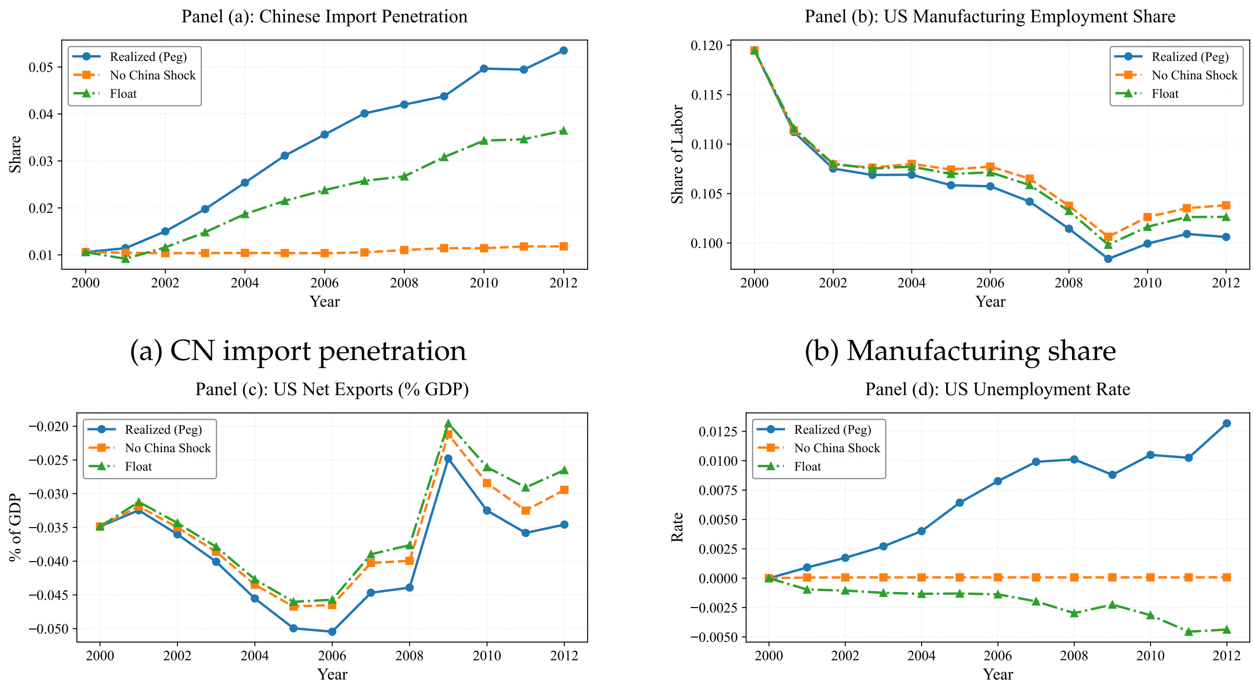


(c) Net exports (% GDP)



(d) Unemployment rate

Figure F.5: Alternative China Shock: Fixed Export Shares.



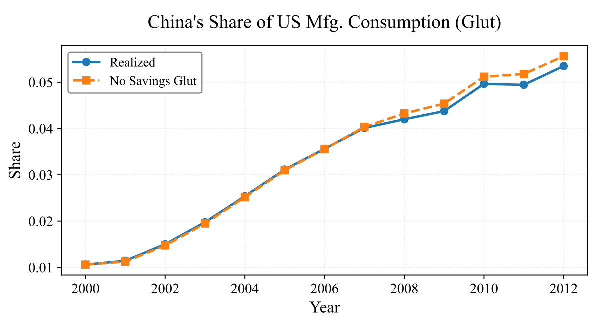
(a) CN import penetration
(b) Manufacturing share
(c) Net exports (% GDP)
(d) Unemployment rate

Figure F.6: Effect of Peg: Fixed Export Shares.

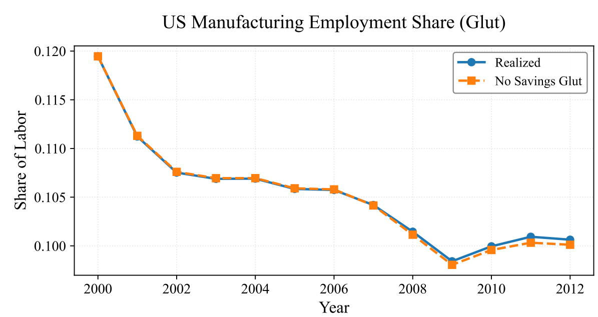
F.3 Isolating China Savings shock

In Section 4 of the main text, we showed that the effects of the China trade shock and of the combined China trade and savings shocks are virtually identical. This result suggests that the savings component plays, at most, a limited role once the exchange rate peg is taken into account. Nevertheless, the preference shifters δ_{it} associated with China's savings behavior are quantitatively important objects in the model, and it is therefore informative to isolate their contribution.

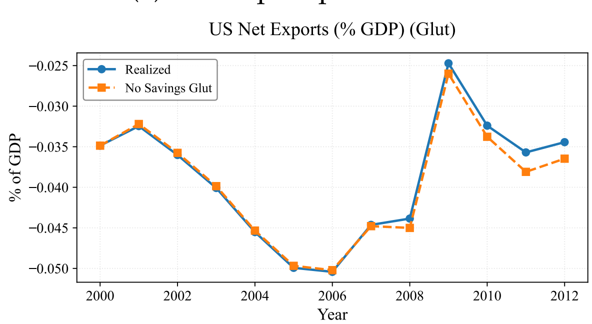
To do so, we construct a counterfactual economy in which China's savings shock is shut down, setting $\hat{\delta}_{CN,t} = 1$, while leaving all other shocks unchanged. Figure F.7 compares the realized economy to this counterfactual. The results indicate that the China savings shock by itself generated an increase of approximately 112 thousand manufacturing jobs, affected the trade deficit by 0.075 percent of GDP, and decreased the unemployment rate by 0.046 percentage points.



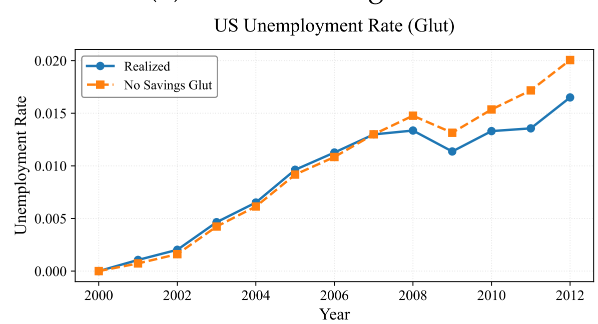
(a) CN import penetration



(b) Manufacturing share



(c) Net exports (% GDP)



(d) Unemployment rate

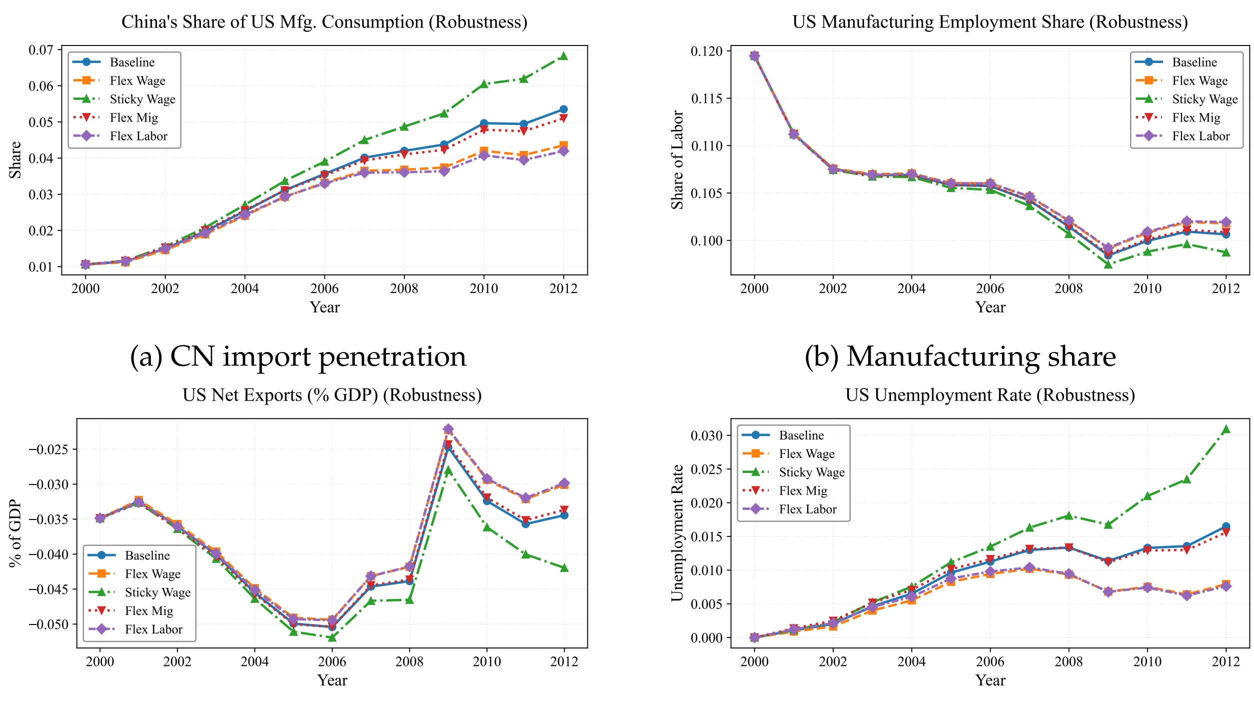
Figure F.7: Effect of Savings Glut.

F.4 Labor Market Frictions

We assess the sensitivity of our results to Chinese labor market rigidities. Figure F.8 first presents a naive comparison where we increase flexibility without recalibrating the underlying shocks. While greater flexibility dampens adverse effects, Panel (a) shows these counterfactuals also lead to smaller rise in import penetration.

To isolate the role of frictions conditional on the *same* trade shock, we recalibrate the model to match the baseline trajectory of Chinese import penetration under four alternative regimes: (1) **flexible Chinese wages** ($\kappa_{w,CN} = 2 \times \kappa_{w,US}$); (2) **reduced migration costs**; (3) **combined labor market flexibility** (both flexible wages and reduced migration costs); and (4) **sticky Chinese wages** ($\kappa_{w,CN} = 0.6 \times \kappa_{w,US}$).

Figures F.9 through F.12 display the results. Across all specifications – whether Chinese labor markets are more flexible or stickier – the effect of the peg remains quantitatively similar to the baseline. This confirms that the amplification of the China shock is driven primarily by the interaction of the currency peg with global trade imbalances, rather than by specific rigidities within the Chinese labor market.



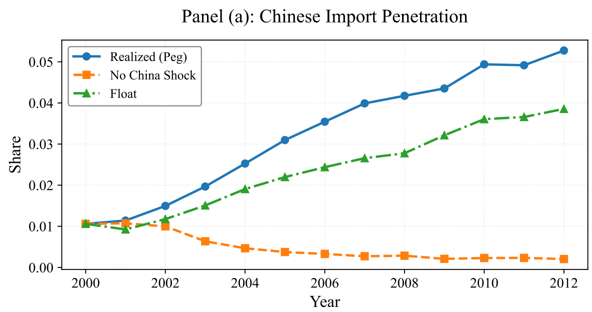
(a) CN import penetration

(b) Manufacturing share

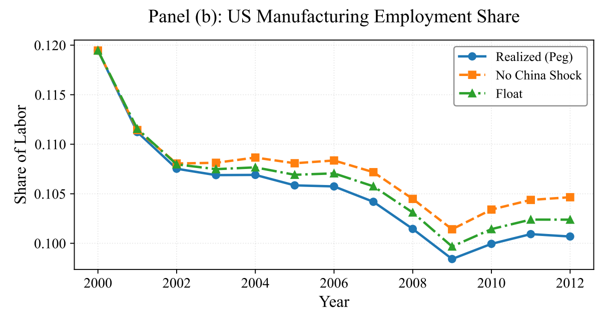
(c) Net exports (% GDP)

(d) Unemployment rate

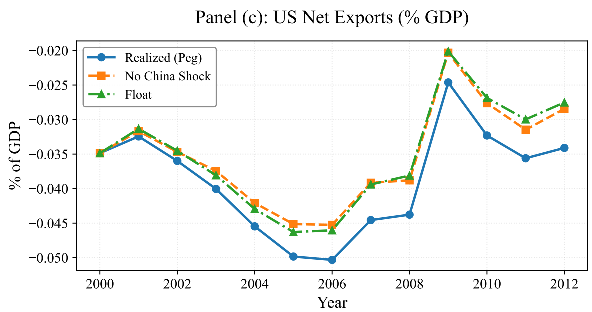
Figure F.8: China Shock: Comparison of Labor Market Parameters.



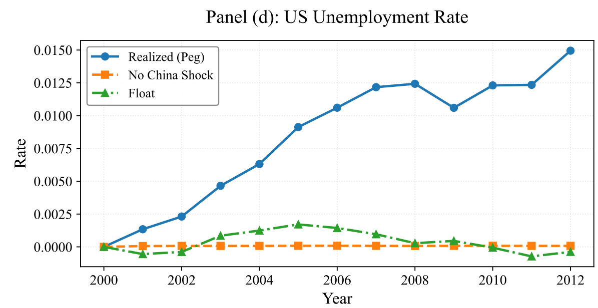
(a) CN import penetration



(b) Manufacturing share

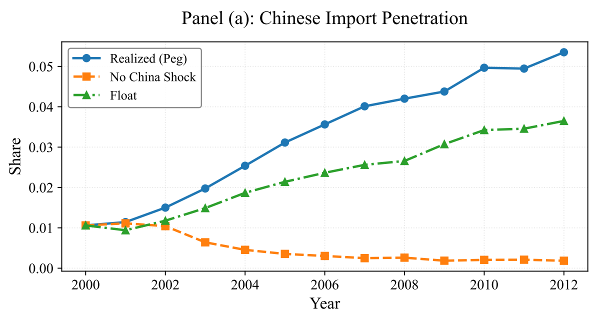


(c) Net exports (% GDP)

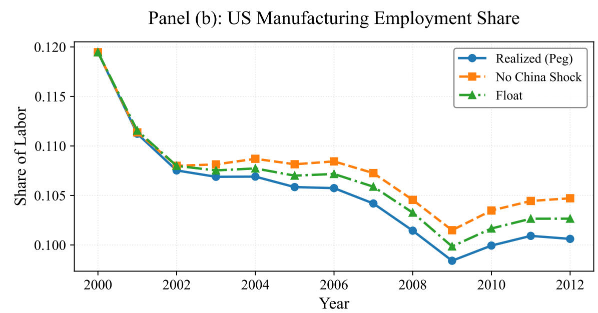


(d) Unemployment rate

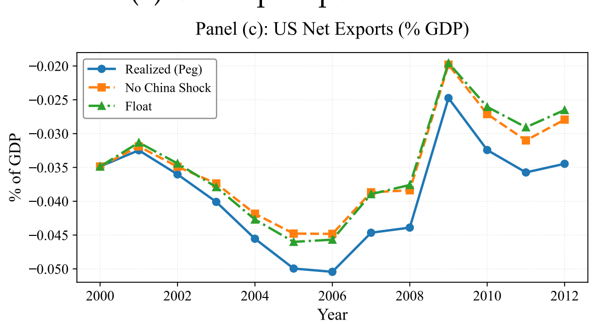
Figure F.9: Effect of Peg: Flexible Chinese Wages.



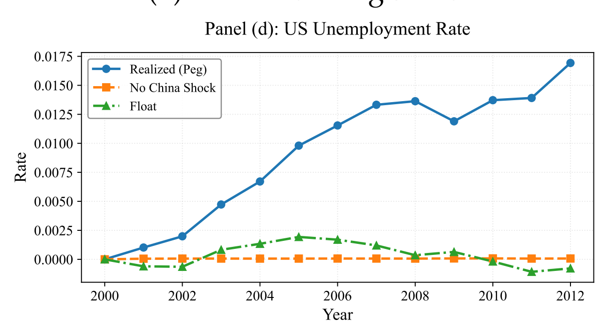
(a) CN import penetration



(b) Manufacturing share

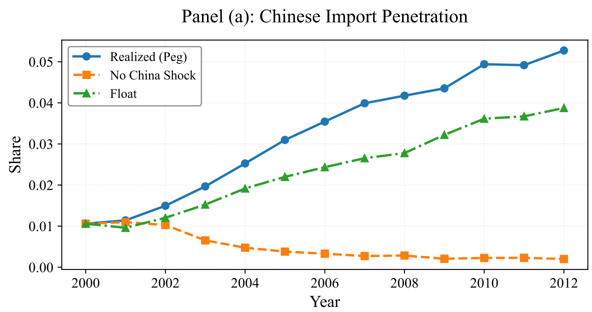


(c) Net exports (% GDP)

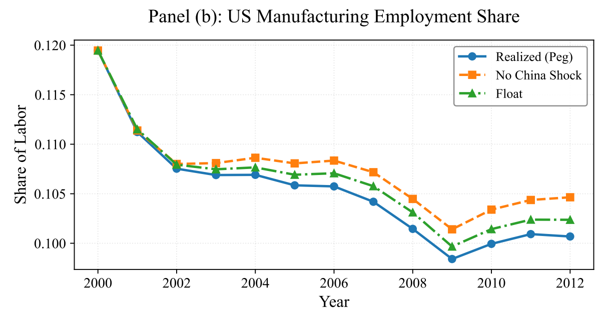


(d) Unemployment rate

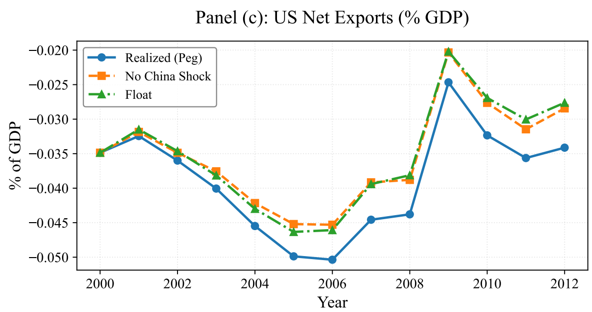
Figure F.10: Effect of Peg: Flexible Chinese Migration.



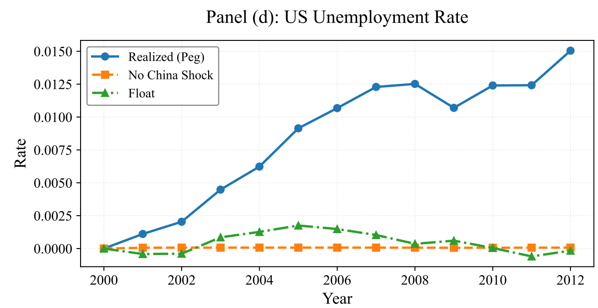
(a) CN import penetration



(b) Manufacturing share

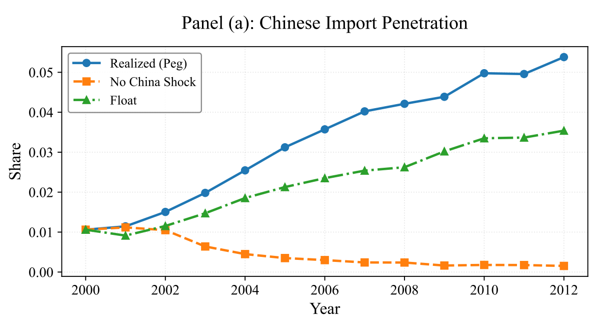


(c) Net exports (% GDP)

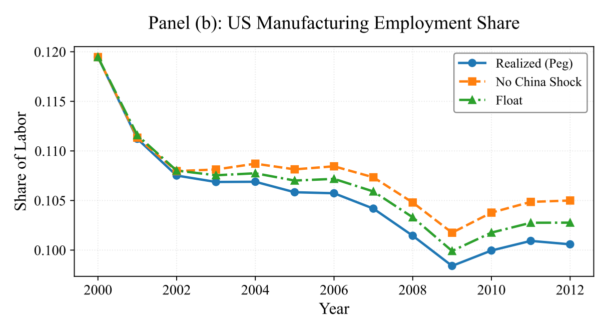


(d) Unemployment rate

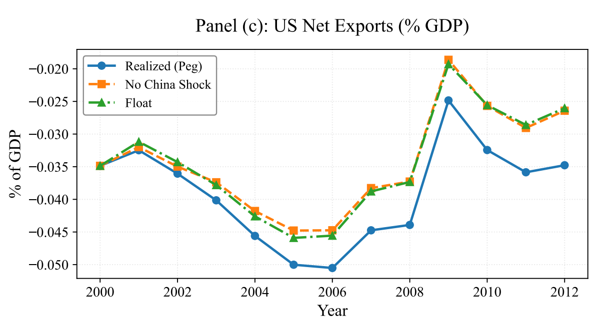
Figure F.11: Effect of Peg: Flexible labor market ($\kappa_w \times 2$, $\chi \times 0.5$.



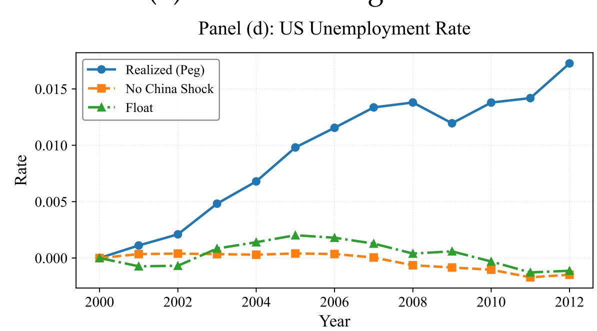
(a) CN import penetration



(b) Manufacturing share



(c) Net exports (% GDP)



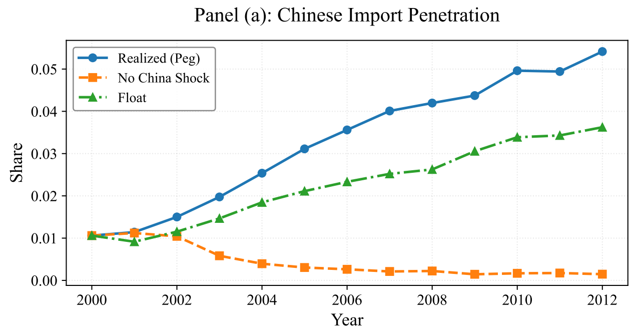
(d) Unemployment rate

Figure F.12: Effect of Peg: Sticky Chinese Wages ($\kappa_{w,CN} = 0.6 \times \kappa_{w,US}$).

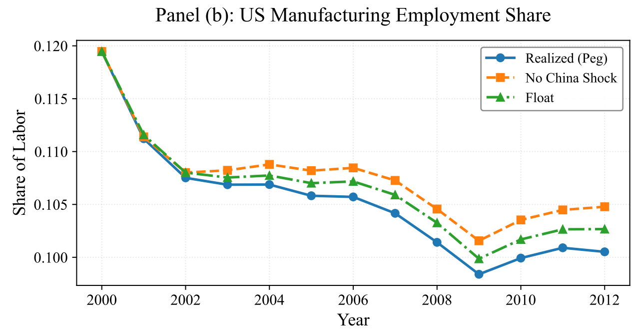
F.5 Different Trade Elasticities

We test the sensitivity of our results to the trade elasticity σ. As shown in Figure F.13, the impact of the China shock and peg remains consistent.

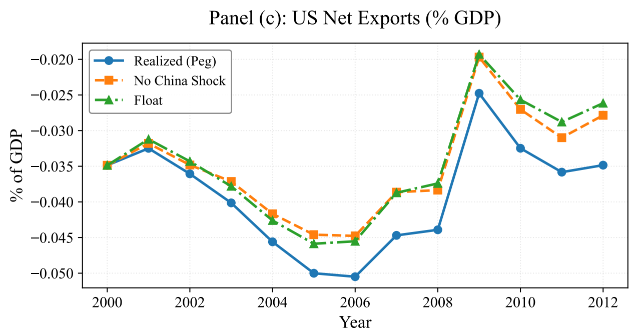
Panel A: High Trade Elasticity



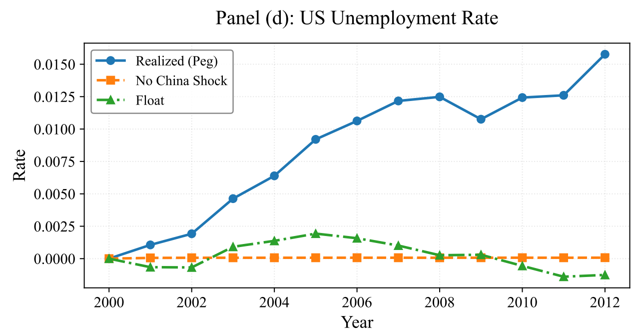
(a) CN import penetration



(b) Manufacturing share

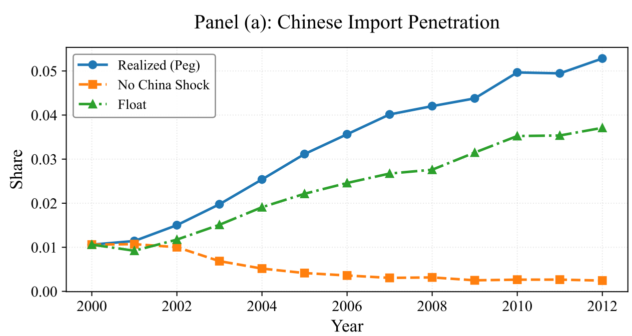


(c) Net exports (% GDP)

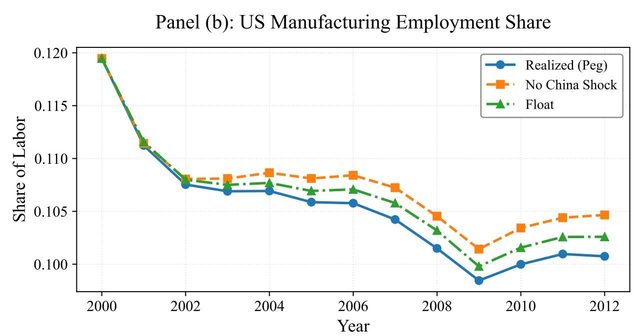


(d) Unemployment rate

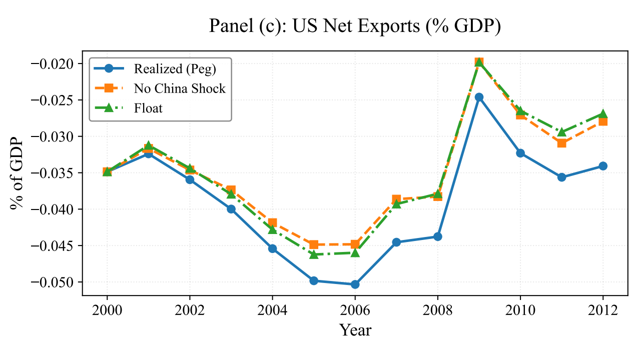
Panel B: Low Trade Elasticity



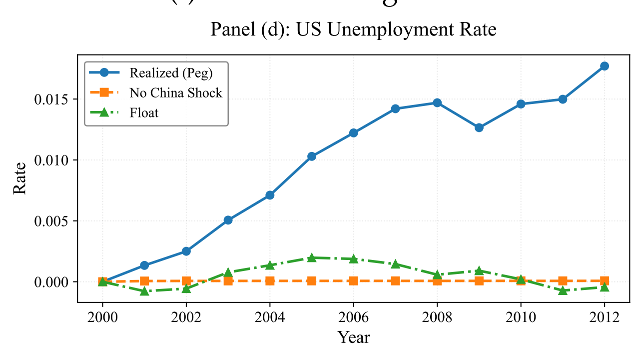
(e) CN import penetration



(f) Manufacturing share



(g) Net exports (% GDP)



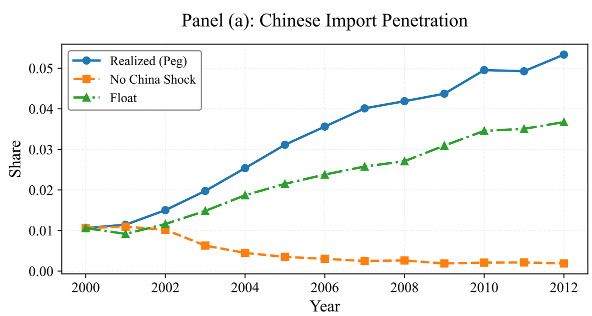
(h) Unemployment rate

Figure F.13: Robustness to Trade Elasticities.

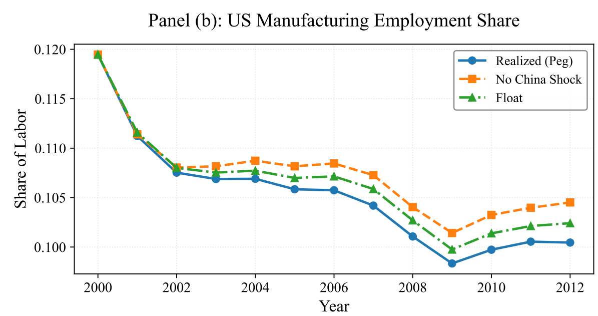
F.6 Zero Lower Bound

We do not impose the zero lower bound (ZLB) because it never binds in our baseline. We calibrate preference shifters δ_{it} to match *relative* current accounts rather than absolute aggregate demand levels; consequently, our model generates relative shifts rather than the synchronized global demand collapses (e.g., the 2008 crisis) that trigger ZLB episodes.

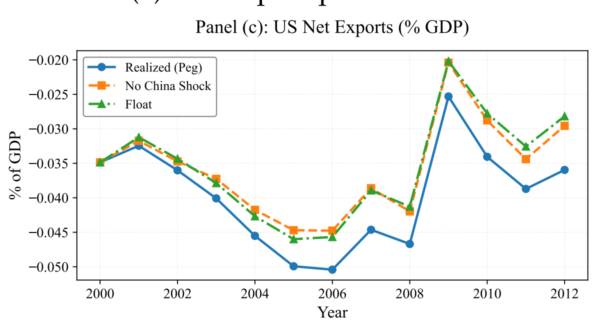
To analyze the effects of the China shock in a constrained monetary environment, we impose a 4% effective lower bound on the US from 2008 to 2014[16]. As shown in Figure F.14, results remain largely analogous to the baseline. While the bound lowers aggregate demand levels and raises unemployment, the marginal effects of the China shock on manufacturing shares, net exports, and unemployment remain identical. This invariance occurs because the constraint binds in both the shock and no-shock scenarios—consistent with the view that the Great Recession would have occurred independently of the China shock.



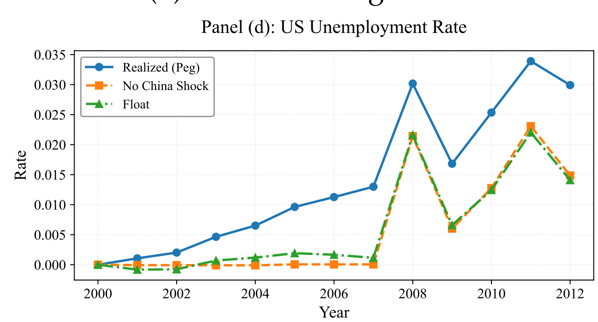
(a) CN import penetration



(b) Manufacturing share



(c) Net exports (% GDP)



(d) Unemployment rate

Figure F.14: Effect of Peg: Zero Lower Bound.

[16] This is 1 percentage point above the average nominal interest rate in our baseline scenario.

F.7 Summary of Robustness Checks

Table F.1 (reproduced below from the main text) summarizes the sensitivity of our main results to alternative calibrations and modeling assumptions. While the preceding subsections explored a comprehensive set of variations (e.g., Dove and Hawk policies, sticky Chinese wages, and savings shocks), this summary table focuses on the distinct classes of robustness checks to provide a concise overview. Below, we provide the detailed definitions of the reported statistics and the specific implementation of the scenarios included in the table.

Row Definitions (Decomposition). Let Y^{Realized} denote the value of an outcome variable in the realized equilibrium (baseline model with calibrated shocks and the exchange rate peg). Let Y^{NoShock} denote the value in the counterfactual economy without the China shock (holding Chinese productivity and trade costs at 2000 levels). Let Y^{Float} denote the value in the counterfactual economy where China follows a floating exchange rate regime (setting $\psi_{CN,t} = 0$ and following a Taylor rule symmetric to the US).

(a) **China Shock:** The total impact of the China shock under the realized policy regime, calculated as:

$$\text{China Shock} = Y^{\text{Realized}} - Y^{\text{NoShock}}$$

(b) **Peg Effect:** The contribution of the exchange rate peg to the total impact, calculated as the difference between the realized outcome and the floating counterfactual:

$$\text{Peg Effect} = Y^{\text{Realized}} - Y^{\text{Float}}$$

(c) **Ratio (%):** The fraction of the total China shock explained by the peg, calculated as:

$$\text{Ratio} = \left(\frac{\text{Peg Effect}}{\text{China Shock}}\right) \times 100$$

Column Definitions (Outcome Variables).

- **Import (pp GDP):** The percentage point increase in the share of Chinese imports in US GDP.
- **MFG Jobs (thousands):** The cumulative decline in US manufacturing employment relative to the counterfactual.
- **Deficit (pp GDP):** The percentage point increase in the US trade deficit relative to GDP.
- **Unemployment (pp):** The percentage point increase in the aggregate US unemployment rate.

- **Welfare (%):** The consumption-equivalent welfare change for the representative US household.

Scenario Specifications. Table F.1 reports the following specifications, which correspond to the experiments discussed in the preceding subsections:

(0) Baseline (CPI Taylor Rule): The standard calibration described in Section 3.3.

(1) Alt. MP Unemp. Targeting: Monetary policy targeting unemployment. See Section F.1 for details.

(2) Alt. MP NGDP Targeting: Monetary policy targeting nominal GDP growth. See Section F.1.

(3) Faster CN Wage: Chinese wage flexibility increased to twice the US level ($\kappa_w^{CN} = 2\kappa_w^{US}$). See Section F.4.

(4) Faster CN Migration: Chinese migration costs halved and elasticity doubled relative to the US. See Section F.4.

(5) Alt. China Shock Definition: Counterfactual holding China's global export shares fixed (rather than productivity). See Section F.2.

(6)–(7) High / Low Sigma: Trade elasticity σ increased/decreased by 25% relative to baseline. See Section F.5.

(8) ZLB (Zero Lower Bound): Imposes a binding lower bound on US interest rates during 2008–2014. See Section F.6.

Scenario		Outcome Variables				
		Import (pp GDP)	MFG Jobs (thousands)	Deficit (pp GDP)	Unemp (pp)	Welfare (%)
0. Baseline	China Shock	4.14	793	0.55	1.77	0.161
(CPI Taylor rule)	Peg Effect	0.66	465	0.52	1.62	0.015
	Ratio (%)	16.0	58.7	95.6	91.7	9.4
1. Alt. MP:	China Shock	4.29	782	0.65	0.41	0.208
u targeting	Peg Effect	0.56	844	0.95	0.34	0.061
	Ratio (%)	13.0	107.9	145.3	82.2	29.3
2. Alt. MP:	China Shock	4.28	774	0.66	0.17	0.211
NGDP targeting	Peg Effect	0.53	892	1.02	0.12	0.074
	Ratio (%)	12.3	115.2	154.8	73.1	35.1
3. Faster CN wage	China Shock	4.06	733	0.46	1.54	0.167
$(2 \times \kappa_w)$	Peg Effect	0.39	392	0.45	1.43	0.010
	Ratio (%)	9.7	53.5	98.1	93.1	5.7
4. Faster CN migration	China Shock	4.12	791	0.55	1.81	0.159
$(0.5 \times \nu, \chi)$	Peg Effect	0.64	467	0.52	1.64	0.014
	Ratio (%)	15.5	59.0	95.1	90.8	8.8
5. Alt. China shock	China Shock	3.31	662	0.40	1.52	0.133
(const import)	Peg Effect	0.67	466	0.52	1.65	0.017
	Ratio (%)	20.1	70.4	131.2	108.5	12.4
6. High Sigma	China Shock	4.12	828	0.59	1.76	0.119
$(\sigma = 6)$	Peg Effect	0.61	487	0.55	1.59	0.015
	Ratio (%)	14.9	58.8	92.6	90.4	12.3
7. Low Sigma	China Shock	4.18	747	0.50	1.82	0.237
$(\sigma = 4)$	Peg Effect	0.70	433	0.48	1.70	0.017
	Ratio (%)	16.7	58.0	96.6	93.2	7.2
8. ZLB Scenario	China Shock	4.14	772	0.53	1.63	0.161
(2008-2014)	Peg Effect	0.67	442	0.50	1.32	0.015
	Ratio (%)	16.2	57.2	94.7	81.2	9.1

Table F.1: Summary of robustness checks.

Note: This table summarizes the key results from the specifications defined in the list above. Please refer to the text in this subsection for the detailed definitions of the Row statistics (China Shock, Peg Effect, Ratio) and Column outcome variables.

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